Balance Function Study

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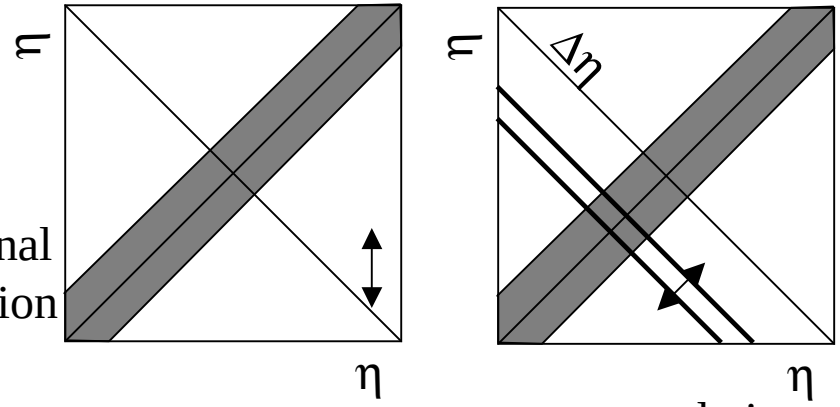
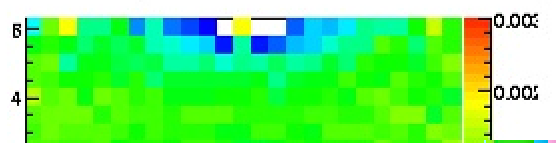
Introduction

- The BF measurement is motivated by theory predictions for the *length scale* of CD correlations
- The critical issue for the BF paper is the interpretability of the *rms* width of the BF, how it is related to that length scale, which depends on several critical aspects
- For context I start with Aya's direct analysis of CD correlations, which are the nominal object of the BF
- I then consider the algebra of the BF, charge mixing and eta mixing as references
- I obtain an eta-mix reference from data for the BF and apply it to Fig. 1 of the draft
- The differential I observe agrees qualitatively with the measured CD autocorrelation
- I then demonstrate that the BF *rms* width is *not monotonic* with correlation length - cannot determine CD correlation-length trends

Charge-dependent η-η Correlations

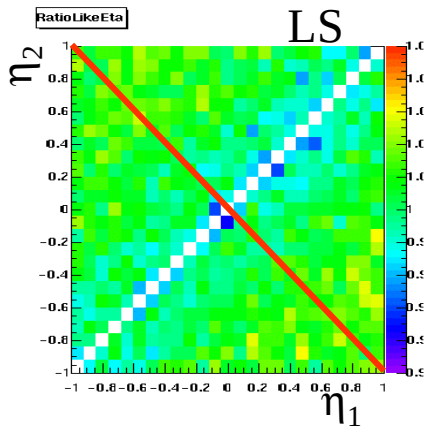
- Balance function *vs* autocorrelation
- Autocorrelation is designed to access correlation length, amplitude separately
- LS, US individual ratios: different physics for each pair type
- Precision references are *essential* for interpretation
- Joint autocorrelation for *each* of ++, −−, +−, LS, US, CI, CD

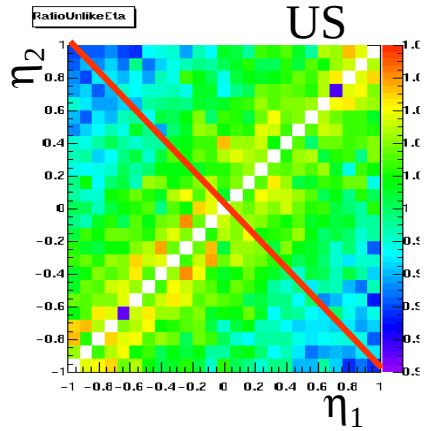


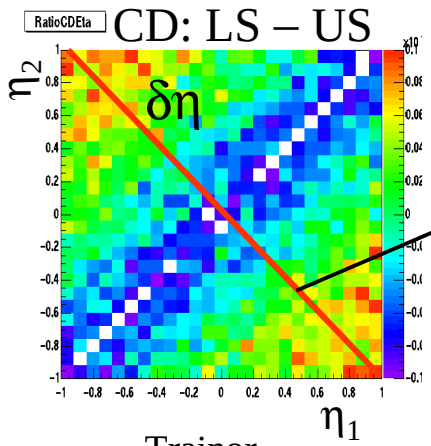


CD

CD autocorrelation on δη





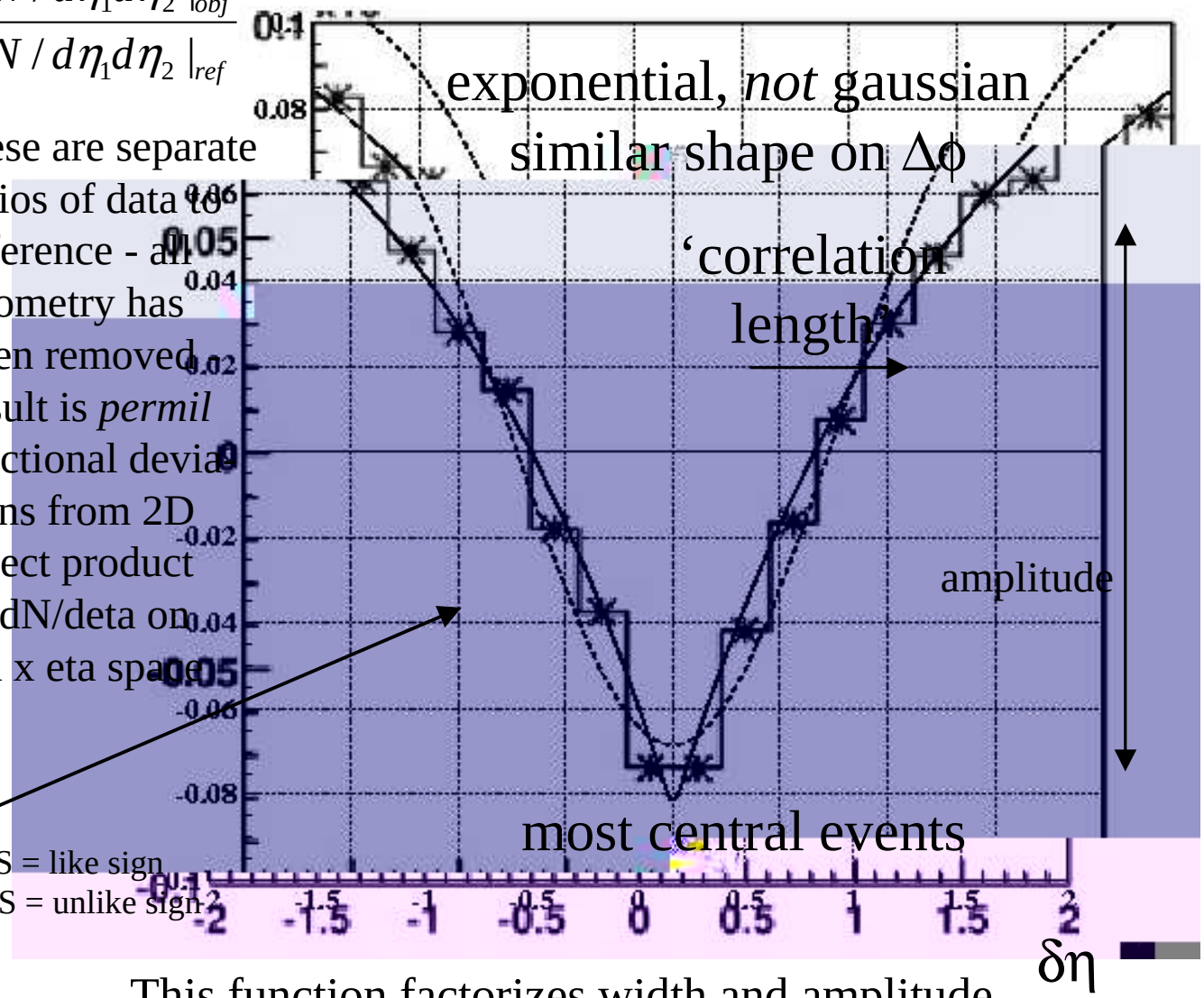


$$\frac{d^2N/d\eta_1 d\eta_2\,|_{obj}}{d^2N/d\eta_1 d\eta_2\,|_{ref}}$$

these are separate ratios of data to reference - all geometry has been removed, result is *permil* fractional deviations from 2D direct product of dN/deta on eta x eta space

LS = like sign
US = unlike sign

this is the basic CD correlation measure



This function factorizes width and amplitude

Aya Ishihara, U. Texas (Austin)

What is the Integral of B(Δy)?

Draft text definition: $$B(\Delta y) = \frac{1}{2}\left\{\frac{\Delta_{+-}(\Delta y) - \Delta_{++}(\Delta y)}{N_+} + \frac{\Delta_{-+}(\Delta y) - \Delta_{--}(\Delta y)}{N_-}\right\}$$

event-wise calculation

- Taking the initial definition at face value one can integrate B event-wise over an acceptance
- If N is the number of either charge sign then number of unlike-sign pairs is N^2, and number of like-sign pairs is N(N-1) - no double counts
- Difference divided by N: = 1
- Int B = 1 for *any* single event
- Therefore, int B = 1 averaged over the event ensemble
- The differential quantity B is then estimated by a unit-area 'triangular' distribution on Δy
- This summary is correct if B is calculated for each event - if B is indeed calculated with even-ensemble averages the argument above is modified

B Integral and Ensemble Averages

$$\int_0^Y B(\Delta y, Y)\, d\Delta y = \frac{1}{2}\left\{ \frac{\overline{N_+ N_-} - (\overline{N_+^2} - \overline{N}_+)}{\overline{N}_+} + \frac{\overline{N_+ N_-} - (\overline{N_-^2} - \overline{N}_-)}{\overline{N}_-} \right\}$$

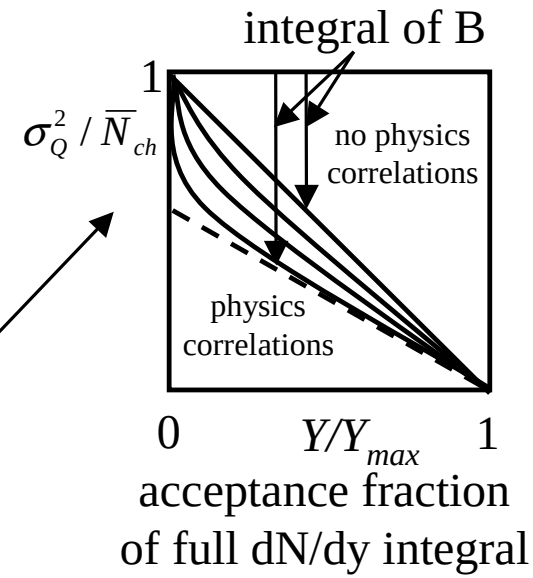
ensemble average integral over given rapidity acceptance Y

$$\overline{N_+ N_-} - (\overline{N_+^2} - \overline{N}_+) + \overline{N_+ N_-} - (\overline{N_-^2} - \overline{N}_-) = \overline{N}_+ + \overline{N}_- - \overline{(N_+ - N_-)^2}$$

$$= \overline{N}\left\{1 - \sigma_Q^2 / \overline{N}\right\}$$

$$\int_0^Y B(\Delta y, Y)\, d\Delta y \cong 1 - \sigma_Q^2(Y) / \overline{N}(Y)$$

net-charge fluctuations

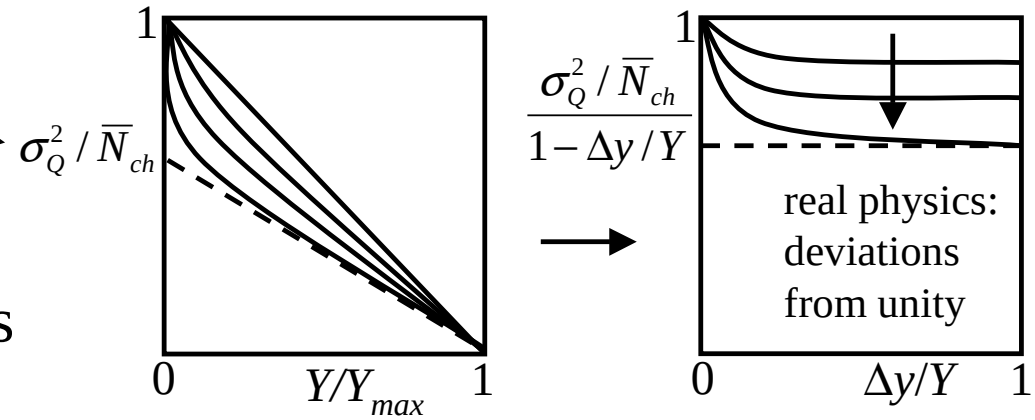


- If the balance function is instead determined by event ensemble averages the integral is not unity but is determined by the acceptance fraction
- The integral is simply related to net-charge fluctuations
- Given the upper right plot the integral of B for given acceptance fraction is obtained
- The integral is *never zero* for finite acceptance, and is *not zero* in the absence of charge correlations

B Integral and Net-charge Fluctuations

$$\frac{<(Q-<Q>)^2>}{<N_{ch}>} = 1 - \int_0^Y d\Delta y\, B(\Delta y \,|\, Y)$$

Eq. (7) - Jeon and Pratt



- The Jeon-Pratt algebra agrees that int B over 4π is unity, contradicting at the source the assertion that ‘int B = 0’ is relevant for data analysis
- The integral of B is dominated by canonical suppression, with a smaller contribution from charge correlations, if any

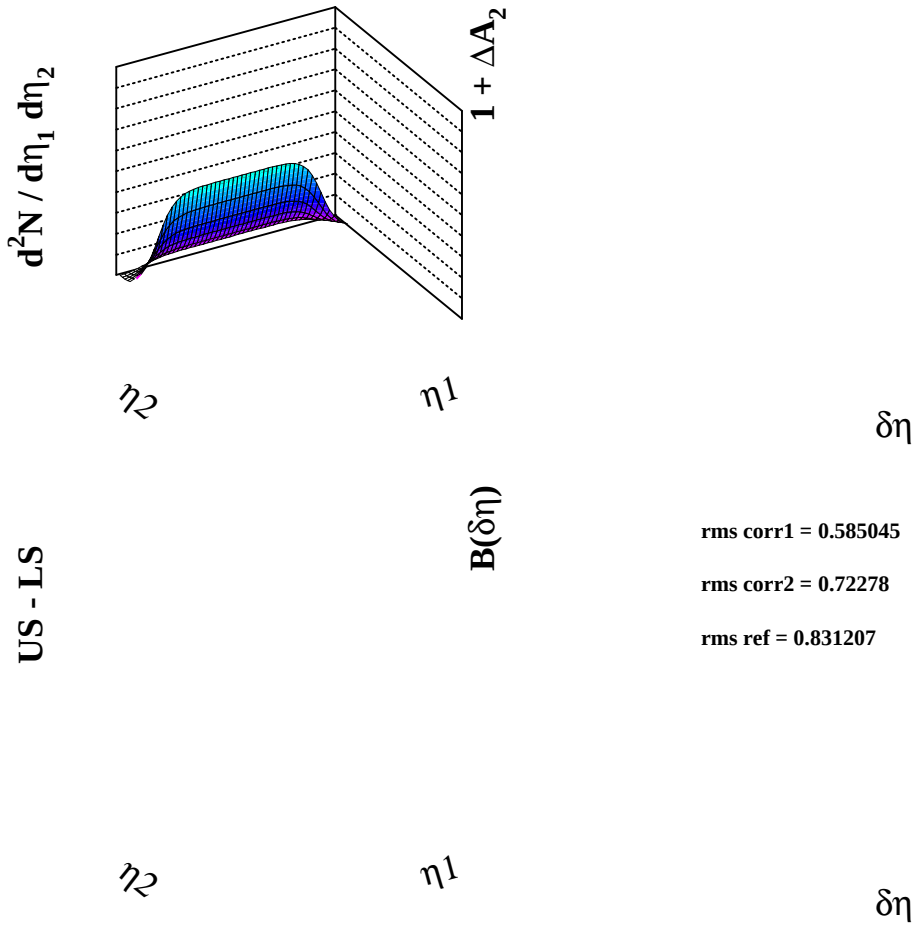
Charge Mixing Reference

- Charge mixing to construct a B reference results in int B = 0
- Charge mixing does two things
  - it does indeed remove charge correlations - the nominal goal
  - it violates the net-charge conservation manifested in canonical suppression
- It is net-charge conservation that dominates the B integral for real data
- Canonical suppression reflects the reality that for a fixed total net charge (2x79) net-charge fluctuations in 4π acceptance go to zero $\rightarrow$ int B = 1
- Randomizing charges within events violates the fixed net-charge constraint - net-charge fluctuations return to the full Poisson value (unity) - charge mixing is not appropriate

‘charge correlations’ is apparently confused with net-charge conservation

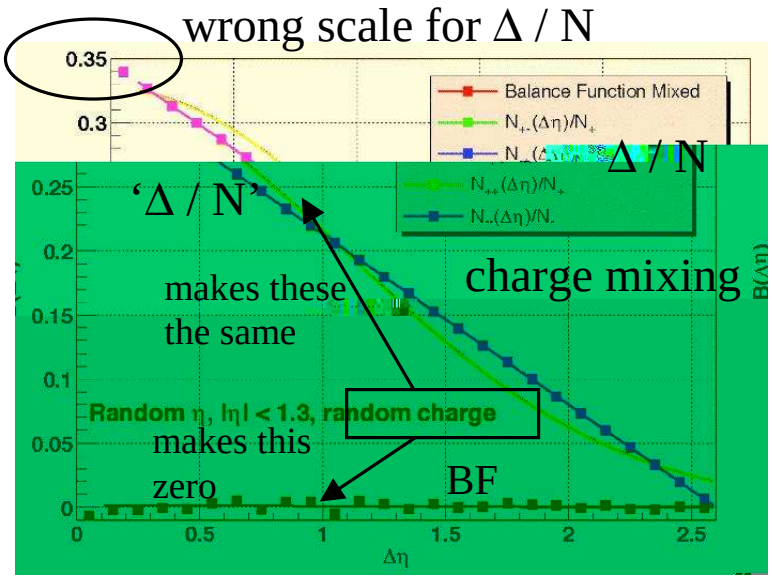
B(Δy) Reference Distribution

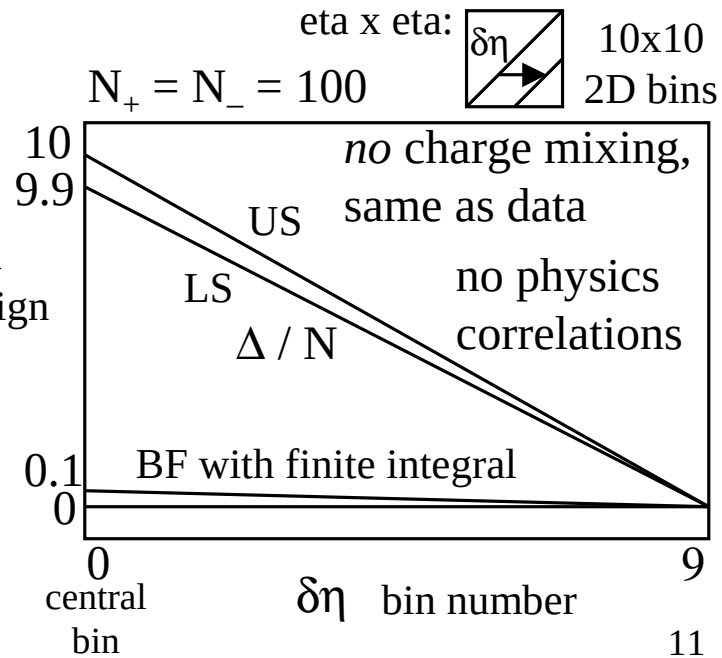
- The reference definition for a precision correlation measure is the *most difficult* aspect
- The language associated with reference construction for B is confusing: redundantly defined terms are used inconsistently
- Having established the real origin of the B-integral value we require a reference for the B distribution on Δy
- Charge mixing is not relevant
- What is relevant is eta mixing, with several options
- Preserve charge association
  - integral B from acceptance
  - direct product dN/deta x dN/deta
  - Random eta exchange within events
  - Random eta selection weighted by data dN/deta
  - Random eta selection from uniform dist. within boundary
  - Random selection of eta from event ensemble


$d^2N / d\eta_1 d\eta_2$
η_2
η_1
$1 + \Delta A_2$
US - LS
$B(\delta\eta)$
η_2
η_1
$\delta\eta$
rms corr1 = 0.585045
rms corr2 = 0.72278
rms ref = 0.831207
$\delta\eta$

Event-wise Balance Components

- Using the toy example below I calculate the components of B
- 10k US pairs, 9900 LS pairs
- 100 and 99 pairs per 2D bin
- Δ / N for central BF bin (sum of 10 2D bins) is therefore 10 , 9.9 = (100 , 99) x 10 / 100
- I sketch the *event-wise* LS and US components Δ / N of BF and the resultant BF
- The plot above right is for mixed charge, not data
- Charge mixing forces int BF = 0

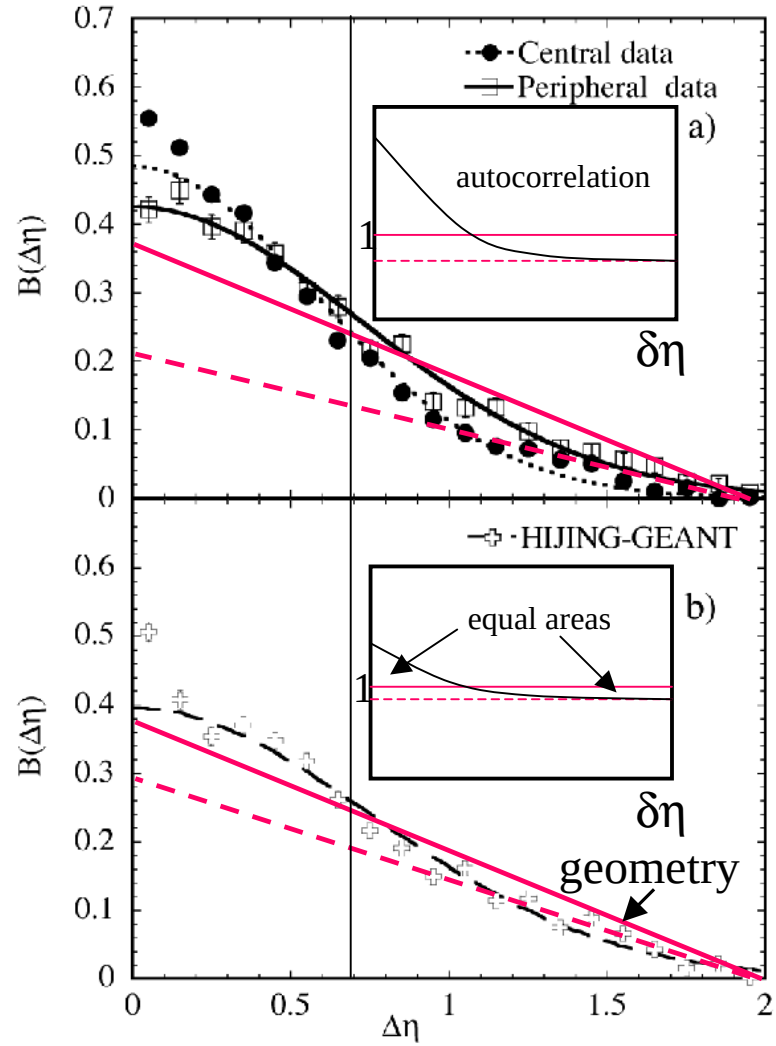




LS = like sign
US = unlike sign

Initial Sketch of Geometry Reference

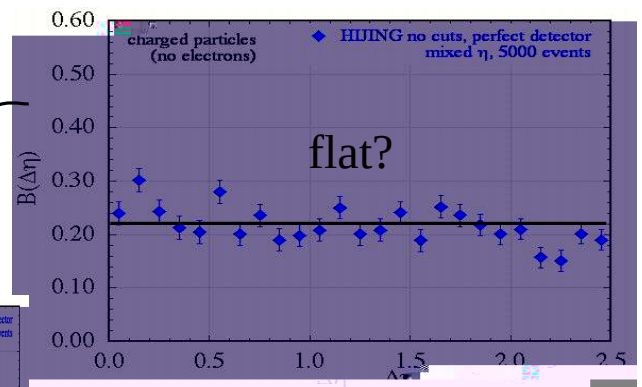
- This was my initial markup of Fig. 1 for Steve, to illustrate the relation between physics correlations and the geometry reference
- I guessed at the latter (solid red line) and sketched the inferred autocorrelation
- This compares qualitatively with Aya's measurement
- An exponential multiplied by a triangle looks like a gaussian



η Mixing and Reference Choices

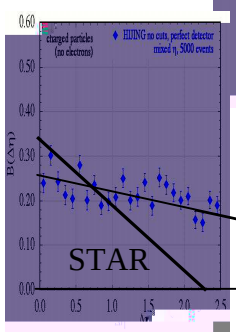
Here I attempt to infer a geometry reference directly from data

from BF web pages:



squeeze

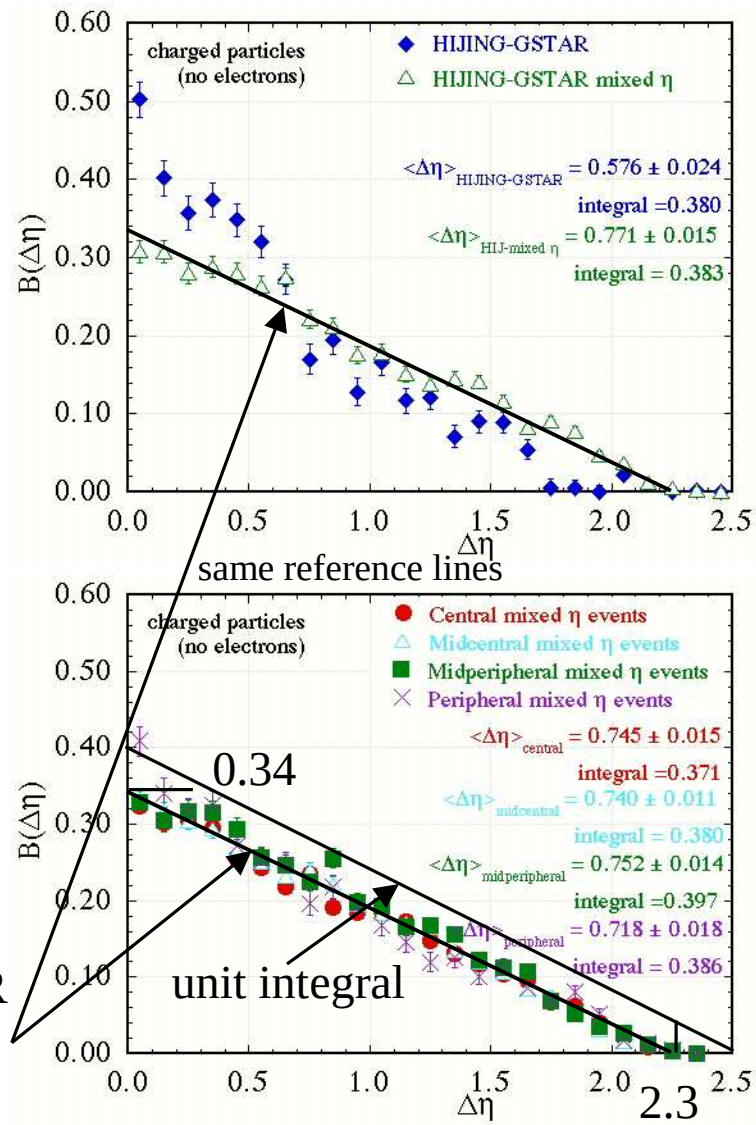
no!



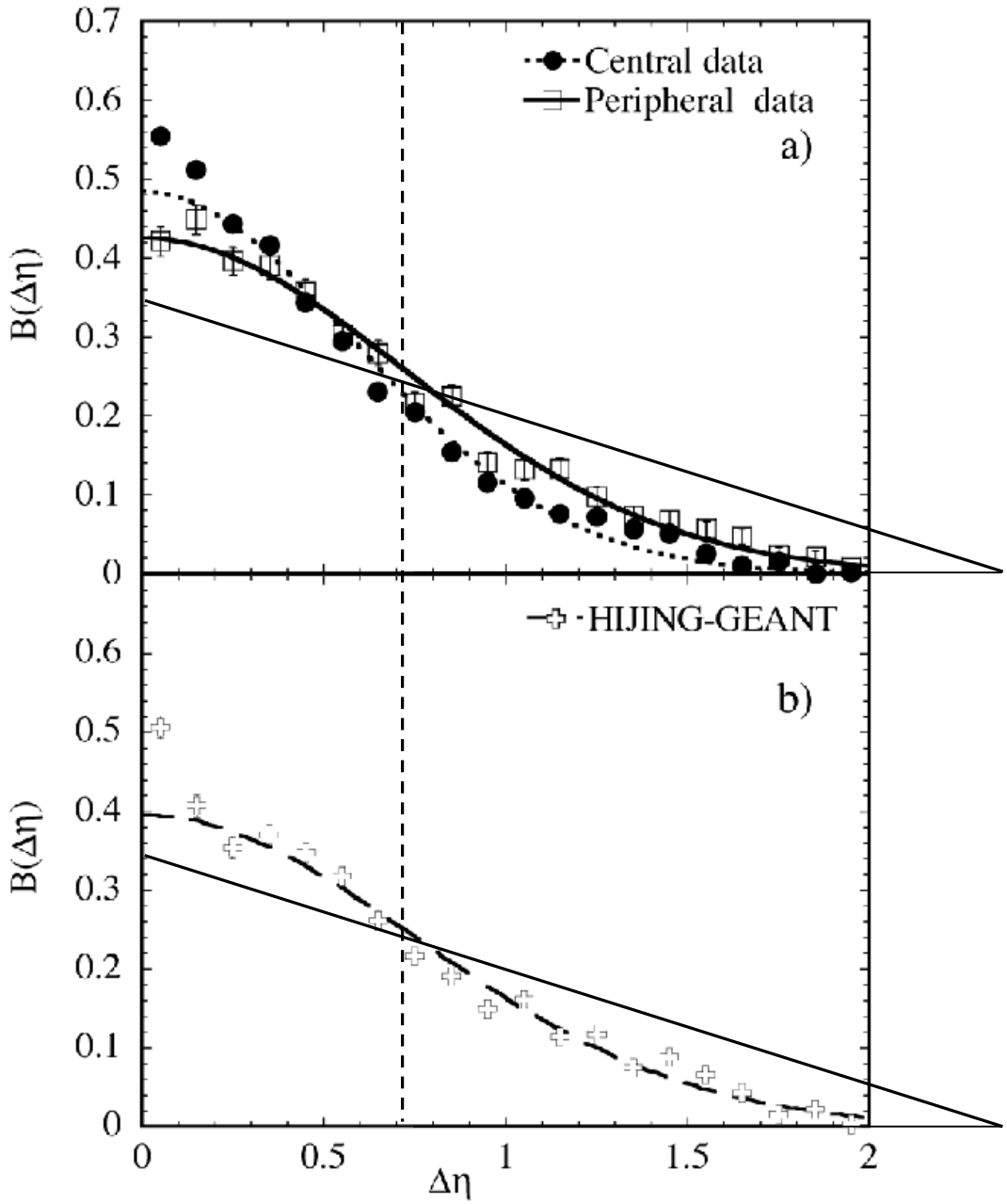
kinematic bound on dN/deta

?

reference estimate for 4π Monte Carlo

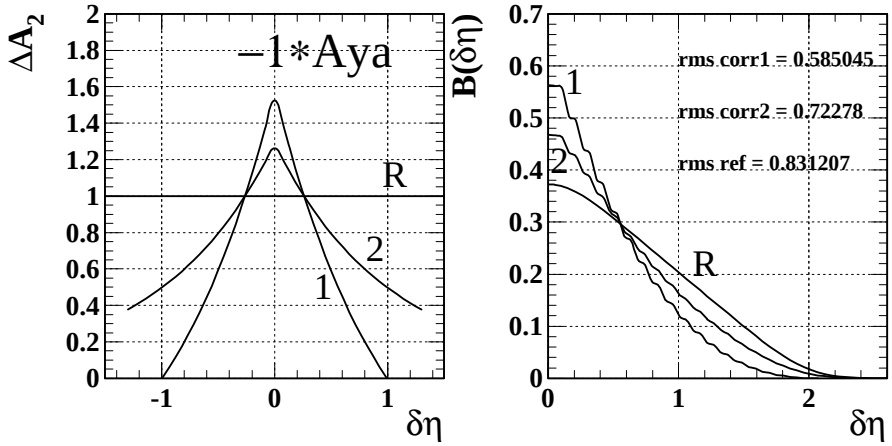


best estimate of STAR geometry reference → apply to Fig. 1

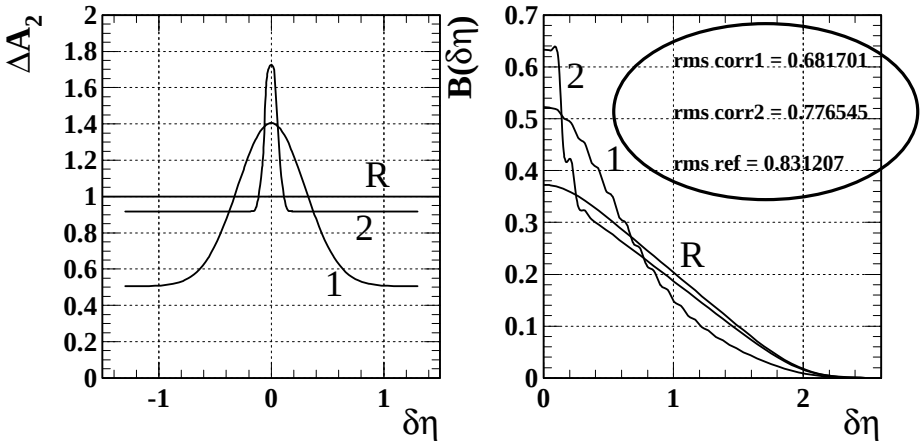

Central data
Peripheral data
a)
HIJING-GEANT
b)
B(Δη)
Δη
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1
1.5
2

0.1
0.05

Relative Sensitivity to Width and Amplitude



exponentials → ‘gaussians’



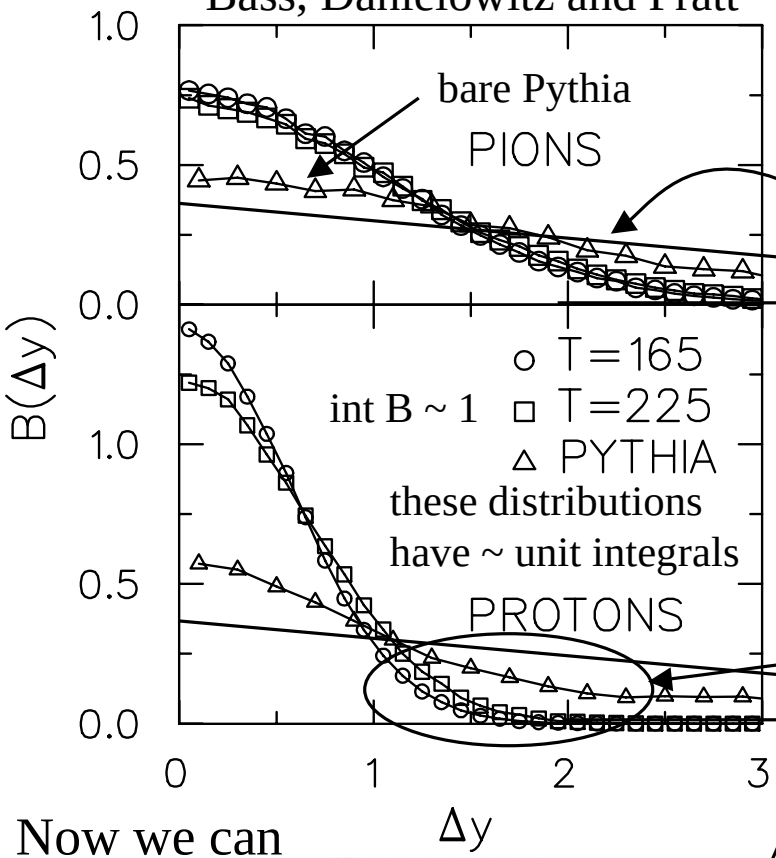
gaussians → ‘gaussians’

- Autocorrelation width is held constant, amplitude changes 2x
- *rms* width of BF monotonically decreases with increasing autocorrelation amplitude
- Pairs are conserved, correlation is transport on scale δη

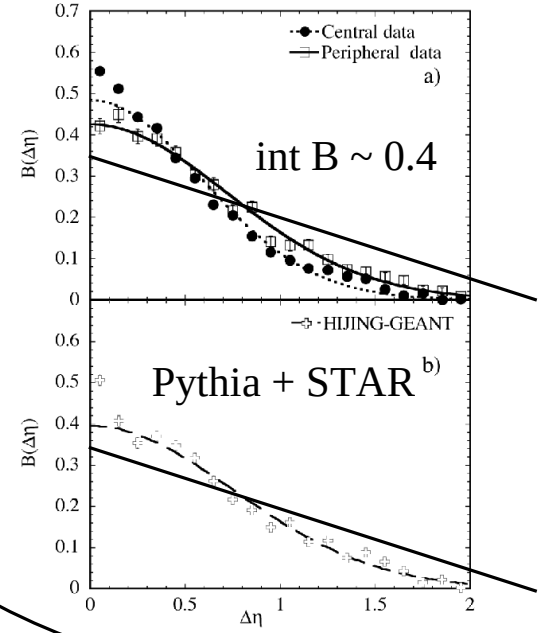
- Amplitude is held constant, width changes by a factor 6x (0.05 → 0.3)
- *rms* width of BF *increases* with decreasing auto width - non-monotonic relationship between autocorrelation width and BF width

Pythia and Other Models

Bass, Danielowitz and Pratt



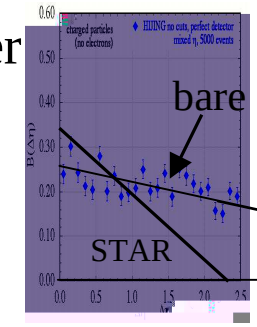
amplitudes are changing, not correlation lengths



compare

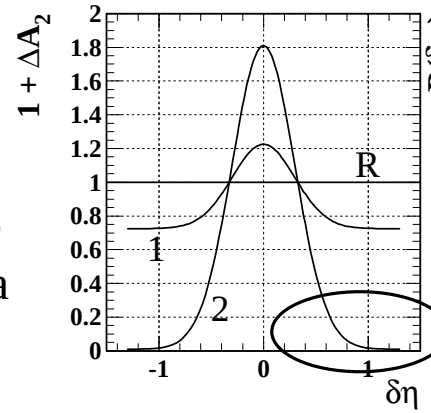
unrealistic correlation amplitudes in model

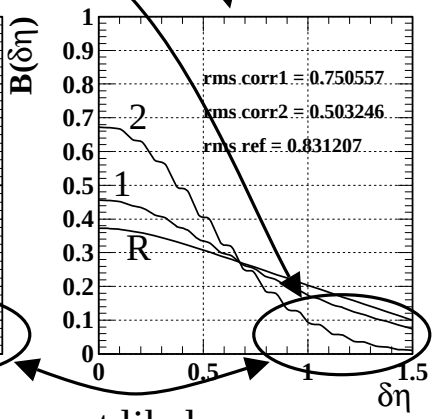
Now we can interpret other results with better under-standing



rough estimate of bare dN/deta as reference

?





not likely

BF Summary

- int BF = 1 for data if the BF is calculated event-wise
- int BF ~ acceptance fraction if the BF is calculated with ensemble means
- Charge correlations then affect B in two aspects
- Charge-mixed references with int B = 0 are not relevant for STAR data
- The BF *rms* width mixes correlation length and amplitude of the auto-correlation, which also determine its integral
- The BF *rms* width is not monotonic with respect to correlation length
- The BF *rms* width is therefore not interpretable