Hard probes in AdS/CFT

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1. Introduction

AdS/CFT is a correspondence between certain strongly-coupled supersymmetric gauge theories and dual supergravity solutions. We can compute expectation values, correlation functions, etc. by solving classical GR problems.

One of the simplest examples is the duality between $\mathcal{N} = 4$ super-Yang-Mills (SYM) and gravity solutions on $AdS_5 \times S^5$.

- $\mathcal{N} = 4$ SYM is an $SU(N)$ gauge theory. The AdS/CFT approximations we're using work when both $N \gg 1$ and $g_{YM}^2 N \gg 1$, where g_{YM} is the gauge coupling constant.
- At RHIC, $\alpha_s \approx 1/2$, so if $\alpha_s = g_{YM}^2/4\pi$, then $g_{YM}^2 N \approx 19$. Other arguments yield a t'Hooft coupling $g_{YM}^2 N$ as small as 5.5 [Gubser 2006c].
- Strongly-coupled $\mathcal{N} = 4$ theory has small viscosity: $\eta/s = 1/4\pi$ [Policastro et al. 2001].
- There are many differences between $\mathcal{N} = 4$ and QCD, but one hopes that there are some universal behaviors that we can learn something about.

Using AdS/CFT, we can examine:

- properties of the $\mathcal{N}=4$ plasma itself;
- properties of particles (quarks, mesons, gluons) going through the plasma.

2. The stress tensor of a quark in $\mathcal{N}=4$ thermal plasma

The gravity dual for a thermal plasma on $\mathbf{R}^{3,1}$ is AdS_5-Schwarzschild in Poincaré coordinates:

$$ds^2 = G^{(0)}_{\mu\nu} dx^\mu dx^\nu = \frac{L^2}{z^2}\left[-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h}\right] \qquad h \equiv 1 - \frac{z^4}{z_H^4}. \tag{1}$$

External quarks (i.e infinitely massive, fundamentally charged, and pointlike) can be introduced as strings on the boundary of AdS ($z=0$).

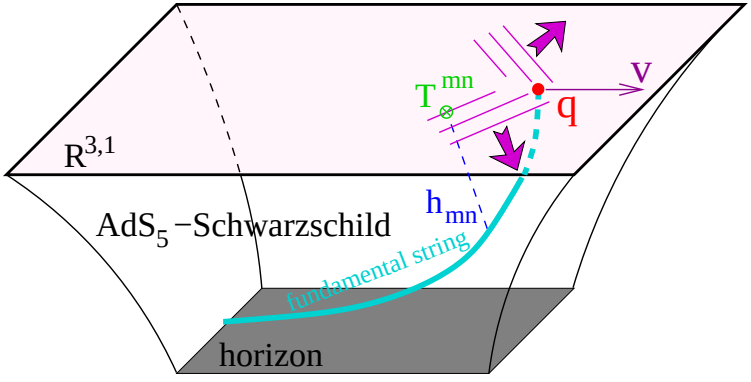


Figure 1: *In blue: the trailing string of an external quark. The dashed line shows classical propagation of a graviton from the string to the boundary, where its behavior can be translated into the stress-energy tensor $\langle T_{mn} \rangle$ of the boundary gauge theory [Friess et al. 2006].*

Let's assume we added an external quark moving with constant velocity $\vec{v}$ and compute $\langle T_{mn} \rangle$.

The computation consists of five steps:

1. Find the shape of the string representing the quark by varying the action

$$S = \int d^4x dz \left[\frac{\sqrt{-G}(R + 12/L^2)}{2\kappa_5^2} - \frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-g} \delta^5(x^\mu - X^\mu(\sigma^\alpha)) \right] . \tag{2}$$

The answer is [Gubser 2006a; Herzog et al. 2006]

$$X^\mu(t,z) \equiv \begin{pmatrix} t & vt+\xi(z) & 0 & 0 & z \end{pmatrix}$$
$$\xi(z) = -\frac{z_H v}{4i}\left(\log\frac{1-iz/z_H}{1+iz/z_H} + i\log\frac{1+z/z_H}{1-z/z_H}\right) . \tag{3}$$

2. Derive the equations satisfied by the gravitational perturbations $h_{\mu\nu}$. Schematically,
$$\mathcal{D}^{\mu\nu\rho\sigma} h_{\rho\sigma} = J^{\mu\nu} . \tag{4}$$
If we fix a gauge, (4) consists of 10 independent second order differential equations and 5 first order constraints [Friess et al. 2006]. One can also work with gauge invariant combinations [Gubser and Pufu 2007; Chesler and Yaffe 2007].

3. Solve the equations (4) in Fourier space by imposing appropriate boundary conditions.

4. Read off the Fourier components of $\langle T_{mn}\rangle$ from the small z behavior of h_{mn}: in

a gauge where $h_{\mu z} = 0$,

$$\begin{aligned} h_{mn} &= p_{mn} z + \frac{\kappa_5^2}{2L} \langle \delta T_{mn} \rangle z^2 + \mathcal{O}(z^3) \\ \langle T_{mn} \rangle &= \frac{\pi^2}{8} N^2 T^4 \mathrm{diag}\{3, 1, 1, 1\} + \langle \delta T_{mn} \rangle \,. \end{aligned} \tag{5}$$

5. Fourier transform back to position space [Gubser et al. 2007ba]. This is hard because the Fourier components of $\langle T_{mn} \rangle$ are singular at both small and large k.

   - At large k the leading singularities are due to the Coulombic field of the quark, but there are also singular terms suppressed by a factor of T^2/k^2.
   - At small k, the singularities are given by the "sonic boom" and the "diffusion wake."

We work with dimensionless quantities:

$$\vec{X} \equiv \pi T \vec{x} \qquad \vec{K} \equiv \vec{k}/\pi T \,. \tag{6}$$

So if $T = 1/\pi\,\mathrm{GeV} \approx 318\,\mathrm{MeV}$, then $X = 1$ corresponds to $0.2\,\mathrm{fm}$.

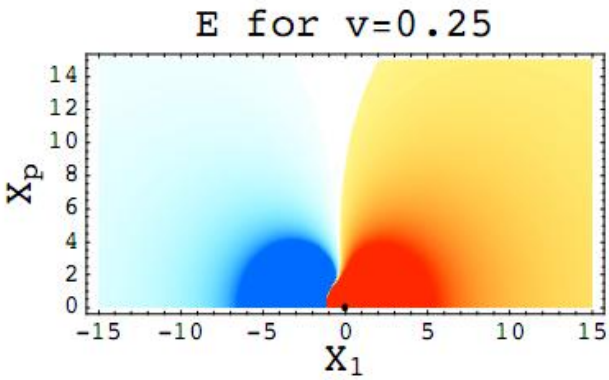
We also focus on the rescaled, subtracted quantity

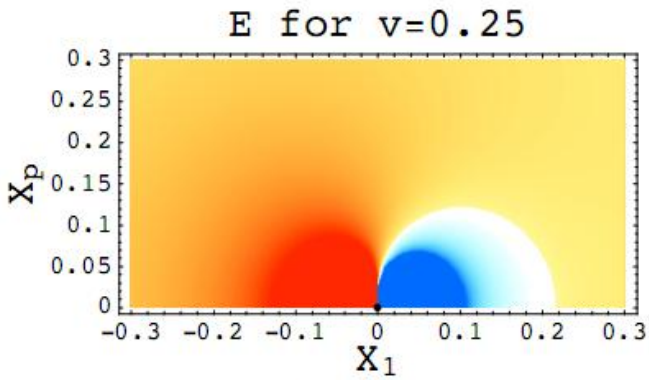
$$\mathcal{T}^{mn}(\vec{X}) \equiv \frac{\sqrt{1-v^2}}{(\pi T)^4\sqrt{g_{YM}^2 N}}\Big(\langle T^{mn}(0,\vec{x})\rangle - \langle T^{mn}\rangle_{\rm bath}\Big)$$
$$\langle T^{mn}\rangle_{\rm bath} \equiv \frac{\pi^2}{8}(N^2-1)T^4 {\rm diag}\{3,1,1,1\}\,. \tag{7}$$

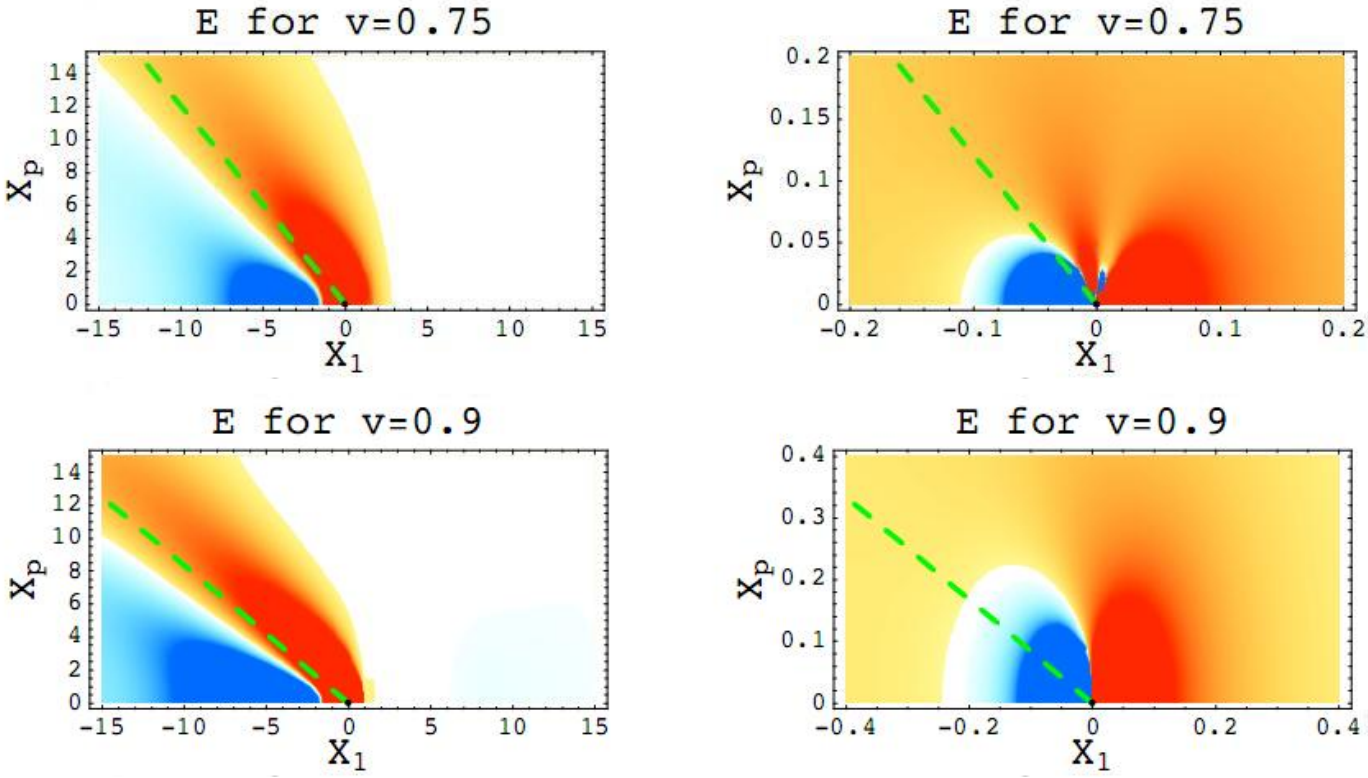
In plots it is also useful to subtract the Coulombic contribution to the near-field:

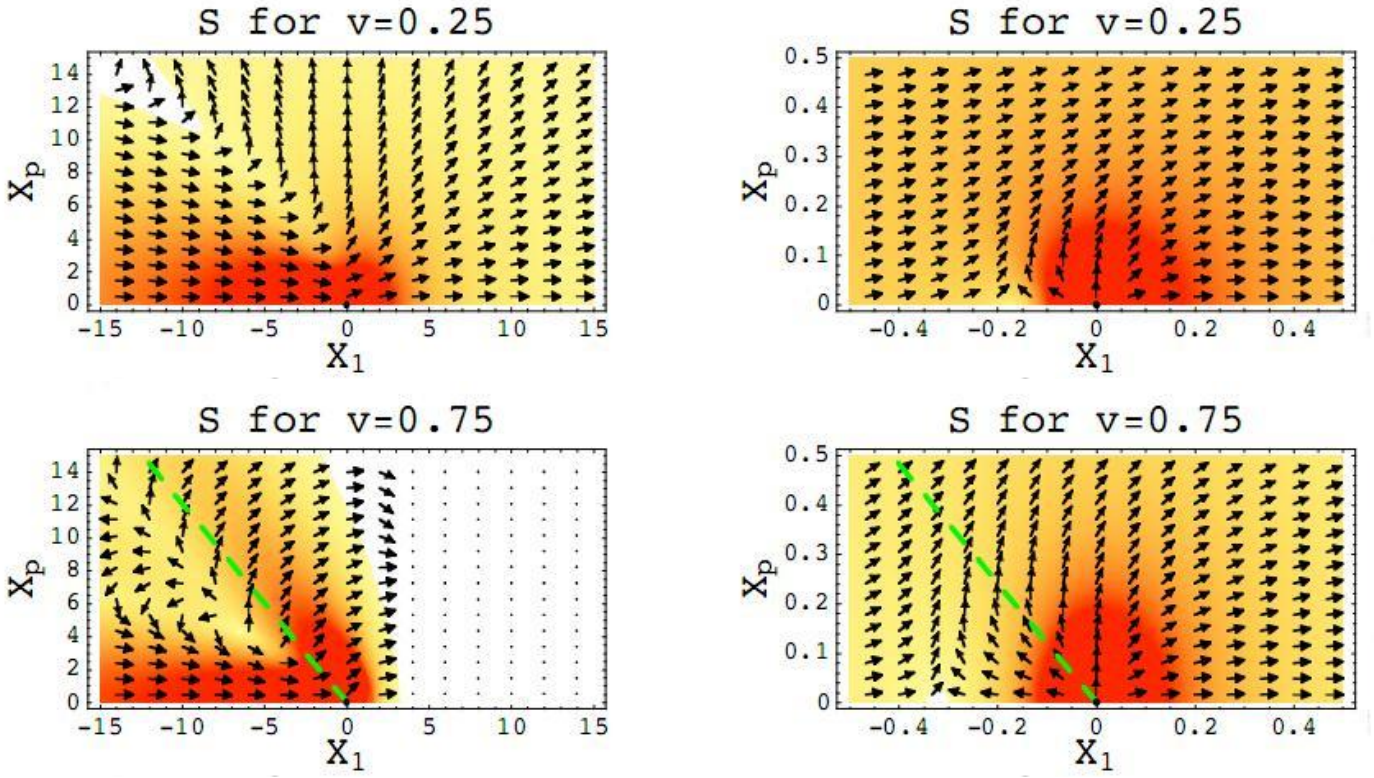
$$\mathcal{T}^{0m} \equiv \begin{pmatrix}\mathcal{E} & \mathcal{S}_1 & \mathcal{S}_2 & \mathcal{S}_3\end{pmatrix} \tag{8}$$

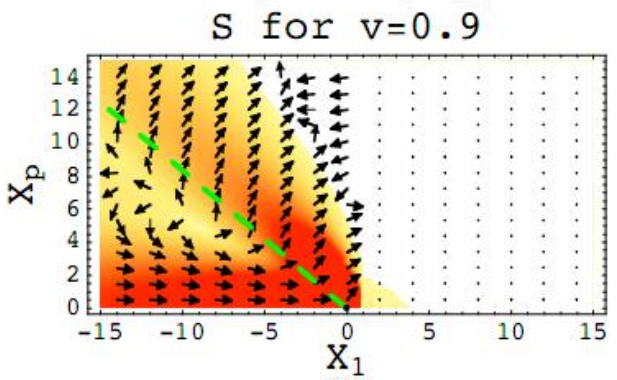
$$E \equiv \mathcal{E} - \mathcal{E}_{\rm Coulomb} \qquad \vec{S} \equiv \vec{\mathcal{S}} - \vec{\mathcal{S}}_{\rm Coulomb}\,. \tag{9}$$






E for v=0.75
E for v=0.75
E for v=0.9
E for v=0.9
X_p
X_1


S for v=0.25
S for v=0.25
S for v=0.75
S for v=0.75
X_p
X_1


S for v=0.9
X_p
X_1
14
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8
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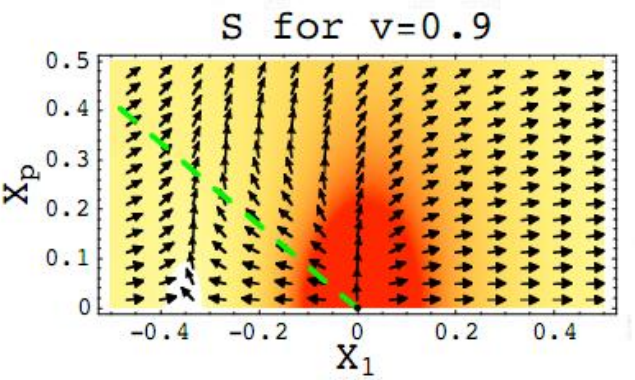
4
2
0
-15
-10
-5
0
5
10
15


S for v=0.9
X_p
X_1
0.5
0.4
0.3
0.2
0.1
0
-0.4
-0.2
0
0.2
0.4

But where exactly do the sonic boom and diffusion wake come from? In Fourier space, the IR asymptotics read

$$\begin{aligned}
\mathcal{E}^{\rm IR} &= -\frac{1}{2\pi}\frac{3ivK_1(1+v^2)-3v^2K_1^2}{K^2-3v^2K_1^2-ivK^2K_1} \\
\mathcal{S}_1^{\rm IR} &= -\frac{1}{2\pi}\frac{i(1+v^2)K_1+vK^2-2v^3K_1^2}{K^2-3v^2K_1^2-ivK^2K_1}+\frac{2v}{\pi}\frac{1+iK_1/4v}{K^2-4ivK_1} \\
\mathcal{S}_2^{\rm IR} &= -\frac{1}{2\pi}\frac{i(1+v^2)K_2+v^3K_1K_2}{K^2-3v^2K_1^2-ivK^2K_1}+\frac{1}{2\pi}\frac{iK_2}{K^2-4ivK_1}\,.
\end{aligned} \tag{10}$$

The structure of the denominators is consistent with linearized hydrodynamics (see for example [Chesler and Yaffe 2007]):

$$\begin{aligned}
\left(-\partial_0^2+c_s^2\nabla^2+\gamma\nabla^2\partial_0\right)\,\vec{\mathcal{S}}^{\rm IR}_{\rm sound} &= \text{sources} \\
\left(\partial_0-D\nabla^2\right)\,\vec{\mathcal{S}}^{\rm IR}_{\rm diffusion} &= \text{sources}\,,
\end{aligned} \tag{11}$$

where in general $c_s^2=\partial p/\partial\epsilon$, $\gamma=4\eta/3(\epsilon+p)$, and $D=\eta/(\epsilon+p)$. For $\mathcal{N}=4$ SYM, $\epsilon=3p=\frac{3}{4}Ts$ and $\eta/s=1/4\pi$, so

$$c_s^2=\frac{1}{3}\qquad \gamma=\frac{1}{3\pi T}\qquad D=\frac{1}{4\pi T}\,. \tag{12}$$

How does one estimate the relative strengths of the sonic boom and diffusion wake? $\mathcal{T}^{mn}$ is conserved except at the location of the quark:

$$\frac{\partial}{\partial X^m}\mathcal{T}^{mn} = -\mathcal{F}^n_{\text{drag}}\delta(X^1 - vX^0) \qquad \mathcal{F}^m_{\text{drag}} = \frac{-1}{2\pi}\begin{pmatrix} v^2 & v & 0 & 0 \end{pmatrix}, \tag{13}$$

where the drag force $\mathcal{F}^n_{\text{drag}}$ was computed in [Gubser 2006a] by looking at the momentum flow down the string. In Fourier space, (13) becomes

$$iK_m\mathcal{T}^{mn}(\vec{K}) = -\mathcal{F}^n_{\text{drag}} \qquad K_m = \begin{pmatrix} -vK_1 & K_1 & K_2 & K_3 \end{pmatrix}. \tag{14}$$

Estimates of the energy that goes into the sonic boom and the diffusion wake are then

$$\begin{aligned} \lim_{K\to 0} iK_m\mathcal{T}^{0m}_{\text{sound}} &= \frac{1+v^2}{2\pi} = -\left(1+\frac{1}{v^2}\right)\mathcal{F}^0_{\text{drag}} \\ \lim_{K\to 0} iK_m\mathcal{T}^{0m}_{\text{diffuse}} &= -\frac{1}{2\pi} = \frac{1}{v^2}\mathcal{F}^0_{\text{drag}}. \end{aligned} \tag{15}$$

So the energy *lost* into the sound modes is $1+v^2$ times greater than the energy *fed in* by the diffusion wake.

- In [Gubser and Yarom 2007] it is argued that this ratio is universal to a large class of gravity backgrounds.

3. Heavy-quark mesons

Heavy-quark mesons are represented as strings with both ends on the boundary of AdS.

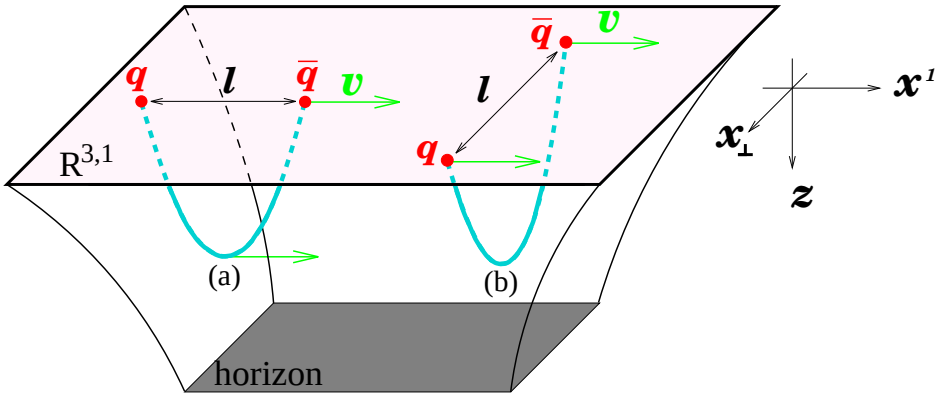


Figure 2: *An inline meson (a) and a perpendicular one (b) moving with velocity $\vec{v}$ in the x^1 direction.*

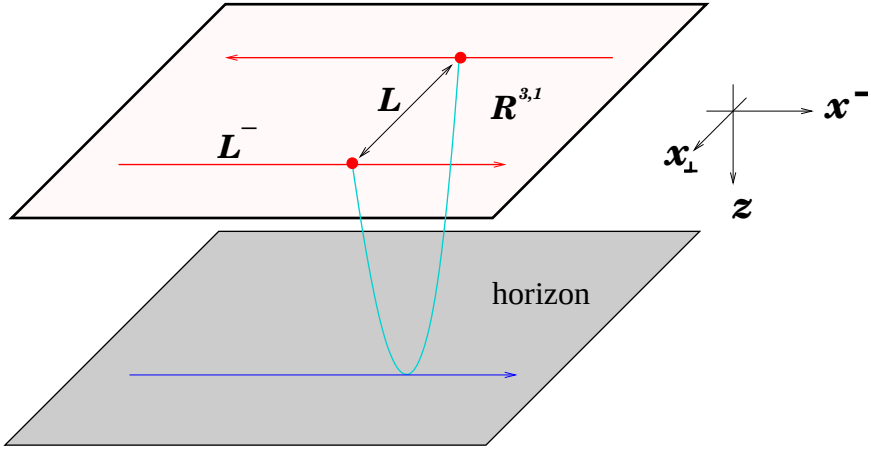
In [Gubser et al. 2007c] we computed $\langle T_{mn}\rangle$ at small k:

$$\begin{aligned}\langle \delta T_{\perp mn}\rangle &= \frac{\Pi_\perp}{k^2-3k_1^2v^2}\left(\ell\tau_{mn}^{(\perp)}+v\sigma_\perp\tau_{mn}^{(\sigma)}\right)+\mathcal{O}(k)\\ \langle \delta T_{\parallel mn}\rangle &= \frac{\Pi_\parallel}{k^2-3k_1^2v^2}\left(\ell\tau_{mn}^{(\parallel)}+2v\sigma_\parallel\tau_{mn}^{(\sigma)}\right)+\mathcal{O}(k)\,,\end{aligned} \tag{16}$$

where $\Pi_\perp$, $\Pi_\parallel$, $\sigma_\perp$, and $\sigma_\parallel$ are constants, while $\tau_{mn}^{(\perp)}$, $\tau_{mn}^{(\parallel)}$, and $\tau_{mn}^{(\sigma)}$ are $\mathcal{O}(k^2)$ expressions.

Since there are no $1/k_1$ singularities in (16), we conclude that these configurations don't generate a diffusion wake, but do generate a sonic boom.

4. Review of LRW



In [Liu et al. 2006 2007], Liu, Rajagopal, and Wiedemann calculate jet-quenching parameter from string theory and get

$$\hat{q}_{\rm LRW} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)}\sqrt{g_{YM}^2 N}\, T^3 \approx 3.6\,\frac{\rm GeV^2}{\rm fm}\left(\frac{T}{280\,\rm MeV}\right)^3 . \qquad (17)$$

But there are some potential issues:

- Defining $\hat{q}$ through

$$\langle W^A(\mathcal{C})\rangle \approx \exp\left\{-\frac{1}{4\sqrt{2}}\hat{q}L^- L^2\right\} \qquad \text{for small } L \qquad (18)$$

relies on dipole approximation and eikonal approximation, and it doesn't have to be related to energy loss or momentum diffusion in strongly coupled $\mathcal{N} = 4$ gauge theories.

- Argyres and collaborators criticize the choice of saddle point [Argyres et al. 2007 2008] and find $\log\langle W^A(\mathcal{C})\rangle \sim L$ not L^2.
- Independent calculations of

$$\hat{q}_T \equiv \langle p_\perp^2\rangle/\lambda \tag{19}$$

for heavy quarks [Gubser 2006b; Casalderrey-Solana and Teaney 2007] lead to larger values than (18): larger by $\sim \sqrt{\gamma}$ as $v \to 1$.

LRW realize that QCD should have lower $\hat{q}$ than $\mathcal{N} = 4$, because there are roughly $1/3$ as many degrees of freedom. They suggest

$$\hat{q}_{\rm QCD} \approx \sqrt{\frac{47.5}{120}}\,\hat{q}_{\mathcal{N}=4} \approx 2.3\,\frac{\rm GeV^2}{\rm fm}\,. \tag{20}$$

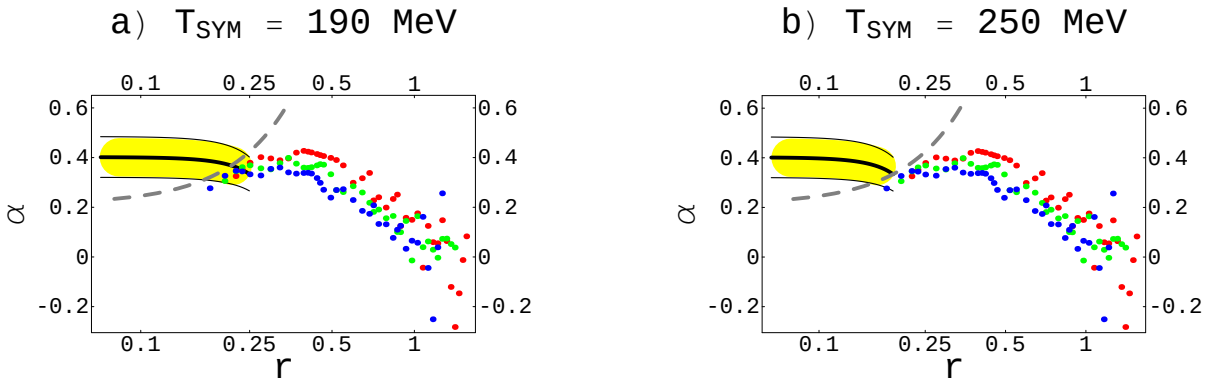
But to get (17) or (20), they use

$$\text{obvious scheme:} \quad g_{YM}^2 N = 6\pi \qquad T_{\mathcal{N}=4} = T_{\rm QCD}\,. \tag{21}$$

(6π corresponds to $\alpha_s = 1/2$ if $g_{YM} = g_s$.) But it may be more reasonable to use

$$\text{alternative scheme:} \quad g_{YM}^2 N = 5.5 \qquad T_{\mathcal{N}=4} = \frac{1}{3^{1/4}} T_{\text{QCD}}\,, \tag{22}$$

which means we compare at fixed energy density and get the best match we can for $V_{q\bar{q}}(r)$ between string theory and lattice, using $\alpha(r) \propto r^2 V'_{q\bar{q}}(r)$ [Gubser 2006c]:



Using (22) one finds

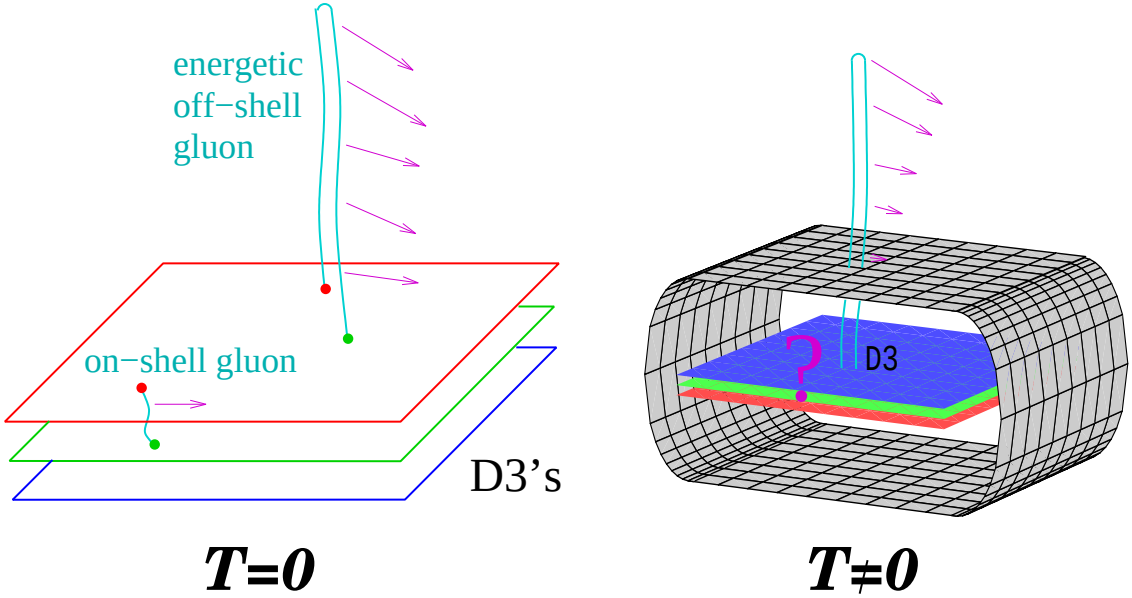
$$\hat{q}_{\text{LRW alternative}} = 0.86\,\frac{\text{GeV}^2}{\text{fm}} \tag{23}$$

(still at $T = 280\,\text{MeV}$), basically the same as perturbative estimate

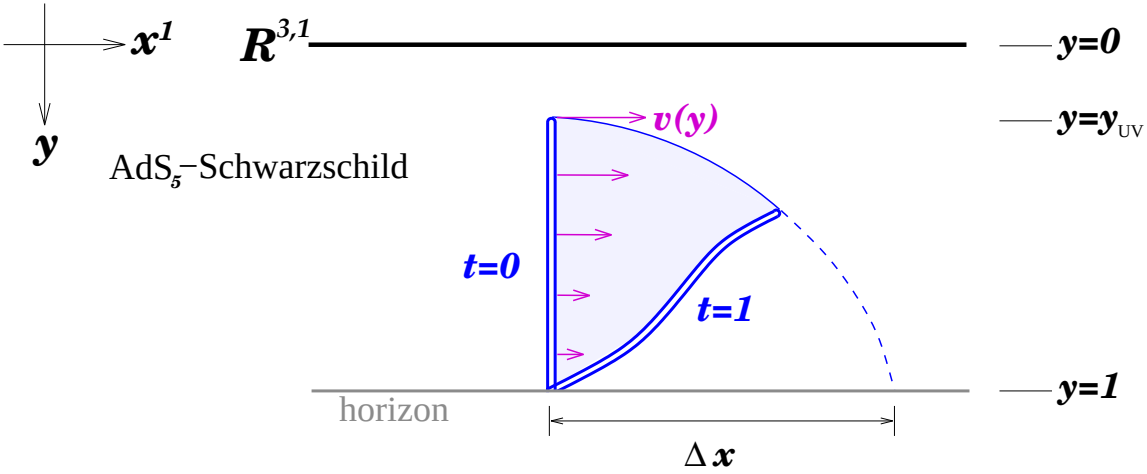
$$\hat{q}_{\text{pert}} = \frac{8}{\pi}\zeta(3)\alpha_s^2 N^2 T^3 \approx 0.77\,\frac{\text{GeV}^2}{\text{fm}}\,. \tag{24}$$

5. Falling strings

Let's consider energy loss by a gluon *ab initio* in string theory. We know that gluons are strings stretched between branes. Recent work [Alday and Maldacena 2007] shows that in energetic scattering processes, off-shell gluons are described by extended strings in AdS_5.



At finite temperature, something funny happens: where the string crosses the horizon, it can't move! (Infinite red-shifting.)

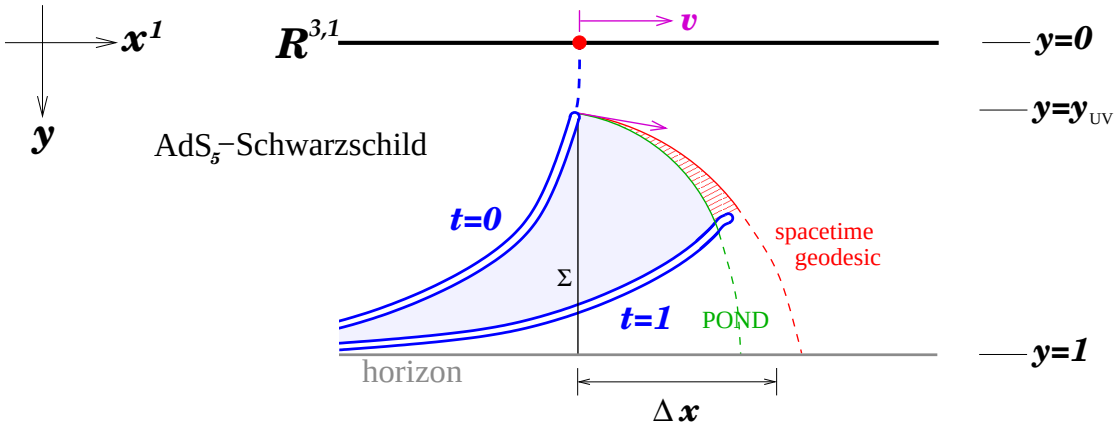


The obvious strategy is to compute the energy of the doubled string at $t = 0$:

$$p_\mu = \int dz \sqrt{-g}\, P^t_\mu \approx \frac{L^2}{\alpha'} \frac{1}{\sqrt{1-v^2}} \frac{T}{y_{\rm UV}}\,, \tag{25}$$

and then solve classical equations of motion to find Δx. But this is hard work...

A segment of the trailing string of [Herzog et al. 2006; Gubser 2006a] "holds its shape" better as it falls into the horizon:



$$X^1 = v\,[t + \xi(z)] \qquad \text{where} \qquad \xi(z) = -\frac{z_H}{4i}\left(\log\frac{1-iy}{1+iy} + i\log\frac{1+y}{1-y}\right). \tag{26}$$

Doubling the string over instead of ending it on $\mathbf{R}^{3,1}$ is the difference between a heavy quark and a gluon—we propose.

There's still the problem of computing E and Δx. Unfortunately, we can only give some estimates.

Spacetime geodesic is the path the tip of the string *would* have taken if the string weren't pulling it down and back.

POND (Point of No Disturbance) is the point below which the string's shape is *exactly* (26).

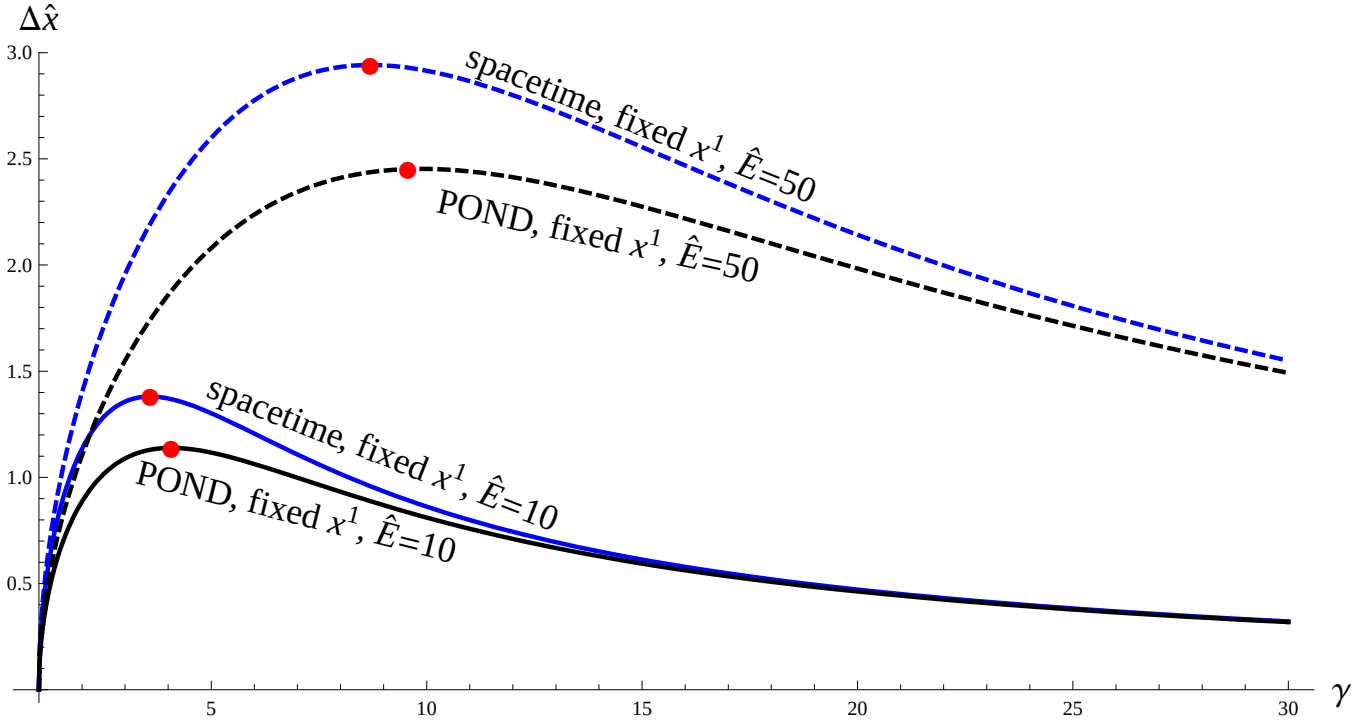
So we know the true Δx satisfies

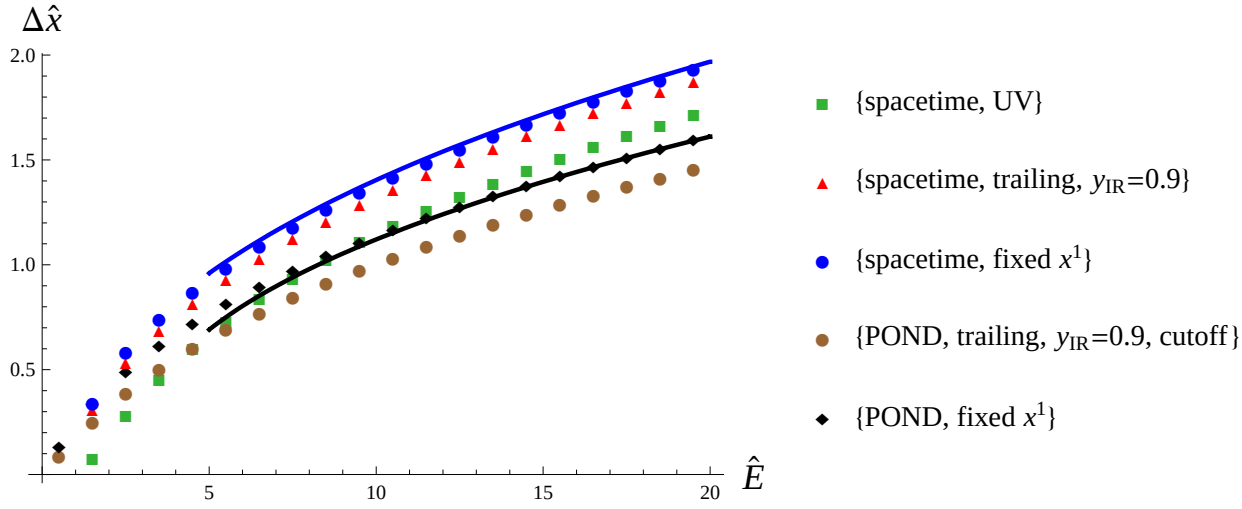
$$\Delta x_{\text{POND}} < \Delta x < \Delta x_{\text{spacetime}} \,. \tag{27}$$

- The good news: Δx_{POND} and $\Delta x_{\text{spacetime}}$ are approximately equal for highly energetic gluon-strings.
- The bad news: there is not a unique way to compute E of the initial gluon.
- The strategy:
  - Decide on a definition of E. Fix E.
  - Decide whether to estimate $\Delta x = \Delta x_{\text{POND}}$ or $\Delta x = \Delta x_{\text{spacetime}}$.
  - Vary y_{UV} and $\gamma = 1/\sqrt{1-v^2}$ until you find the maximum Δx.

In order to work with dimensionless quantities, define

$$\hat{x} = \pi T x \qquad \hat{E} = \frac{1}{\sqrt{g_{YM}^2 N}} \frac{E}{T}\,, \tag{28}$$





At large $\hat{E}$, one finds

$$\begin{aligned} \Delta\hat{x}_{\text{spacetime,fixed } x^1} &\approx 1.0185\hat{E}^{1/3} - 0.8180 + 0.052\hat{E}^{-1/3} \\ \Delta\hat{x}_{\text{POND,fixed } x^1} &\approx 0.8798\hat{E}^{1/3} - 0.8252 + 0.058\hat{E}^{-1/3}\,, \end{aligned} \tag{29}$$

where fixed x^1 refers to our preferred way of computing E.

This is not too different from $\Delta x_{\text{BDMPS}} \propto E^{1/2}$. So let's try to extract an effective value of $\hat{q}$. We're not completely sure we understand how to do this, but here's what we tried...

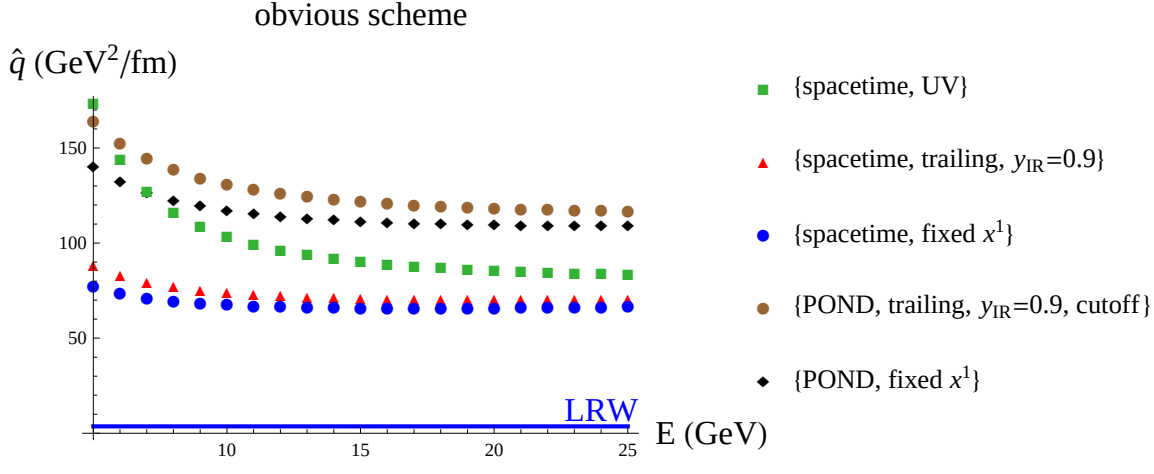
Start with

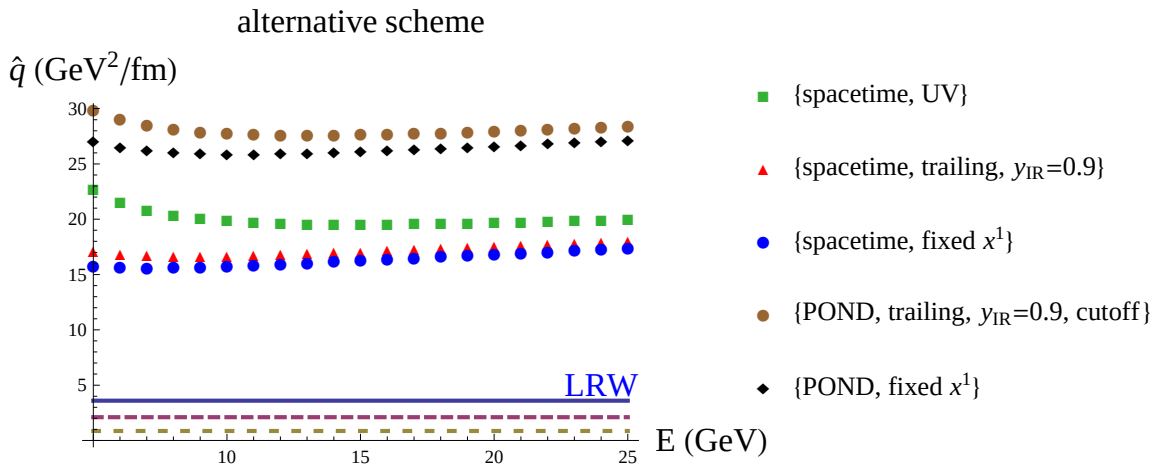
$$\Delta E_{\rm BDMPS} = \frac{1}{4}\alpha_s C_R \hat{q}(\Delta x)^2 \,, \tag{30}$$

and set $E = \Delta E_{\rm BDMPS}$ to express the idea that the gluon is supposed to lose all its energy. Solve for $\hat{q}$:

$$\hat{q}_{\rm rough} \equiv \frac{4E}{3\alpha_s(\Delta x)^2} \,. \tag{31}$$

The value of $\hat{q}$ depends weakly on E and strongly on the comparison scheme, (21) or (22):





The solid blue line is $\hat{q}_{\rm LRW}$ as computed in (18).

Punchline: Energy loss for energetic gluons represented as doubled strings is much faster than one gets from BDMPS with the LRW computation of $\hat{q}$. Instead, in the alternative scheme, one finds $\hat{q} \approx 21\,{\rm GeV}^2/{\rm fm}$ for $E \approx 15\,{\rm GeV}$.

Our calculations could use some improvement:

- Doubled string representation is probably in the right ballpark for representing an off-shell gluons, but it is not understood in detail.
- We maximized Δx with E held fixed. Should we instead have averaged it over

some measure in γ representative of off-shell gluons as produced in energetic collisions?

- We only estimated E and Δx, and the justification for using the trailing string was loose.
- Was it at all right to set $E = \Delta E_{\mathrm{BDMPS}}$ to get $\hat{q}$ in terms of $E/(\Delta x)^2$?
- We ignored thermal fluctuations of the string, which seemed to be a strong effect in [Gubser 2006b; Casalderrey-Solana and Teaney 2007].
- We haven't computed $\langle T_{mn} \rangle$, which might help establish what the calculation means.

6. Conclusions

- We used AdS/CFT to provide and all-scales description of the energy flow around heavy quarks moving with constant velocity $\vec{v}$ through $\mathcal{N} = 4$ thermal plasma.
- The energy that goes into the sound waves is $1+v^2$ times greater than the energy fed in by the diffusion wake.
- There is no diffusion wake for mesons.
- Gluons can be represented as strings ending behind the horizon.
- The maximum penetration depth achieved by certain string configurations grows like $\Delta x \propto E^{1/3}$ at large energies, and corresponds roughly to $\hat{q} \approx 21\,\mathrm{GeV}^2/\mathrm{fm}$ for $E \approx 15\,\mathrm{GeV}$. This calculation suffers from large uncertainties. But in any case, the energy loss seems to be stronger than LRW and BDMPS seem to indicate.

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