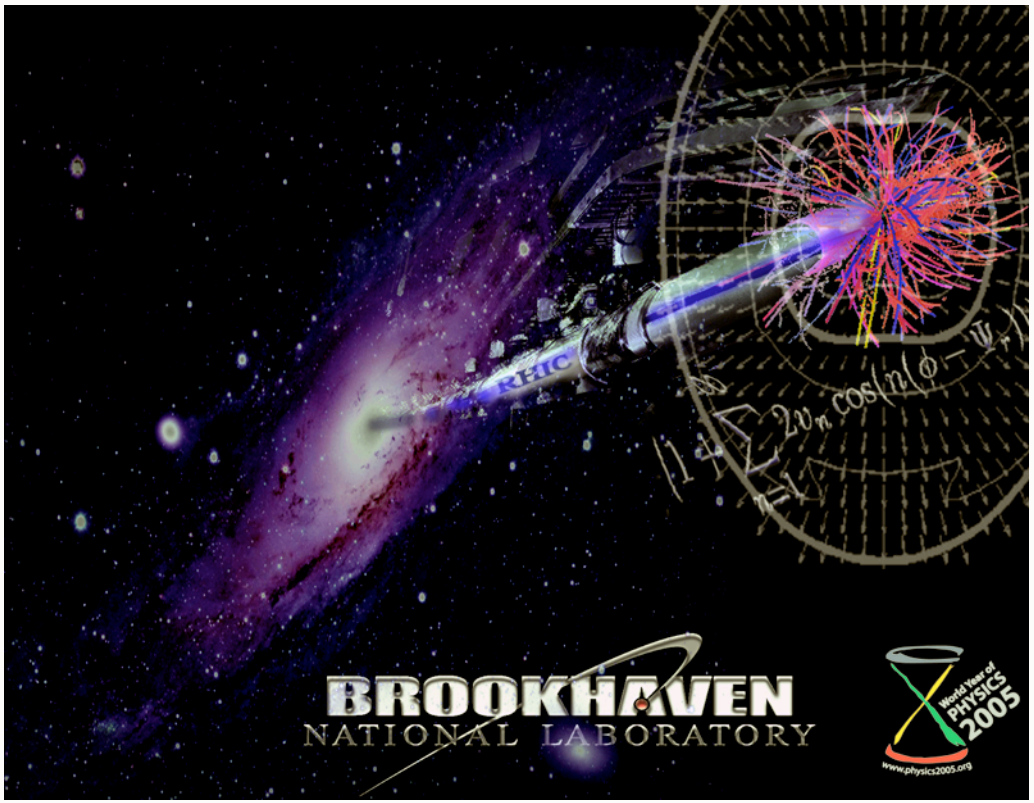
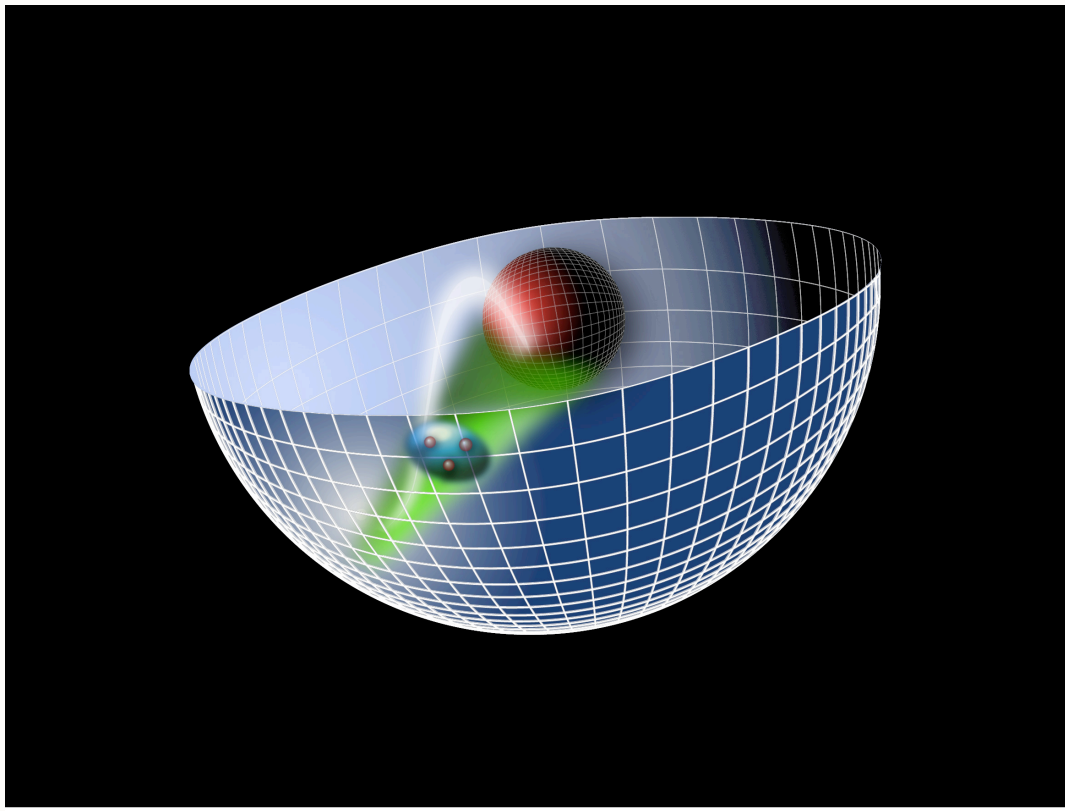
New Perspectives on QCD from AdS/CFT





Nuclear Physics Seminar
BNL, November 20, 2007

Stan Brodsky
SLAC, Stanford University

QCD Lagrangian

gluon dynamics

quark kinetic energy + quark-gluon dynamics

mass term

$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \overline{\psi_f} D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi_f} \psi_f$$

QCD color charge

field strength tensor

covariant derivative

quark field

Yang-Mills Gauge Principle: Invariance under Color Rotation and Phase Change at Every Point of Space and Time

Dimensionless Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.

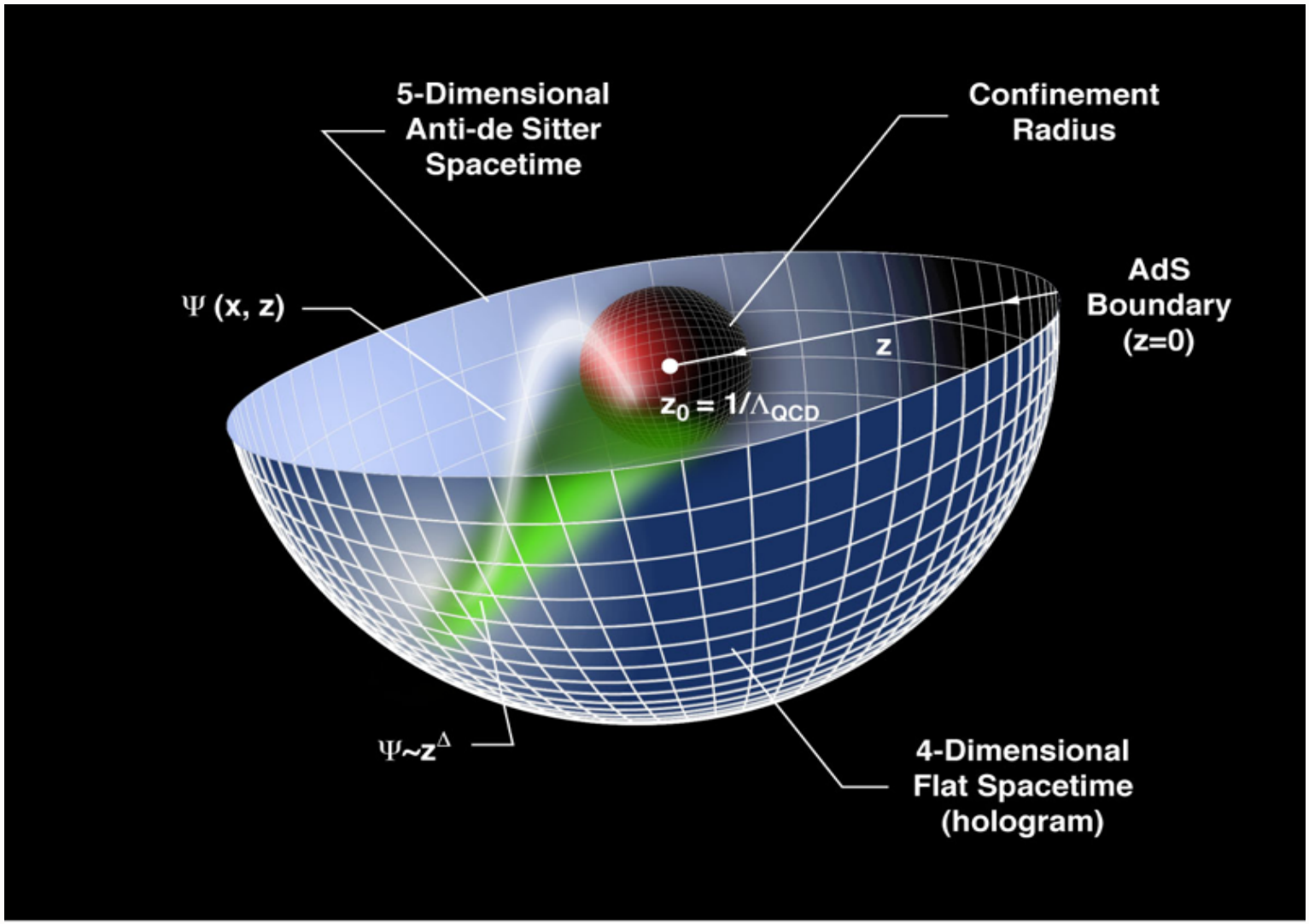
- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing, QGP, CGC, ...

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities. —Mark Twain

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Holographic Model*

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

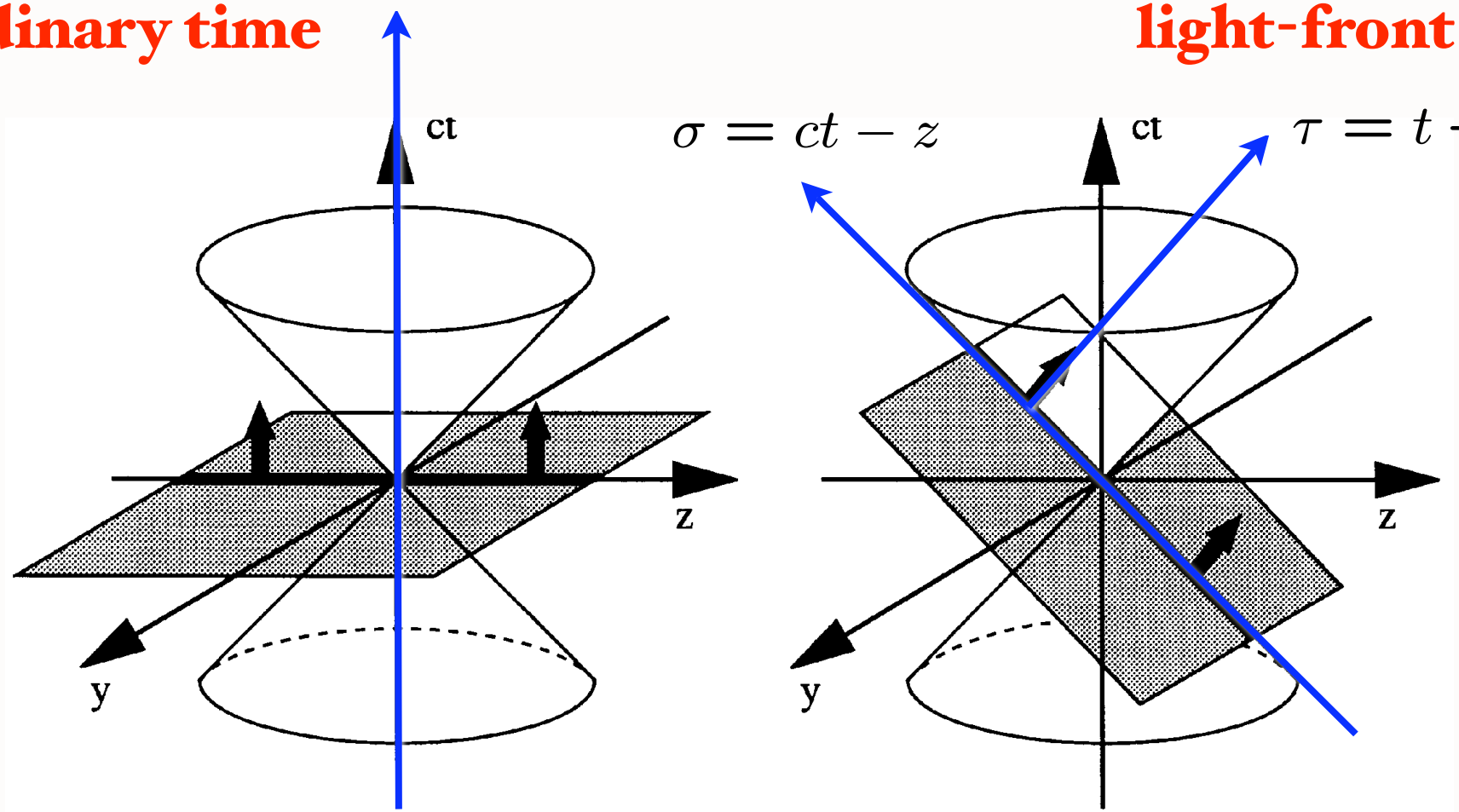
in collaboration with Guy de Teramond

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

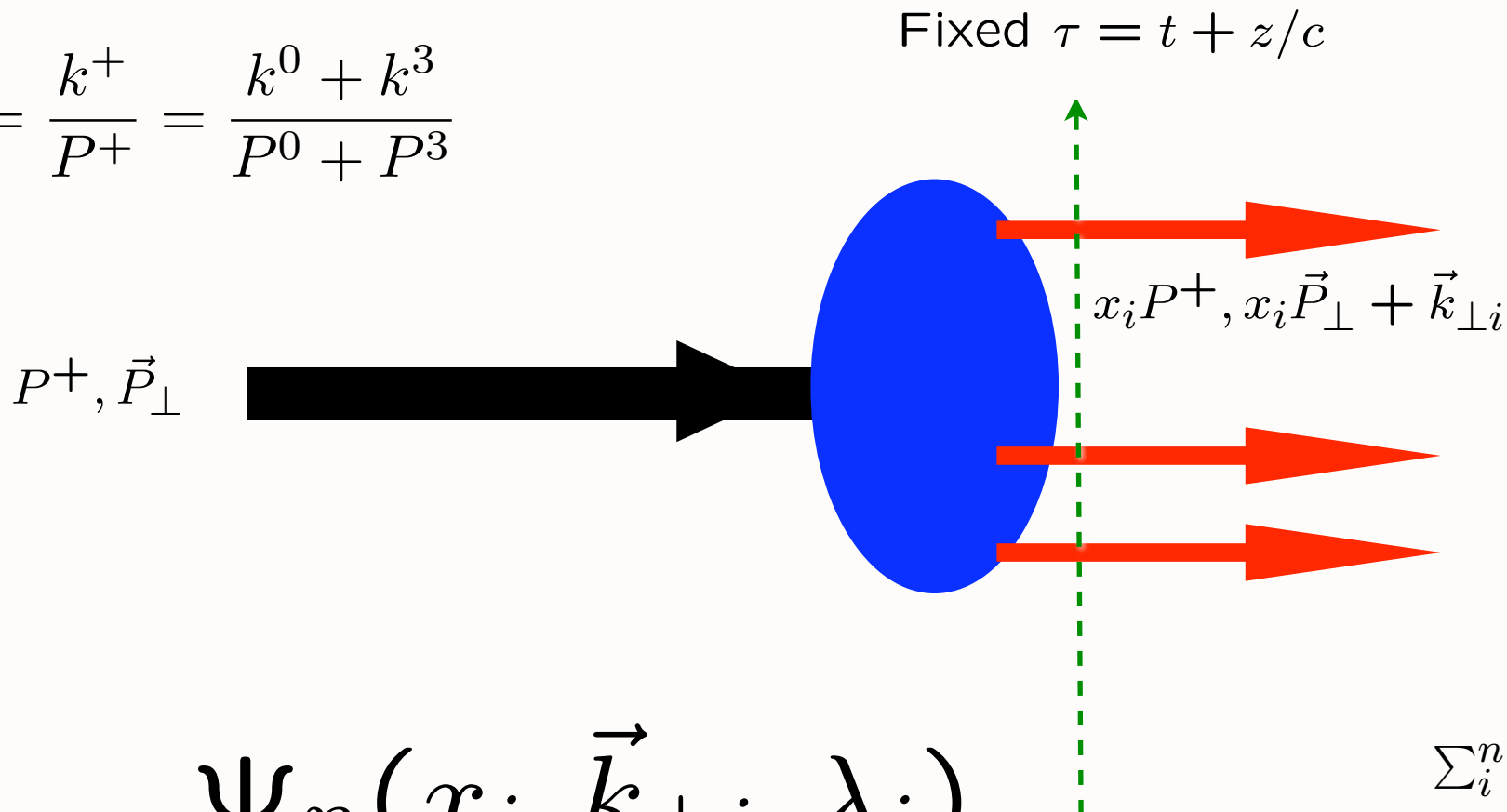
Evolve in light-front time!



Instant Form **Front Form**

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

'Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
'tis an immortal thing

...A moment standing still for ever.

James Montgomery 1833

Sed fugit, interea, fugit irreparabile tempus.
VIRG. *Georg.* iii. 284.

Light-Front Dynamics and Mapping AdS Modes

Light-Front Wave Functions in QCD

- Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h\rangle = \sum_n \psi_{n/h}|\psi_h\rangle$ of the LF Hamiltonian $H_{LF} = P^2 = P^+P^- - \mathbf{P}_\perp^2$, $H_{LF}|P\rangle = \mathcal{M}^2|P\rangle$, at fixed LF time $\tau = t + z/c$ (Dirac '49; Pauli and Pinsky, sjb Phys. Rept. 1988).

- Fock components

$$\psi_{n/h}(x_i, \mathbf{k}_{\perp i}) = \left\langle n;\ x_i, \mathbf{k}_{\perp i}, \middle|\psi_h(P^+, \mathbf{P}_\perp)\right\rangle,$$

  frame independent and encode hadron properties in high momentum-transfer collisions.

- Momentum fraction $x_i = k_i^+/P^+$ and $\mathbf{k}_{\perp i}$ are the relative coordinates of parton i in Fock-state n

$$\sum_{i=1}^{n} x_i = 1 \quad \sum_{i=1}^{n} \mathbf{k}_{\perp i} = 0.$$

- Define transverse position coordinates $x_i \mathbf{r}_{\perp i} = x_i \mathbf{R}_\perp + \mathbf{b}_{\perp i}$

$$\sum_{i=1}^{n} \mathbf{b}_{\perp i} = 0, \quad \sum_{i=1}^{n} x_i \mathbf{r}_{\perp i} = \mathbf{R}_\perp.$$

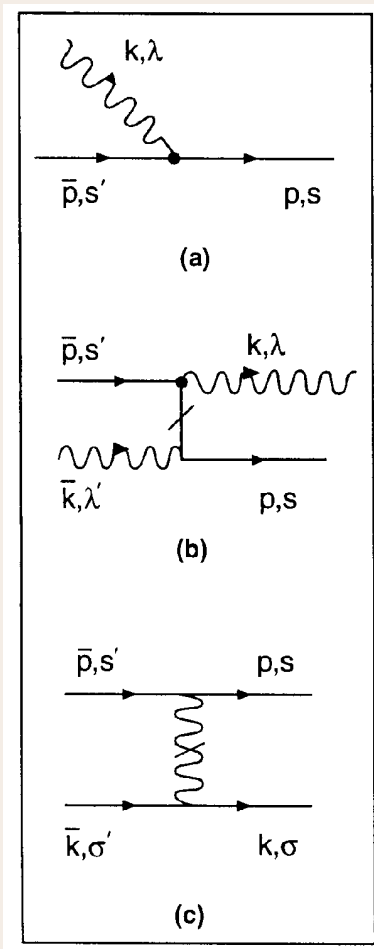
Light-Front QCD

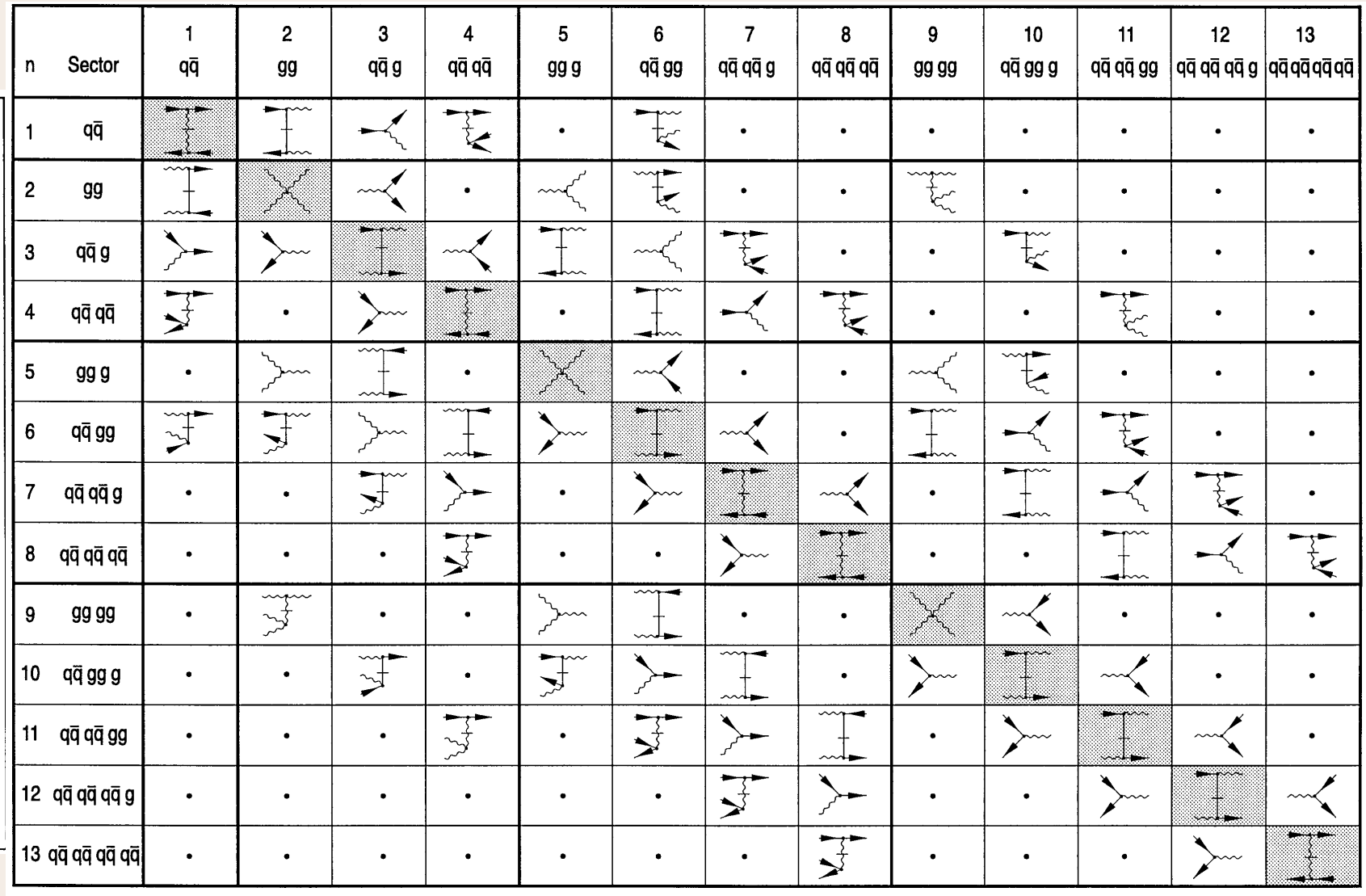
Heisenberg Matrix Formulation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

DLCQ

Discretized Light-Cone Quantization



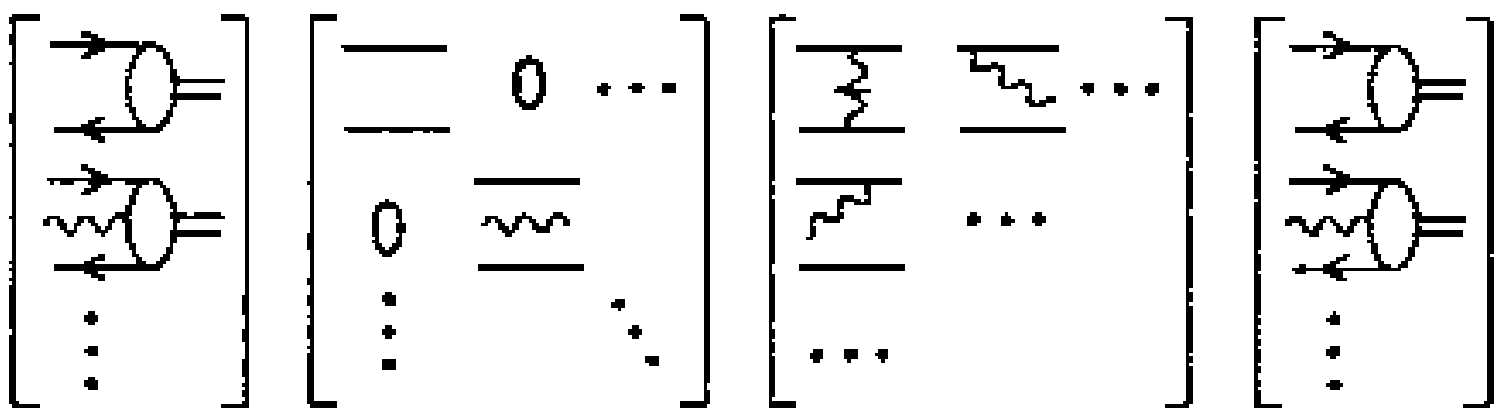


Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Hans Christian Pauli & sjb

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V\, |q\bar{q}\rangle & \langle q\bar{q}|\, V\, |q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V\, |q\bar{q}\rangle & \langle q\bar{q}g|\, V\, |q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G. P. Lepage, sjb

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

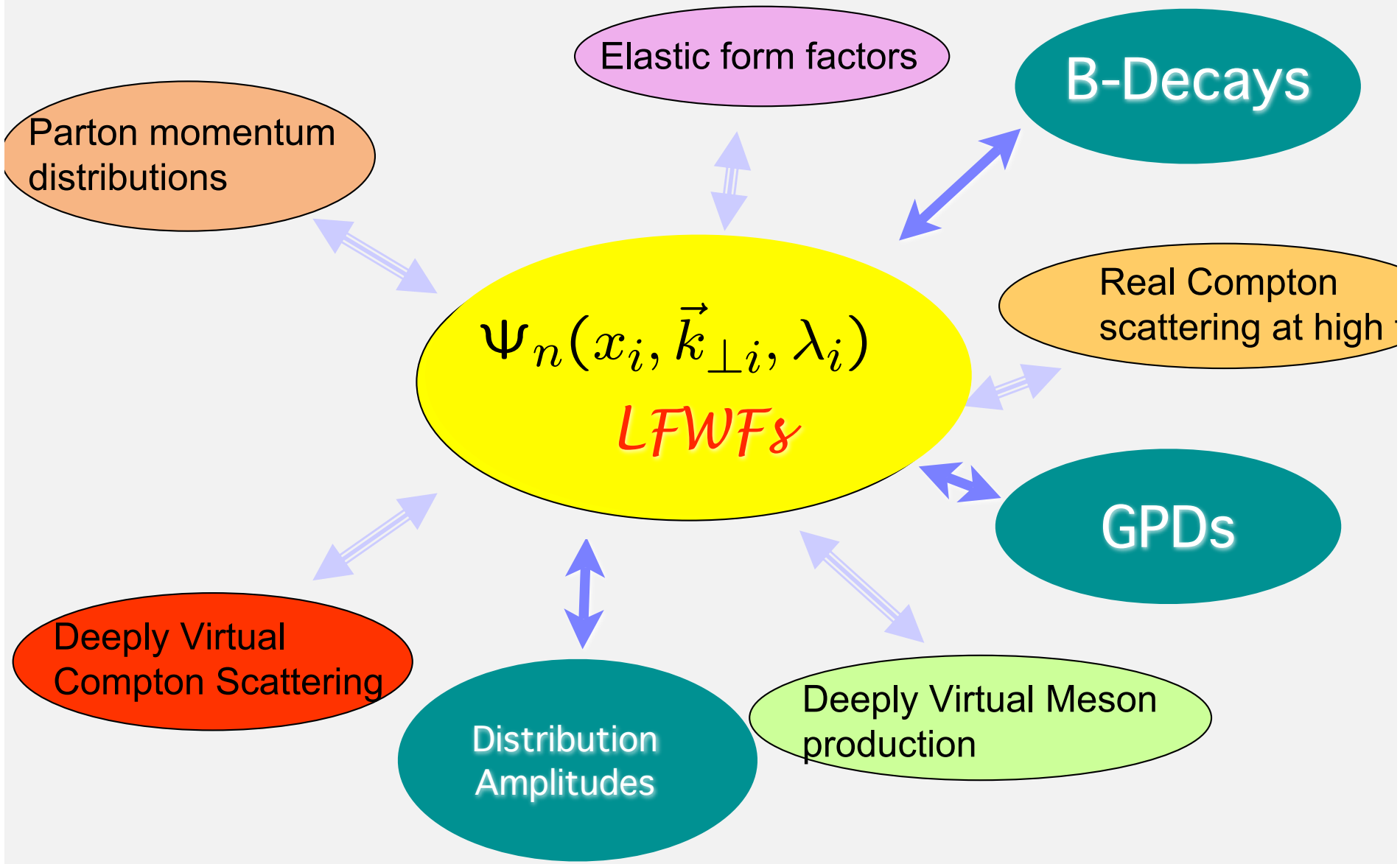
Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

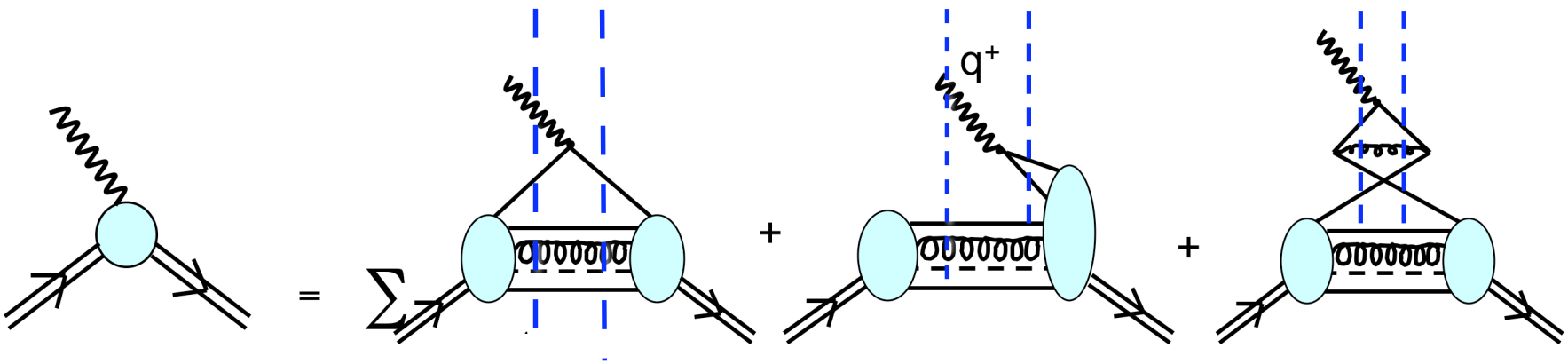
n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

A Unified Description of Hadron Structure

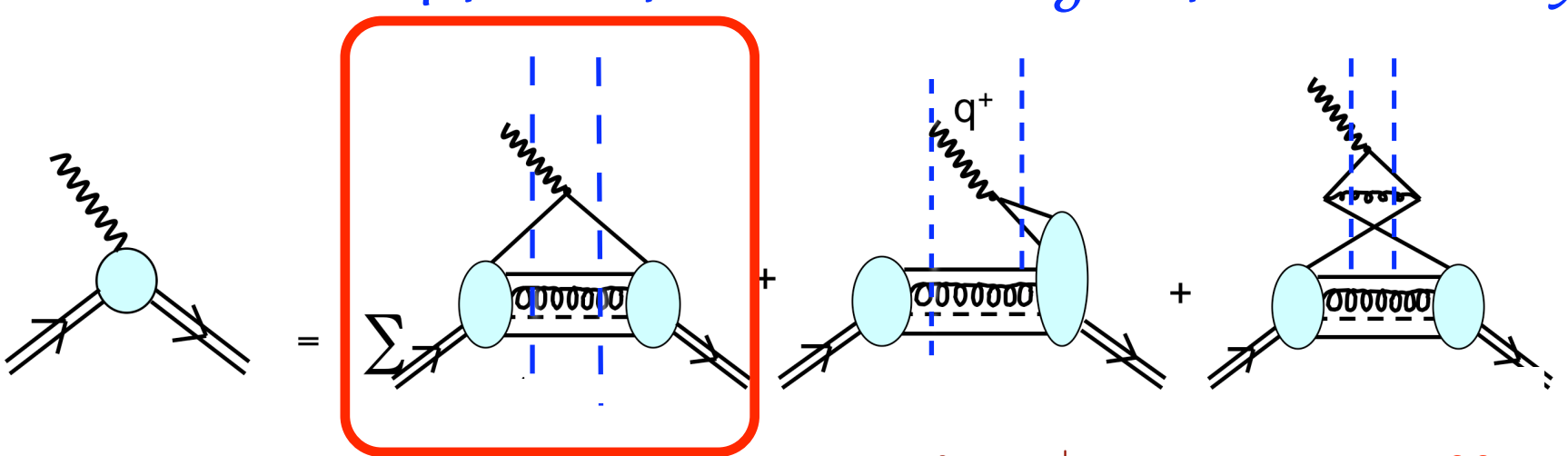


Calculation of Form Factors in Equal-Time Theory



Need vacuum fluctuations

Calculation of Form Factors in Light-Front Theory



zero for $q^+ = 0$ ***zero !!***

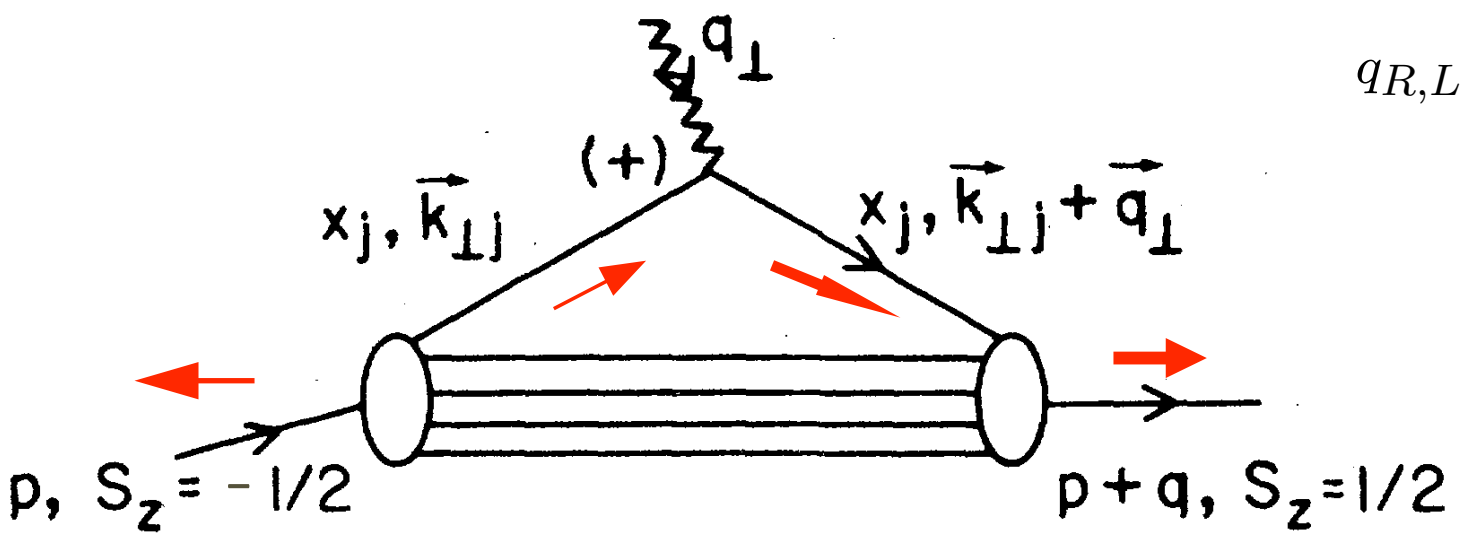
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



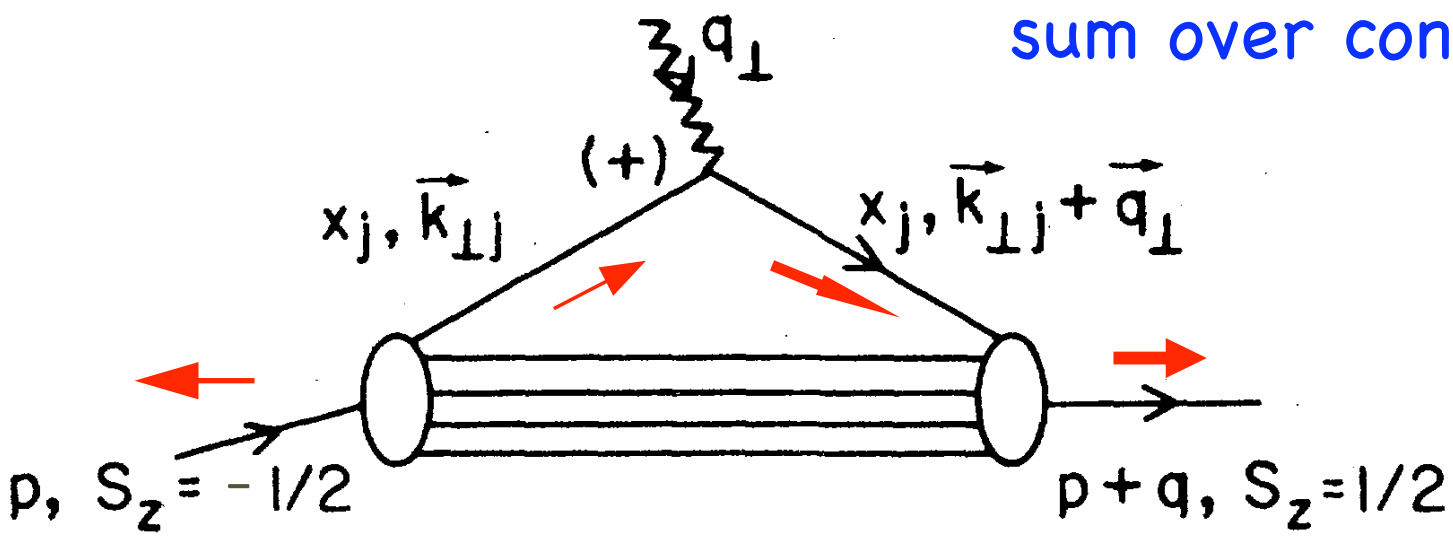
Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Anomalous gravitomagnetic moment B(0)

Okun, Kobzarev, Teryaev: *B(0) Must vanish because of Equivalence Theorem*

graviton

sum over constituents



Hwang, Schmidt, sjb; Holstein et al

$$B(0) = 0$$

Each Fock State

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

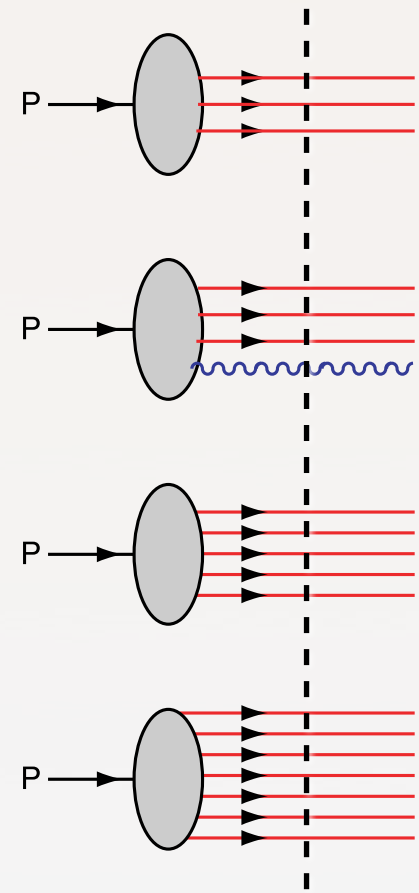
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks $\bar{u}(x) \neq \bar{d}(x)$

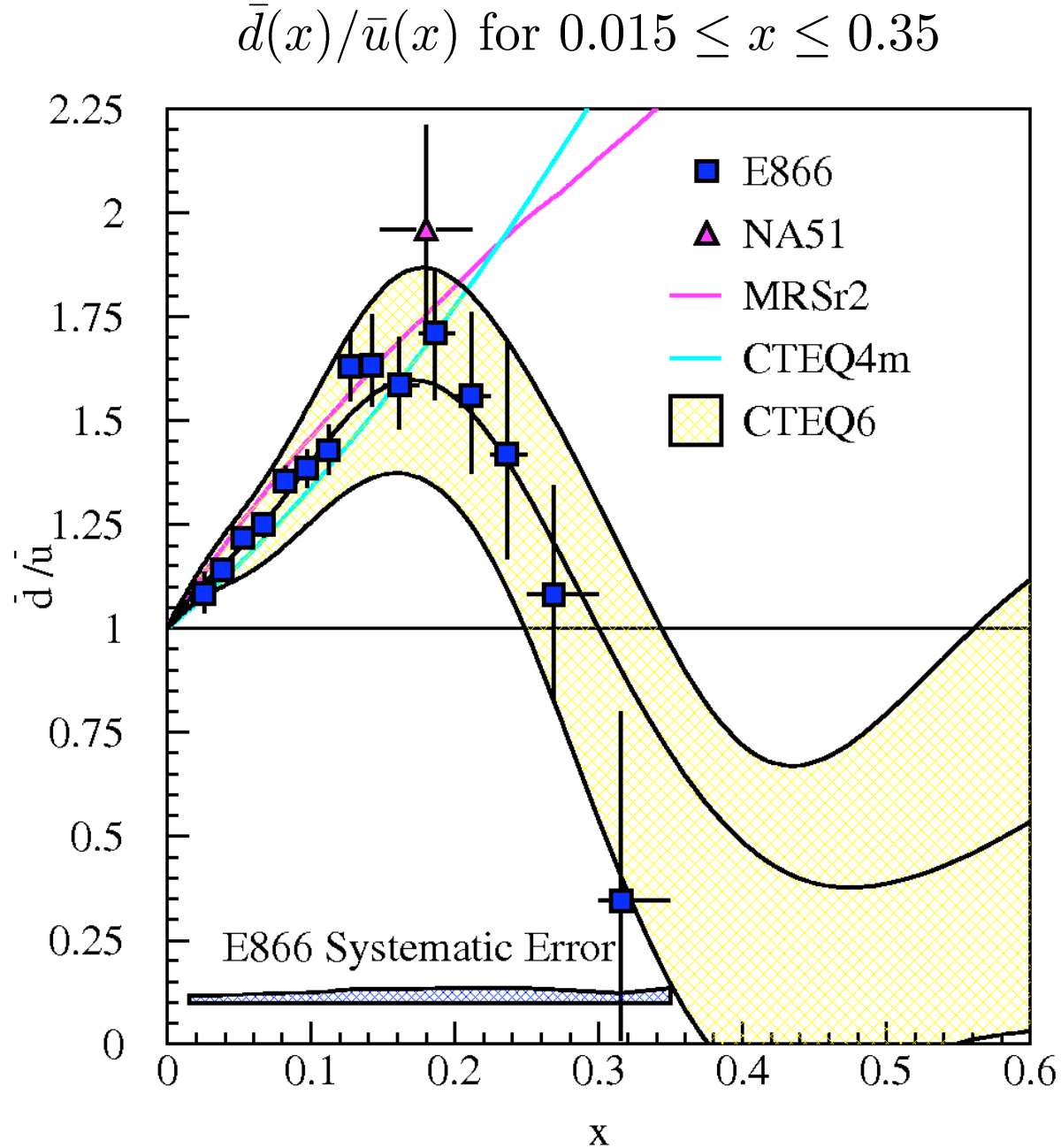
Mueller: BFKL DYNAMICS $\bar{s}(x) \neq s(x)$

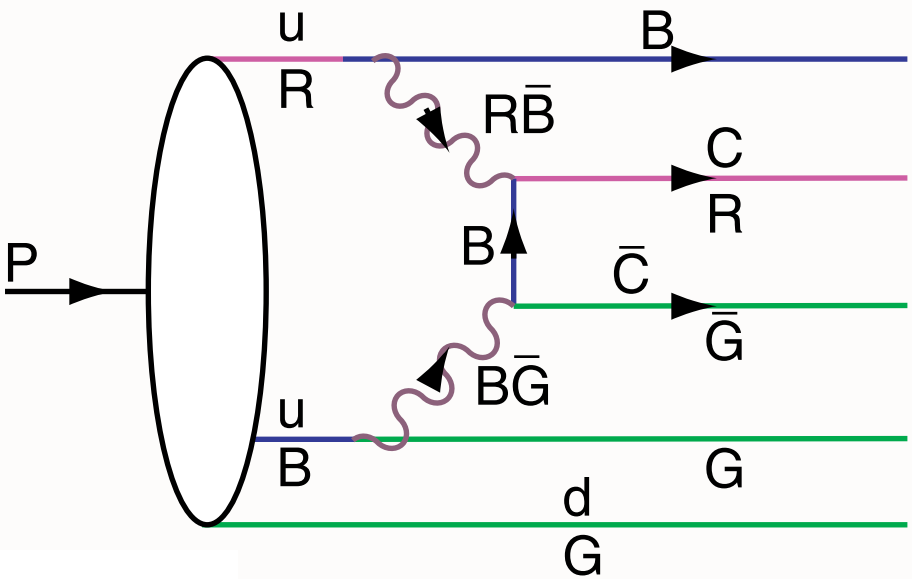
Light Antiquark Flavor Asymmetry

- Naïve Assumption from gluon splitting:

$$\bar{d}(x) = \bar{u}(x)$$

- E866/NuSea (Drell-Yan)





$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p>$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p>$

$c\bar{c}$ in Color Octet

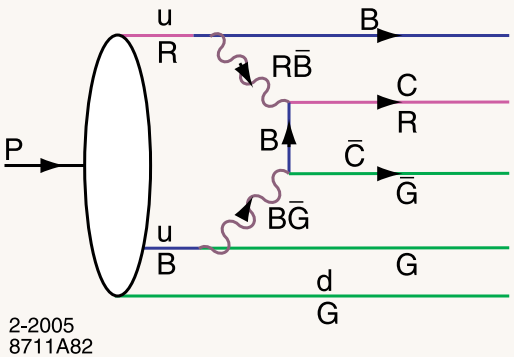
Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$\widehat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

High x charm!

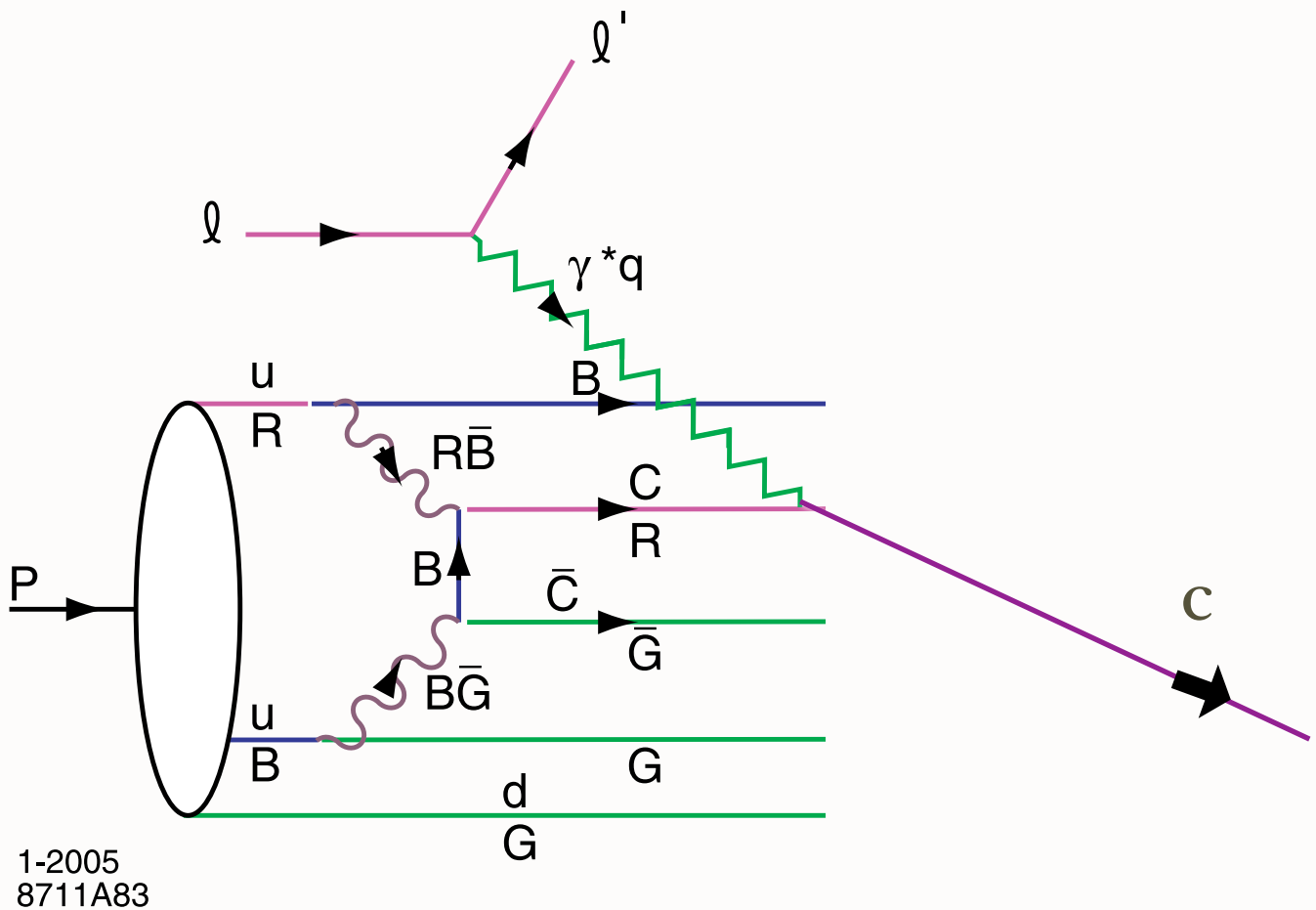
Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States



- Rigorous prediction of QCD, OPE
- Color - Octet + Color - Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

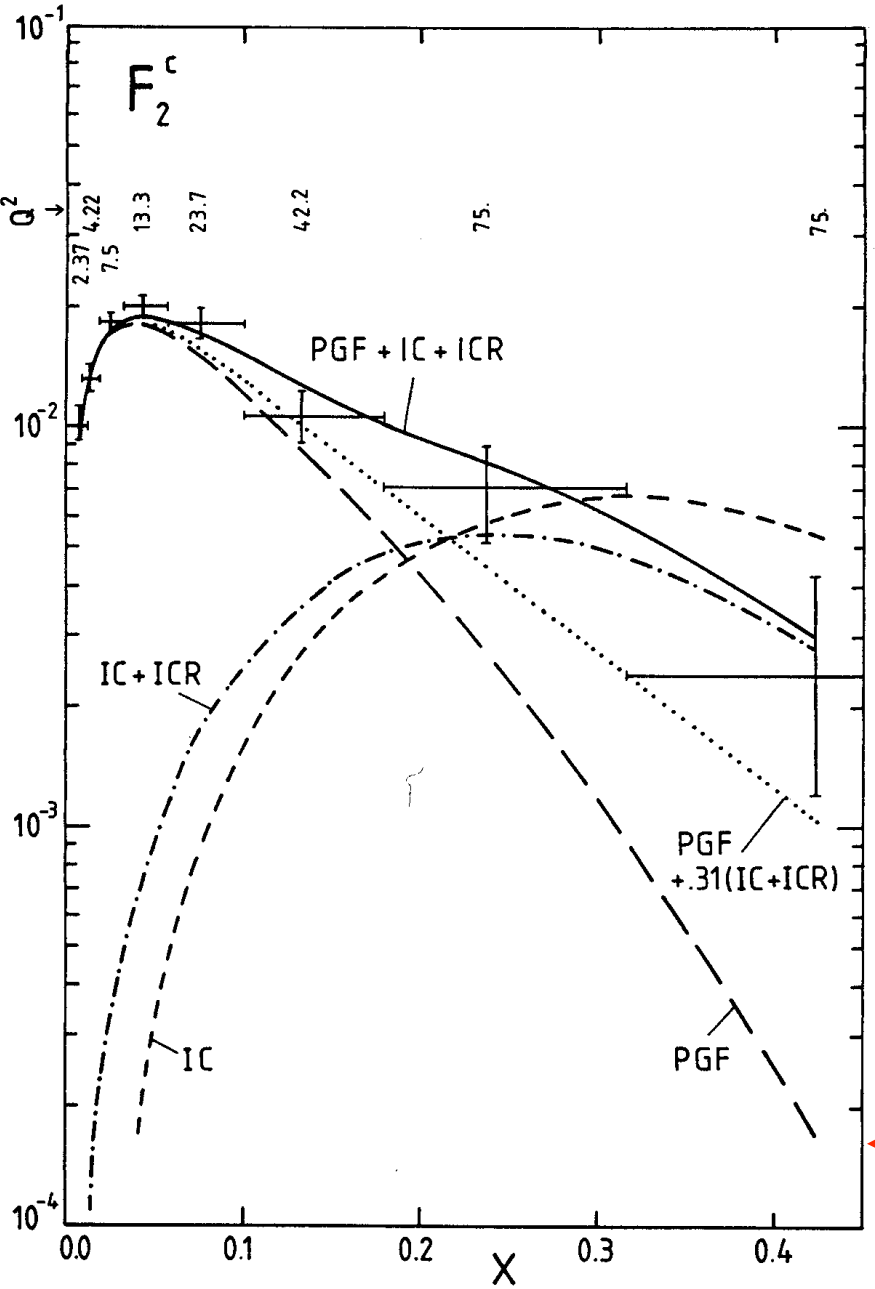
Measure c(x) in Deep Inelastic Lepton-Proton Scattering



Hoyer, Peterson, SJB

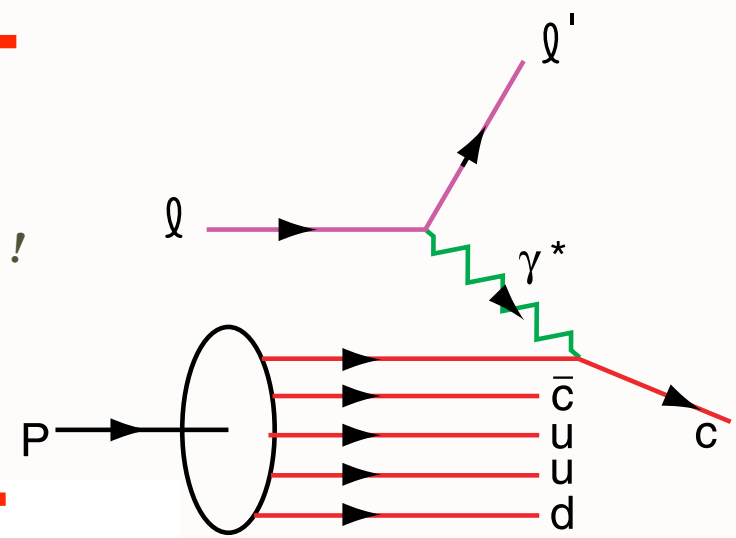
Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



First Evidence for Intrinsic Charm

factor of 30 !



DGLAP / Photon-Gluon Fusion: factor of 30 too small

- EMC data: $c(x, Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \to J/\psi X$

- High x_F $pp \to J/\psi J/\psi X$

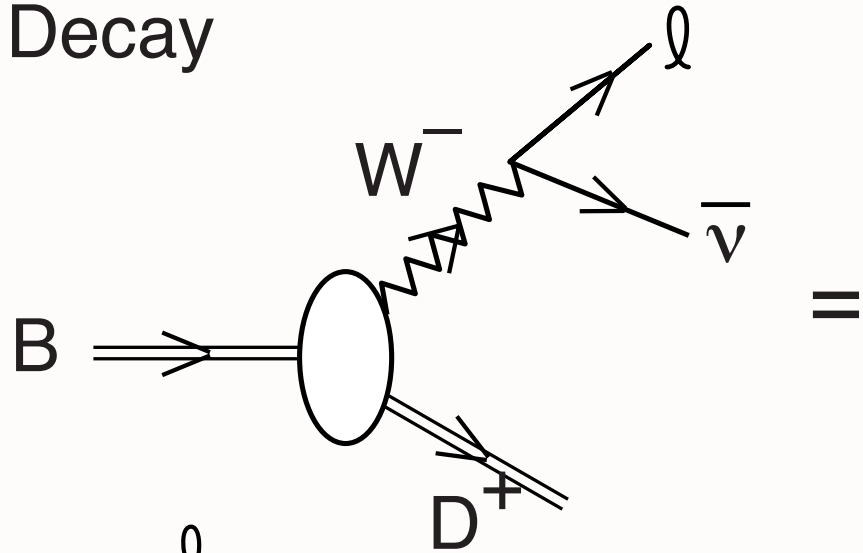
- High x_F $pp \to \Lambda_c X$

- High x_F $pp \to \Lambda_b X$

- High x_F $pp \to \Xi(ccd) X$ (SELEX)

Weak Exclusive Decay

$\langle D | J^+(0) | B \rangle$



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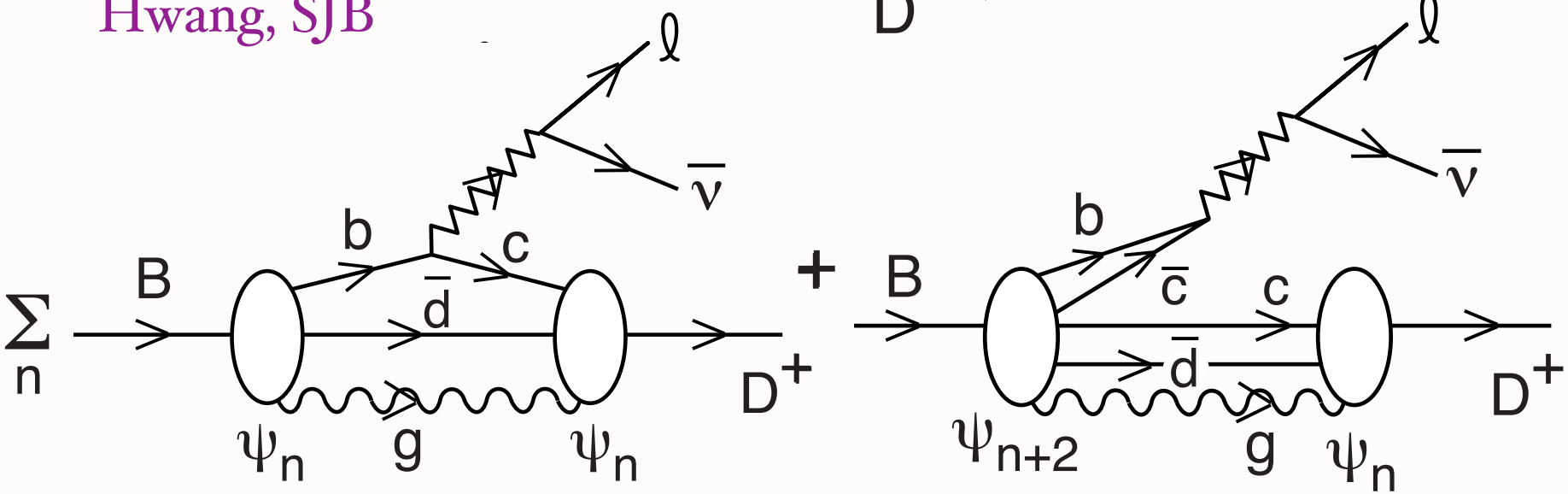
Exact Formula!

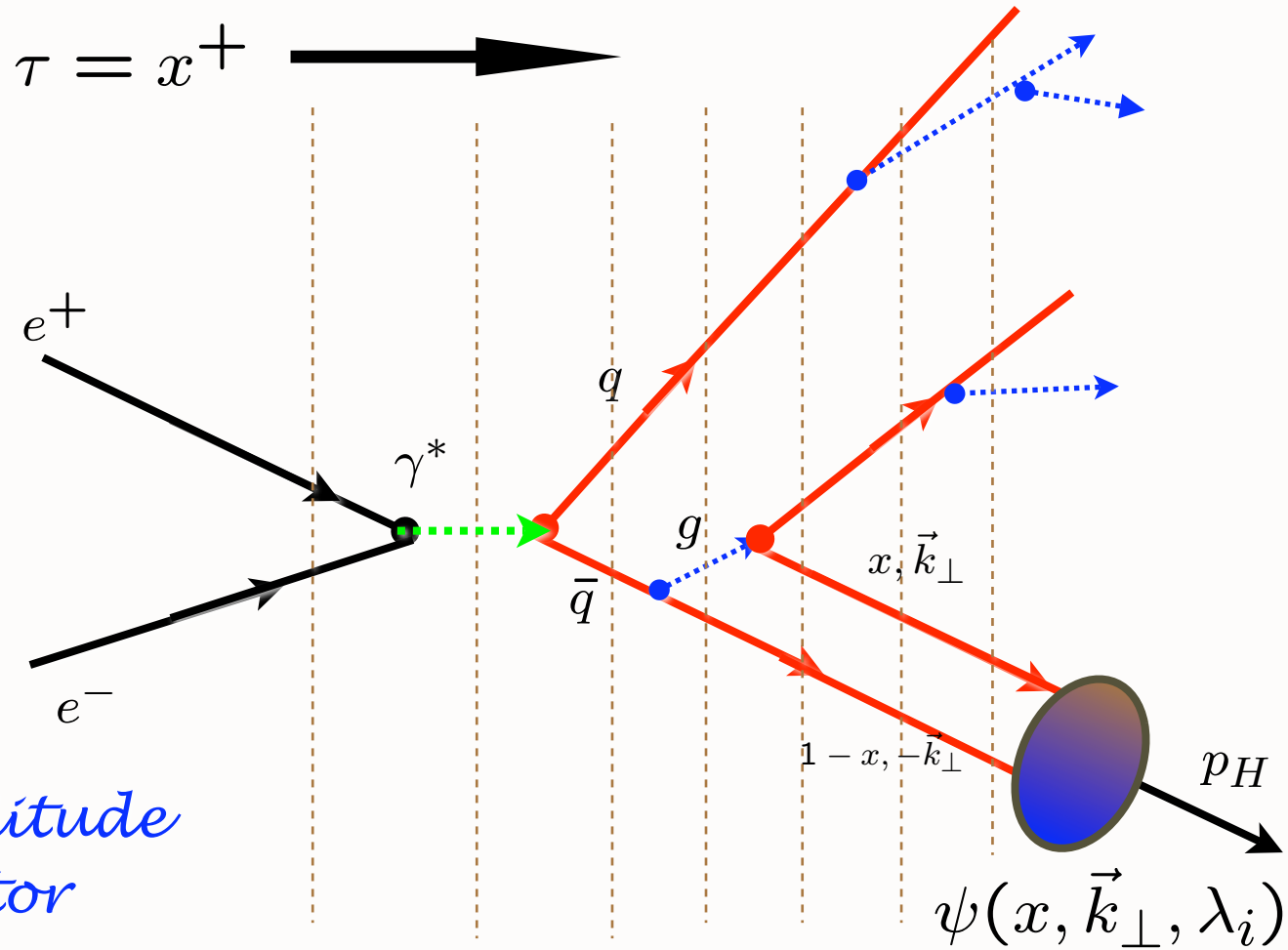
Hwang, SJB



Annihilation amplitude needed for Lorentz Invariance

Non-perturbative complication from IC Fock states!

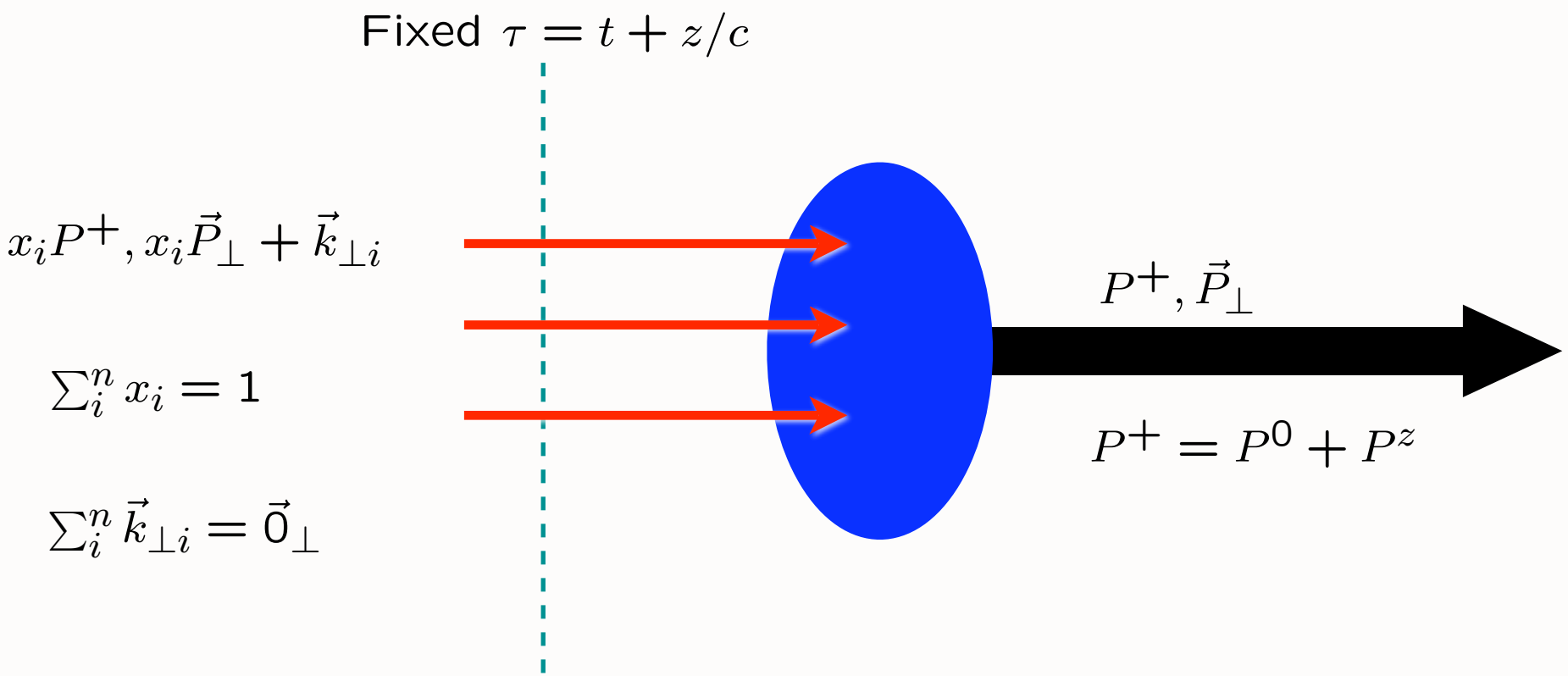
Hadronization at the Amplitude Level



Event amplitude generator

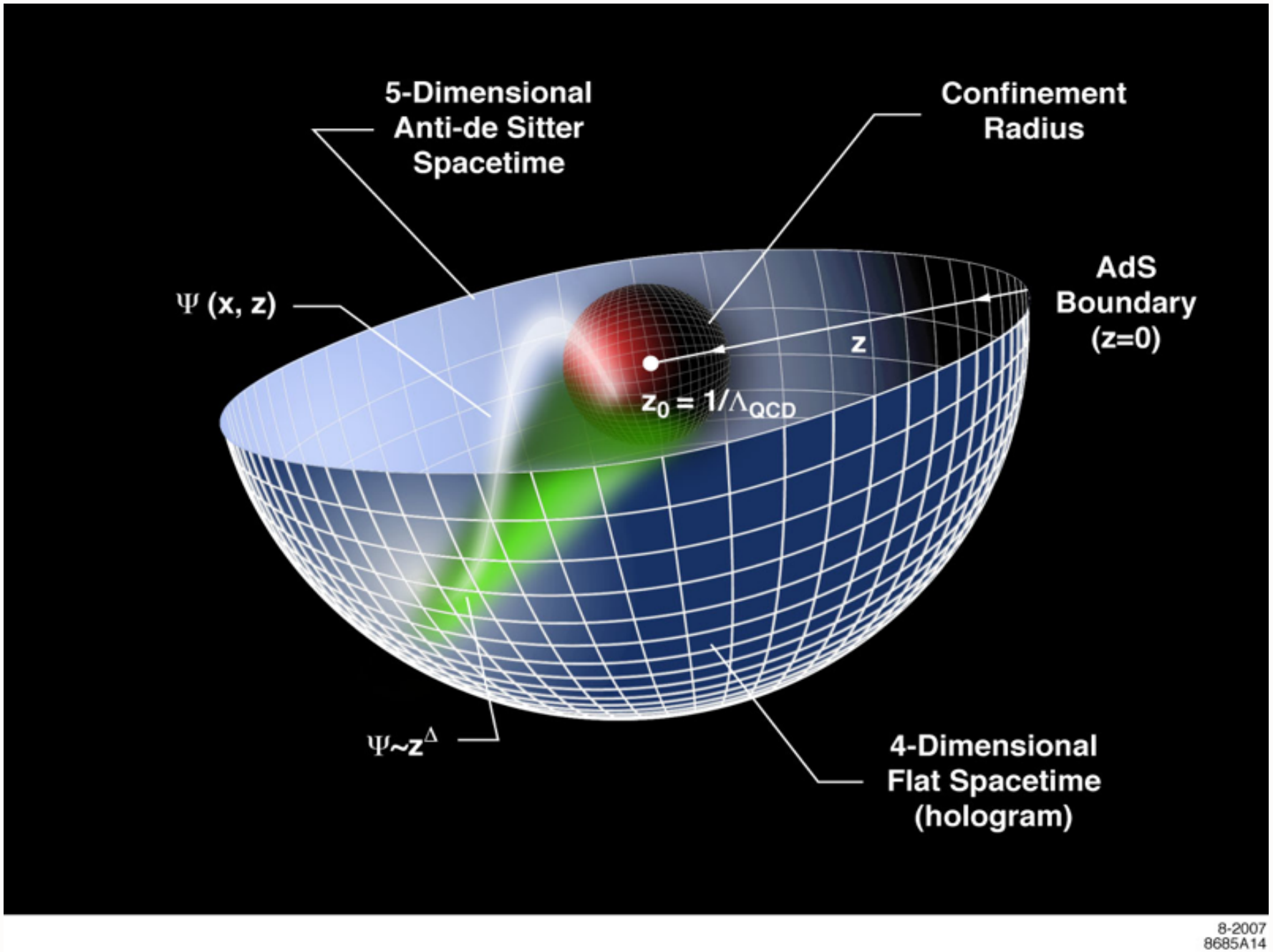
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front Wavefunctions



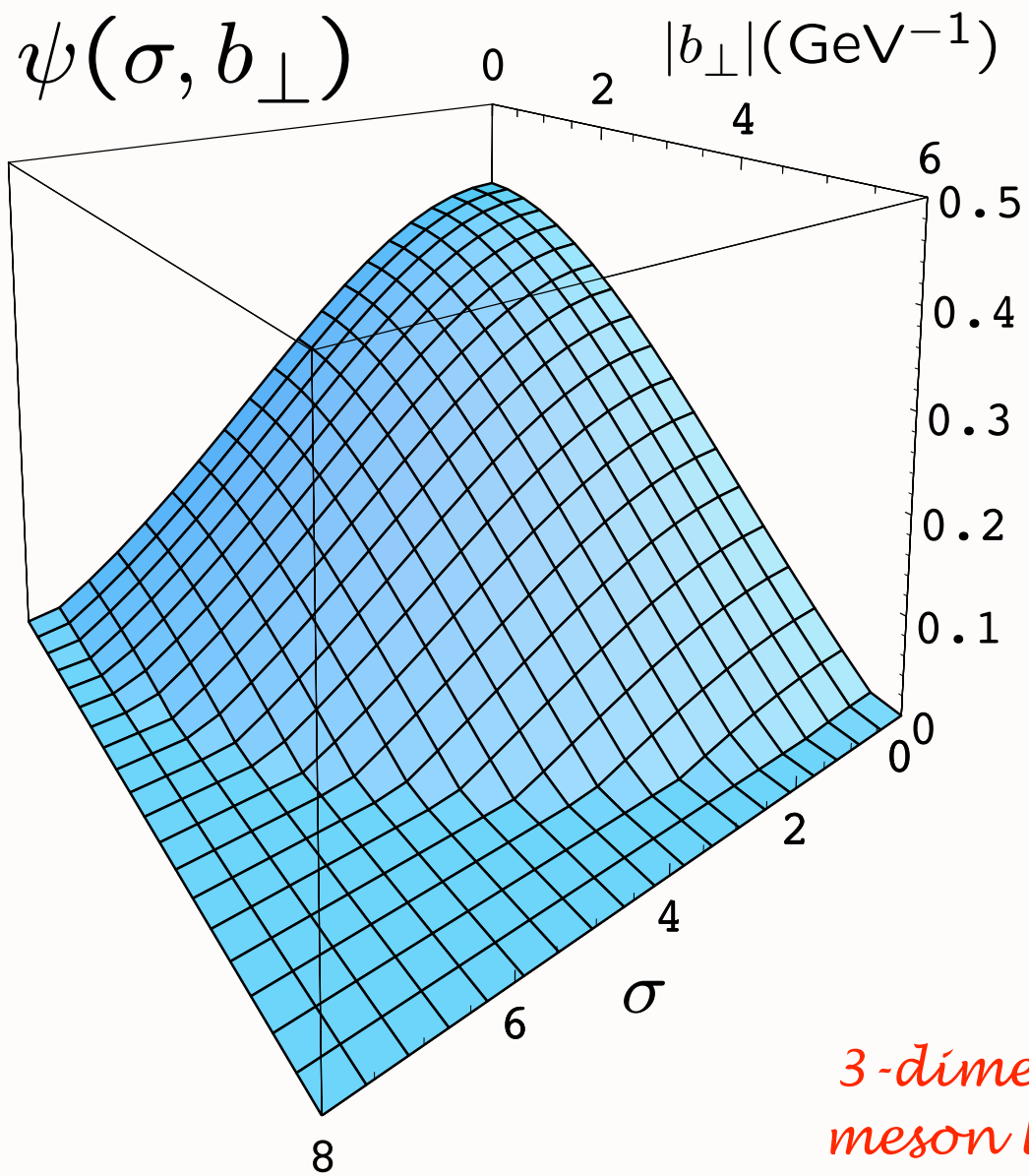
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

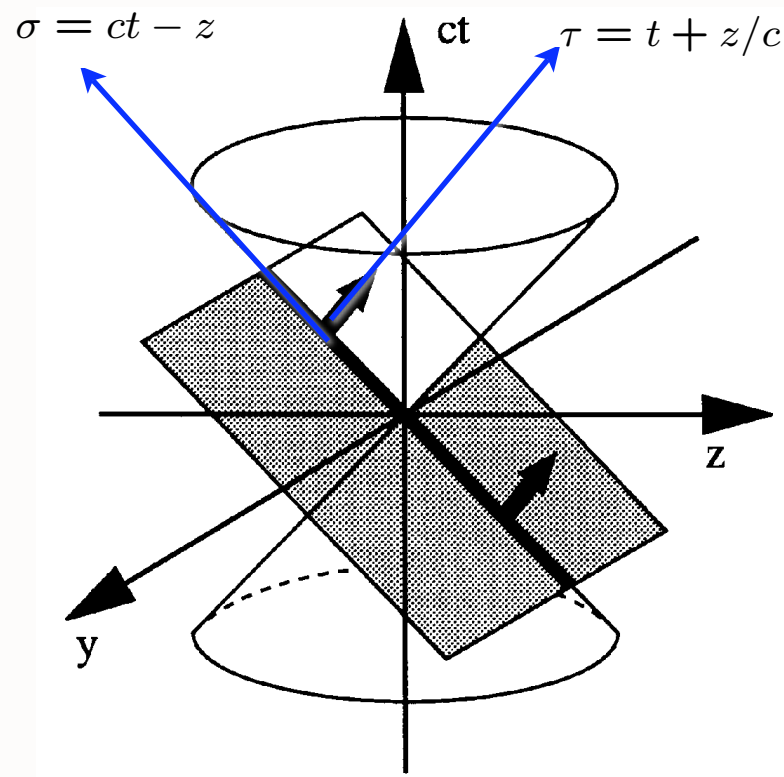
Invariant under boosts! Independent of P^μ


5-Dimensional
Anti-de Sitter
Spacetime
Confinement
Radius
Ψ (x, z)
AdS
Boundary
(z=0)
z
$z_0 = 1/\Lambda_{QCD}$
$\Psi \sim z^{\Delta}$
4-Dimensional
Flat Spacetime
(hologram)
8-2007
8685A14

AdS/CFT Holographic Model

G. de Teramond
SJB

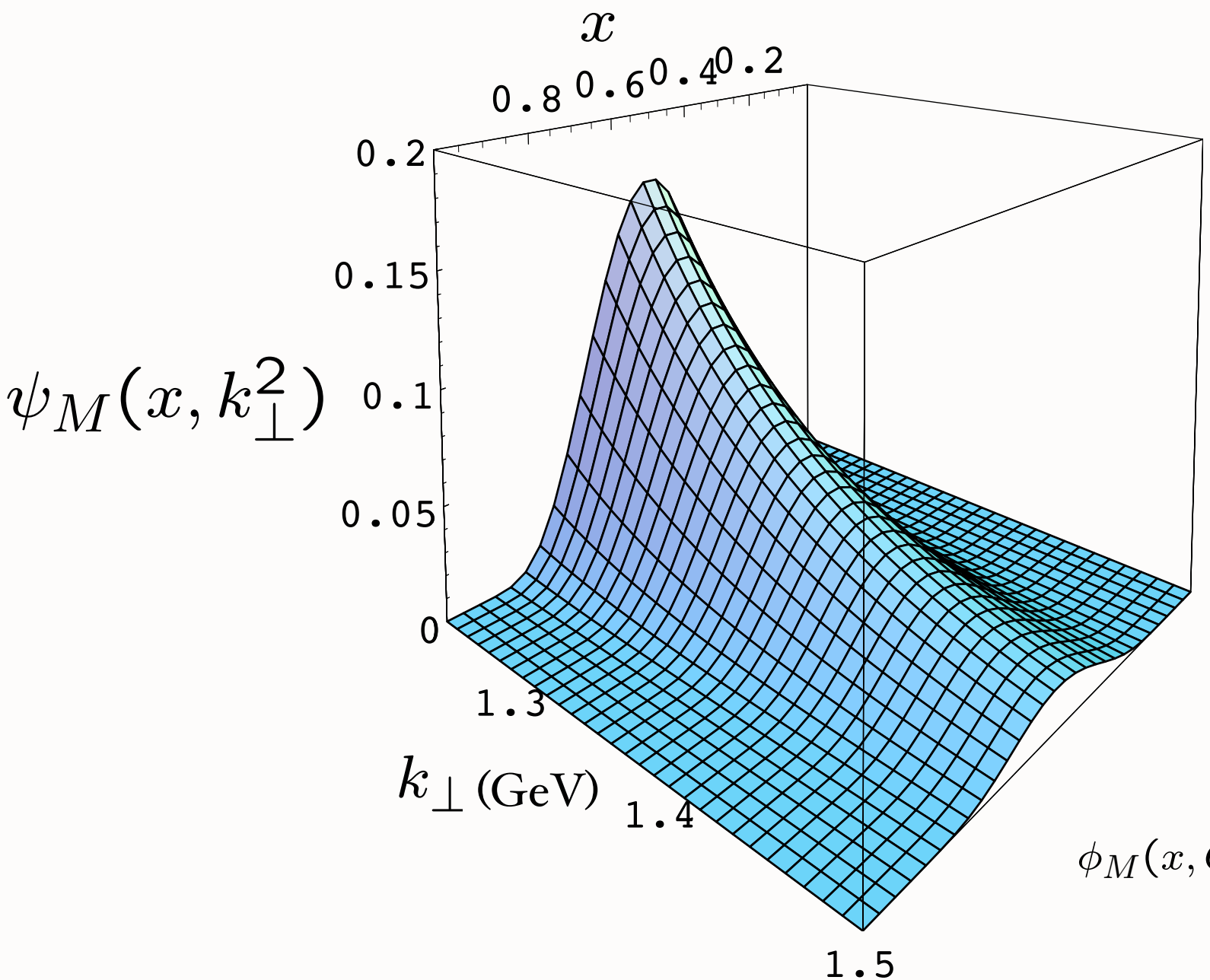




The front form

3-dimensional photograph:
meson LFWF at fixed LF Time

Prediction from AdS/CFT: Meson LFWF



"Soft Wall" model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathrm{M}^{\mu\nu}, \mathrm{P}^{\mu}, \mathrm{D}, \mathrm{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

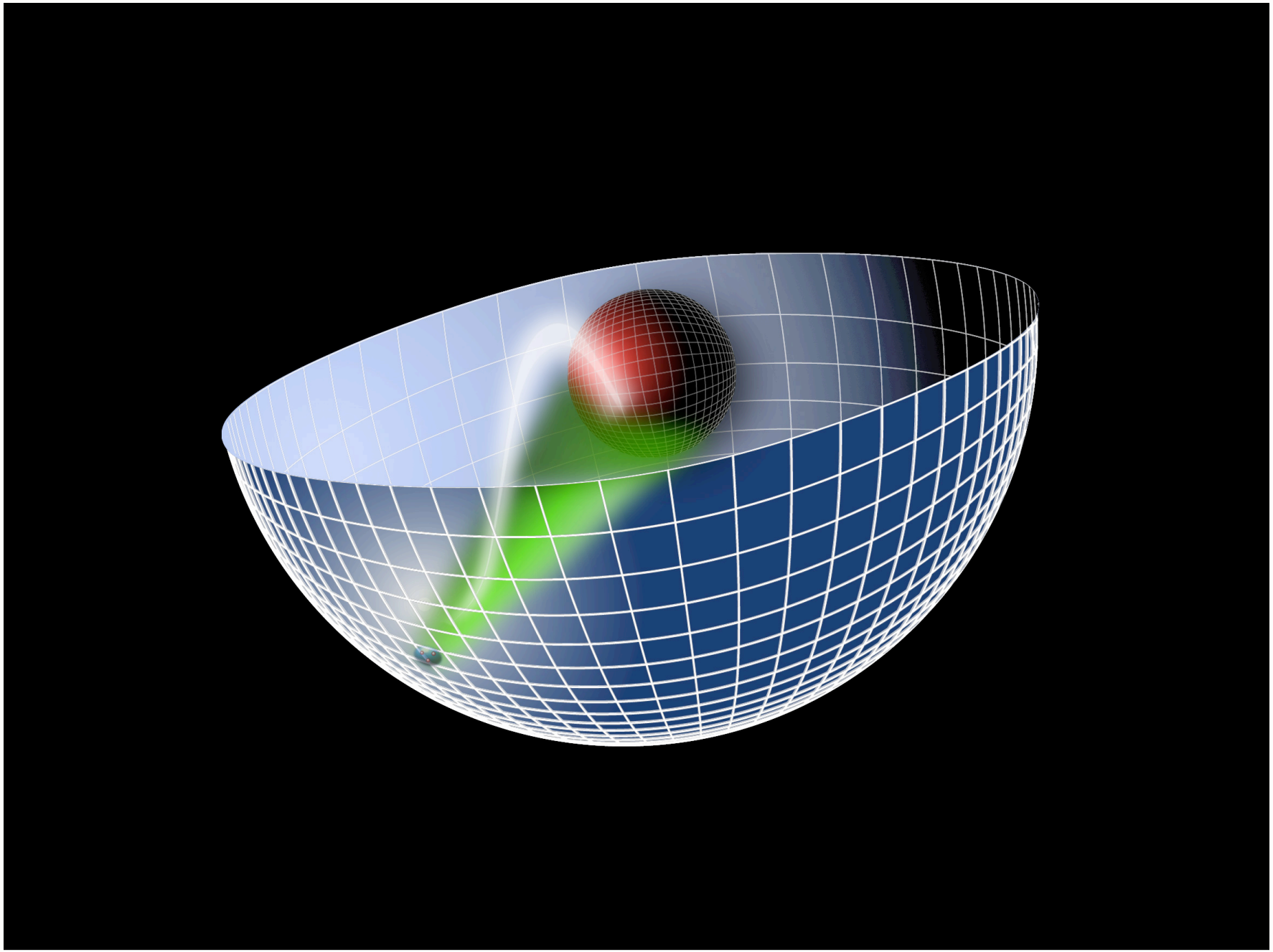
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

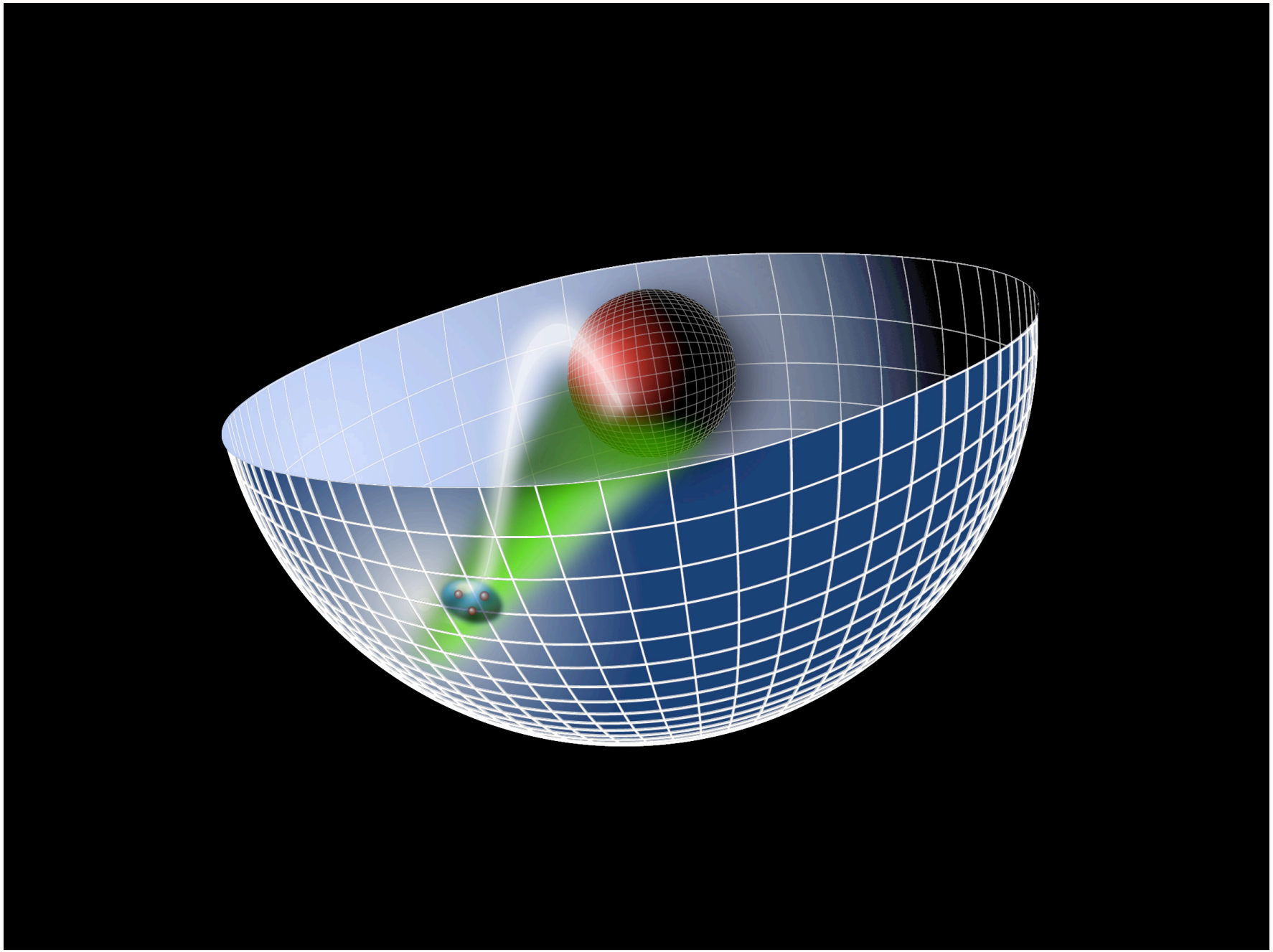
- Different values of z correspond to different scales at which the hadron is examined.

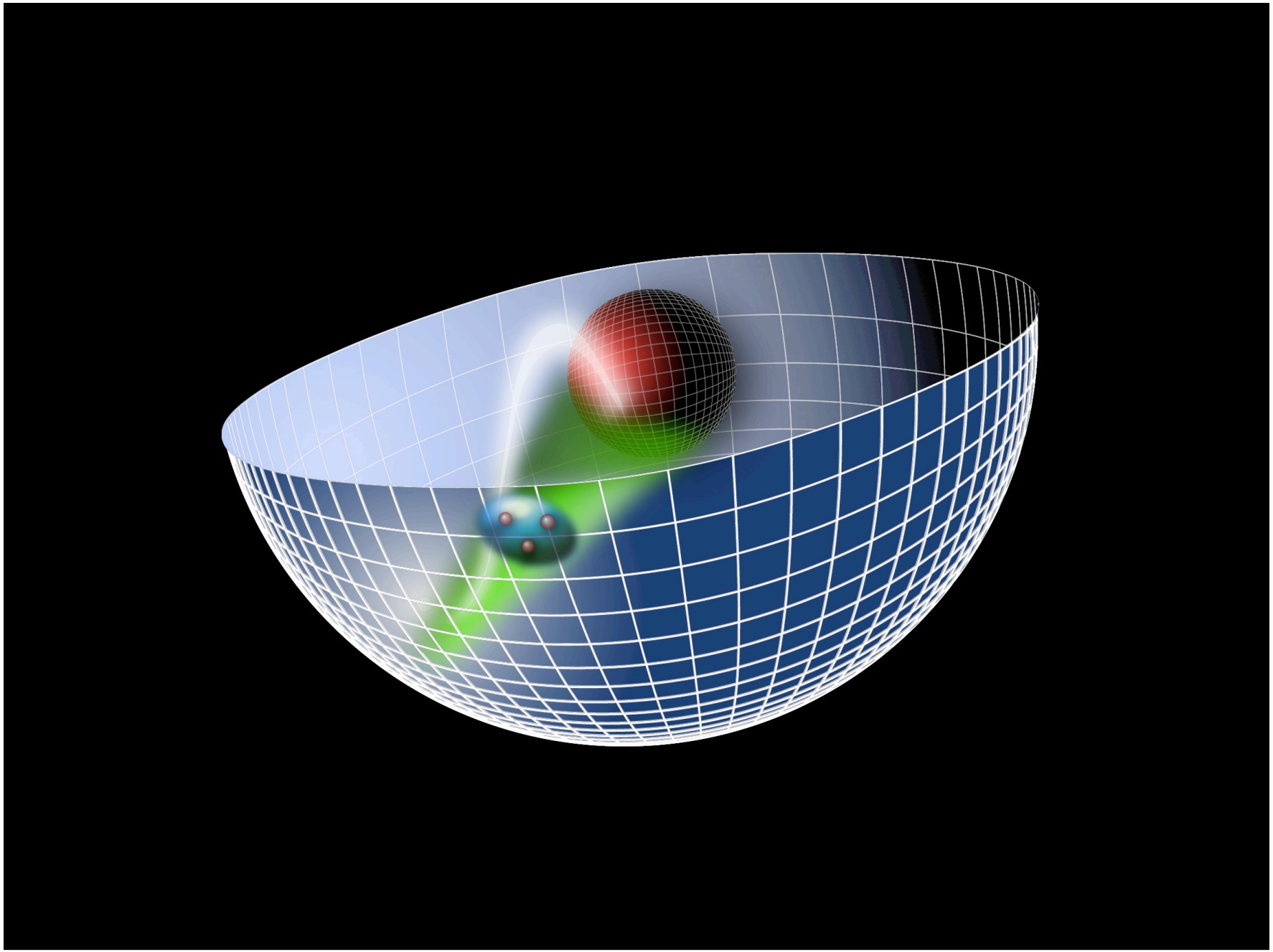
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

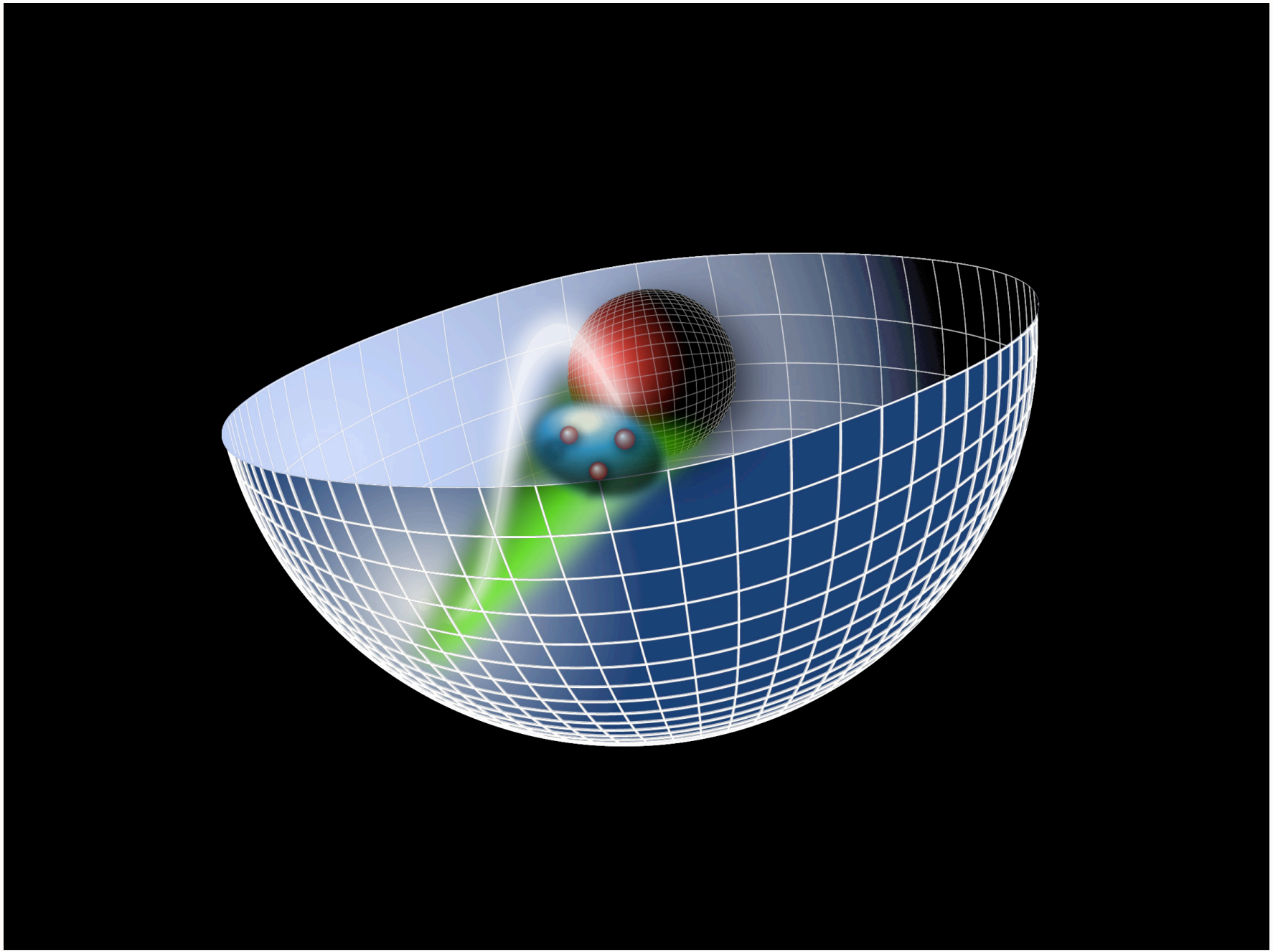
$x^2 = x_\mu x^\mu$: invariant separation between quarks

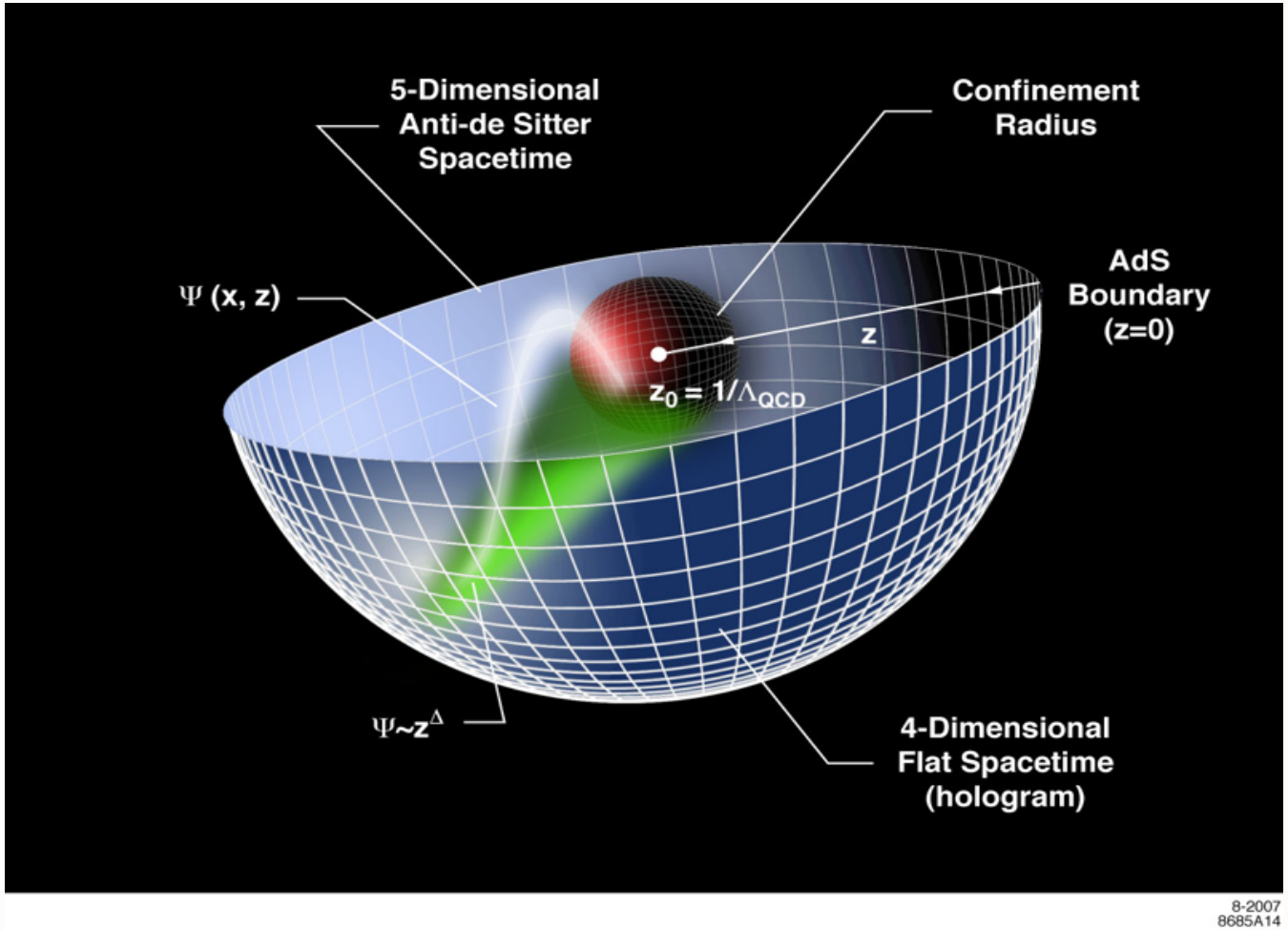
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.











- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

We will consider both holographic models

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

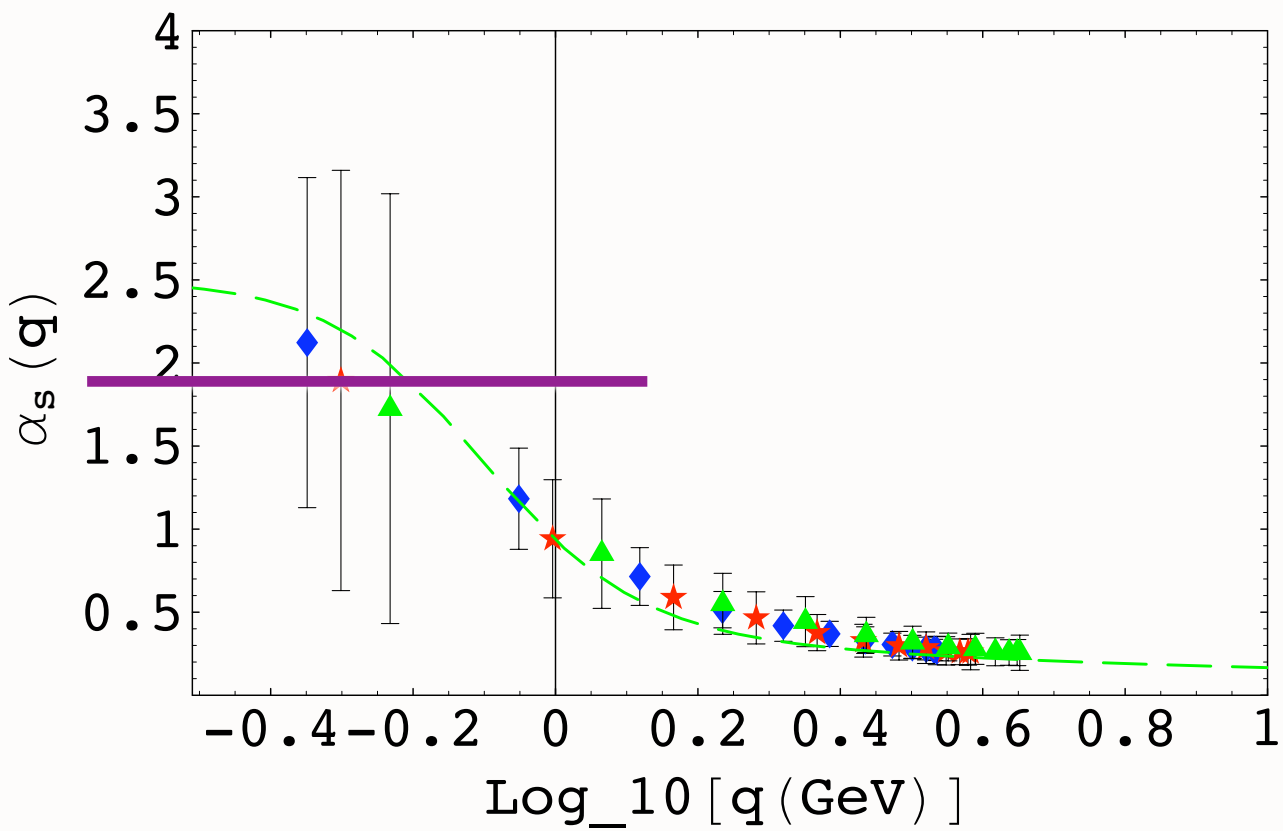
Maldacena:

Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq$ const at small Q^2
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

Conformal QCD Window in Exclusive Processes

- Does α_s develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur . . .
- Recent lattice simulations: evidence that α_s becomes constant and is not small in the infrared Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).

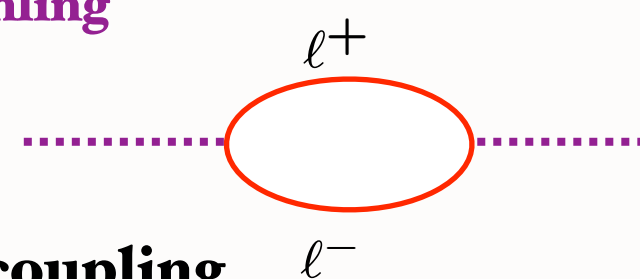


IR Fixed-Point for QCD?

- *Dyson-Schwinger Analysis:* **QCD Coupling has IR Fixed Point**
- *Evidence from Lattice Gauge Theory*
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**
- Confined or massive gluons: **Decoupling of QCD vacuum polarization at small Q^2**

Serber-Uehling

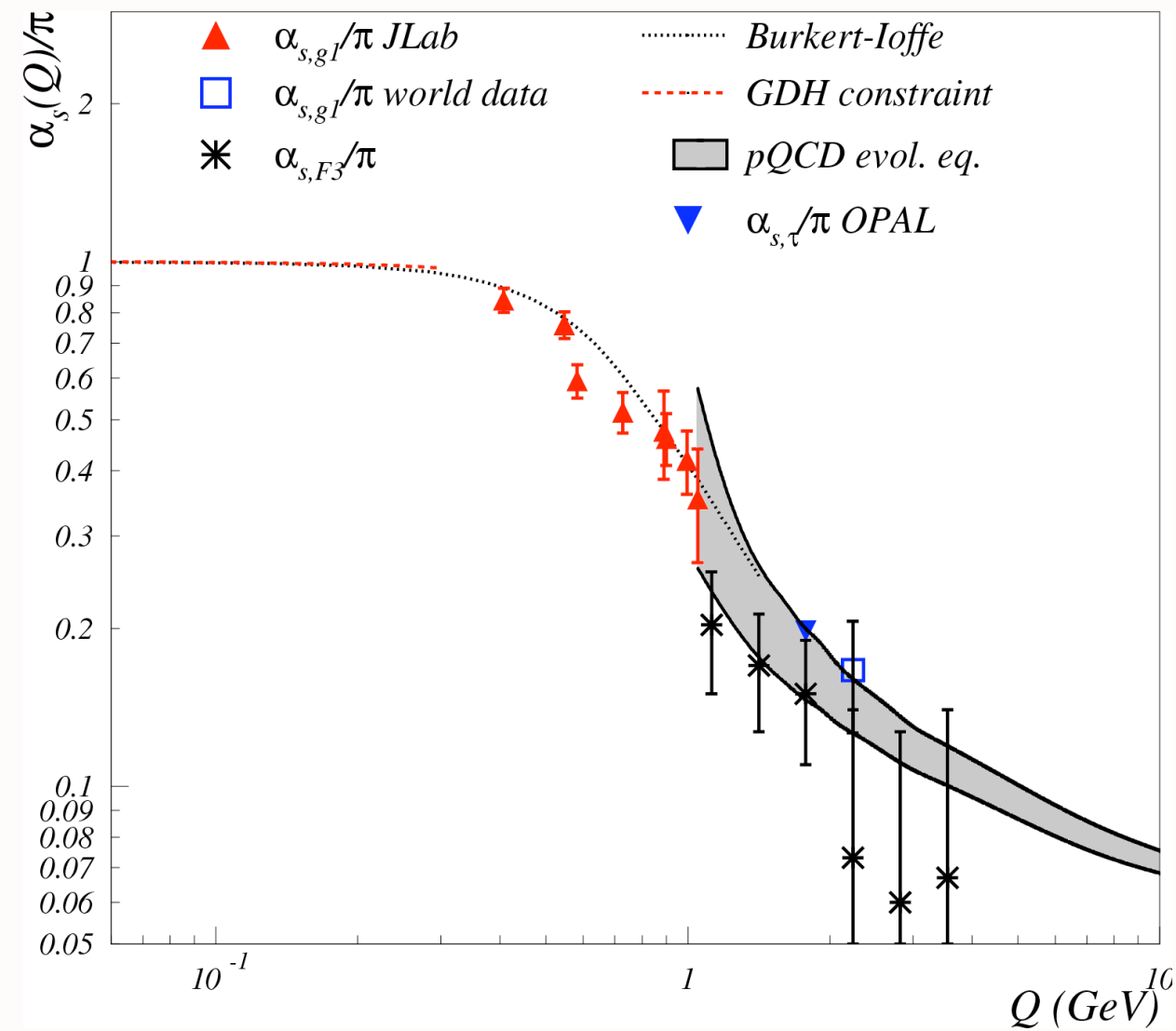
$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



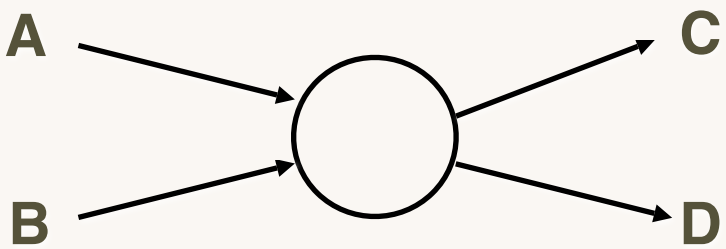
- **Justifies application of AdS/CFT in strong-coupling conformal window**

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$



Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\tfrac{1}{Q^2}]^{n_H - 1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

Farrar & sjb; Matveev, Muradyan, Tavkhelidze

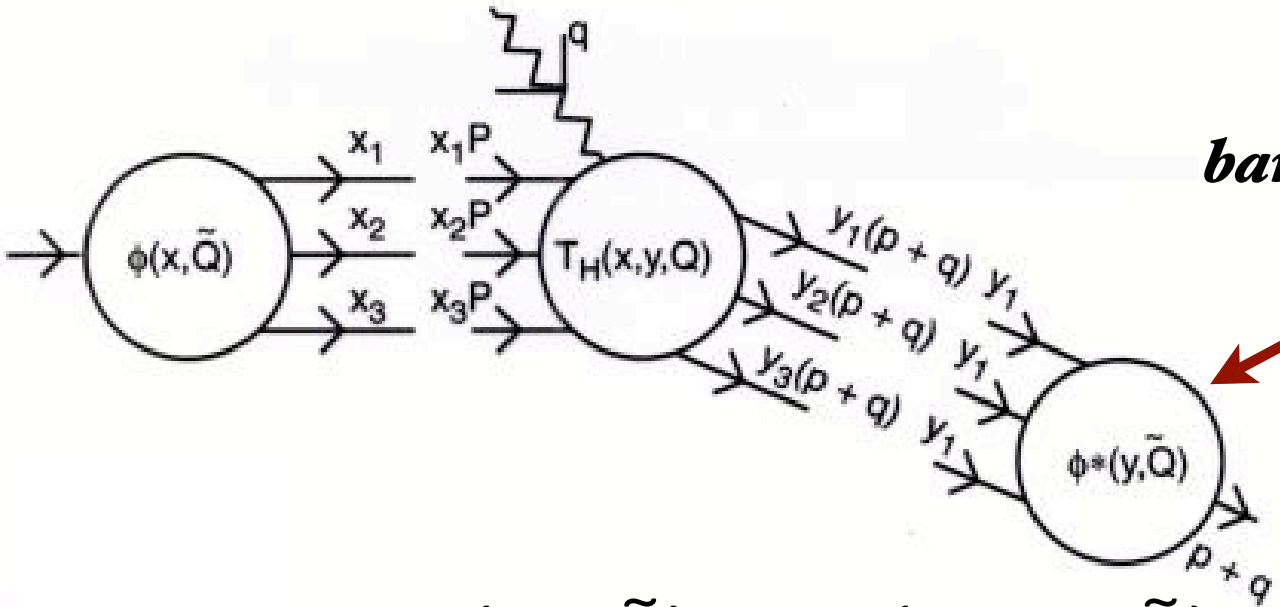
Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

Many new J-PARC, GSI, J-Lab, Belle, Babar tests

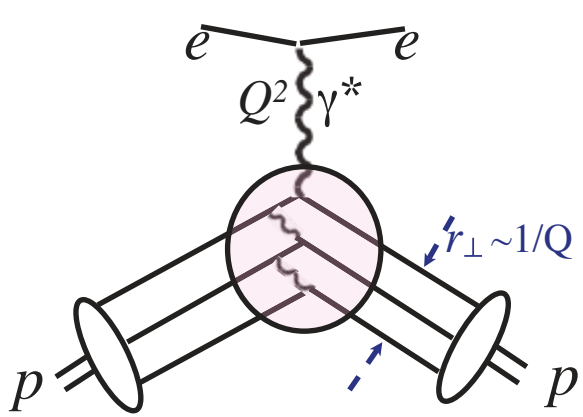
Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

Lepage, sjb



baryon distribution amplitude

$$M = \int \Pi dx_i dy_i \phi_F(x_i, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \times \phi_I(y_i, \tilde{Q})$$



If $\alpha_s(\tilde{Q}^2) \simeq$ constant

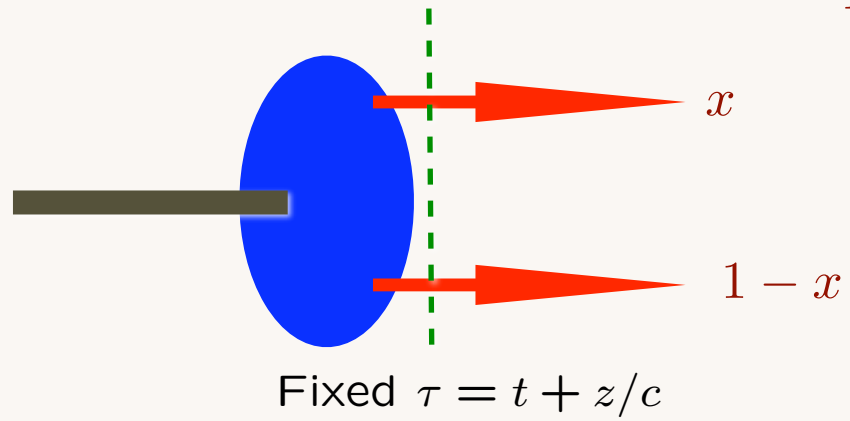
$$Q^4 F_1(Q^2) \simeq \text{constant}$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

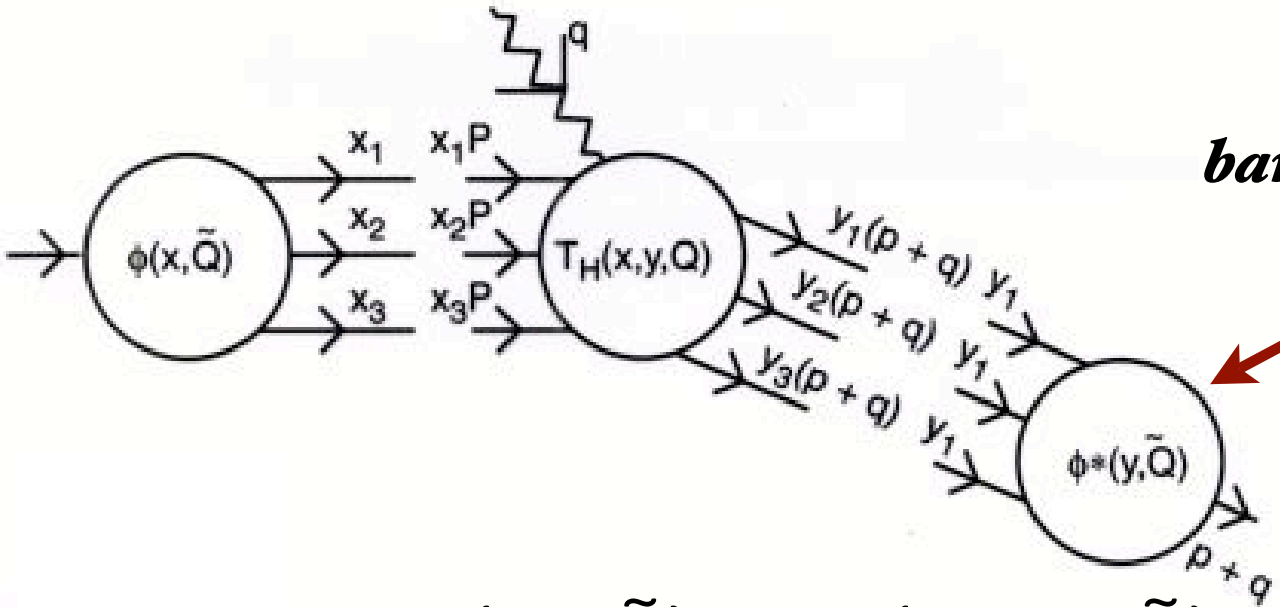
Lepage, sjb
Frishman, Lepage, Sachrajda, sjb
Peskin Braun
Efremov, Radyushkin Chernyak etal

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

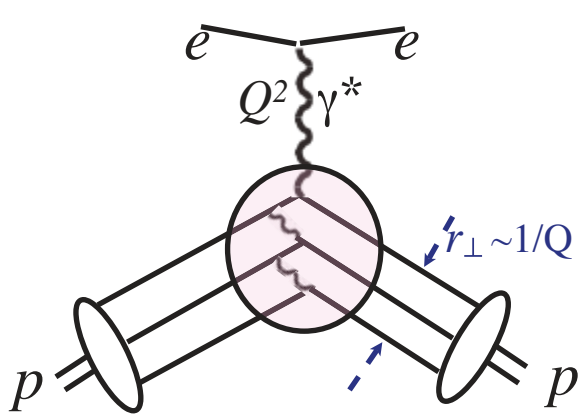
Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

Lepage, sjb



baryon distribution amplitude

$$M = \int \Pi dx_i dy_i \phi_F(x_i, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \times \phi_I(y_i, \tilde{Q})$$



If $\alpha_s(\tilde{Q}^2) \simeq$ constant

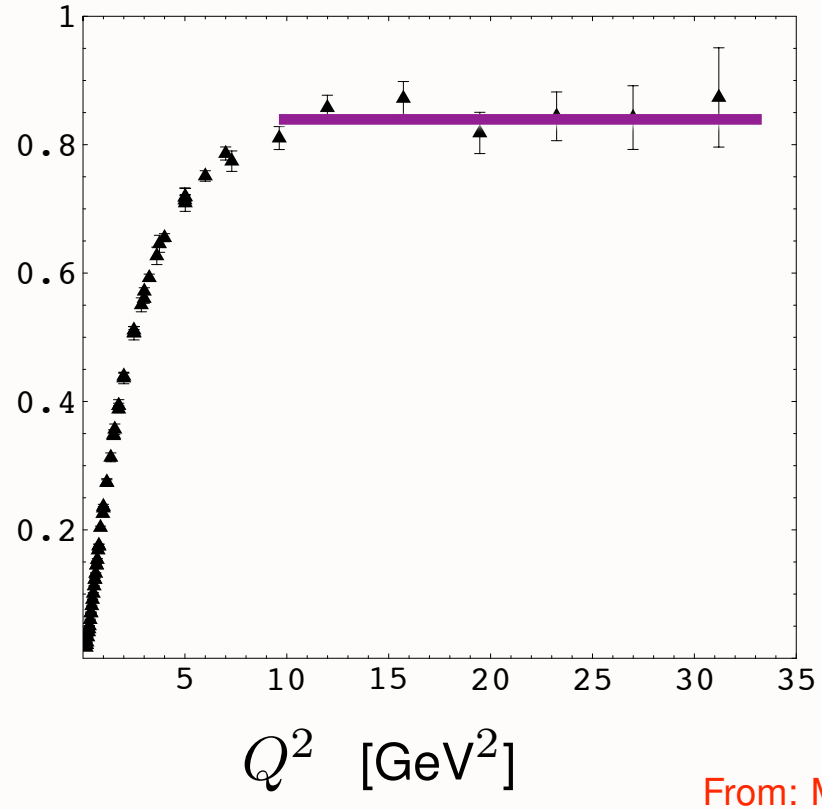
$$Q^4 F_1(Q^2) \simeq \text{constant}$$

Features of Hard Exclusive Processes in PQCD

Lepage, sjb; Duncan, Mueller

- Factorization of perturbative hard scattering subprocess amplitude and nonperturbative distribution amplitudes $M = \int T_H \times \Pi \phi_i$
- Dimensional counting rules reflect conformal invariance: $M \sim \frac{f(\theta_{CM})}{Q^{N_{tot}-4}}$
- Hadron helicity conservation: $\sum_{initial} \lambda_i^H = \sum_{final} \lambda_j^H$
- Color transparency Mueller, sjb;
- Hidden color Ji, Lepage, sjb;
- Evolution of Distribution Amplitudes Lepage, sjb; Efremov, Radyushkin

$Q^4 F_1^p(Q^2)$ [GeV^4]



$$F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \quad n = 3$$

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Phenomenological success of dimensional scaling laws for exclusive processes

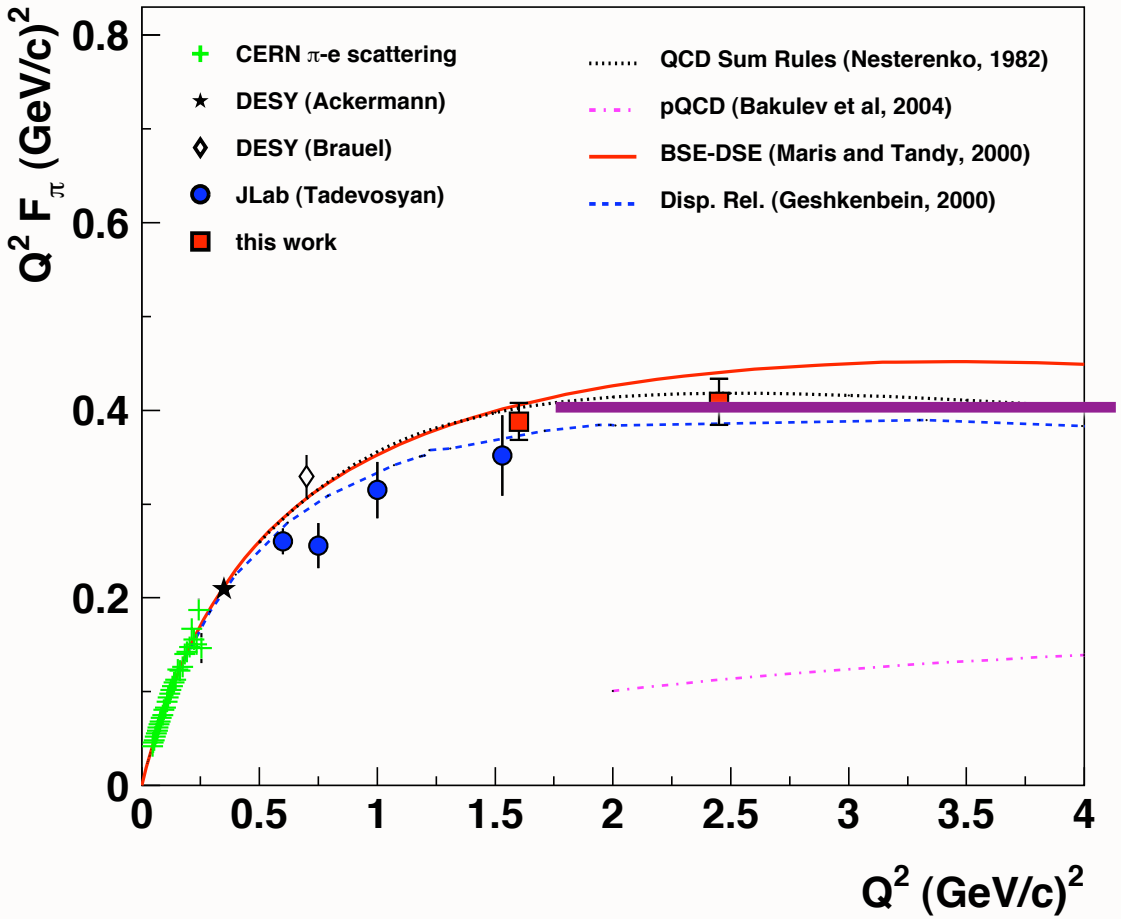
$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev *et al.* (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

Conformal behavior: $Q^2 F_\pi(Q^2) \to$ const

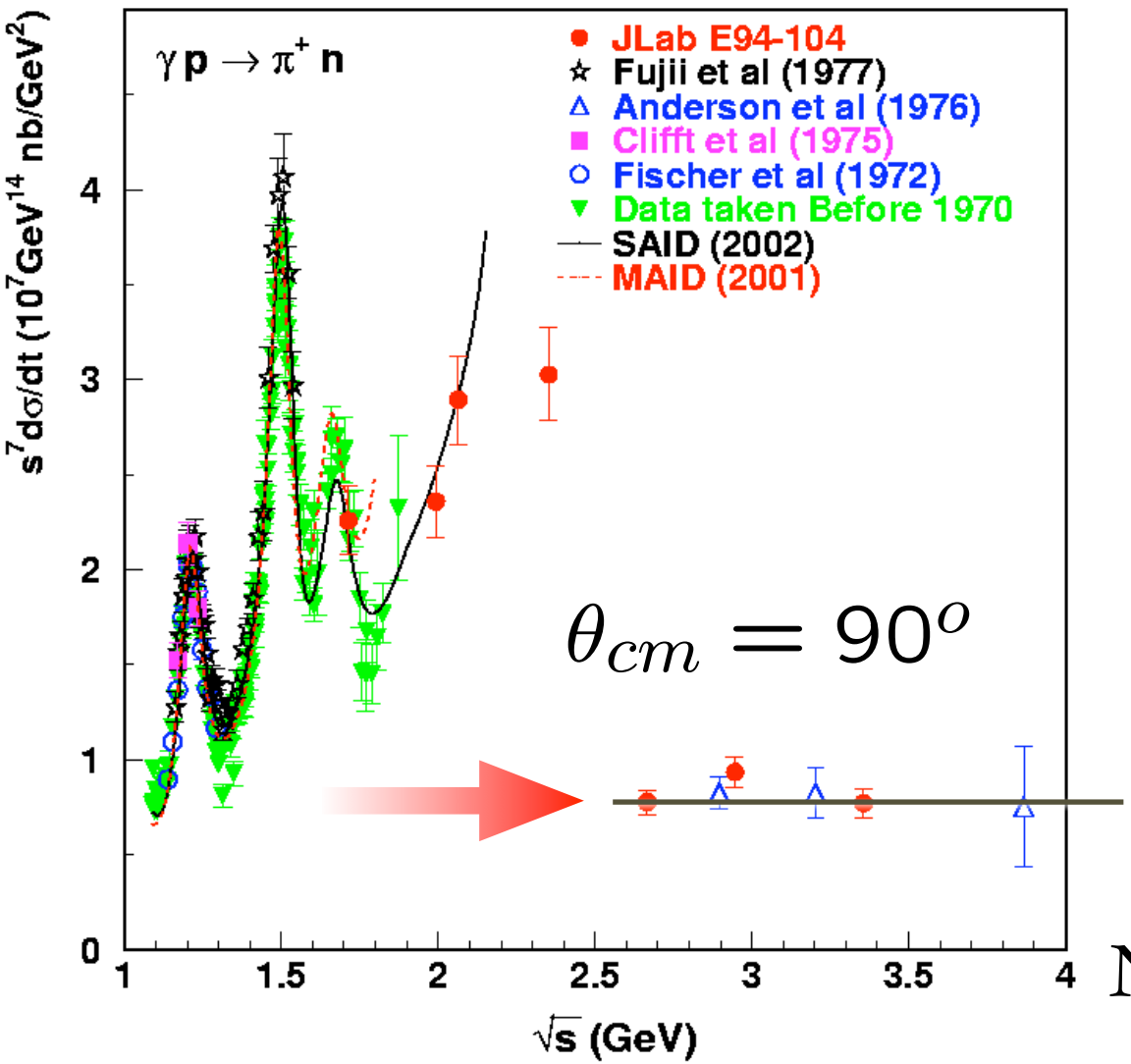


Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Tavkelidze



$$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$$

fixed θ_{CM} scaling

PQCD and AdS/CFT:

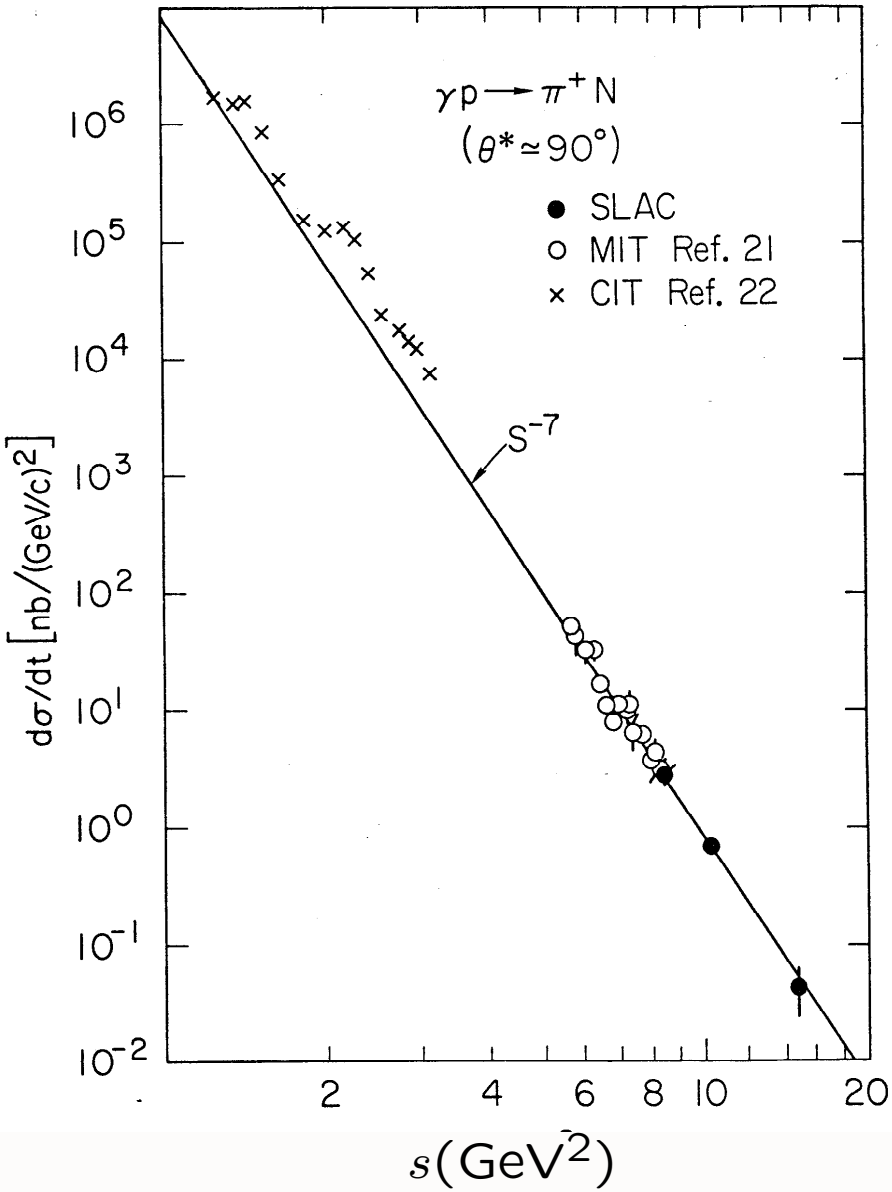
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

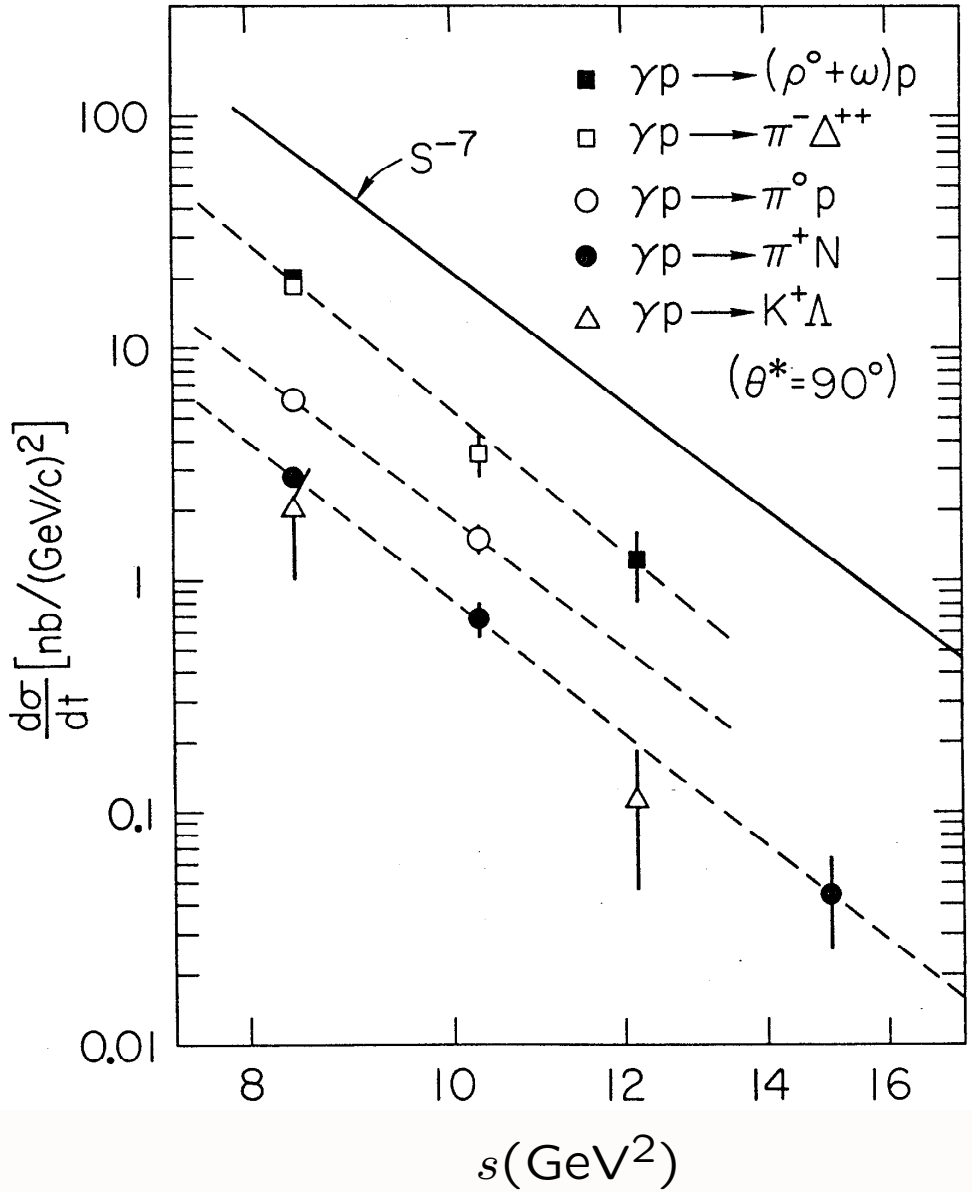
$$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

No sign of running coupling

Conformal invariance

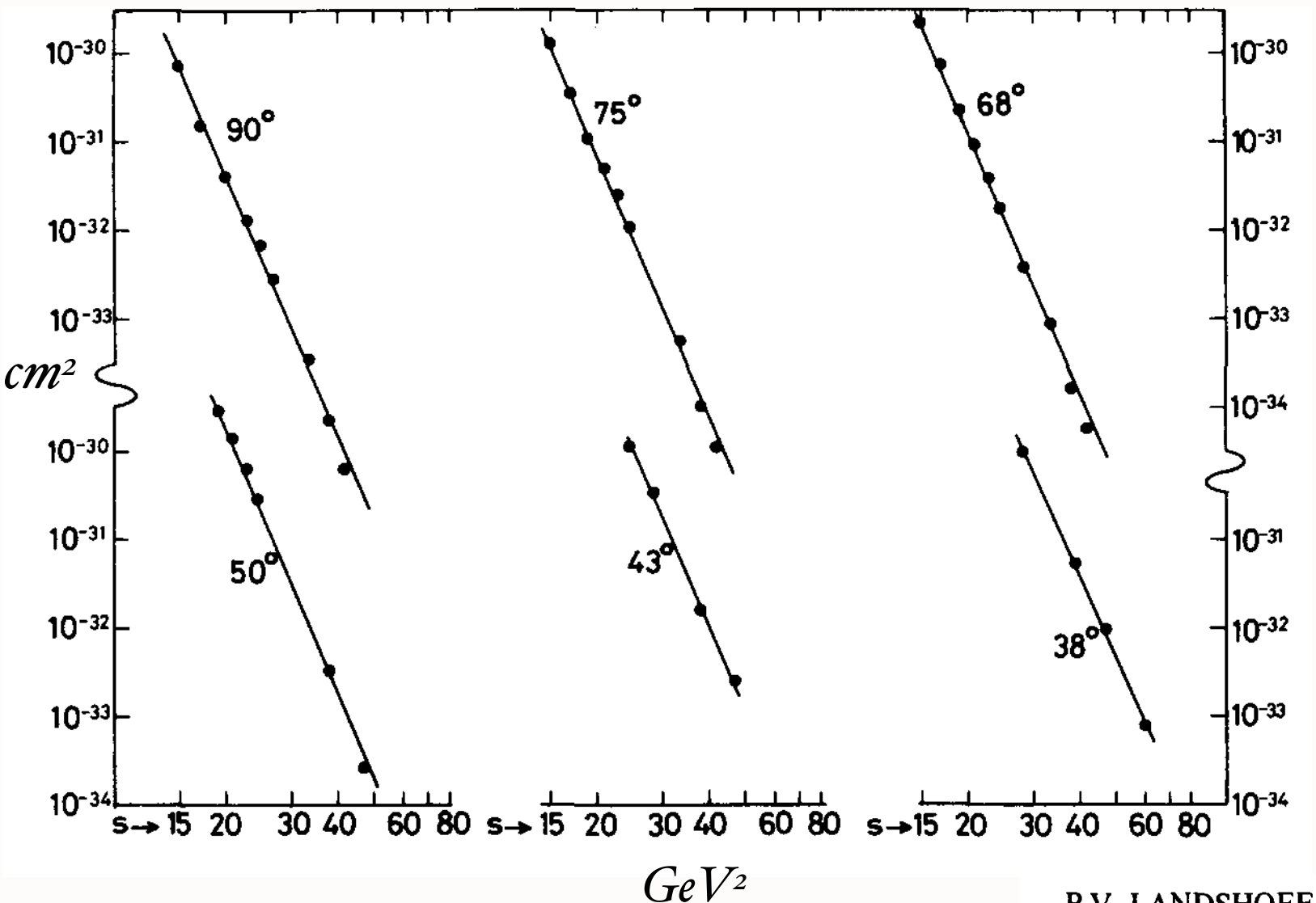




Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting: $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $\quad n = 4 \times 3 - 2 = 10$



P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD} ; variational methods

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2 \qquad z \to \lambda z$
- Hard wall model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

$d = 4$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(x,z) = C z^{\frac{d}{2}} J_{\Delta - \frac{d}{2}} (z\mathcal{M}), \quad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

$$V(z) = -\frac{1-4L^2}{4z^2}$$

Interpret L as orbital angular momentum

Derived from variation of Action in AdS_5

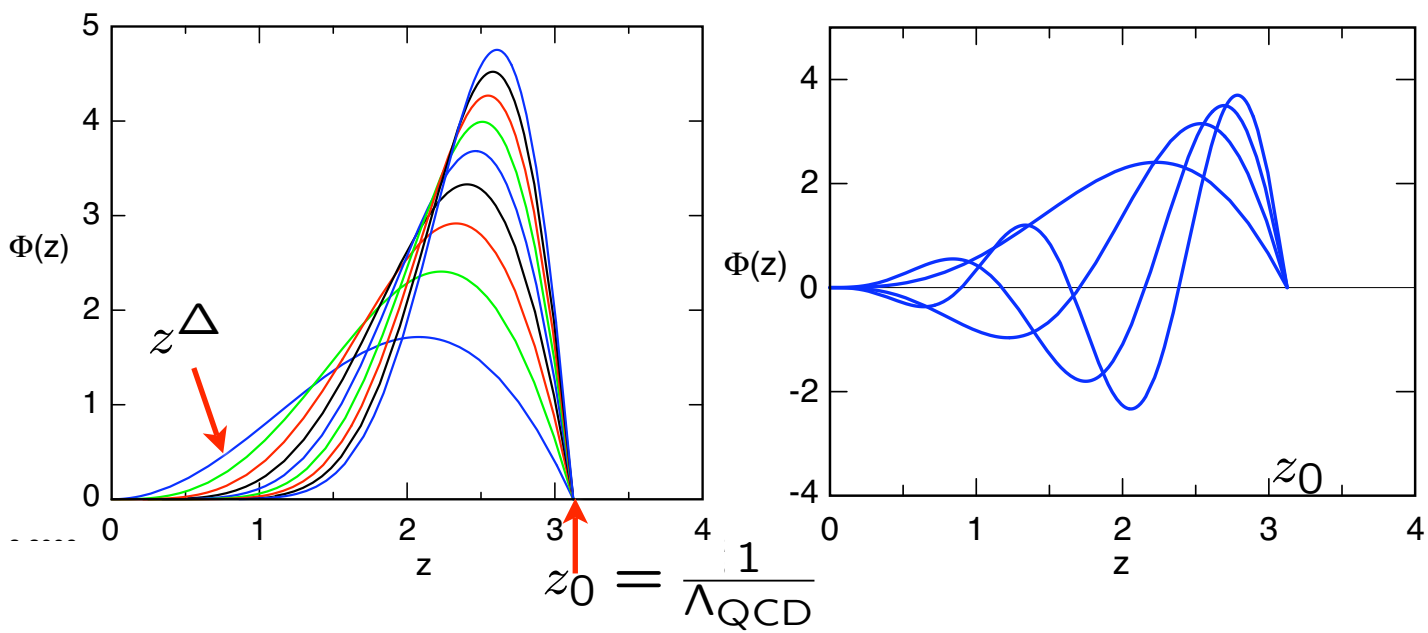
Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \ldots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

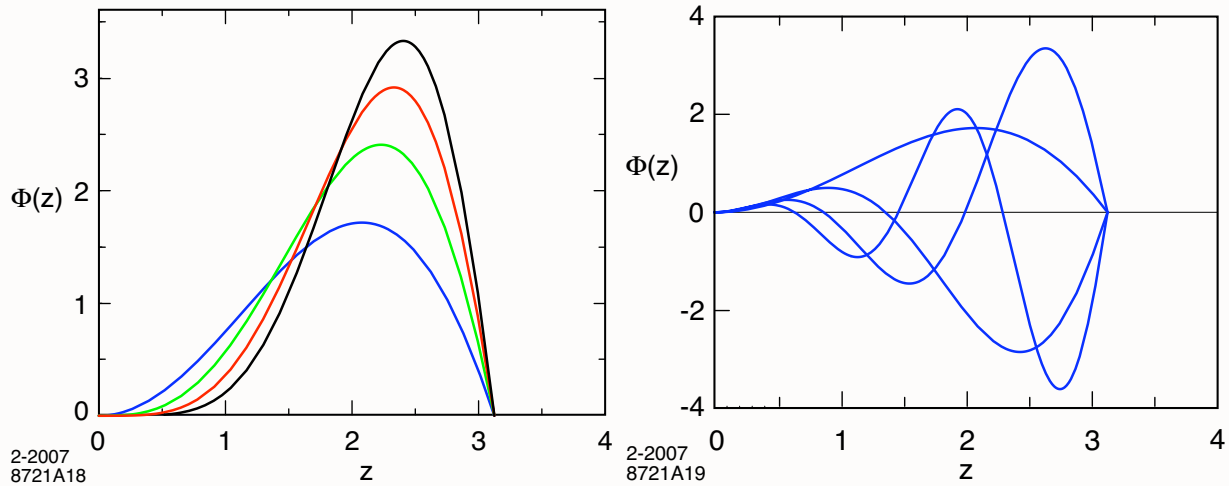


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

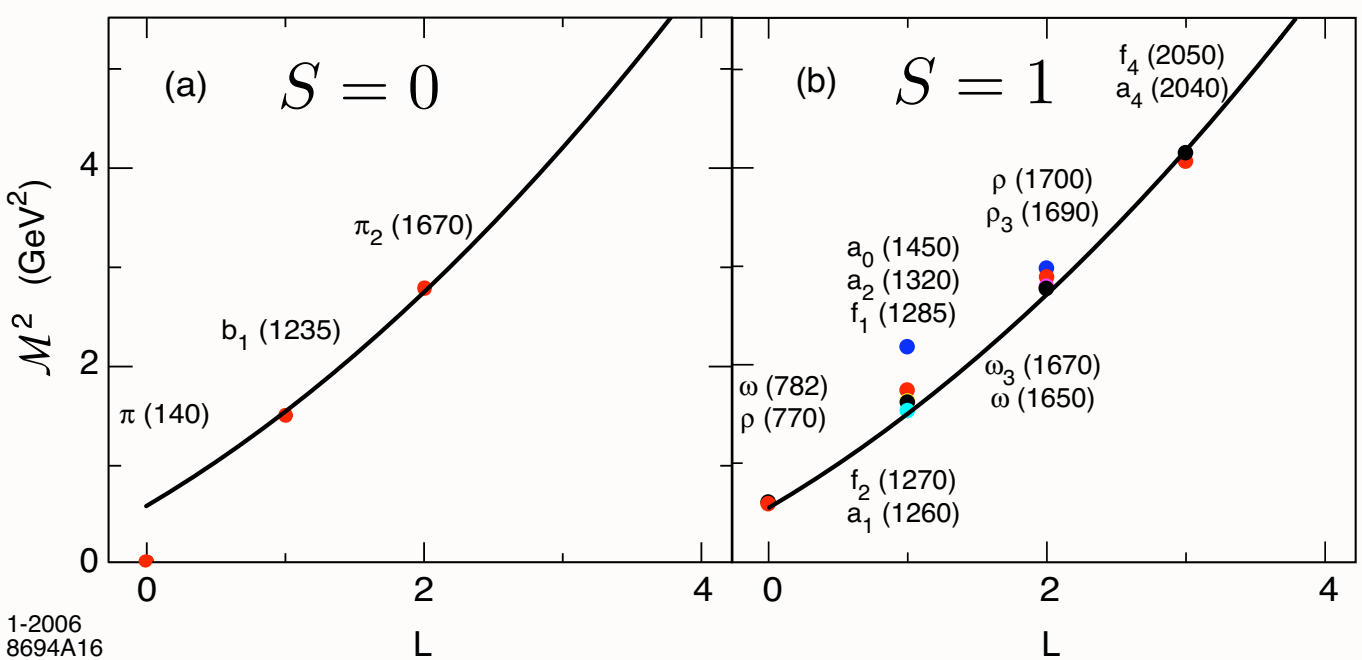


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

State	I	J^P	L	S	$\mathcal{O}$
$\pi(140)$	1	0^-	0	0	$\overline{q}\gamma_5 \frac{1}{2}\vec{\tau} q$
$b_1(1235)$	1	1^+	1	0	$-i\overline{q}\gamma_5 \vec{\partial}\frac{1}{2}\vec{\tau} q$
$\pi_2(1670)$	1	2^+	2	0	$-\overline{q}\gamma_5 \frac{1}{2}(3\partial_i\partial_j - \delta_{ij}\vec{\partial}^2)\frac{1}{2}\vec{\tau} q$
$\ldots$					
$\rho(770)$	1	1^-	0	1	$q^\dagger \vec{\alpha}\frac{1}{2}\vec{\tau} q$
$\omega(782)$	0	1^-	0	1	$q^\dagger \vec{\alpha}\, q$
$a_1(1260)$	1	1^+	1	1	$-iq^\dagger(\vec{\alpha}\times\vec{\partial})\frac{1}{2}\tau q$
$f_2(1270)$	0	2^+	1	1	$-iq^\dagger[\frac{3}{2}(\alpha_i\partial_j + \alpha_j\partial_i) - \vec{\alpha}\cdot\vec{\partial}\delta_{ij}]q$
$f_1(1285)$	0	1^+	1	1	$-iq^\dagger(\vec{\alpha}\times\vec{\partial})q$
$a_2(1320)$	1	2^+	1	1	$-iq^\dagger[\frac{3}{2}(\alpha_i\partial_j + \alpha_j\partial_i) - \vec{\alpha}\cdot\vec{\partial}\delta_{ij}]\frac{1}{2}\vec{\tau} q$
$a_0(1450)$	1	0^+	1	1	$-iq^\dagger\vec{\alpha}\cdot\vec{\partial}\frac{1}{2}\vec{\tau} q$
$\ldots$					

Tensor decomposition of total angular momentum interpolating operators $\mathcal{O}$, $[O] = 2 + L$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

Hard wall model: truncated space

$$V(z) = -\frac{1-4L^2}{4z^2} \qquad \phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

Soft wall model: Harmonic oscillator confinement

$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2$$

Derived from variation of Action in AdS_5

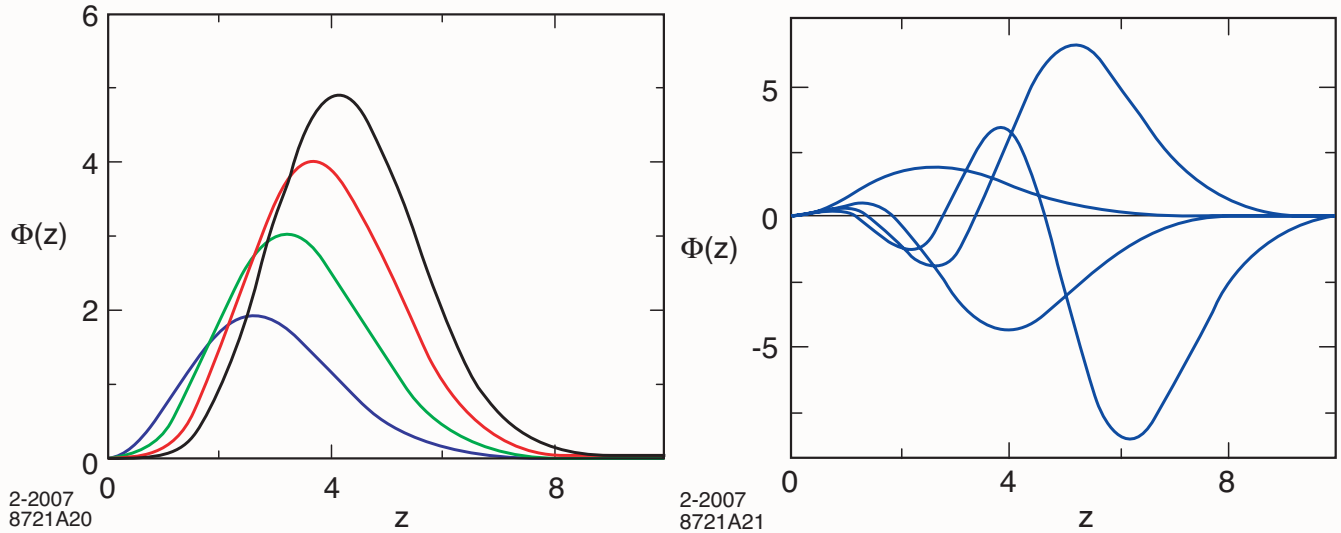
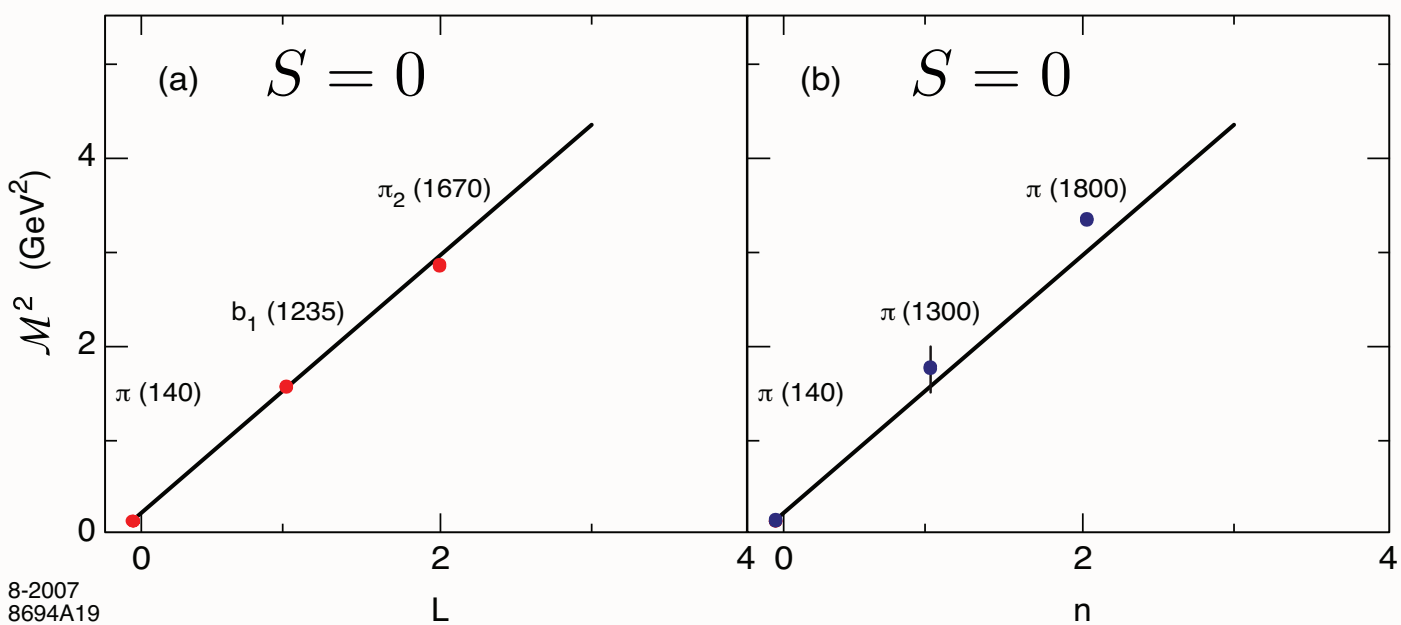


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

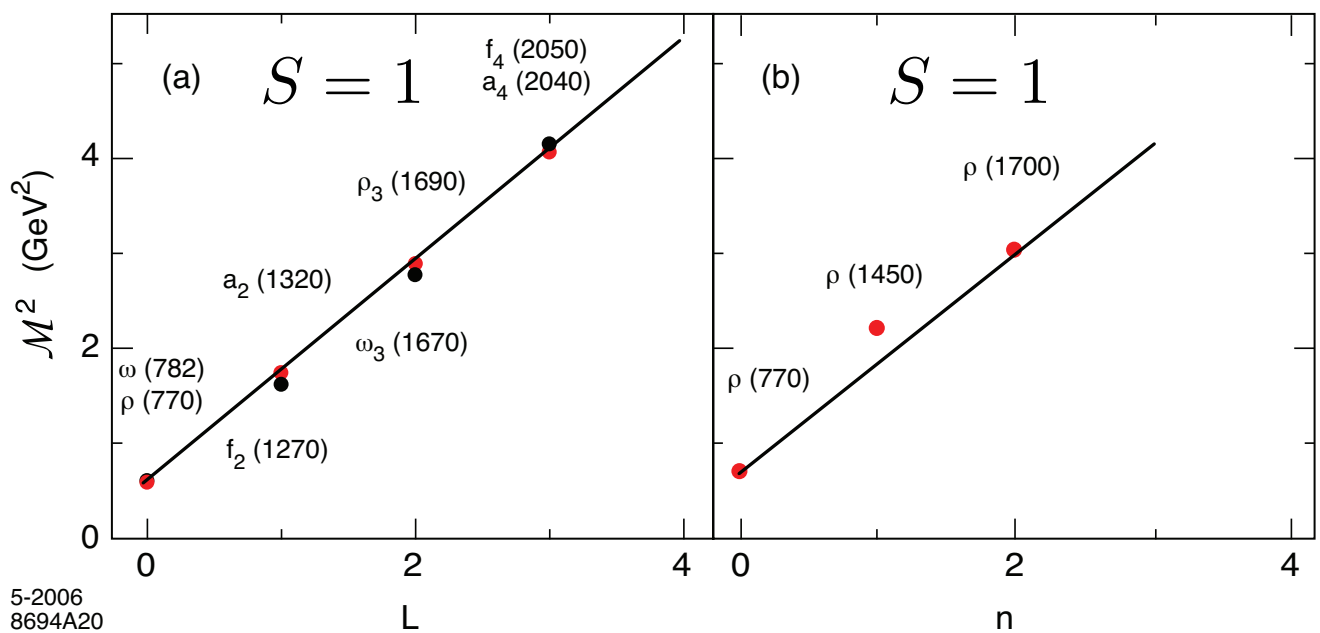
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2\phi_S(z)$$

with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n+2L+S)$.

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\,(n+L+1/2)\,.$

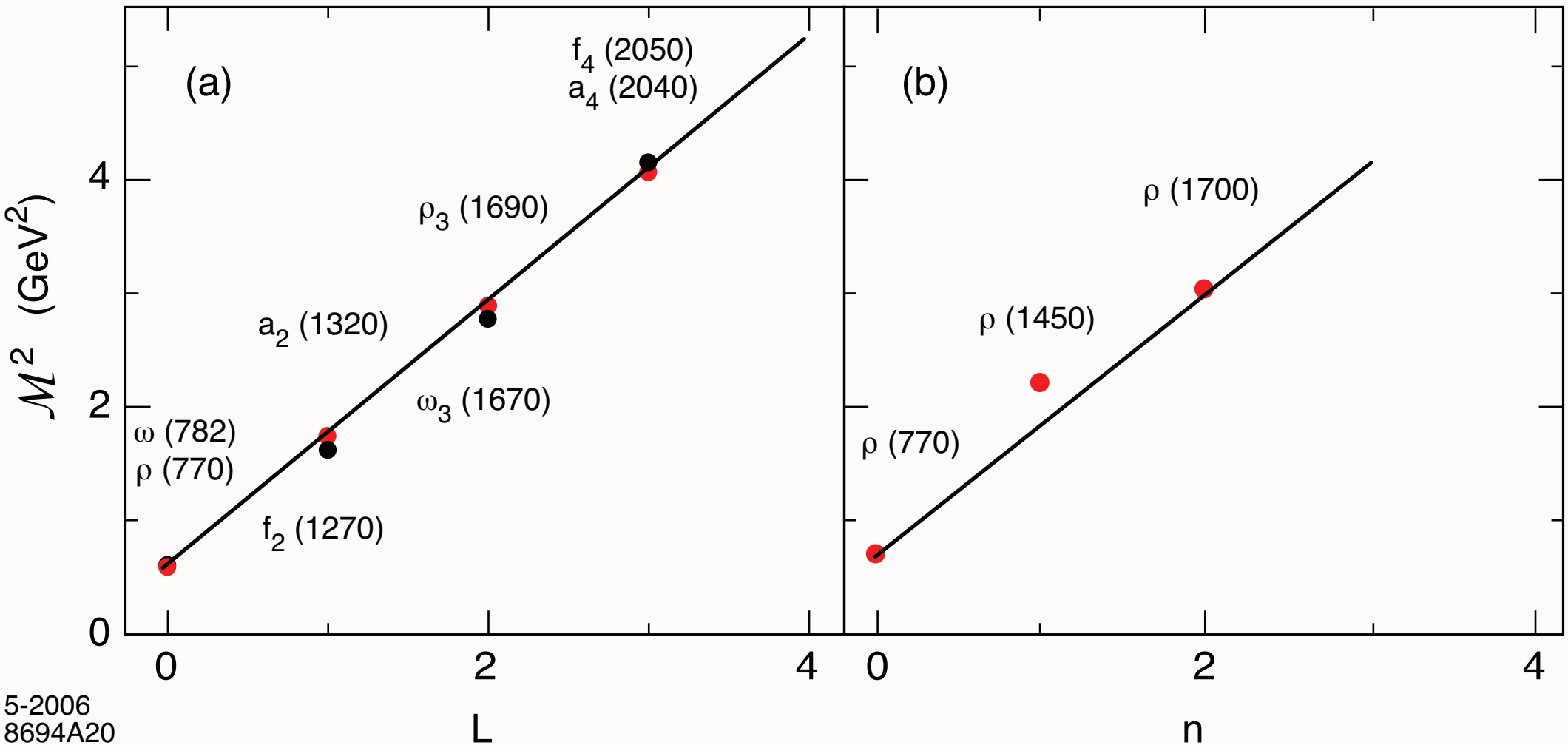


Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri(2007).

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$



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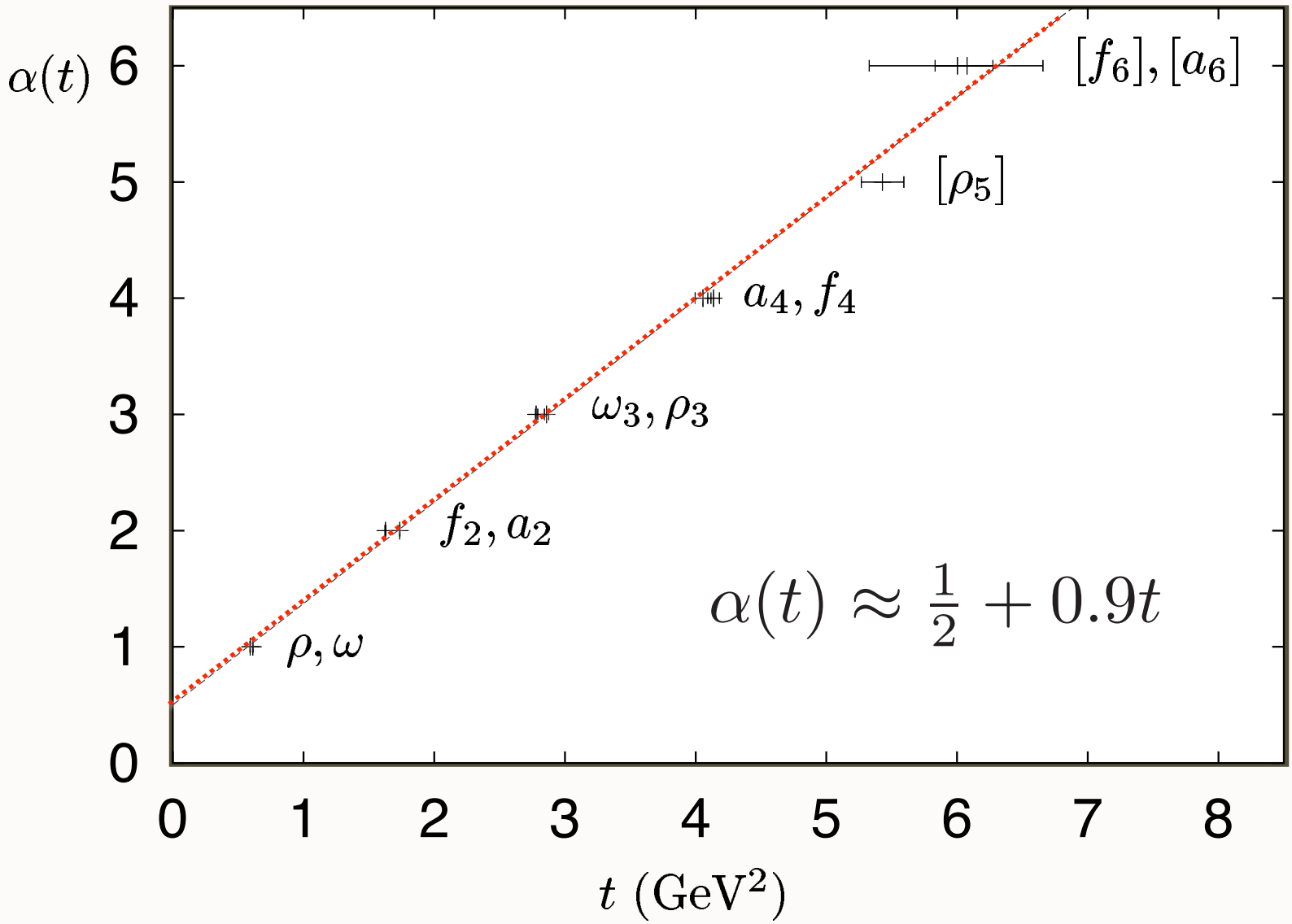


AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$\begin{aligned} F(Q^2)_{I\to F} &= R^{3+2\sigma}\int_0^\infty \frac{dz}{z^{3+2\sigma}}\, e^{(3+2\sigma)A(z)}\Phi_F(z)\, J(Q,z)\, \Phi_I(z) \\ &\simeq R^{3+2\sigma}\int_0^{z_o} \frac{dz}{z^{3+2\sigma}}\, \Phi_F(z)\, J(Q,z)\, \Phi_I(z), \end{aligned}$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.$$

- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z\to 0$:

$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell),\ x^\ell = (x^\mu, z)$

$$i g_5 \int d^4x\, dz\, \sqrt{g}\, A^\ell(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_\ell \Phi_P(x,z).$$

- Electromagnetic probe polarized along Minkowski coordinates $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z), \quad A_z = 0.$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z\, \partial_z - z^2 Q^2\right] J(Q,z) = 0,$$

subject to boundary conditions $J(Q=0,z) = J(Q, z=0) = 1.$

- Solution

$$J(Q,z) = zQK_1(zQ).$$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x}\, \Phi(z), \quad \Phi(z) \to z^\Delta, \ \ z \to 0.$$

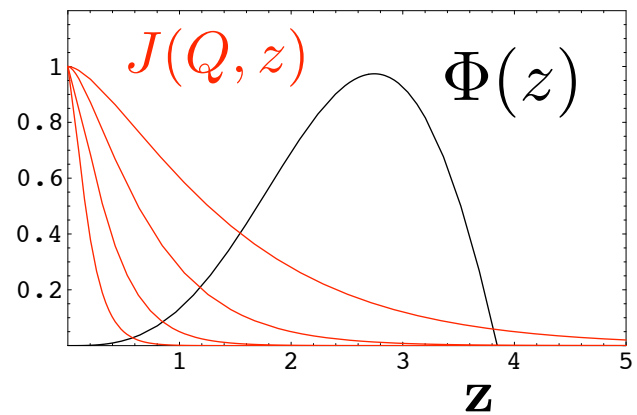
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ
scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rule
General result from
AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Current Matrix Elements in AdS Space (SW)

sjb and GdT,
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1+2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1+\frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

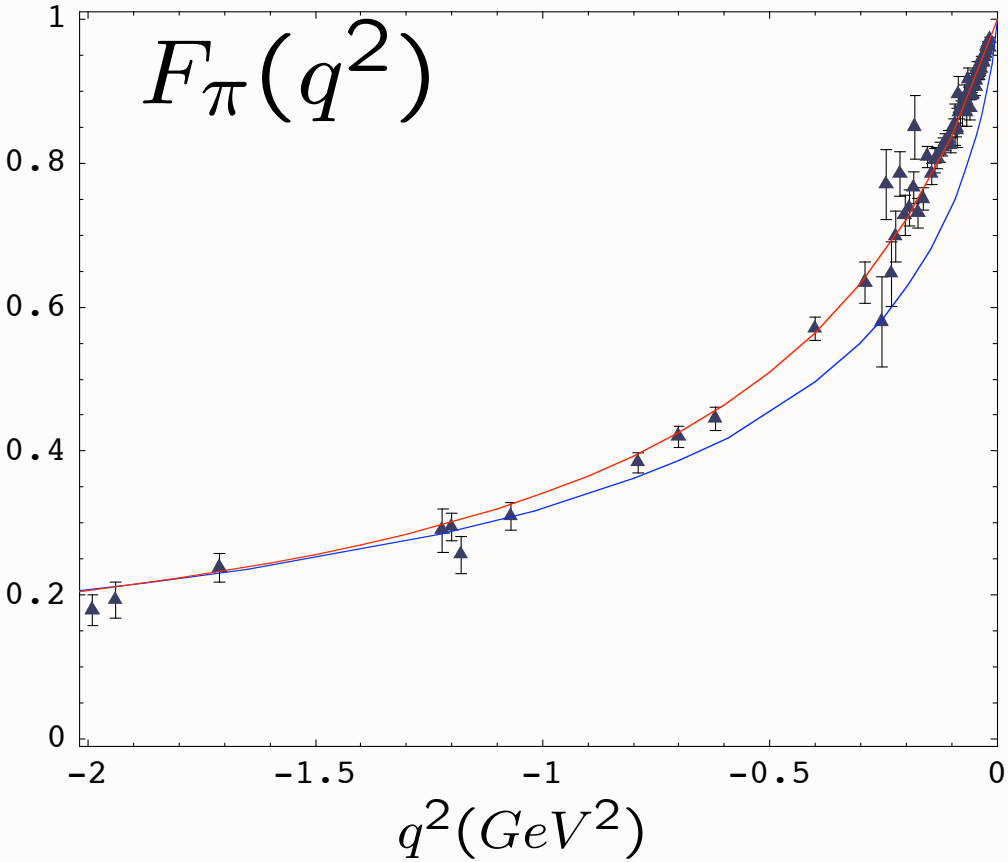
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

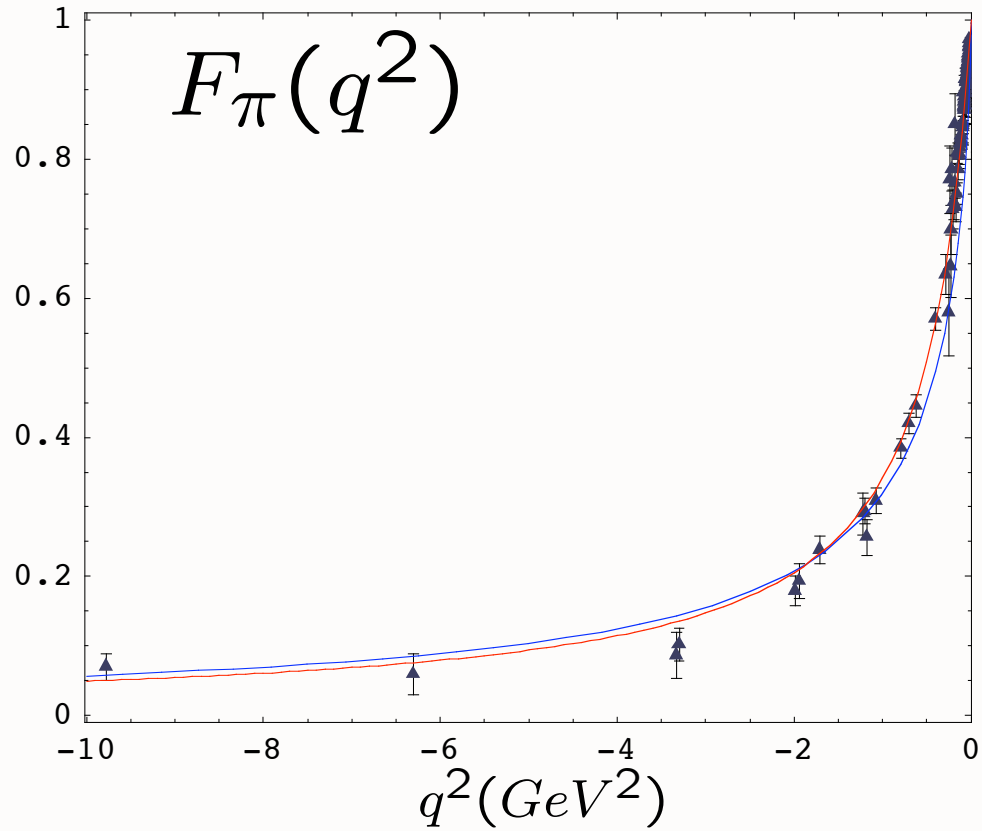
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Spacelike pion form factor from AdS/CFT





Data Compilation from Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Space and Time-Like Pion Form Factor

- Hadronic string modes $\Phi_\pi(z) \to z^2$ as $z \to 0$ (twist $\tau = 2$)

$$\Phi_\pi^{HW}(z) = \frac{\sqrt{2}\Lambda_{QCD}}{R^{3/2} J_1(\beta_{0,1})} z^2 J_0\left(z\beta_{0,1}\Lambda_{QCD}\right),$$

$$\Phi_\pi^{SW}(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2.$$

- F_π has analytical solution in the SW model $F_\pi(Q^2) = \frac{4\kappa^2}{4\kappa^2+Q^2}$.

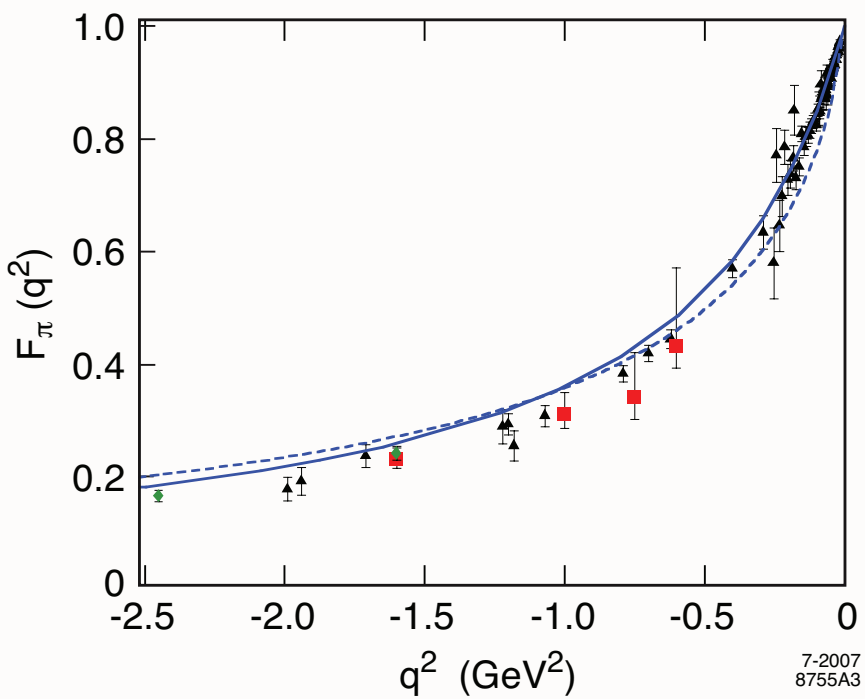


Fig: $F_\pi(q^2)$ for $\kappa = 0.375$ GeV and $\Lambda_{QCD} = 0.22$ GeV. Continuous line: SW, dashed line: HW.

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1+\frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau+\frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N, \quad \Gamma(N+z) = (N-1+z)(N-2+z)\ldots(1+z)\Gamma(1+z).$

- Form factor expressed as $N-1$ product of poles

$$\begin{aligned}
F(Q^2) &= \frac{1}{1+\frac{Q^2}{4\kappa^2}}, \quad N=2, \\
F(Q^2) &= \frac{2}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)}, \quad N=3, \\
&\ldots \\
F(Q^2) &= \frac{(N-1)!}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)\cdots\left(N-1+\frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

- For large Q^2:

$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\,(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

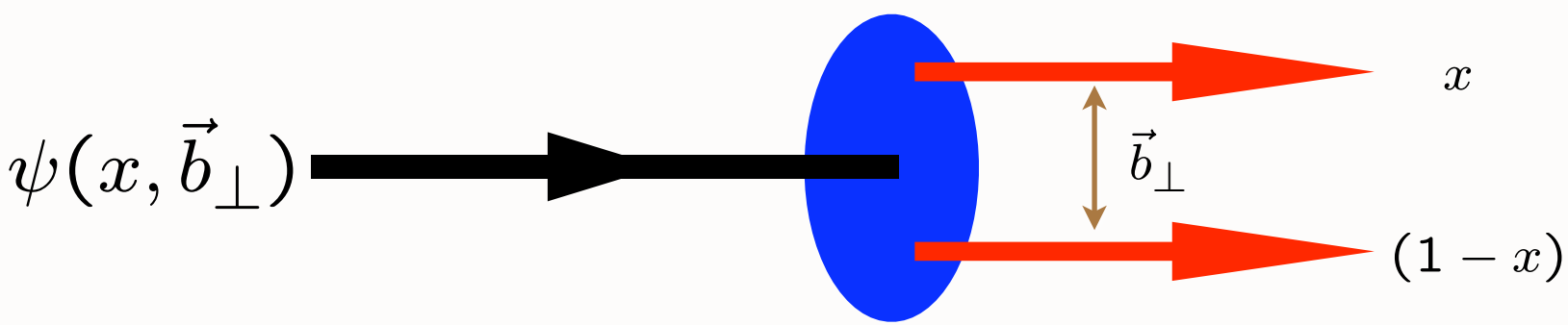
$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

LF(3+1) $\qquad$ AdS_5

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \longleftrightarrow \quad z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Holography: Map AdS/CFT to 3+1 LF Theory

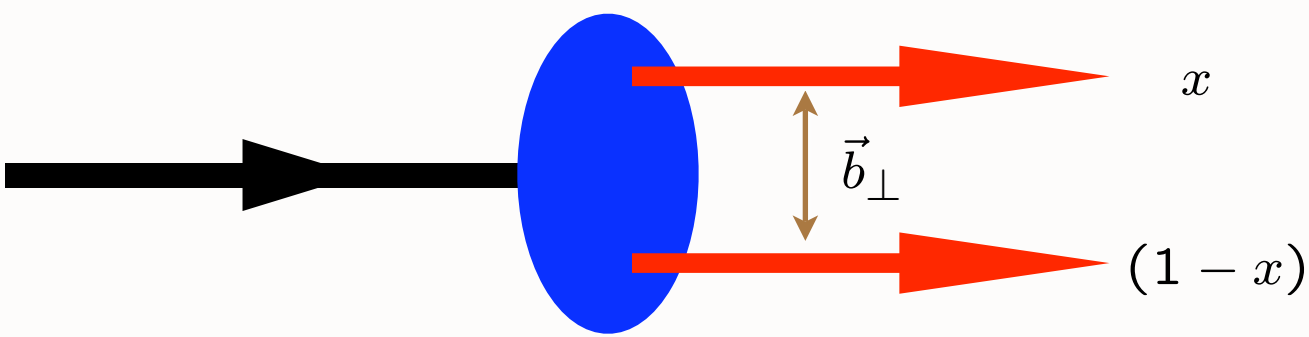
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$

confining potential:

Consider the AdS_5 metric:

$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$.

ds^2 invariant if $x^\mu \to \lambda x^\mu,\ z \to \lambda z$,

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu}dx_\mu dx_\nu = dx_0{}^2 - dx_3{}^2 - dx_\perp{}^2 = dx^+dx^- - dx_\perp{}^2$

and

$ds^2 = -\frac{R^2}{z^2}(dx_\perp{}^2 + dz^2)$ for $x^+ = 0$.

Light-Front AdS_5 Duality

- ds^2 is invariant if $dx_\perp{}^2 \to \lambda^2 dx_\perp{}^2$, and $z \to \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L+N-2)$].

- Light-front Hamiltonian equation

$$H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle,$$

leads to effective LF Schrödinger wave equation (KKSS)

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L-1)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

with eigenvalues $\mathcal{M}^2 = 4\kappa^2(n+L)$ and eigenfunctions

$$\phi_L(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L\left(\kappa^2\zeta^2\right).$$

- Transverse oscillator in the LF plane with $SO(2)$ rotation subgroup has Casimir L^2 representing rotations for the transverse coordinates $\mathbf{b}_\perp$ in the LF.

- SW model is a remarkable example of integrability to a non-conformal extension of AdS/CFT [Chim and Zamolodchikov (1992) - Potts Model.]

Example: Pion LFWF

- Two parton LFWF bound state:

$$\widetilde{\psi}^{HW}_{\bar{q}q/\pi}(x,\mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right),$$

$$\widetilde{\psi}^{SW}_{\bar{q}q/\pi}(x,\mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right).$$

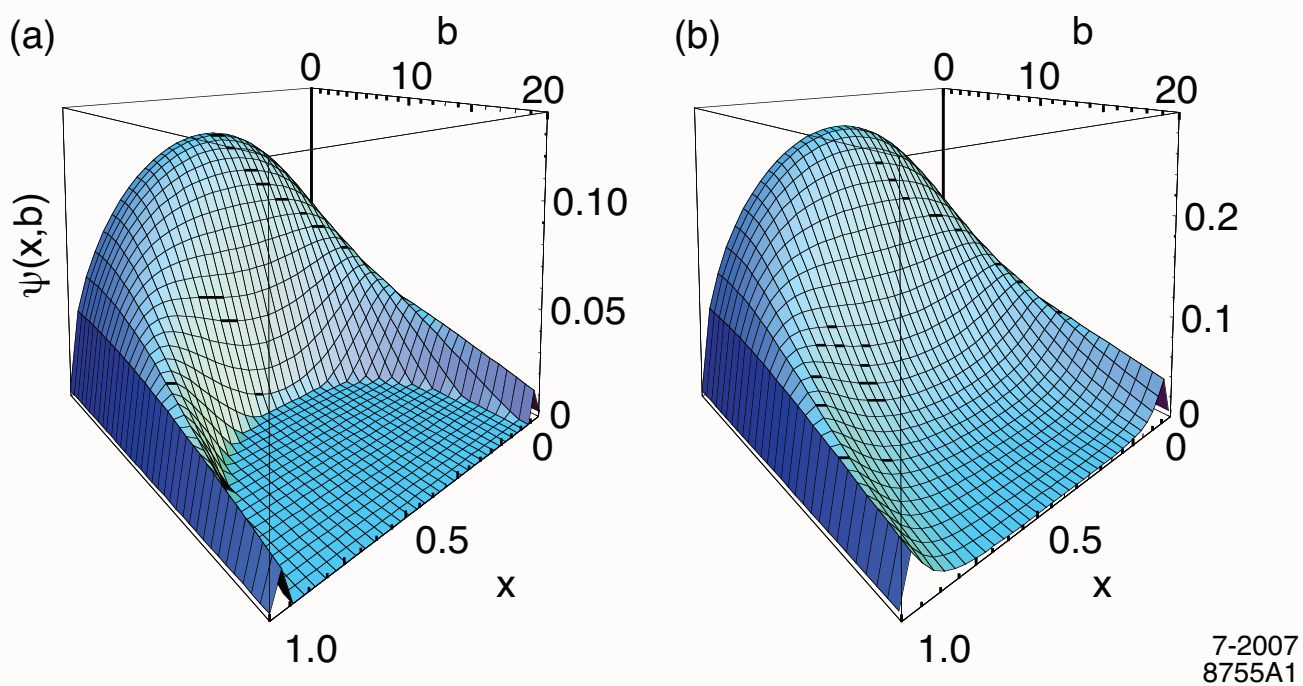
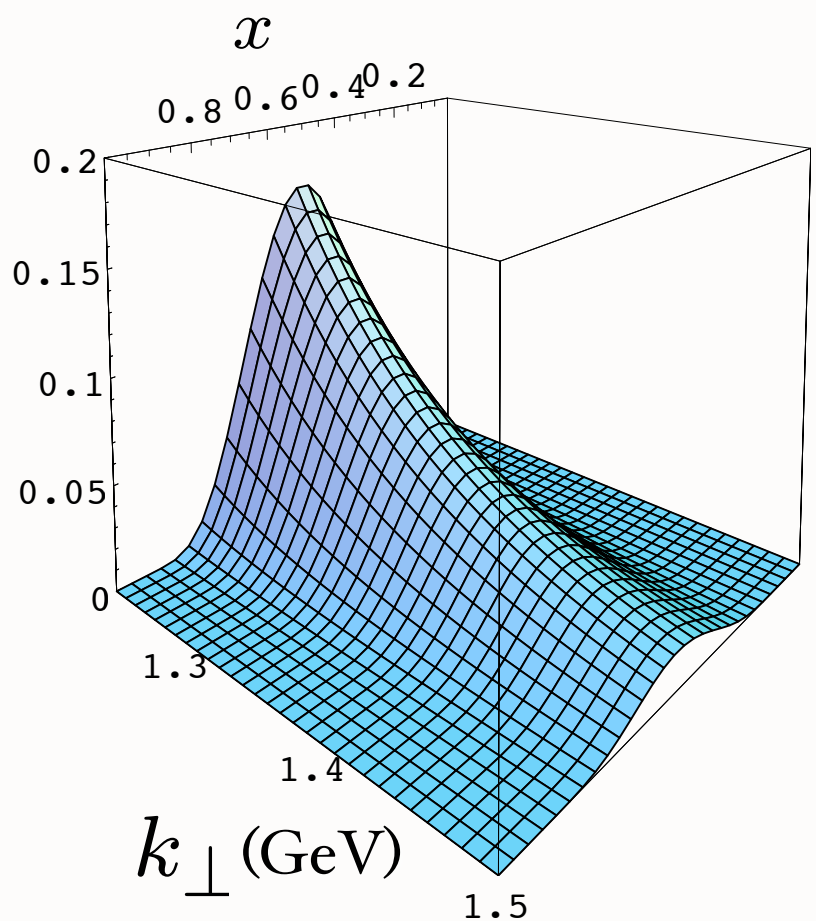


Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

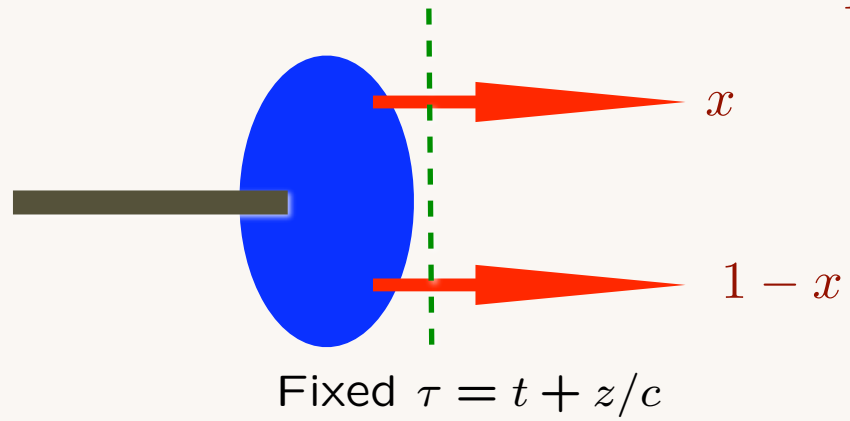
$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$$k_\perp^2 < Q^2$$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for mesons, baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Frishman, Lepage, Sachrajda, sjb
Peskin Braun
Efremov, Radyushkin Chernyak etal

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

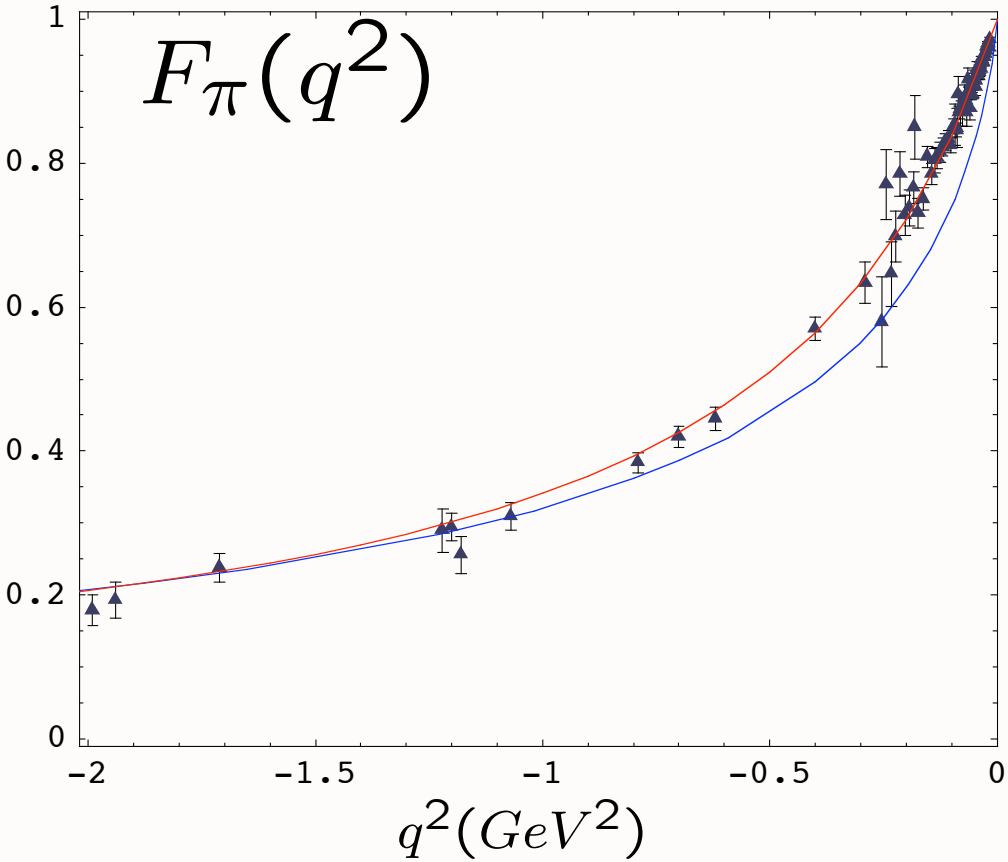
$$\xi = 1 - 2x$$

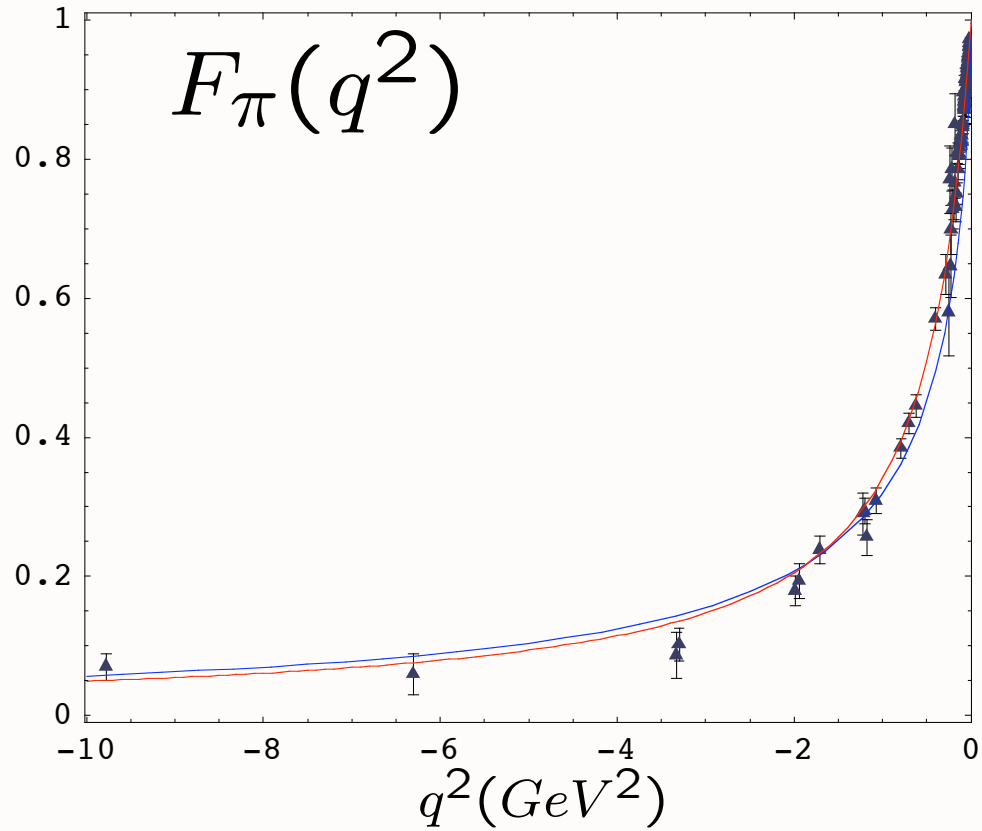
$$< \xi^2 >= 1/5 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >= 1/4 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Sachrajda Lattice: $< \xi^2 >= 0.28 \pm 0.02$

Spacelike pion form factor from AdS/CFT





Data Compilation from Baldini, Kloe and Volmer

SW: Harmonic Oscillator Confinement

HW: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space $\mathbf{b}_\perp$

$$F_\pi(q^2) = \int_0^1 dx \int db^2\, \widetilde{\rho}(x, b, Q),$$

with $\widetilde{\rho}(x, b, Q) = \pi J_0\left[b\, Q(1-x)\right] |\widetilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \to 1$ as Q increases.

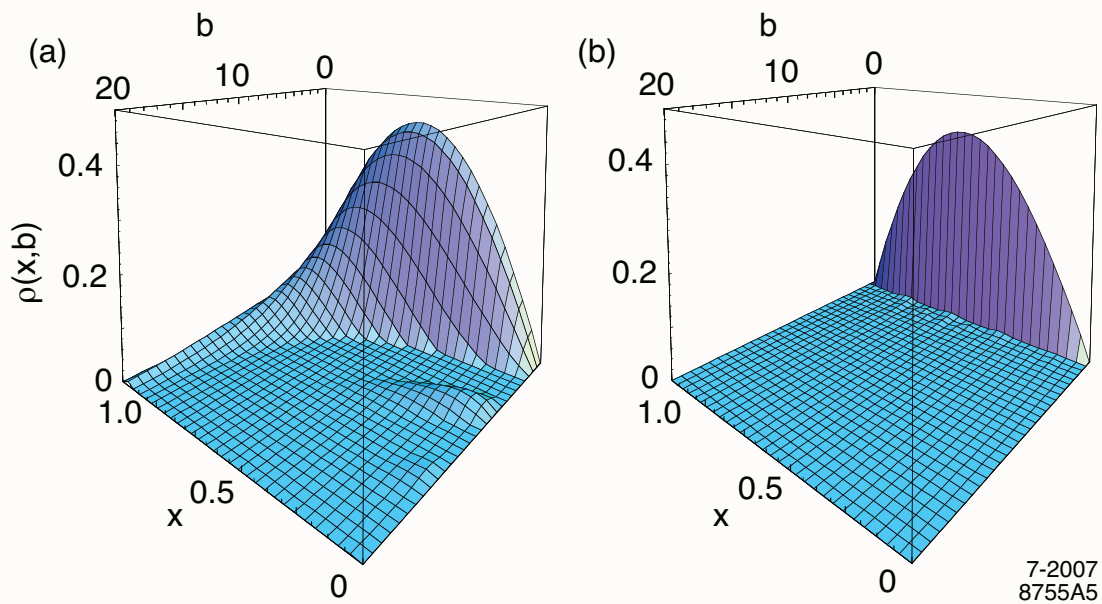


Fig: LF partonic density $\rho(x, b, Q)$: (a) Q = 1 GeV/c, (b) very large Q.

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\,d\downarrow} d^\dagger_{c\,u\uparrow} - b^\dagger_{c\,d\uparrow} d^\dagger_{c\,u\downarrow} \right) |0\rangle.$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

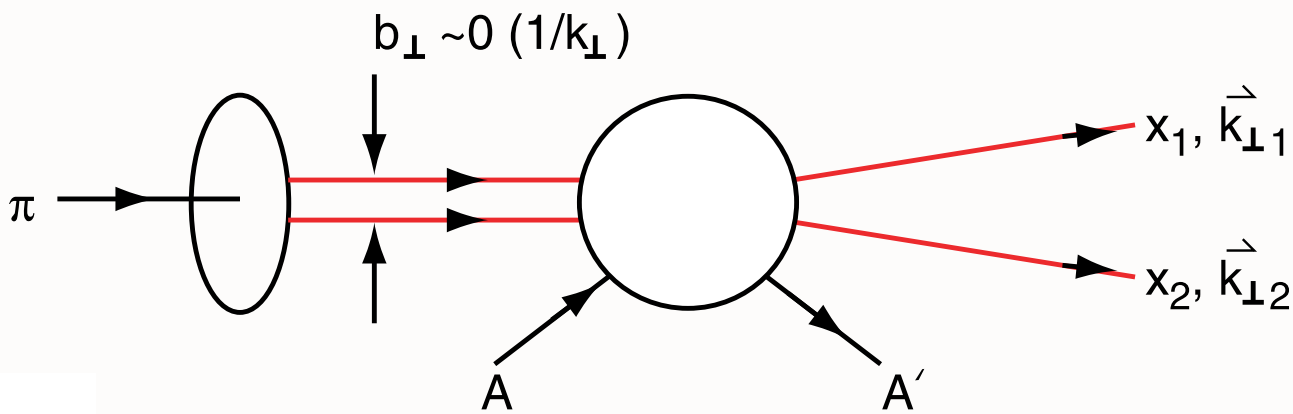
$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}}\, R^{3/2} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

- Holographic result ($\Lambda_{\rm QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})}\, \Lambda_{\rm QCD} = 91.7\,{\rm MeV},\ f_\pi^{SW} = \frac{\sqrt{3}}{8}\,\kappa = 81.2\,{\rm MeV},$$

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



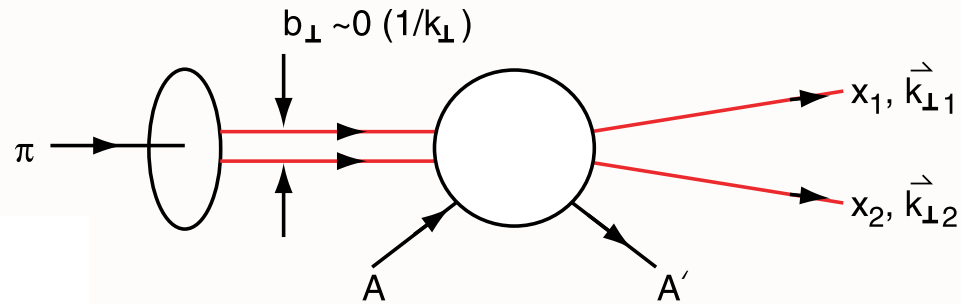
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

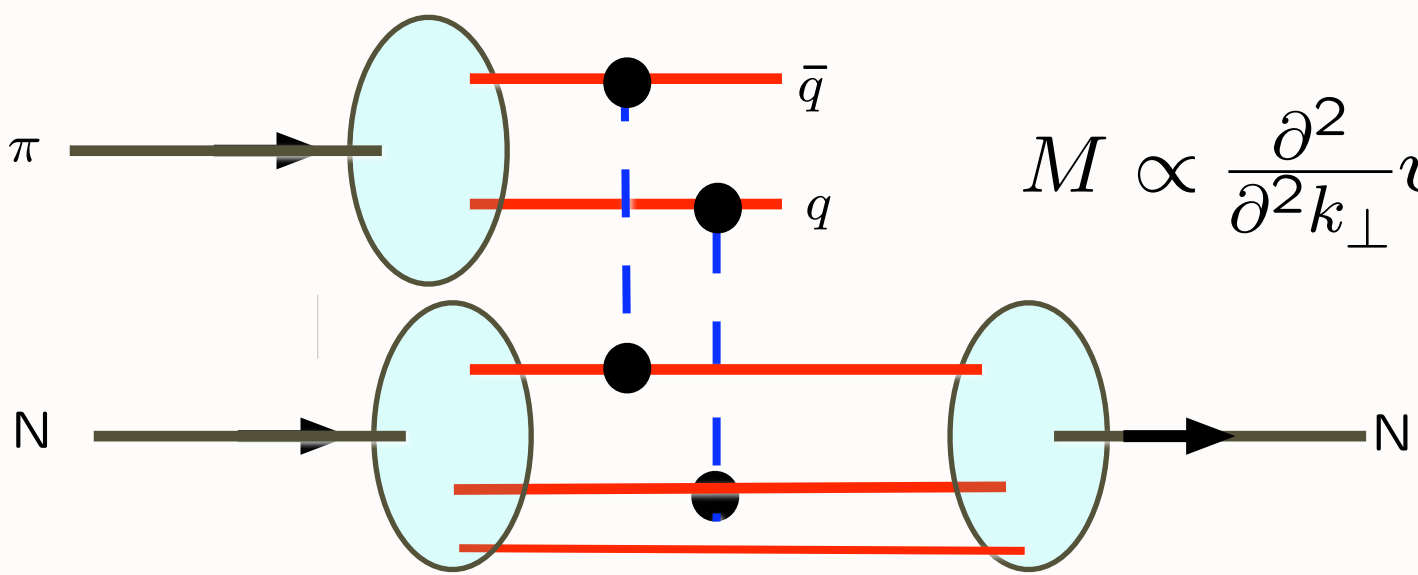
Nucleus left Intact!

E791 FNAL Diffractive DiJet



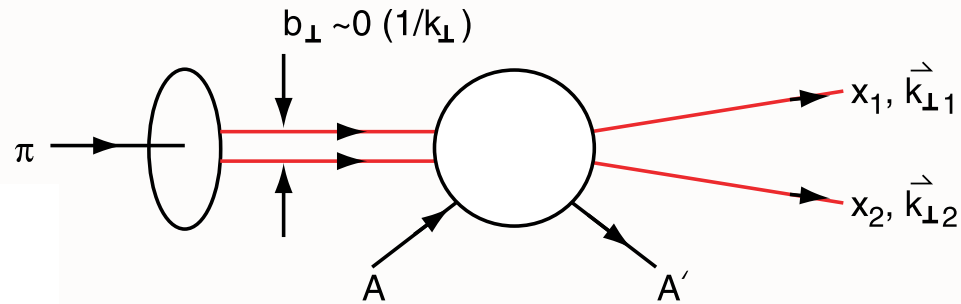
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

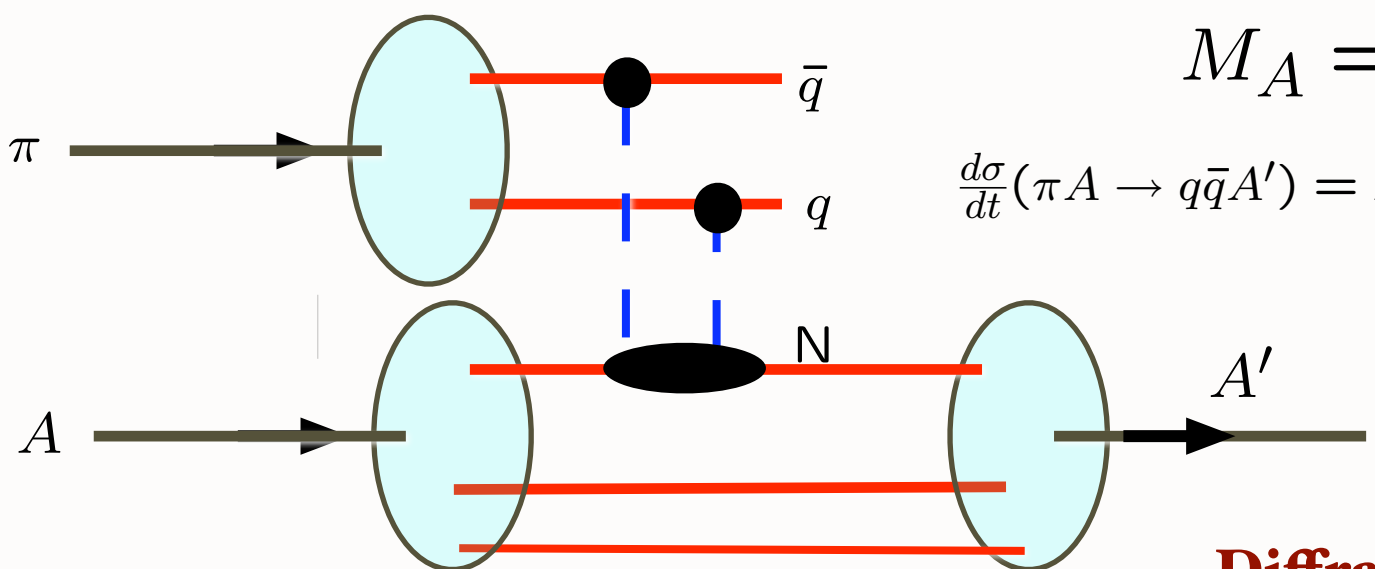
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A \; M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2 \; \frac{d\sigma}{dt}(\pi N \to q\bar{q}N') \; F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

Ashery E791

α (Incoh.) $= 0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out !

Factor of 7

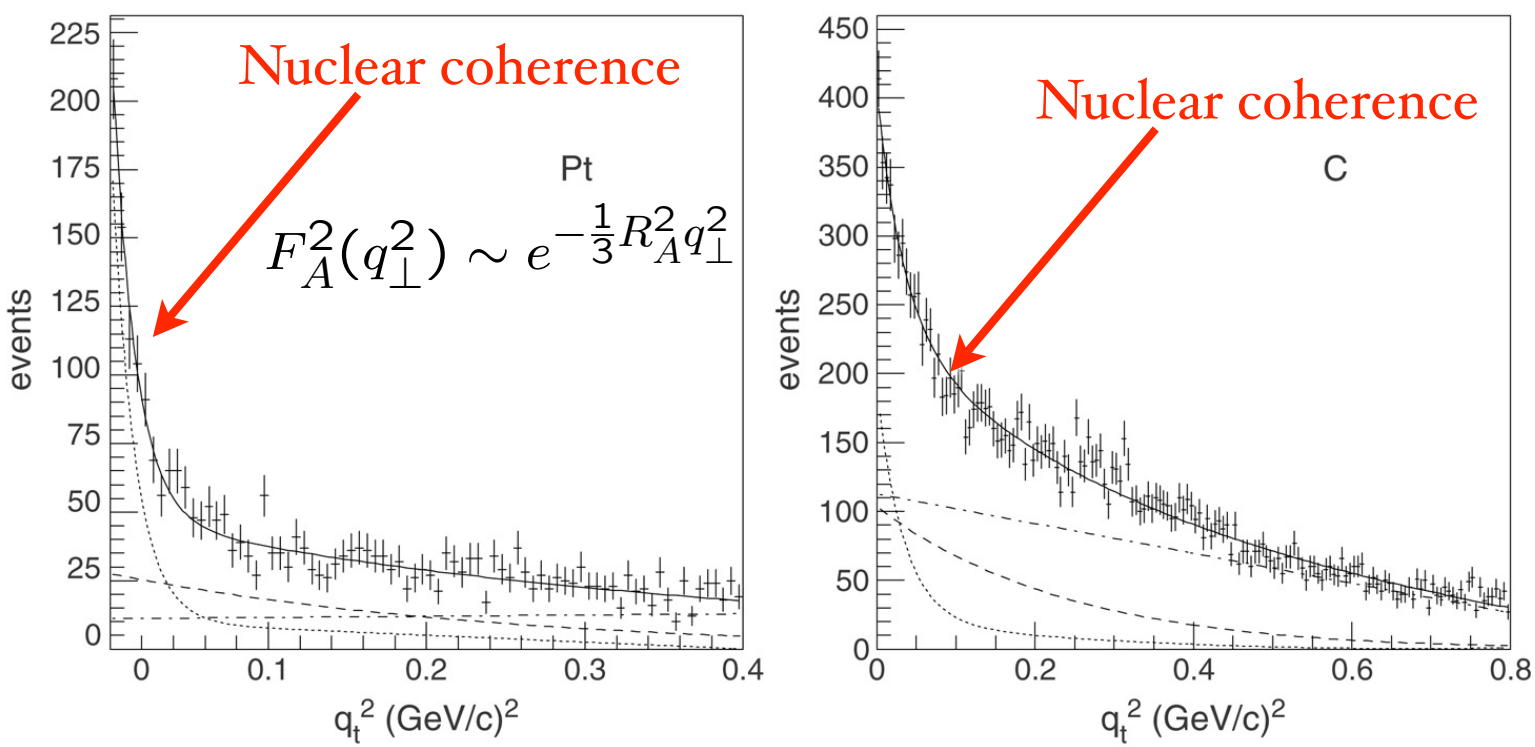
- Fully coherent interactions between pion and nucleons.

- Emerging Di-Jets do not interact with nucleus.

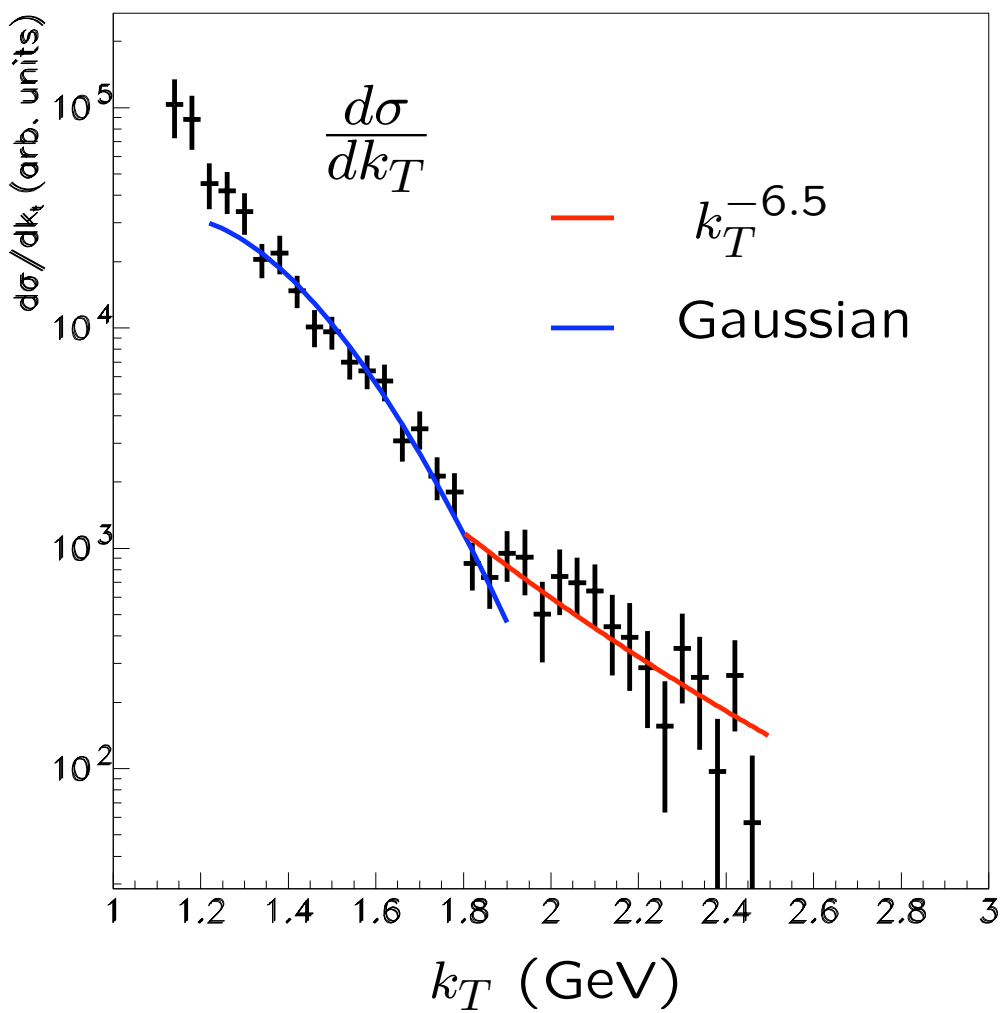
$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



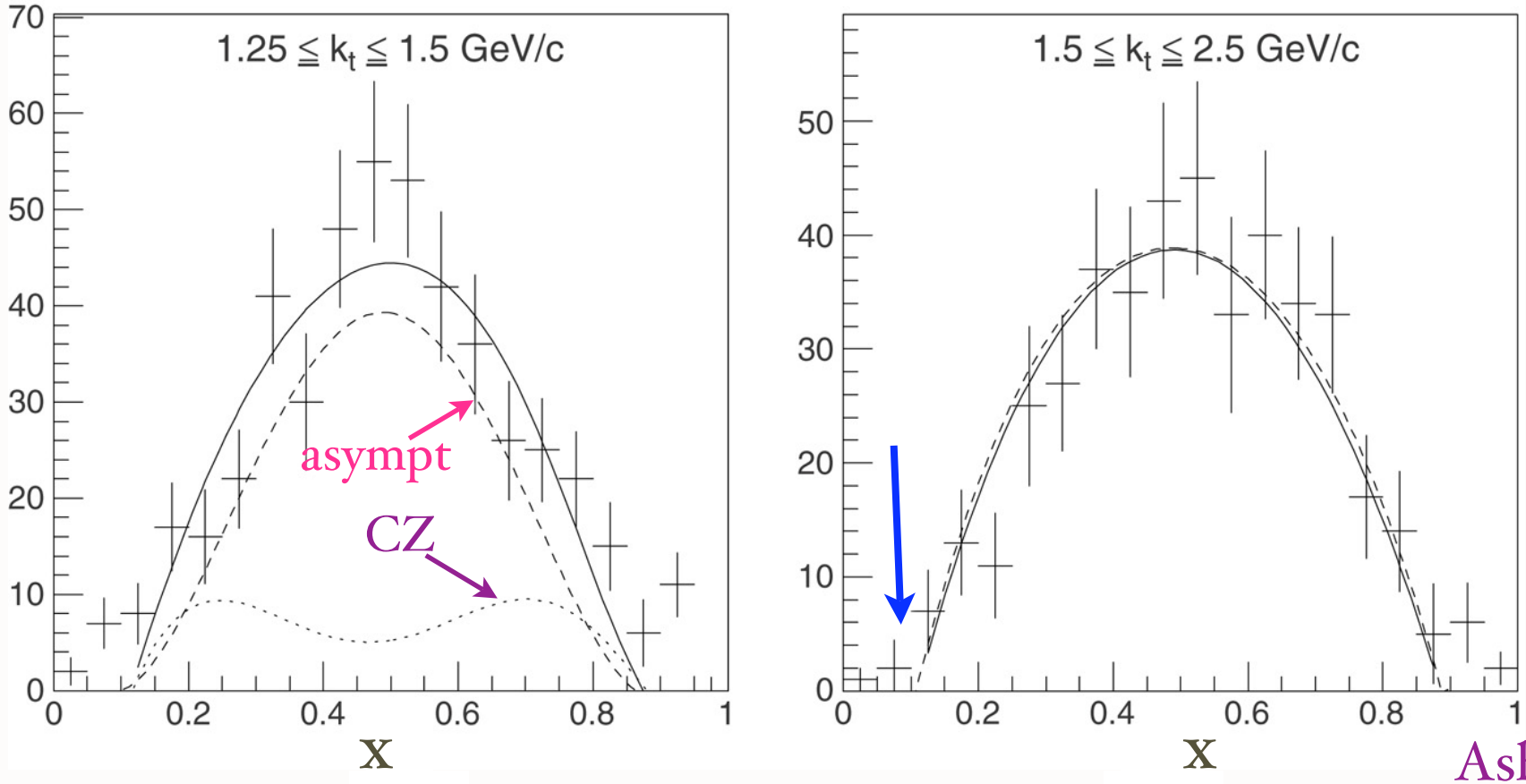
E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence $k_T^{-6.5}$ *consistent with PQCD, ERBL Evolution*

Gaussian component similar to AdS/CFT HO LFWF



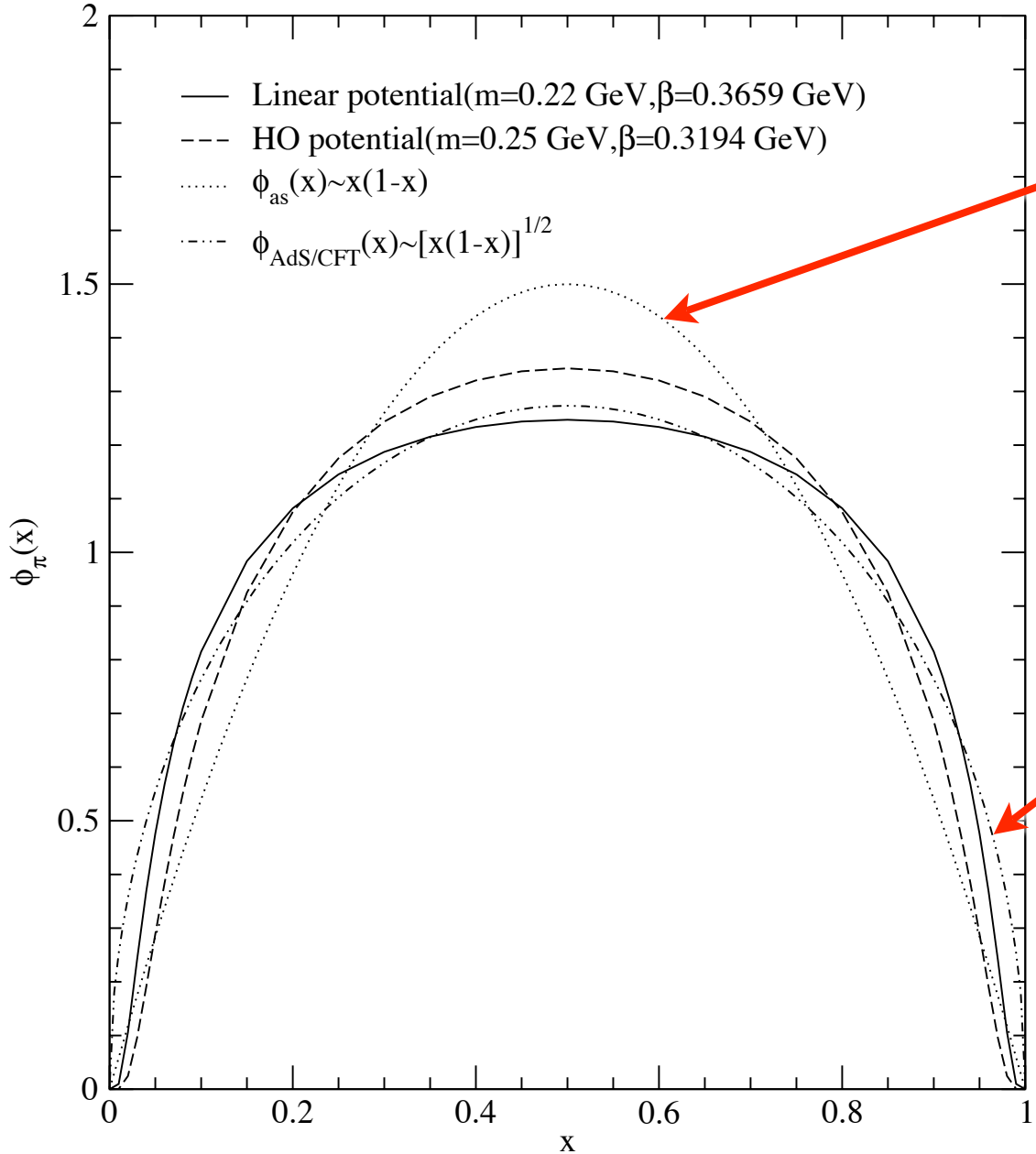
Ashery E791

Narrowing of x distribution at higher jet transverse momentum

x distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and
Perturbative (ERBL)
Evolution to asymptotic distribution

$$\phi(x) \propto \sqrt{x(1-x)}$$



$$\phi_{asympt} \sim x(1-x)$$

AdS/CFT:

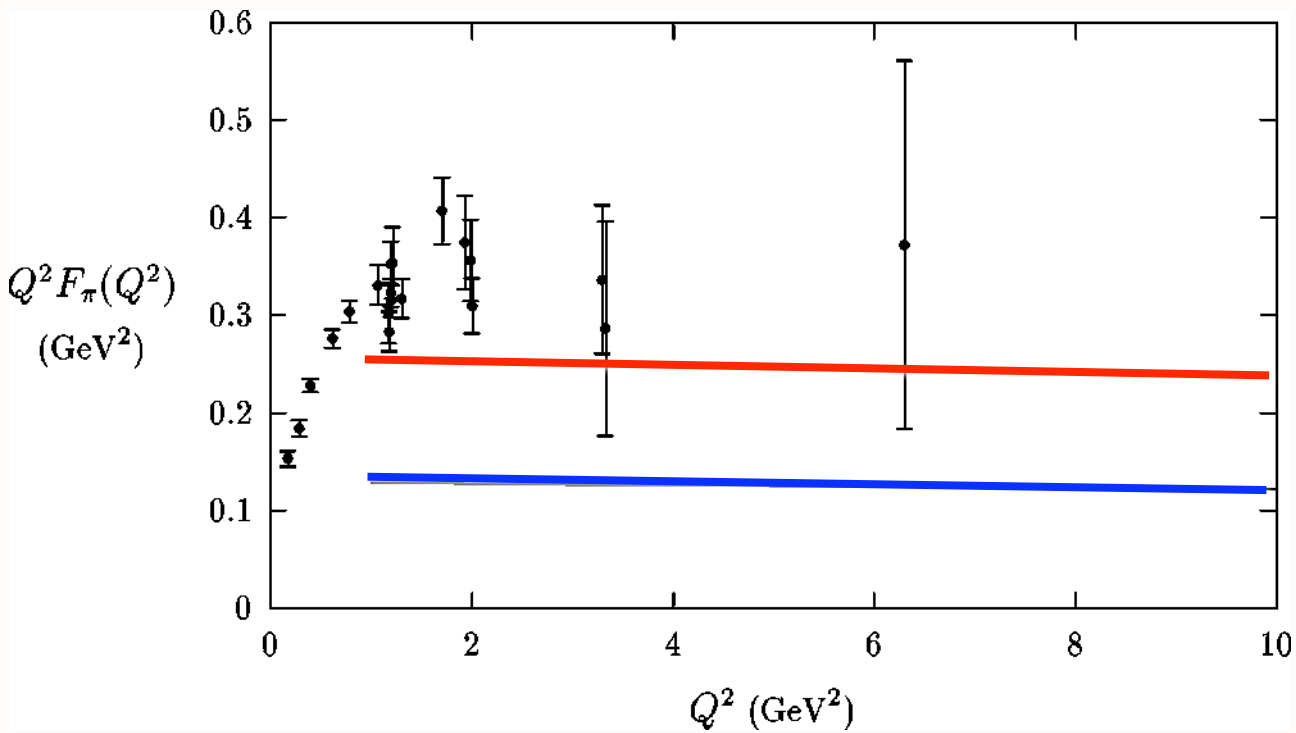
$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD leading twist predictio
$F_\pi(Q^2)$ by factor 16/9

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb
Choi, Ji

$$F_\pi(Q^2) = \int_0^1 dx\, \phi_\pi(x) \int_0^1 dy\, \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

Higher Spin Bosonic Modes HW

- Each hadronic state of integer spin $S \leq 2$ is dual to a normalizable string mode

$$\Phi(x,z)_{\mu_1\mu_2\cdots\mu_S} = \epsilon_{\mu_1\mu_2\cdots\mu_S}\, e^{-iP\cdot x}\, \Phi_S(z).$$

with four-momentum P_μ and spin polarization indices along the 3+1 physical coordinates.

- Wave equation for spin S-mode W. S. I'Yi, Phys. Lett. B **448**, 218 (1999)

$$\left[z^2\partial_z^2 - (d{+}1{-}2S)z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi_S(z) = 0,$$

- Solution

$$\widetilde{\Phi}(z)_S = \left(\frac{z}{R}\right)^S \Phi(z)_S = Ce^{-iP\cdot x} z^{\frac{d}{2}} J_{\Delta-\frac{d}{2}}(z\mathcal{M})\,\epsilon(P)_{\mu_1\mu_2\cdots\mu_S},$$

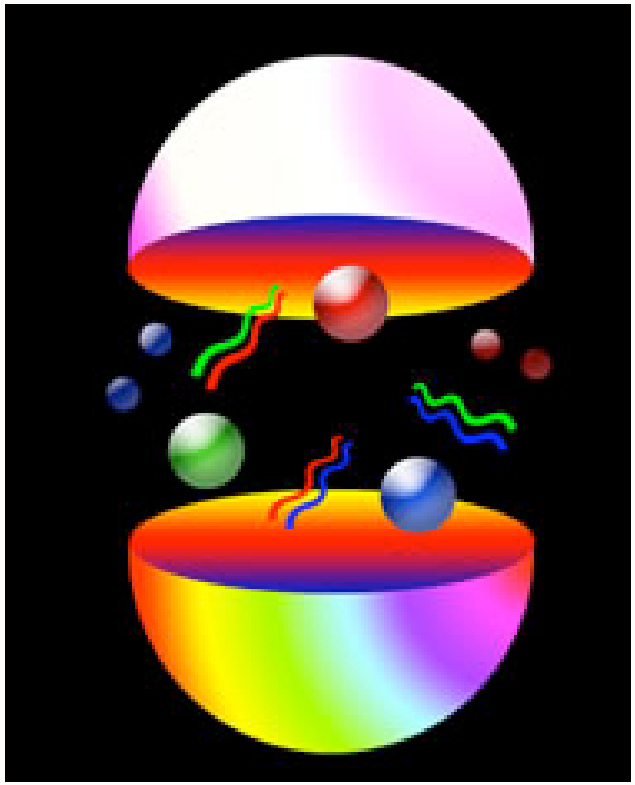
- We can identify the conformal dimension:

$$\Delta = \frac{1}{2}\left(d + \sqrt{(d-2S)^2 + 4\mu^2R^2}\right).$$

- Normalization:

$$R^{d-2S-1}\int_0^{\Lambda_{\rm QCD}^{-1}} \frac{dz}{z^{d-2S-1}}\,\Phi_S^2(z) = 1.$$

Baryons in Ads/CFT



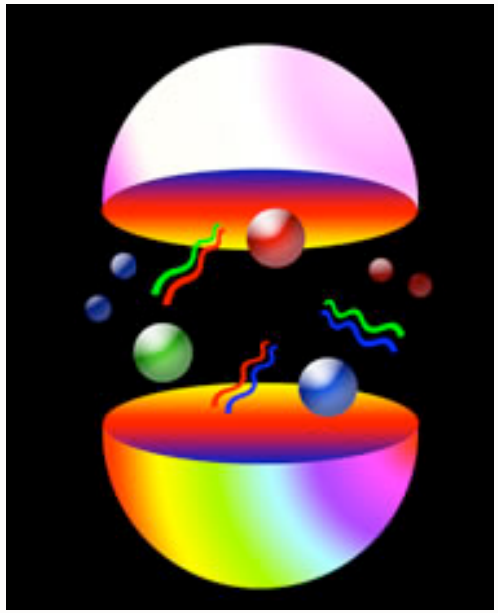
Baryons in "bottom-up" approach to holographic QCD:
{ GdT and SJB (2004)}.

"Top-down" Sakai-Sugimoto model:
{ Hong, Rho, Yee and Yi (2007);
Hata, Sakai, Sugimoto, Yamato (2007)}.

- Baryons Spectrum in "bottom-up" holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in AdS/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi},\Psi] = \int d^{d+1}x\,\sqrt{g}\,\overline{\Psi}(x,z)\left(i\Gamma^\ell D_\ell - \mu\right)\Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right)\Psi(x,z) = 0$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R\right]\Psi(x^\ell) = 0.$$

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Supersymmetric QM between bosonic and fermionic modes in AdS?

- Note: in the Weyl representation ($i\alpha = \gamma_5\beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

with a scale independent mass ratio.

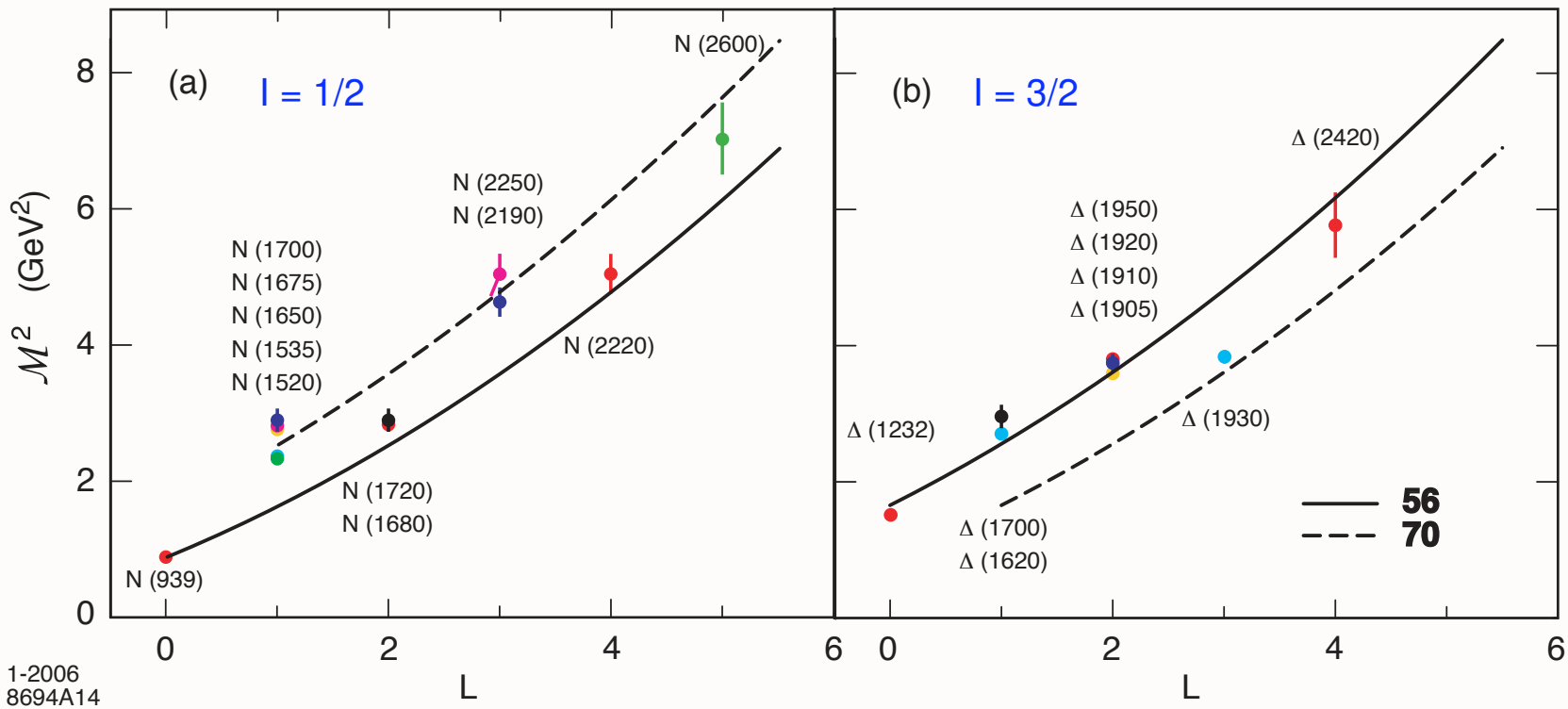


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

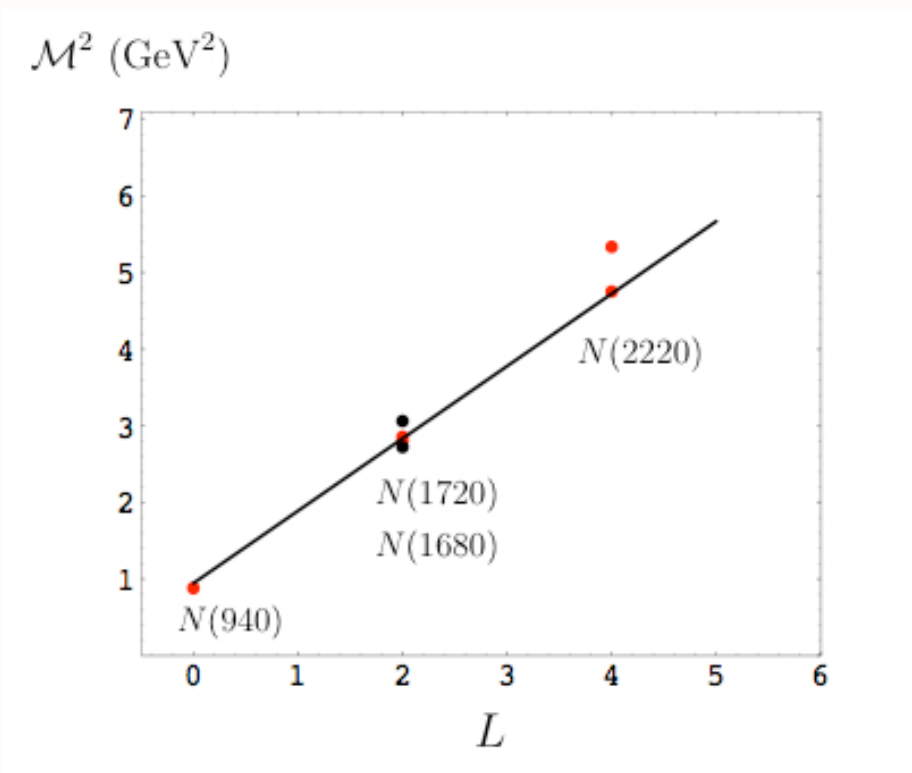
$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1).$$



Proton Regge Trajectory $\quad \kappa = 0.49$GeV

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta)|\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

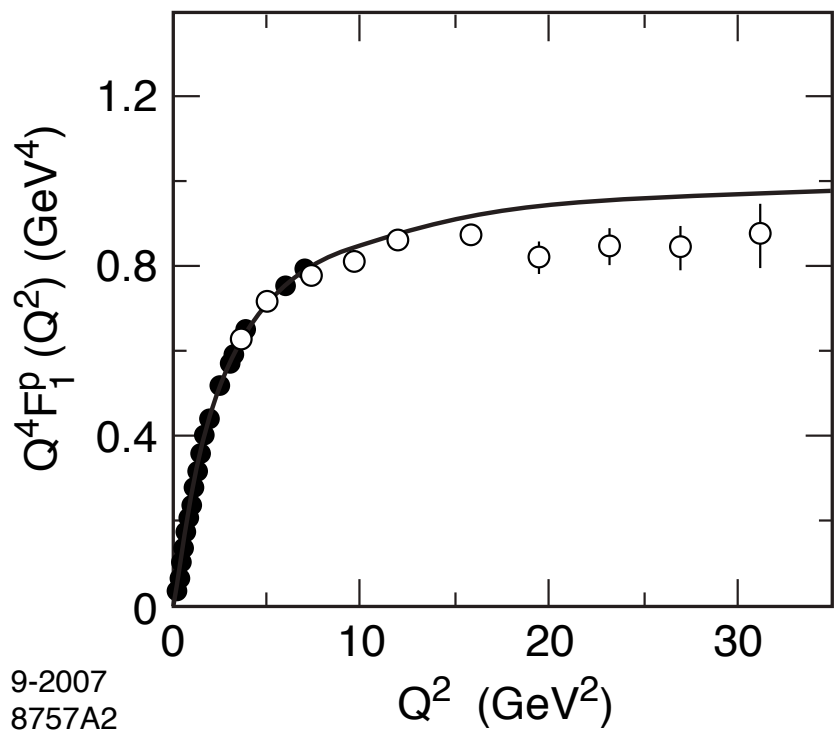
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3}\int d\zeta\, J(Q,\zeta)\left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

where $F_1^p(0) = 1,\ F_1^n(0) = 0.$

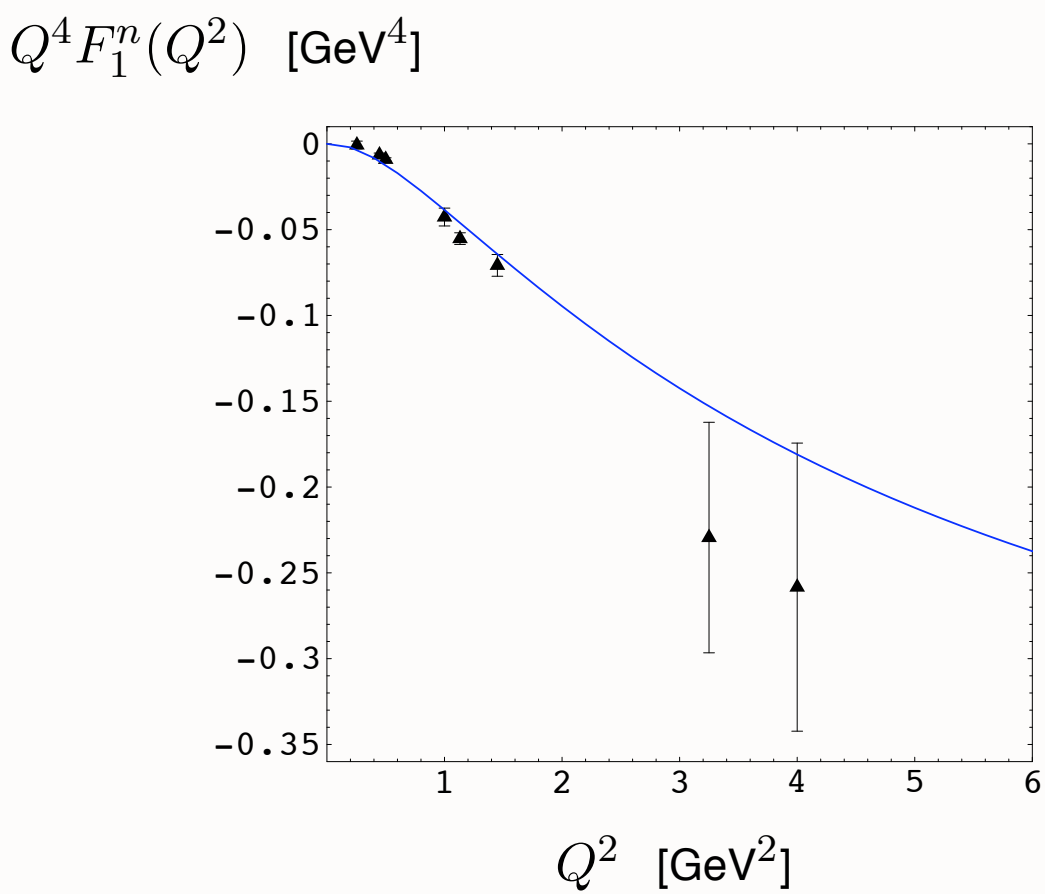
- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

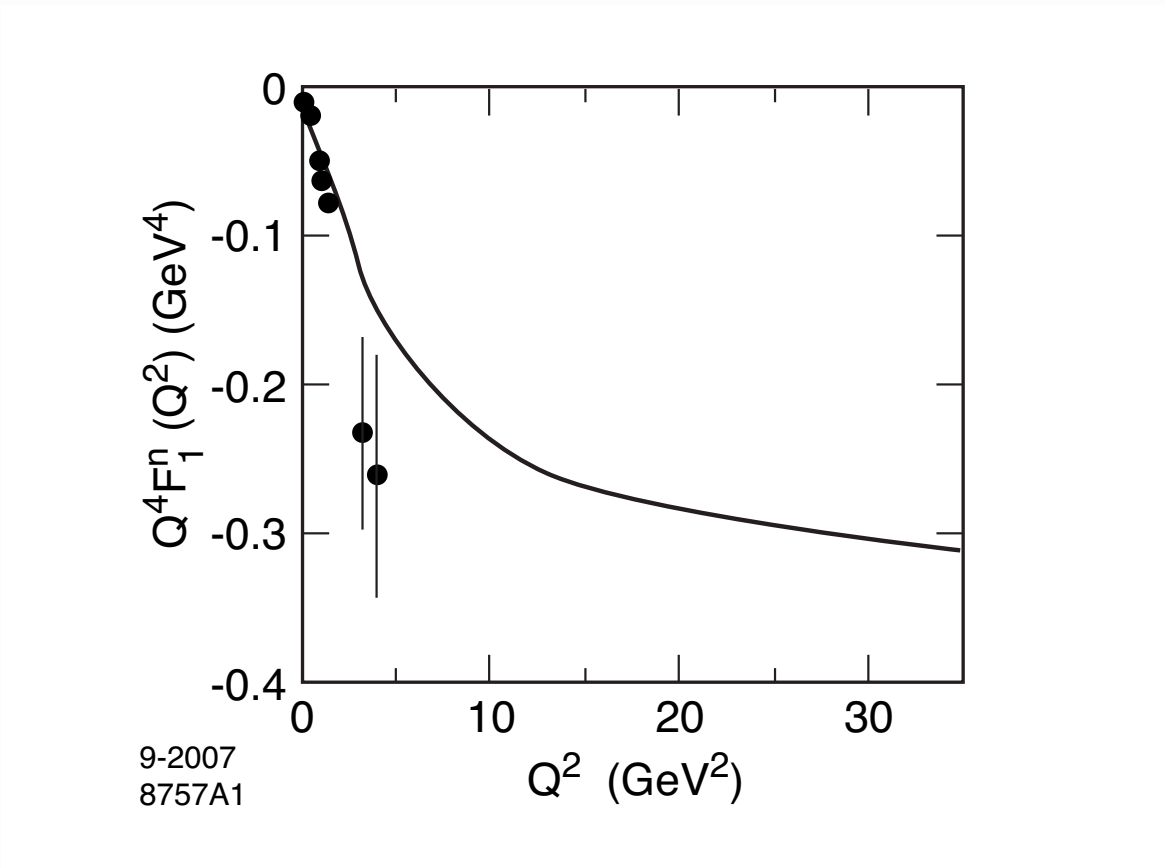
Dirac Neutron Form Factor
(Valence Approximation)

Truncated Space Confinement



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

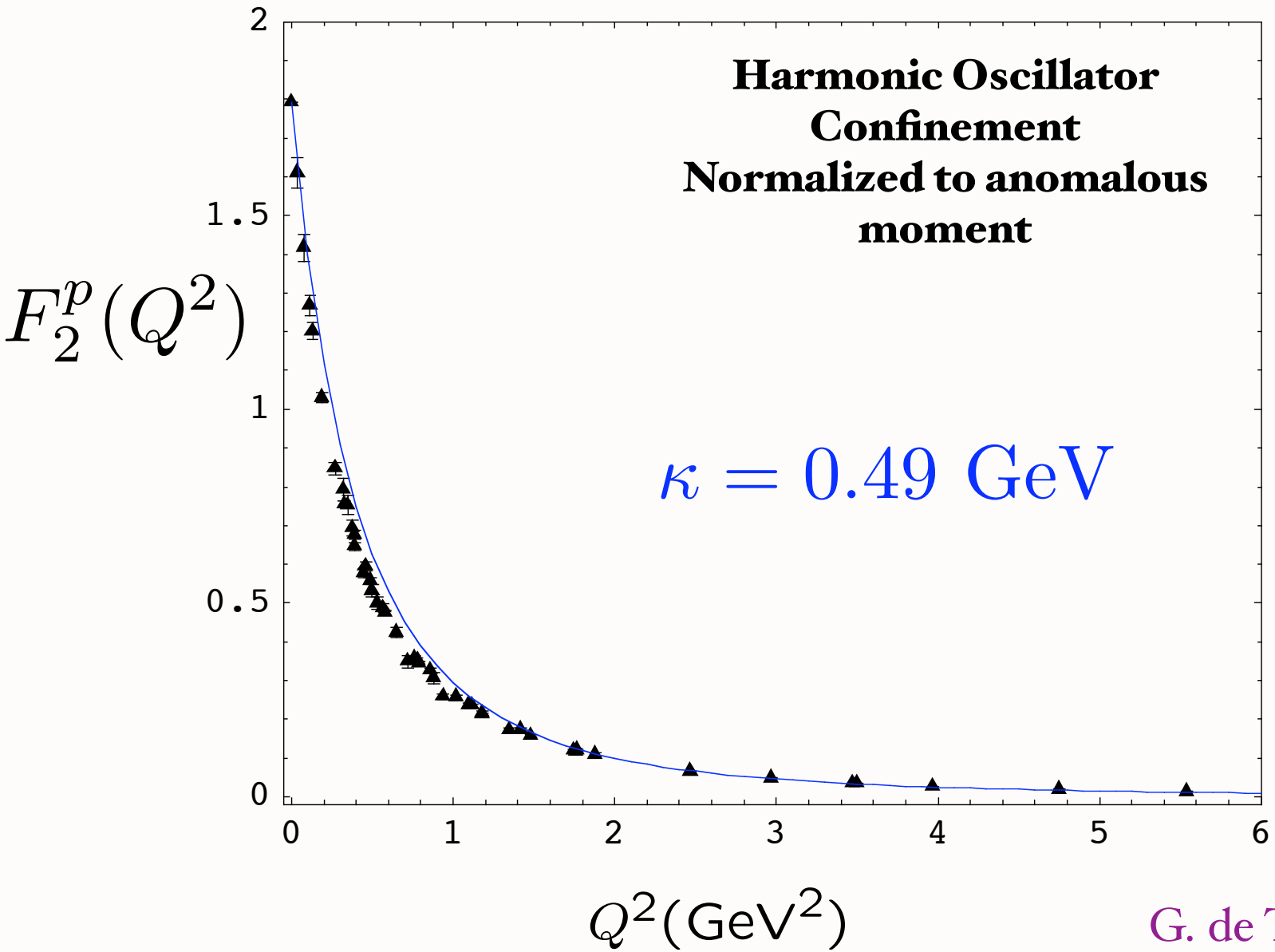


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



G. de Teramond, sjb

Holographic Connection between LF and AdS/CFT

- Predictions for hadronic spectra, light-front wavefunctions, interactions
- Deduce meson and baryon wavefunctions, distribution amplitude, structure function from holographic constraint
- Identification of Orbital Angular Momentum Casimir for SO(2): LF Rotations
- Extension to massive quarks

String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3 +1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

AdS/CFT and Integrability

- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations

Algebraic Structure , Integrability and Stability Conditions (HW Model)

- If $L^2 > 0$ the LF Hamiltonian, H_{LF}, can be written as a bilinear form

$$H^L_{LF}(\zeta) = \Pi^\dagger_L(\zeta)\Pi_L(\zeta)$$

in terms of the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\right),$$

and its adjoint

$$\Pi^\dagger_L(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{L+\frac{1}{2}}{\zeta}\right),$$

with commutation relations

$$\left[\Pi_L(\zeta), \Pi^\dagger_L(\zeta)\right] = \frac{2L+1}{\zeta^2}.$$

- For $L^2 \geq 0$ the Hamiltonian is positive definite

$$\langle\phi\left|H^L_{LF}\right|\phi\rangle = \int d\zeta\,|\Pi_L\phi(z)|^2 \geq 0$$

and thus $\mathcal{M}^2 \geq 0$.

Ladder Construction of Orbital States

- Orbital excitations constructed by the L-th application of the raising operator

$$a_L^\dagger = -i\Pi_L$$

on the ground state:

$$a^\dagger|L\rangle = c_L|L+1\rangle.$$

- In the light-front ζ-representation

$$\begin{aligned}\phi_L(\zeta) &= \langle\zeta|L\rangle = C_L\sqrt{\zeta}\,(-\zeta)^L\left(\frac{1}{\zeta}\frac{d}{d\zeta}\right)^L J_0(\zeta\mathcal{M}) \\ &= C_L\sqrt{\zeta}J_L\left(\zeta\mathcal{M}\right).\end{aligned}$$

- The solutions ϕ_L are solutions of the light-front equation $(L = 0, \pm1, \pm2, \cdots)$

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-L^2}{4\zeta^2}\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta),$$

- Mode spectrum from boundary conditions : $\phi\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0.$

Non-Conformal Extension of Algebraic Integrability (SW Model)

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field $\varphi(z)$.
- Consider the generator (short-distance Coulombic and long-distance linear potential)

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

$$\Pi_L^\dagger(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{L+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2} - 2\kappa^2.$$

- The LF Hamiltonian

$$H_{LF} = \Pi_L^\dagger \Pi_L + C$$

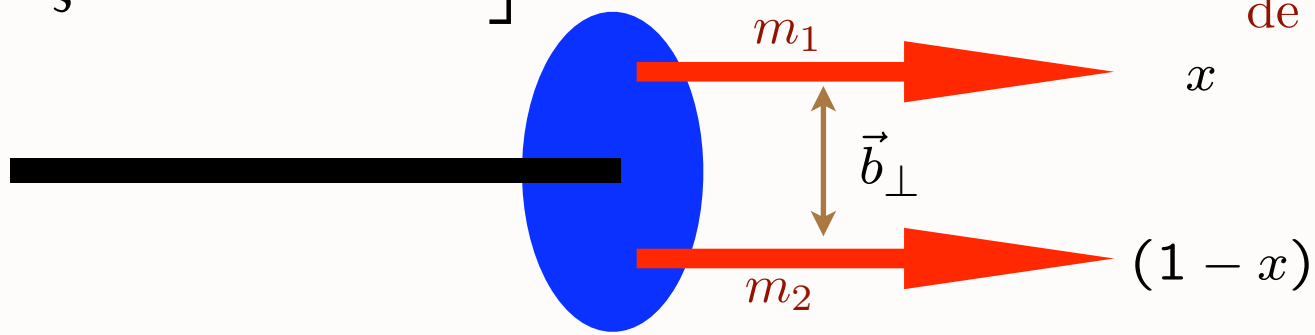
is positive definite $\langle\phi|H_{LF}|\phi\rangle \geq 0$ for $L^2 \geq 0$, and $C \geq -4\kappa^2$.

Integrable !

- Orbital and radial excited states are constructed from the ladder operators from the $L = 0$ state.

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}}\, e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

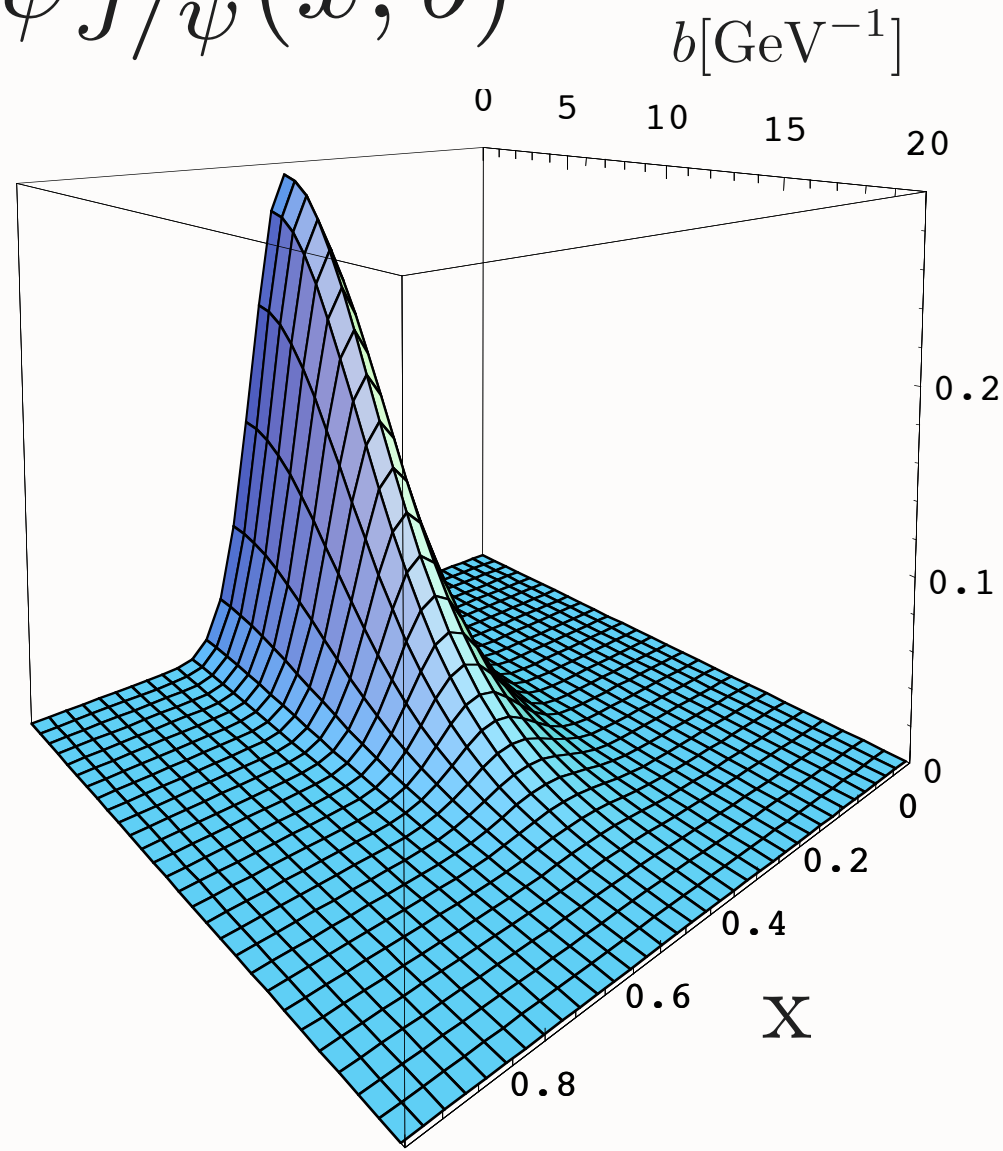
$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

$$\psi_{J/\psi}(x,b)$$



LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

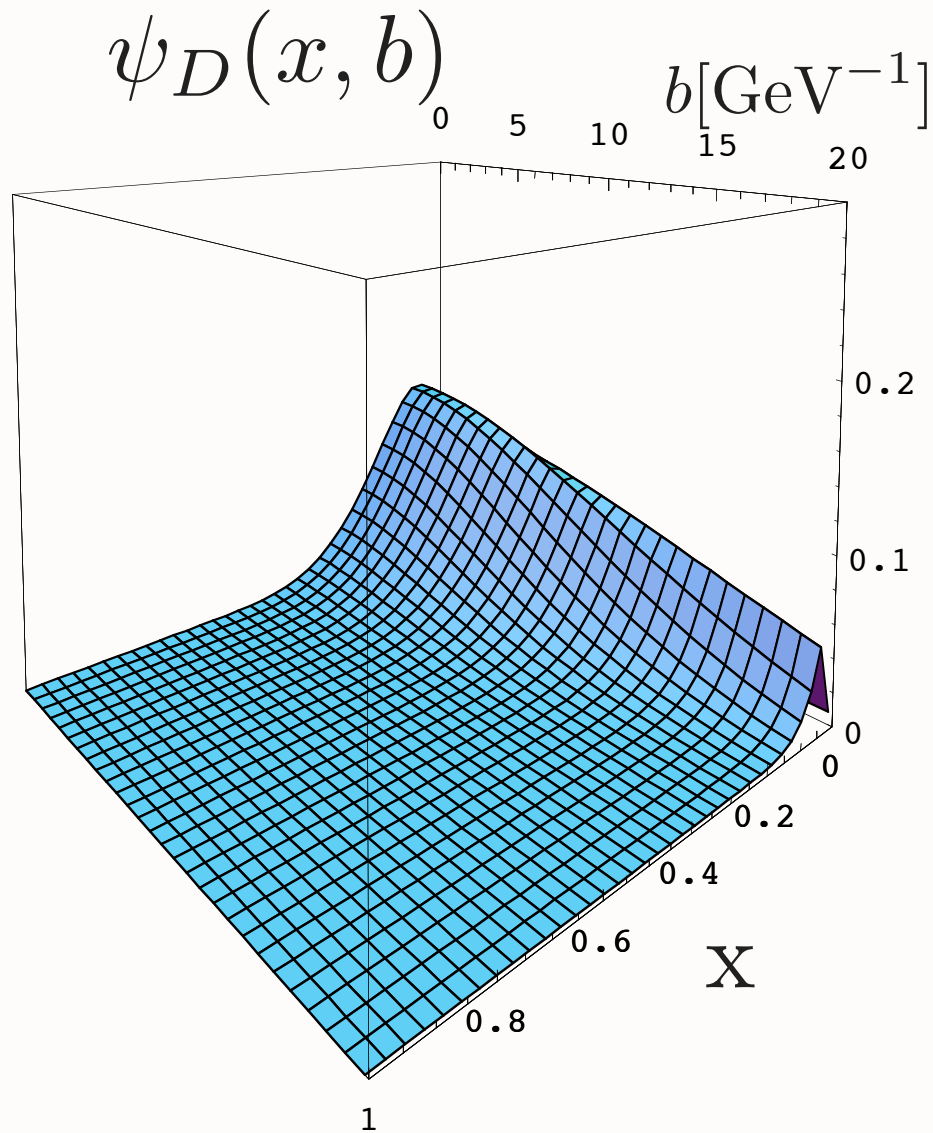
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

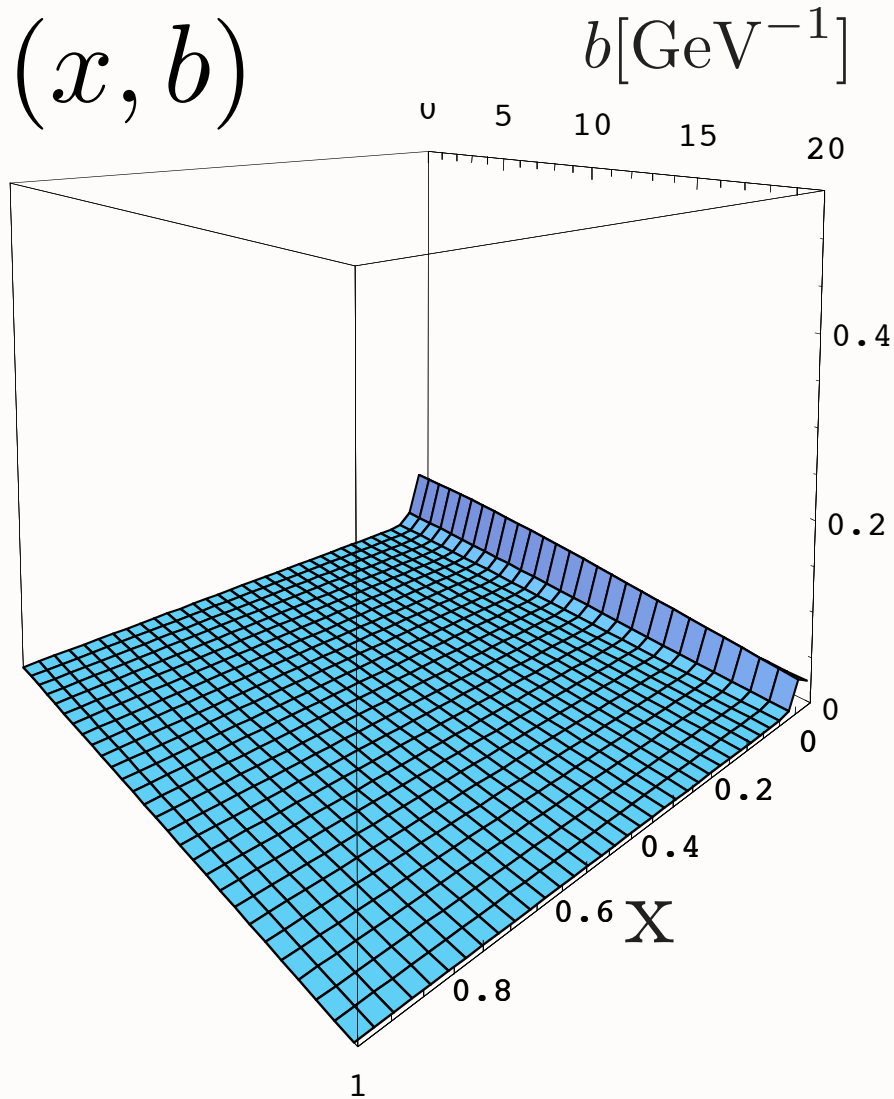
$$\kappa = 0.375 \text{ GeV}$$

$$m_a = m_b = 1.25 \text{ GeV}$$



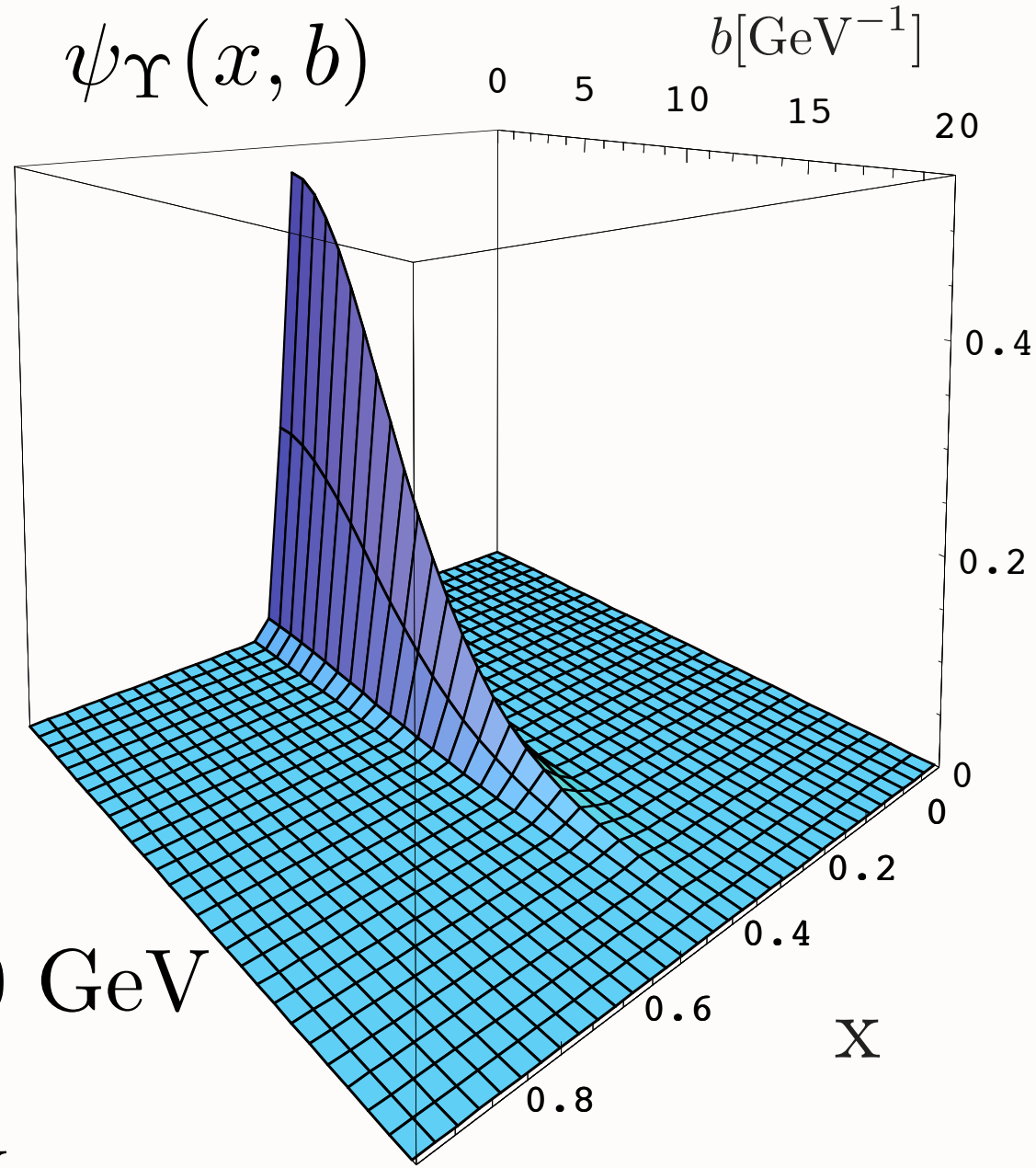
$$\kappa = 0.375 \text{ GeV} \qquad m_q = 0, \; m_c = 1.3 \text{ GeV}$$

$\psi_B(x, b)$



$$\kappa = 0.375 \text{ GeV} \qquad m_1 = 0, m_2 = 4.0 \text{ GeV}$$

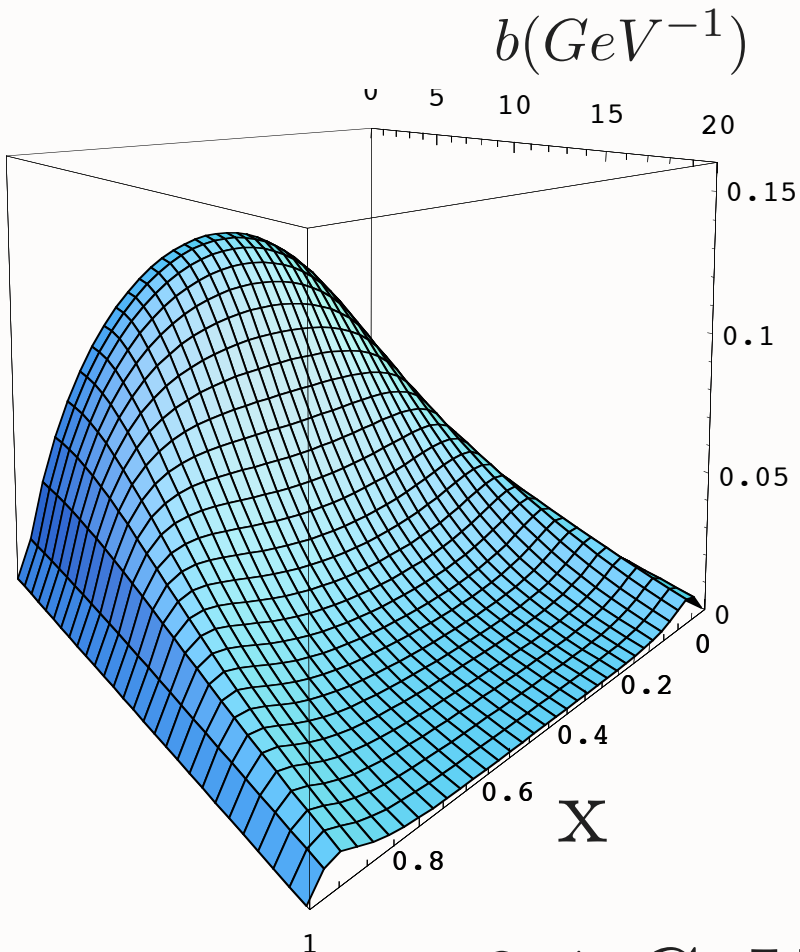
Υ



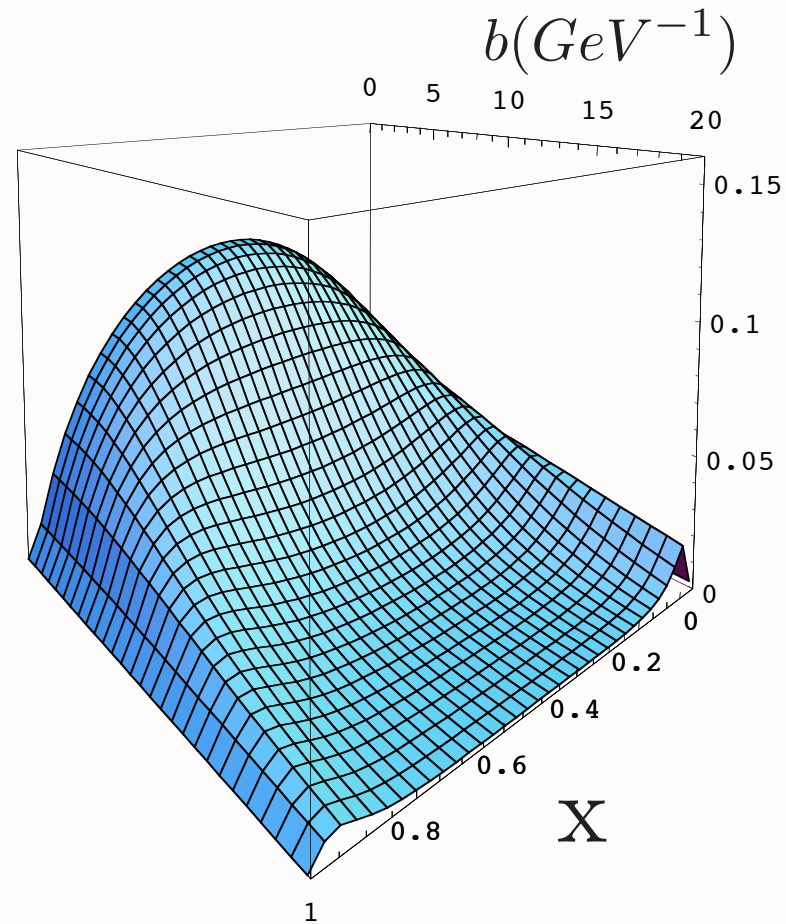
$$m_a = m_b = 4.0 \text{ GeV}$$

$$\kappa = 0.375 \text{ GeV}$$

$\psi_\phi(x, b_\perp)$



$\psi_K(x, b_\perp)$



$m_1 = m_2 = 0.1$ GeV

$m_1 = 0, m_2 = 0.1$ GeV

$\kappa = 0.375$ GeV

LF Schrodinger Equation for soft-wall model with massive quarks

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta),$$

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L-1) + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}$$

$$z \to \zeta \to \chi$$

Eigenvalue $\mathcal{M}^2$ shifted by $[\sum_i m_i]^2$

Consistency at $\kappa = 0$

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis

Pauli, Hornbostel, Hiller, McCartor, sjb

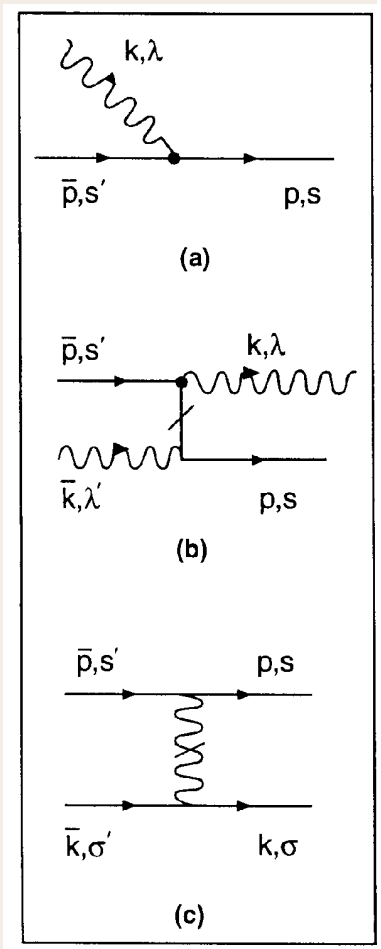
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

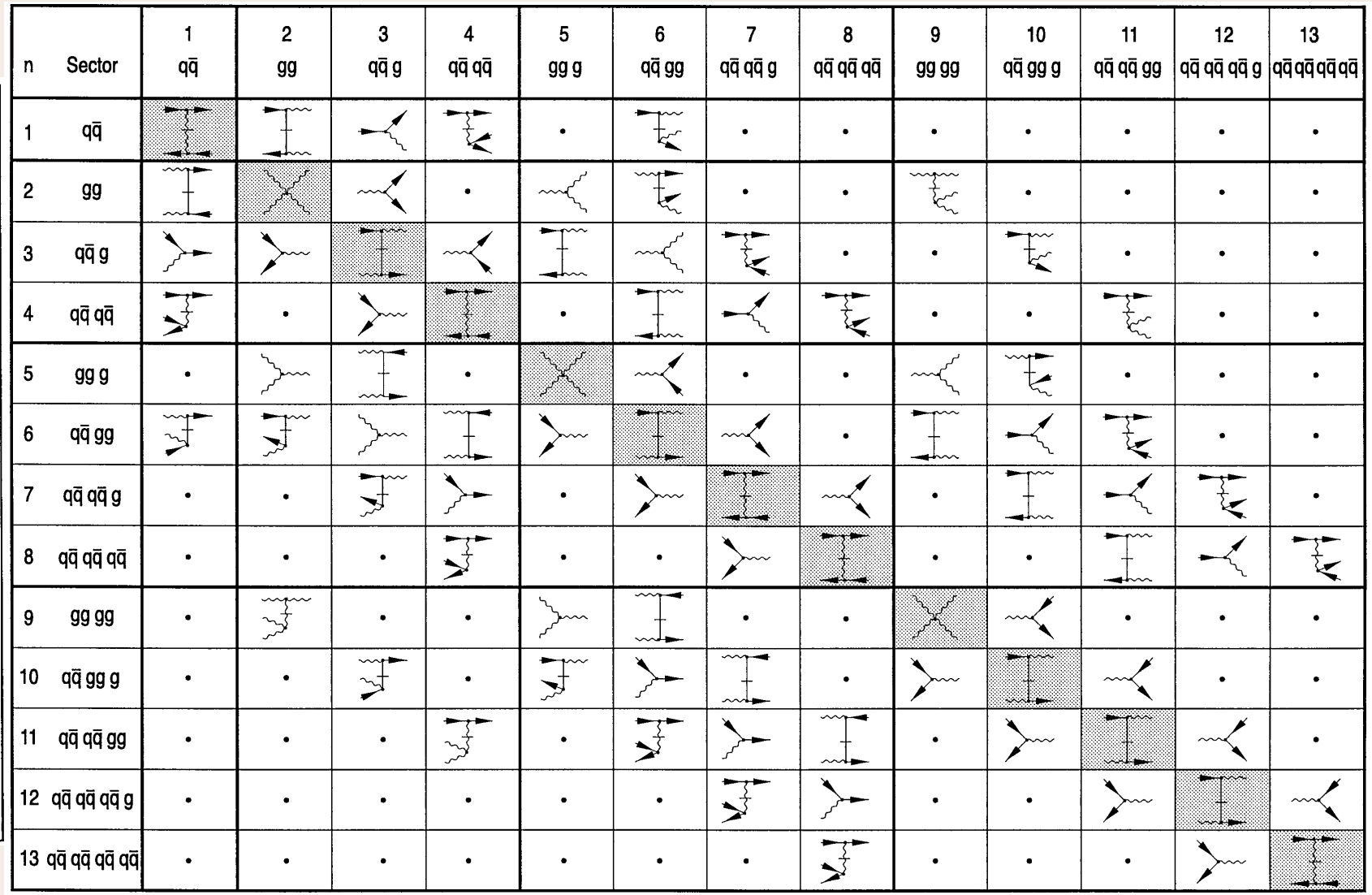
Vary, Harinandrath, Maris, sjb

- Similar to Shell Model calculations

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$



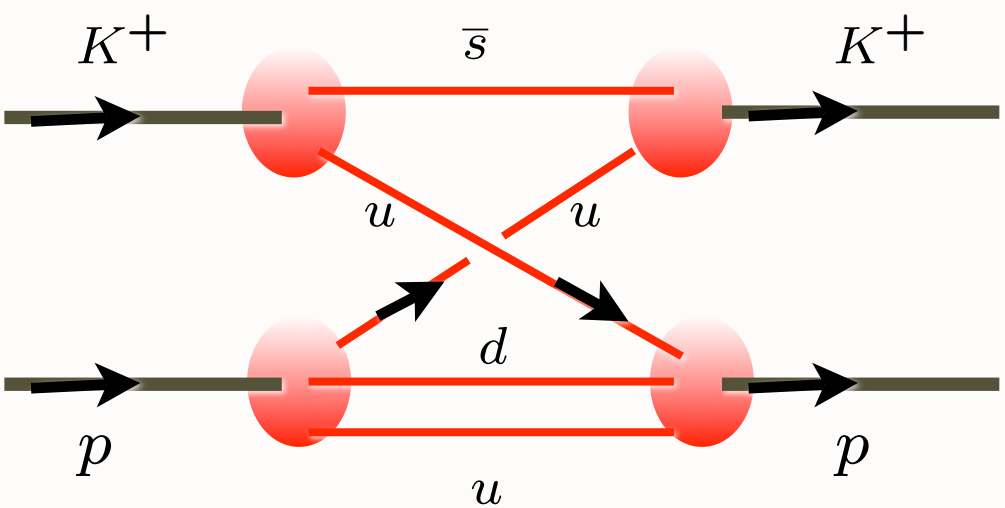


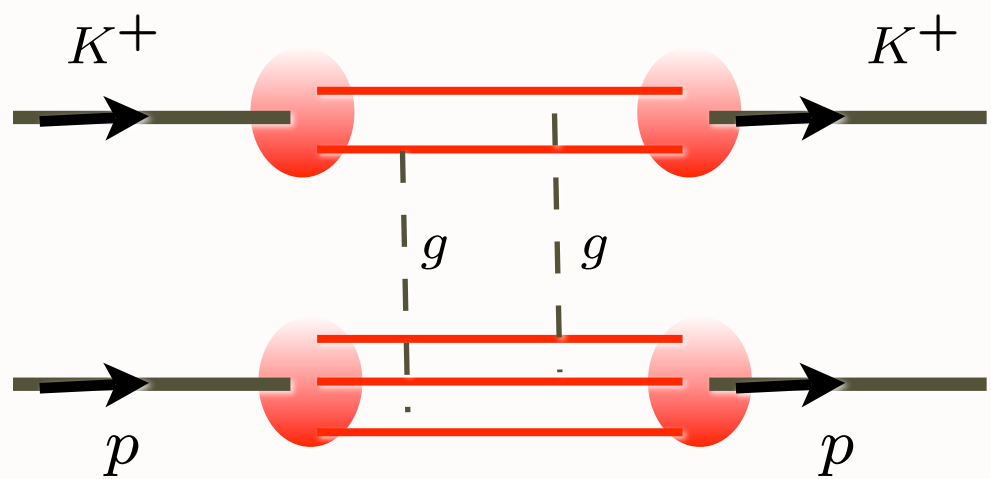
Use AdS/QCD basis functions

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb





Quark Interchange (Spin exchange in atom-atom scattering)

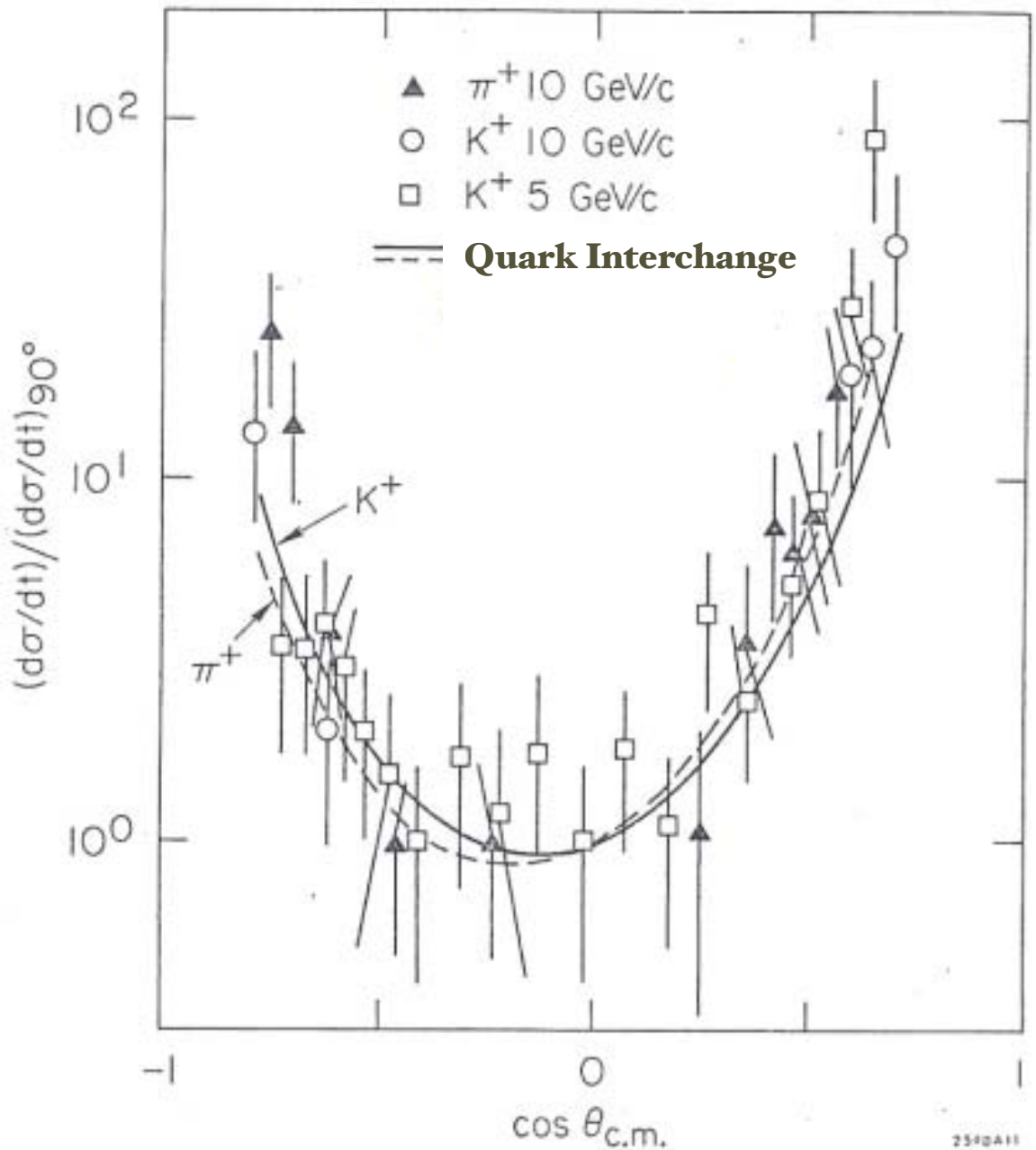
Gluon Exchange (Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \; \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^{0}/\Sigma^{0})K^{0}$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\pm}$,

$K^{\pm}p \to pK^{\pm}$,

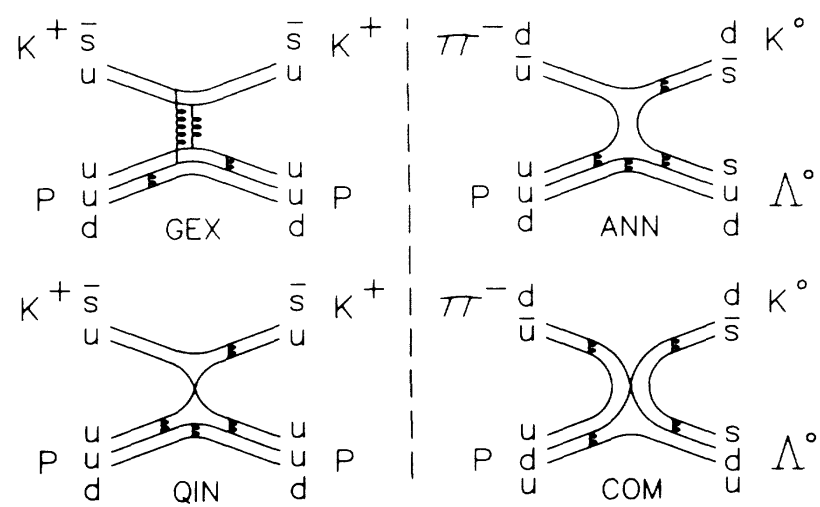
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

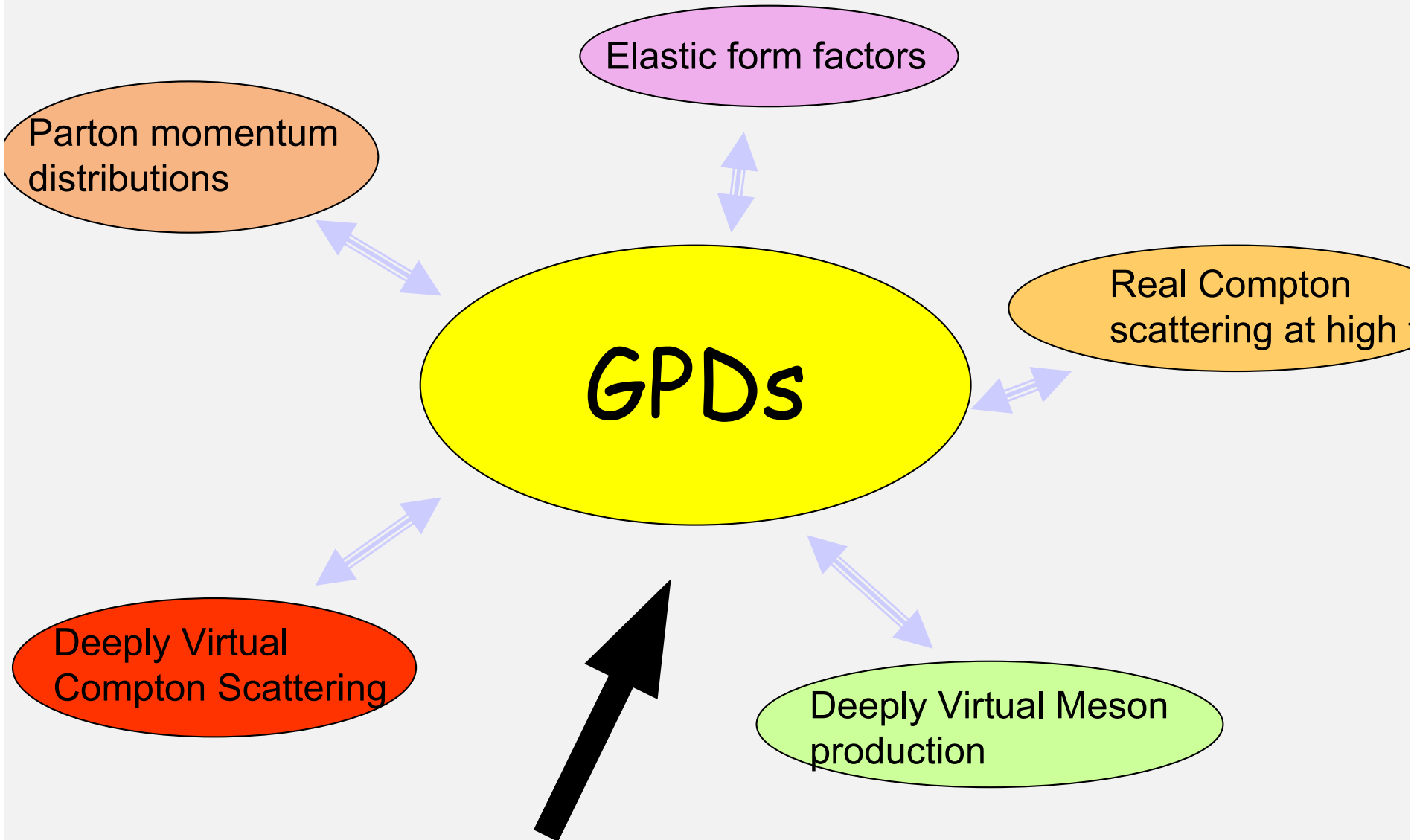
$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^{0}K^{0}, \Sigma^{0}K^{0}$,

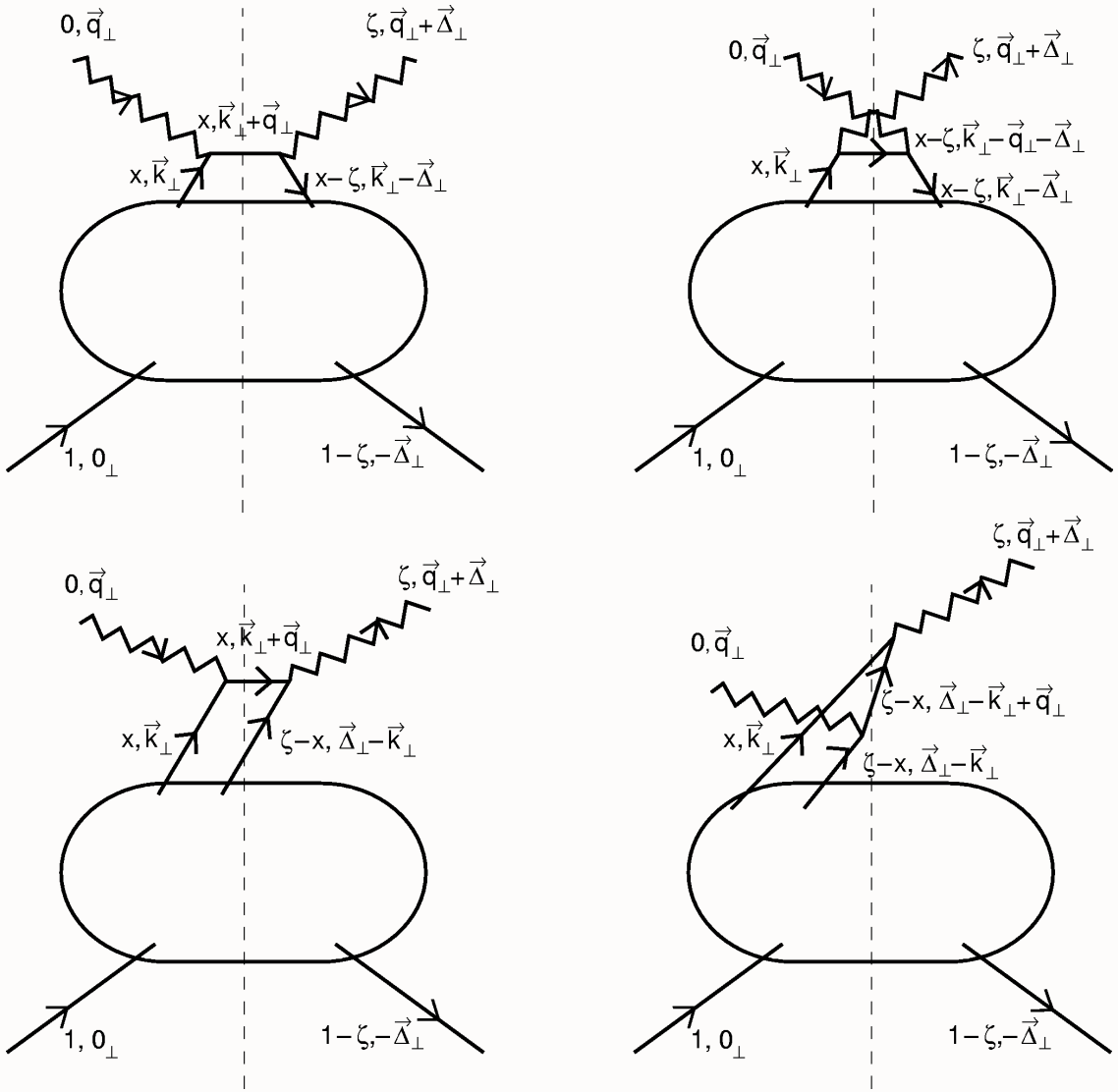
$p^{\pm}p \to pp^{\pm}$.



A Unified Description of Hadron Structure



Light Front Wavefunctions

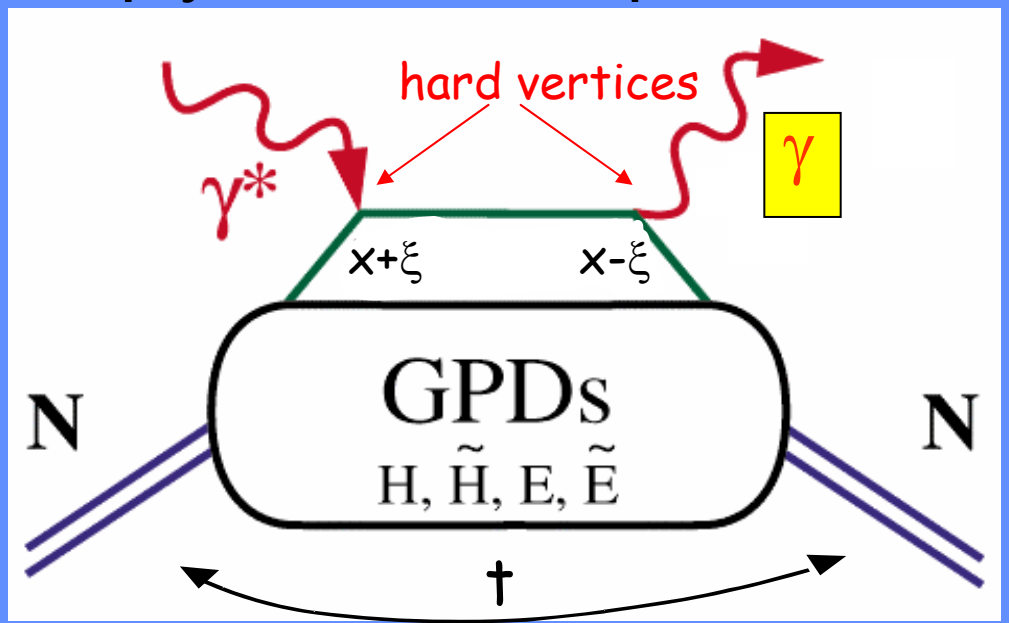


Light-cone wavefunction representation of deeply virtual Compton scattering ☆

Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

GPDs & Deeply Virtual Exclusive Processes
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)



x – quark momentum fraction

ξ– longitudinal momentum transfer

$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

$H(x,\xi,t)$, $E(x,\xi,t)$, . . "Generalized Parton Distributions"

Quark angular momentum (Ji sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

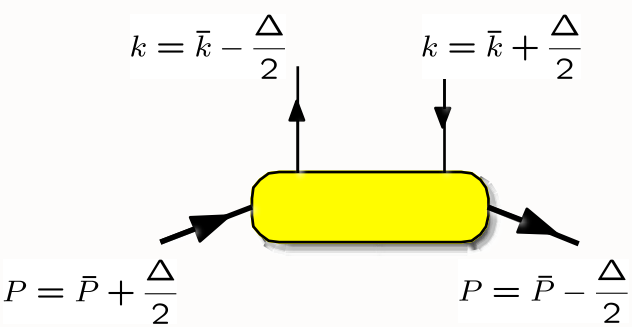
X. Ji, Phy.Rev.Lett.78,610(1997)

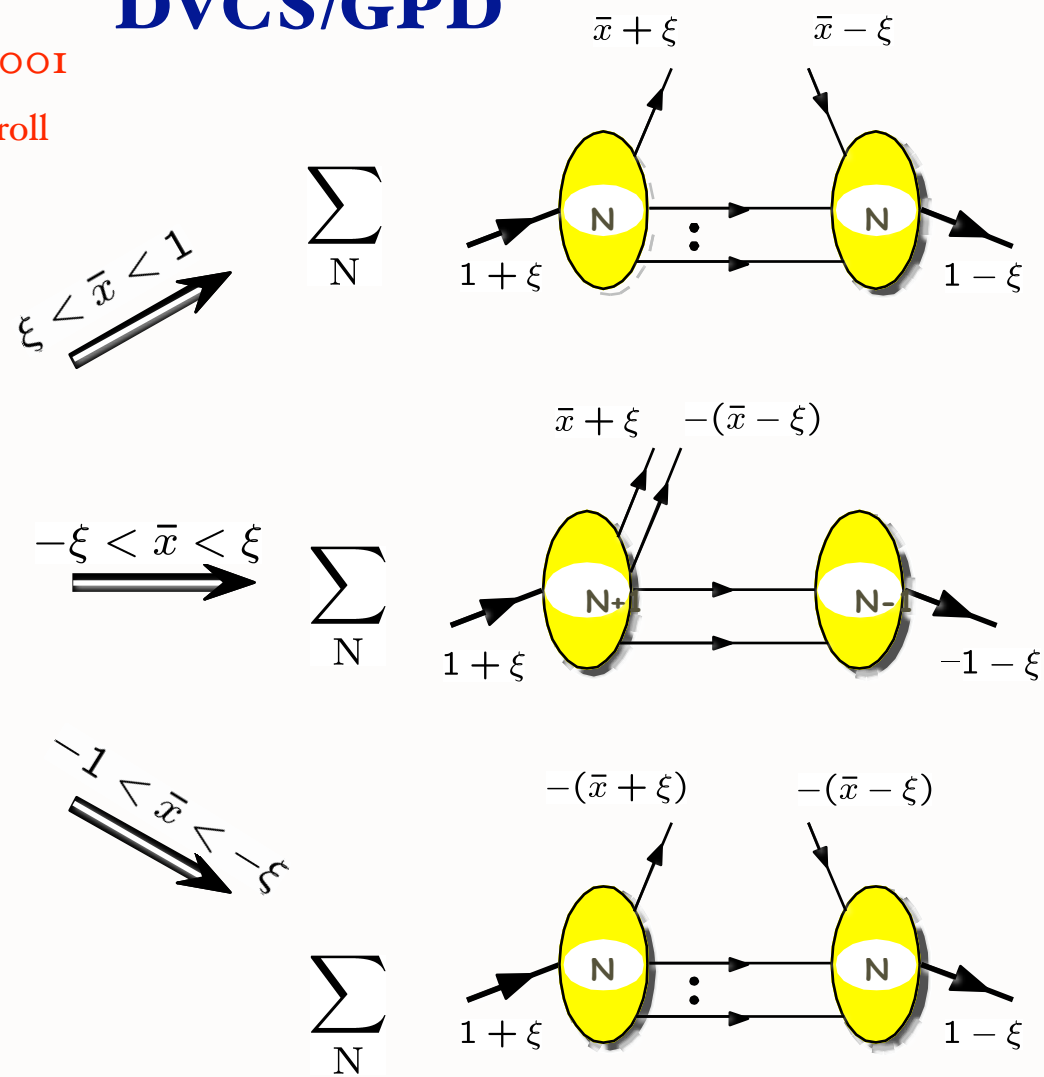
Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll





N=3 VALENCE QUARK ⇒ Light-cone Constituent quark model

N=5 VALENCE QUARK + QUARK SEA ⇒ Meson-Cloud model

Example of LFWF representation of GPDs (n => n)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\Delta^2}{2M}E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n}\sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\,16\pi^3\delta\left(1-\sum_{j=1}^{n}x_j\right)\delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right)$$

$$\times\,\delta(x-x_1)\psi^{\uparrow *}_{(n)}\left(x'_i,\vec{k}'_{\perp i},\lambda_i\right)\psi^{\downarrow}_{(n)}\left(x_i,\vec{k}_{\perp i},\lambda_i\right),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2,\ldots,n.$$

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \tilde{H}^q, \tilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl,Hwang, sjb

Quark angular momentum (Ji's sum rule)

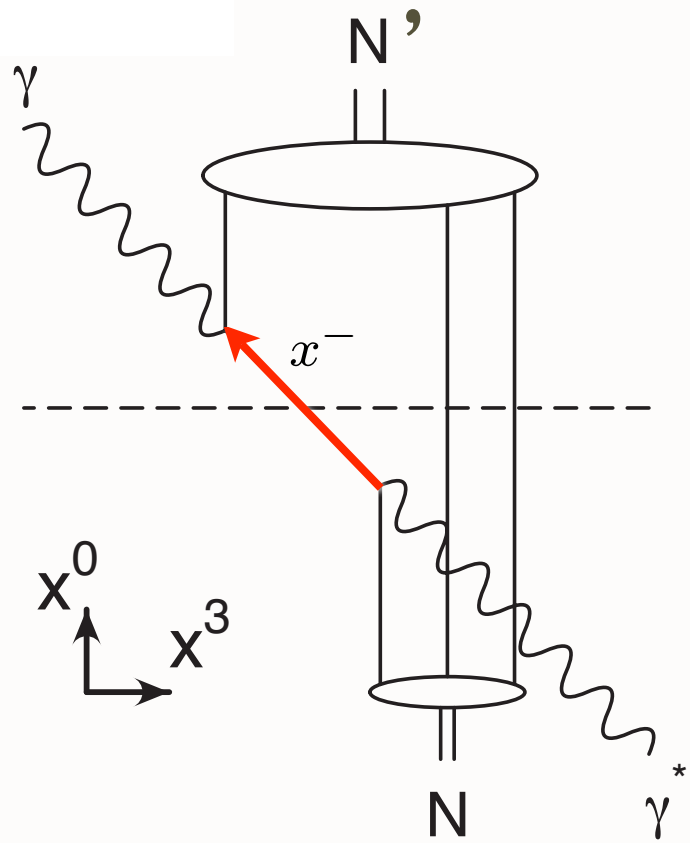
$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x dx \left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

Space-time picture of DVCS

P. Hoyer

$\sigma = \frac{1}{2} x^- P^+$



$x^+ = \mathbf{x}_\perp = 0$

The position of the struck quark differs by x^- in the two wave functions

Measure x- distribution from DVCS:
Take Fourier transform of skewness, $\xi = \frac{Q^2}{2p.q}$
the longitudinal momentum transfer

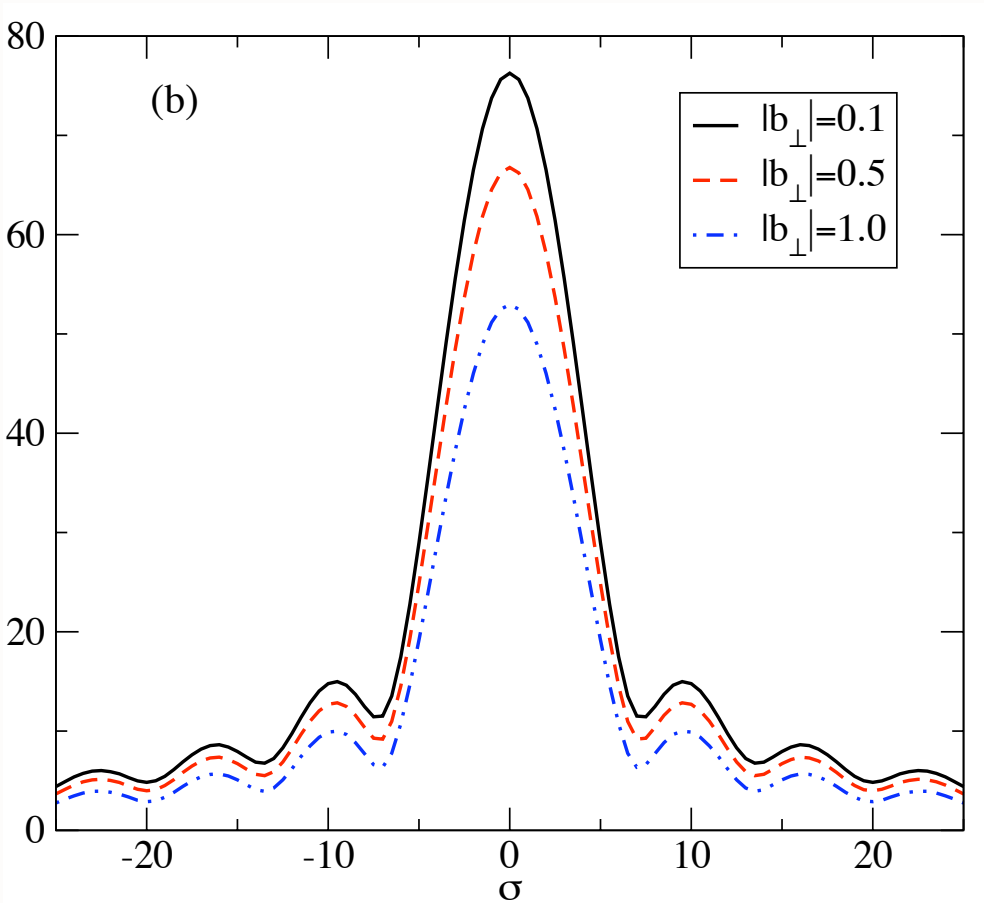
S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, \vec{b}_\perp) = \frac{1}{2\pi} \int d\xi e^{i\frac{1}{2}\xi\sigma} \tilde{A}(\xi, \vec{b}_\perp)$$

$$\sigma = \frac{1}{2} x^- P^+ \qquad \xi = \frac{Q^2}{2p.q}$$

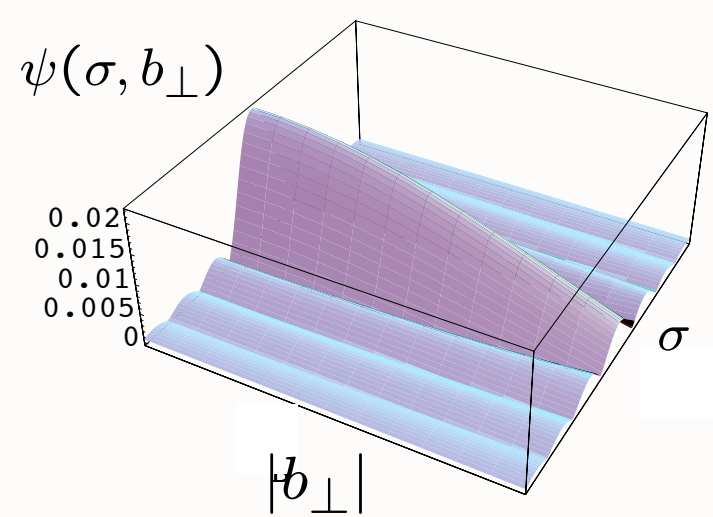


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

$$\Lambda_{QCD} = 0.32$$



Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect

Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role if ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

New Perspectives on QCD Phenomena from AdS/CFT

- AdS/CFT: Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions
- AdS/CFT provides novel insights into the quark structure of hadrons

A Few References: Bottom-up-Approach

- Derivation of dimensional counting rules of hard exclusive glueball scattering in AdS/CFT:
  Polchinski and Strassler, hep-th/0109174.
- Deep inelastic scattering in AdS/CFT:
  Polchinski and Strassler, hep-th/0209211.
- Unified description of the soft and hard pomeron in AdS/CFT:
  Brower, Polchinski, Strassler and Tan, hep-th/0603115.
- Hadron couplings and form factors in AdS/CFT:
  Hong, Yoon and Strassler, hep-th/0409118.
- Low lying meson spectra, chiral symmetry breaking and hadron couplings in AdS/QCD (Emphasis on axial and vector currents)
  Erlich, Katz, Son and Stephanov, hep-ph/0501128,
  Da Rold and Pomarol, hep-ph/0501218, hep-ph/0510268.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 . . .

- Counting rules, low lying meson and baryon spectra and form factors in AdS/CFT, holographic light front representation and mapping of string amplitudes to light-front wavefunctions, integrability and stability of AdS/CFT equations (Emphasis on hadronic quark constituents)
  Brodsky and GdT, hep-th/0310227, hep-th/0409074, hep-th/0501022, hep-ph/0602252, 0707.3859 [hep-ph], 0709.2072 [hep-ph].

1. **"Light-Front Dynamics and AdS/QCD: The Pion Form Factor in the Space- and Time-Like Regions"**
S. J. Brodsky and G. F. de Teramond
arXiv:0707.3859 [hep-ph]
SLAC-PUB-12554(2007) (Submitted to Phys.Rev.D)

2. **"AdS/CFT and QCD"**
S. J. Brodsky and G. F. de Teramond
arXiv:hep-th/0702205
SLAC-PUB-12361(2007)
Invited talk at 2006 International Workshop on the Origin of Mass and Strong Coupling Gauge Theories (SCGT 06), Nagoya, Japan, 21-24 Nov 2006

3. **"Hadronic spectra and light-front wavefunctions in holographic QCD"**
S. J. Brodsky and G. F. de Teramond
Phys. Rev. Lett. **96**, 201601 (2006) [arXiv:hep-ph/0602252]

4. **"Advances in light-front quantization and new perspectives for QCD from AdS/CFT"**
S. J. Brodsky and G. F. de Teramond
Nucl. Phys. Proc. Suppl. **161**, 34 (2006)
Invited talk at Workshop on Light-Cone QCD and Nonperturbative Hadron Physics 2005 (LC 2005), Cairns, Queensland, Australia, 7-15 Jul 2005

5. **"Hadron spectroscopy and wavefunctions in QCD and the AdS/CFT correspondence"**
S. J. Brodsky and G. F. de Teramond
AIP Conf. Proc. **814**, 108 (2006) [arXiv:hep-ph/0510240]
Invited talk at 11th International Conference on Hadron Spectroscopy (Hadron05), Rio de Janeiro, Brazil, 21-26 Aug 2005

6. **"Applications of AdS/CFT duality to QCD"**
S. J. Brodsky and G. F. de Teramond
Int. J. Mod. Phys. A **21**, 762 (2006) [arXiv:hep-ph/0509269]
Invited talk at International Conference on QCD and Hadronic Physics, Beijing, China, 16-20 Jun 2005

7. **"Nearly conformal QCD and AdS/CFT"**
G. F. de Teramond and S. J. Brodsky
arXiv:hep-ph/0507273
SLAC-PUB-11375(2005)
Presented at 1st Workshop on Quark-Hadron Duality and the Transition to pQCD, Frascati, Rome, Italy, 6-8 Jun 2005

8. **"The hadronic spectrum of a holographic dual of QCD"**
G. F. de Teramond and S. J. Brodsky
Phys. Rev. Lett. **94**, 201601 (2005) [arXiv:hep-th/0501022]

9. **"Baryonic states in QCD from gauge / string duality at large N(c)"**
G. F. de Teramond and S. J. Brodsky
arXiv:hep-th/0409074
SLAC-PUB-10693(2004)
Presented at ECT Workshop on Large Nc QCD 2004, Trento, Italy, 5-9 Jul 2004*

10. **"Light-front hadron dynamics and AdS/CFT correspondence"**
S. J. Brodsky and G. F. de Teramond
Phys. Lett. B **582**, 211 (2004) [arXiv:hep-th/0310227]