GlauberFest

A Tale of Two Glauber Calculations:
Monte Carlo vs. Optical Limit Approaches

Mark Baker, Peter Steinberg

Brookhaven National Laboratory

Patrick Decowski

Massachusetts Institute of Technology

BROOKHAVEN NATIONAL LABORATORY

GlauberFest

Theory vs. Experiment

- Theoretical calculations

  Fundamental Input: ***b* is chosen at random**

  $\Rightarrow N_{part}$, N_{coll} are derived

  $\Rightarrow$Particles are generated

- Experimental measurements

  Fundamental Input: **Particles are measured**

  $\Rightarrow$derive N_{part} , N_{coll} , b

- Not a completely theoretical derivation
  - Based on some robust assumptions
  - Can estimate systematics

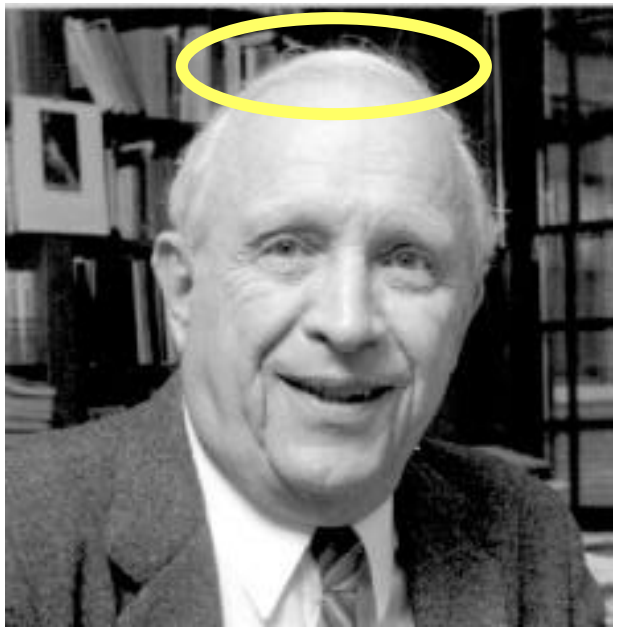
Assumptions

- Monotonicity
  - ***monotonic*** relationship (on average) between X and the measured quantity (N)
    - $d\langle X\rangle/d\langle N\rangle > 0$ (or < 0) for all $\langle N\rangle$
  - A given fraction of events selected from N corresponds to $\langle X\rangle$ for the same fraction
- Total Cross Section
  - Fraction of *what*? (i.e. model has correct σ_{TOT})
  - RHIC experiments have not used the directly measured cross section (see A. Drees talk)
  - Instead trigger efficiency is estimated using models (e.g. HIJING)
  - Systematic error based on this is included in the final results

GlauberFest

Glauber calculations

- Nucleons are distributed according to a density function (e.g. Woods-Saxon)
- Nucleons travel in straight lines and are not deflected as they pass through the other nucleus
- Nucleons interact according to the inelastic cross section measured in pp collisions, even after interacting
  - Participants – counts nucleons which interact
  - Binary collisions – counts collisions
- Two methodologies satisfy basic criteria
  - Probability integrals
  - Monte Carlo approach
- Do they give the same answer?



Roy Glauber

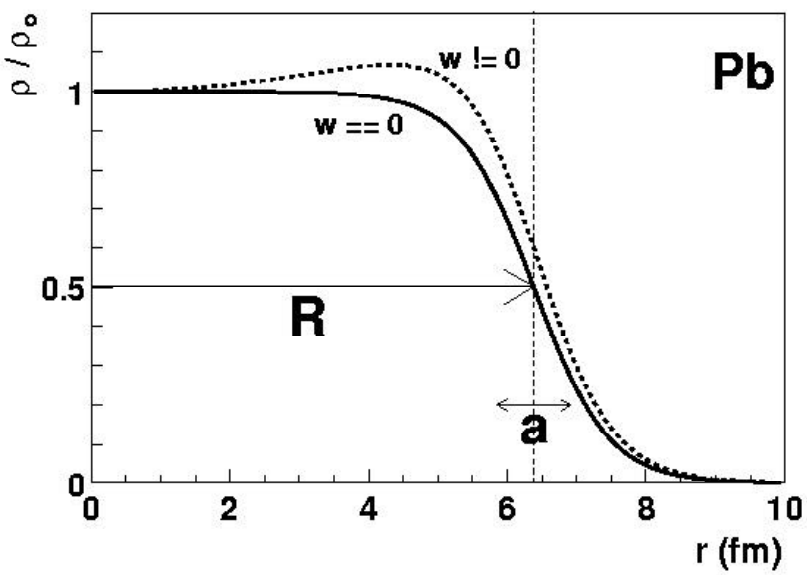
GlauberFest

Nuclear Profile & Thickness

$$r(r) = \frac{r_0\left(1 + wr^2 / R^2\right)}{1 + \exp((r - R)/a)}$$

Nucleus	A	R	a	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	**197**	**6.38**	**0.535**	**0**
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

H. DeVries, C.W. De Jager, C. DeVries, 1987



$$T_A(s) = \int_{-\infty}^{+\infty} r_A(\vec{s}, z)$$

GlauberFest

Total AB Cross Section

$$s_{AB} = \int d^2b \int d^2s_1^A \ldots d^2s_A^A d^2s_1^B \ldots d^2s_B^B \times$$

Configuration Space

$$T_A\left(s_1^A\right)\ldots T_A\left(s_A^A\right) T_B\left(s_1^B\right)\ldots T_B\left(s_B^B\right) \times$$

Nuclear Thickness

$$\left\{1 - \prod_{j=1}^{B} \prod_{i=1}^{A} \left[1 - s\left(b - s_i^A + s_j^B\right)\right]\right\}$$

Interaction Terms

Intractable. Instead, most people use “optical limit”:

$$s_{AB} = \int d^2b \left\{1 - [1 - s_{NN} T_{AB}(b)]^{AB}\right\}$$

where $T_{AB}(b) = \int d^2s T_A(b) T_B\left(\left|\vec{b} - \vec{s}\right|\right)$

Supposedly valid for large A and/or when σ_{NN} is small

GlauberFest

Calculating N_{part} and N_{coll}

- Number of participants

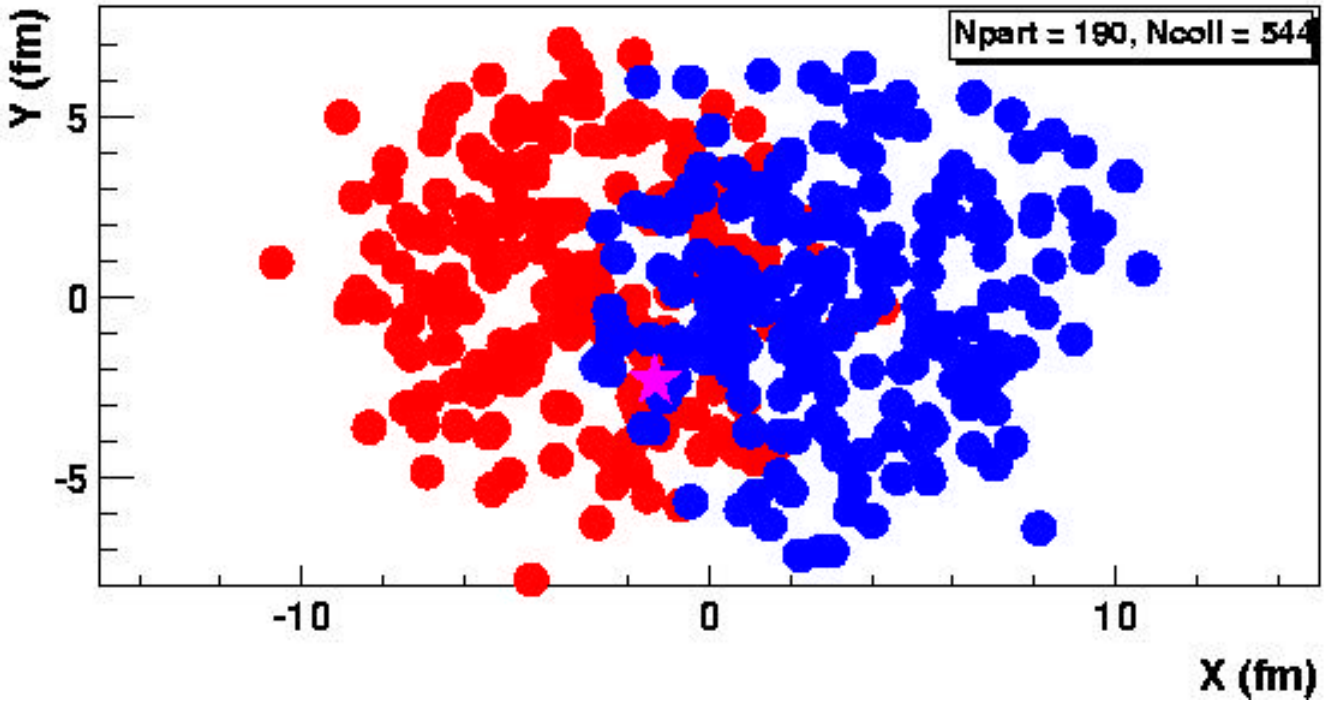
$$N_{part}^{AB}(b) = A\int d^2 s T_A(\vec{s})\left\{1-\left[1-s_{NN}T_B\left(\vec{s}-\vec{b}\right)\right]^B\right\} + (A \Leftrightarrow B)$$

- Number of collisions

$$N_{coll}^{AB}(b) = AB\int d^2 s T_A(\vec{s}) T_A\left(\vec{s}-\vec{b}\right) s_{NN}$$

GlauberFest

Glauber Monte Carlo



- Random impact parameter, nucleon positions
- Interactions occur for $D < sqrt(s_{NN}/p)$
- Can directly count N_{part}, N_{coll} for each event

GlauberFest

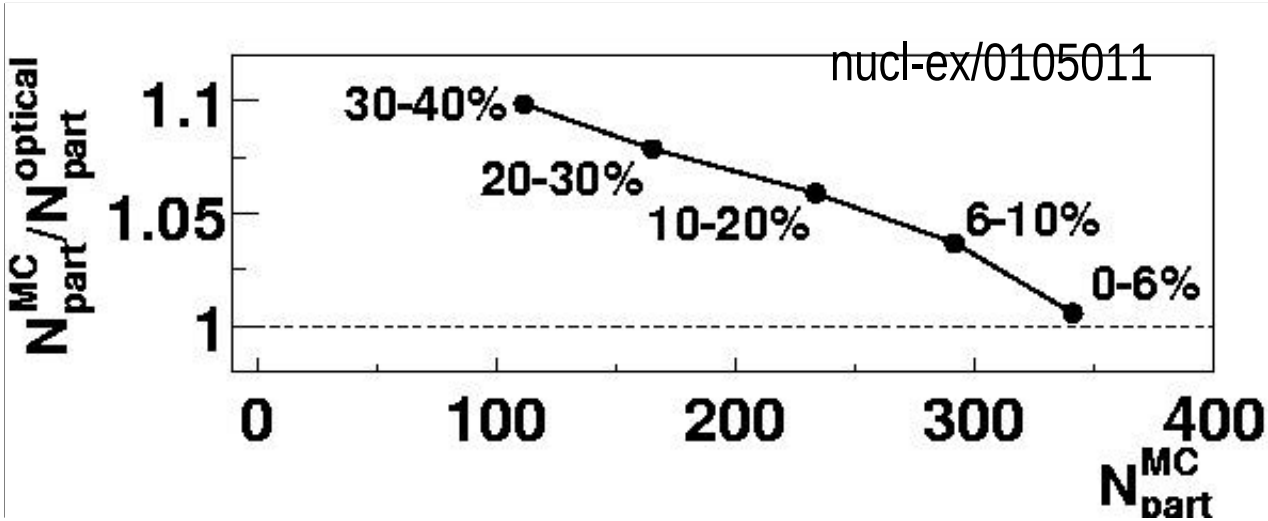
Two Different Answers?

- HIJING 130 GeV
  - Monte Carlo approach
  - Gaussian nucleon profile

%-ile	<Npart>
60-70	112.3
70-80	164.8
80-90	233.3
90-94	294.6
94-100	348

- Kharzeev/Nardi
  - Optical-limit approach
  - Point nuclei

%-ile	<Npart>
60-70	102
70-80	153
80-90	221
90-94	281.5
94-100	339

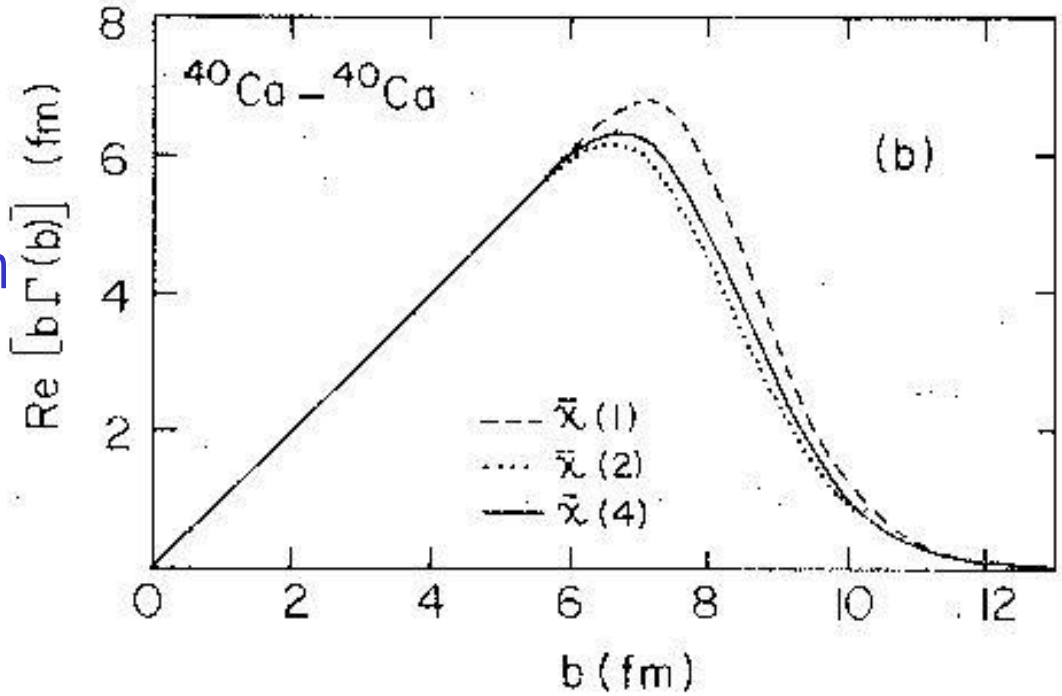


BROOKHAVEN NATIONAL LABORATORY

GlauberFest

Beyond the Optical Limit

- Franco & Varna, 1977
- Include higher order terms in optical limit
  - Next higher order decreases cross section
  - Addition of next term *increases* it again
- Not clear if series is convergent
- Shows difficulty of problem



GlauberFest

“Ball” Glauber

- Keep optical limit formalism but give nucleons finite size
- Modifies nuclear thickness function

$$T_B^{ext}(s) = \int_{s' < \sqrt{s_{NN}/p}} d^2 s' T_B(s + s') \Big/ s_{NN}$$

- Overlap function

$$T_{AB}^{ext}(b) = \int d^2 s T_A(b) T_B^{ext}\left(\left|\vec{b} - \vec{s}\right|\right)$$

- Total cross section

$$s_{AB} = \int d^2 b \left\{1 - \exp(-AB s_{NN} T_{AB}^{ext}(b)\right\}$$

- Number of participants

$$N_w^{AB}(b) = A \int d^2 s T_A(\vec{s}) \left\{1 - \left[1 - s_{NN} T_B^{ext}(s - b)\right]^B\right\} + (A \Leftrightarrow B)$$

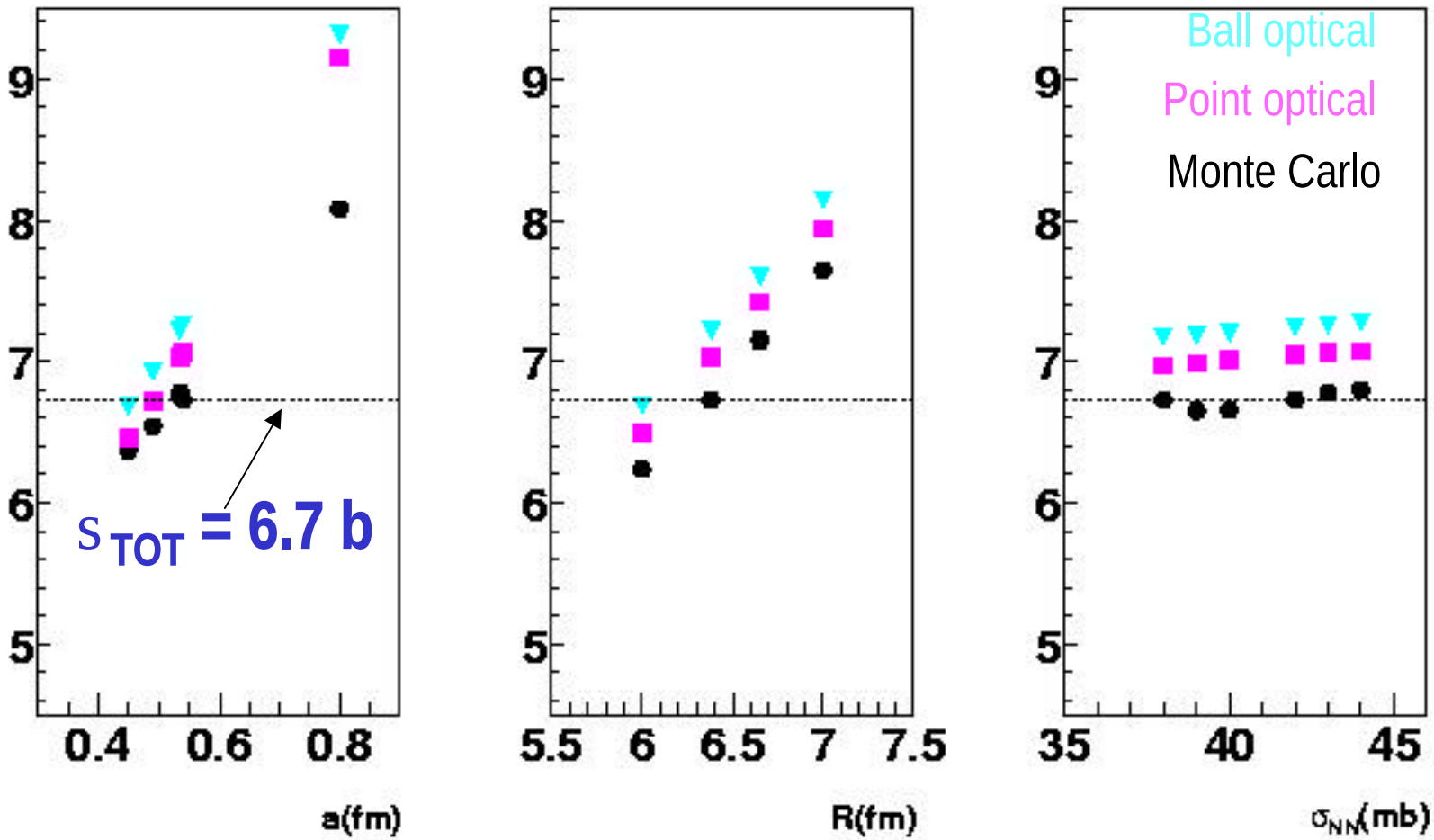
Study of systematics

- We have implemented our own
  - Monte Carlo (MC) Glauber calculation
  - Numerical Integration
    - Point-like nucleons
    - Ball-like nucleons
- Allows us to directly compare quantities
  - Total cross section
  - N_{part} as a function of percentile of cross section
  - What are the sensitivities to R, a, σ_{NN}

GlauberFest

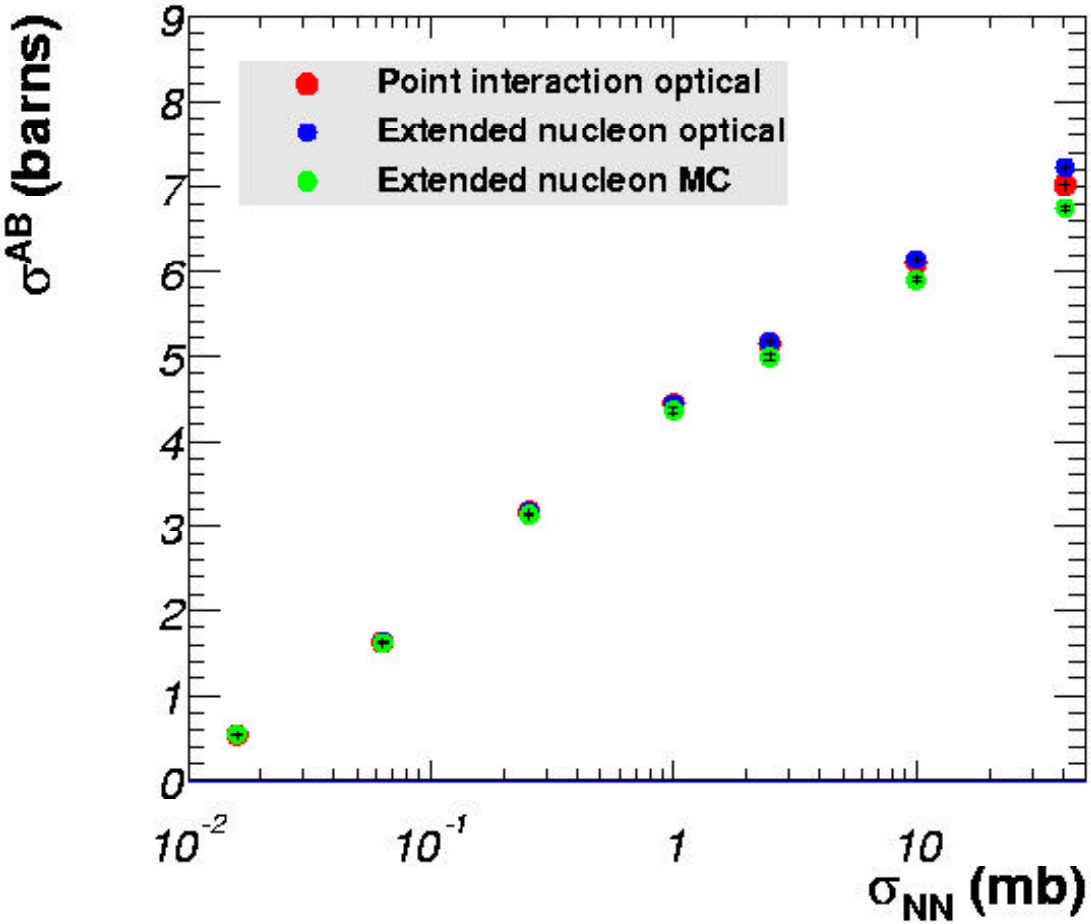
Total Cross Section

Standard Parameters: $R = 6.38\,fm, \quad a = .535\,fm, \quad s_{NN} = 41\,fm$

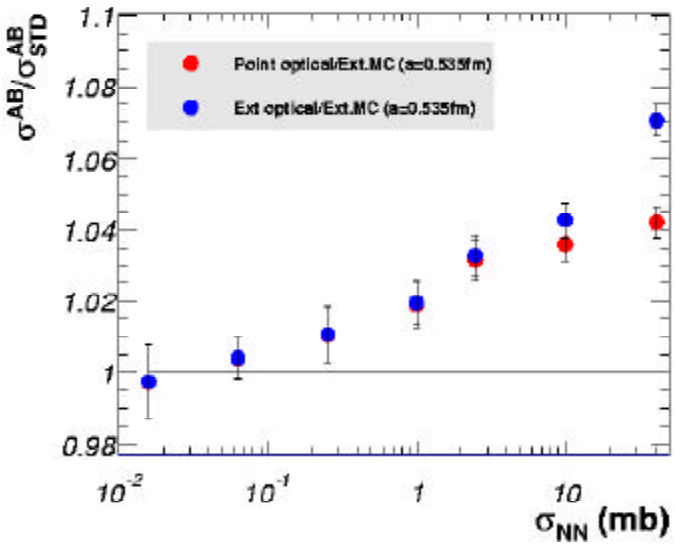


GlauberFest

Dilute Limit: Convergence



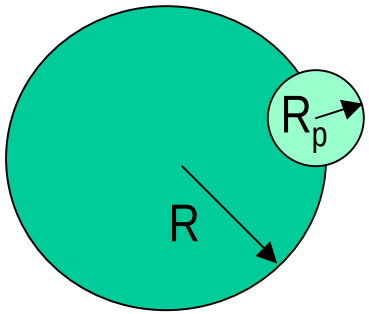
- As we lower the NN cross section, two methods converge!
- Reduces effects of multi-particle collisions

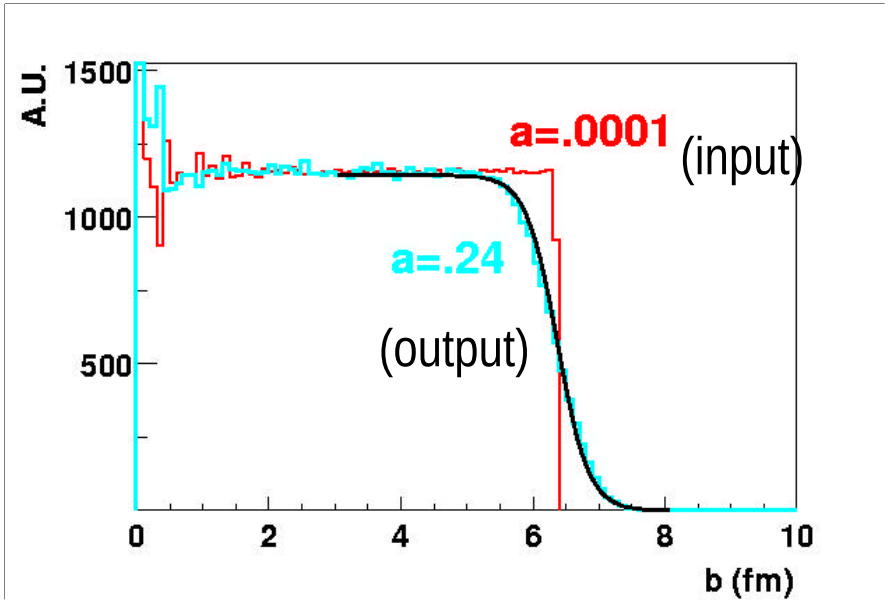


GlauberFest

Proton Charge Distribution

- Electron experiments measure *charge* distribution
- Density ρ(s) tracks nucleons
- Can do calculation
  - Nucleons distributed by 2-parameter Woods-Saxon
  - Nucleon charge distributed in gaussian sphere (R_p~.8fm)
  - Recalculate a' for original a



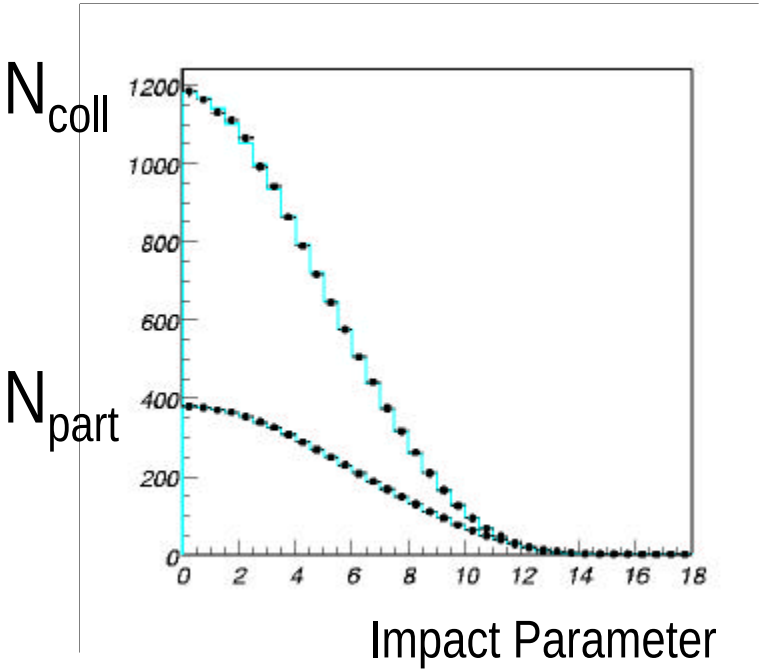


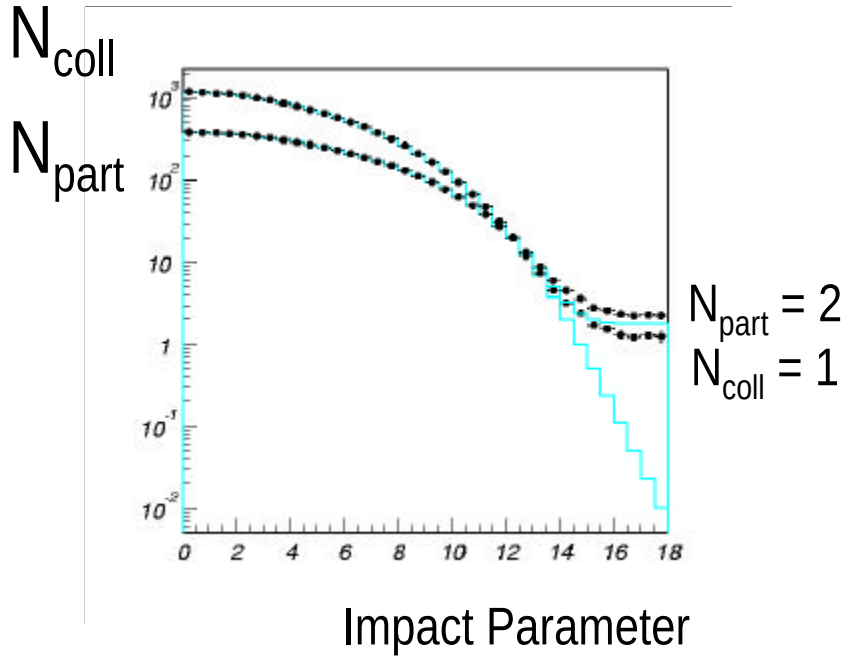
- a~0 induces a ~ .24 fm
  - Adds in quadrature
  - a(input) = .48 gives a~.535
  - Gives s_{TOT} **= 6.5 b**

BROOKHAVEN NATIONAL LABORATORY

GlauberFest

Impact parameter dependence

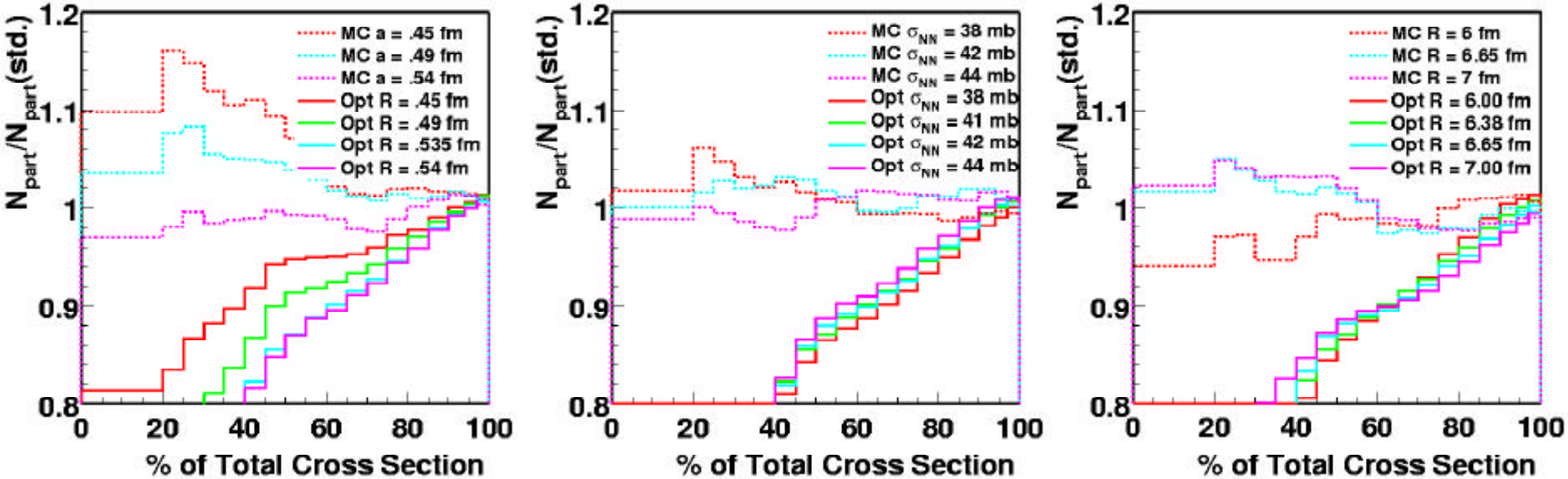




- Using standard parameters
- Both approaches yield same $N_{part}(b)$, $N_{coll}(b)$
- We have fixed N_{part} to prevent $N_{part}<2$, not N_{coll}
  - $N_{part}(b) \times (1-P_0(b))$ where $P_0(b) = \exp(-AB\sigma_{NN}T_{AB})$

GlauberFest

Npart in %-ile bins



- Although $N_{part}(b)$ and $N_{coll}(b)$ agree, bins in %-ile bins disagree systematically
  - Clear separation in reasonable ranges in R,σ
  - Need to lower a substantially to even approach agreement
- Total cross section is clearly not the only controlling parameter for N_{part} in bins

Conclusions

- Two kinds of Glauber calcuations are done
  - Optical Limit & Monte Carlo
- Agree on certain features
  - $N_{part}(b)$, $N_{coll}(b)$
- Disagree on others
  - Total cross section – 6.5 b vs. 7-7.2 b
  - Extraction of N_{part} vs. fraction
  - Connection between these is not completely clear
- Two approaches converge in dilute limit ($\sigma_{NN} \rightarrow 0$)
- Side-point about skin thickness
  - a ~ .48 fm might be better than the standard
  - Not critical, but could use some clarification