Jet Energy Loss

– Putting it all together –

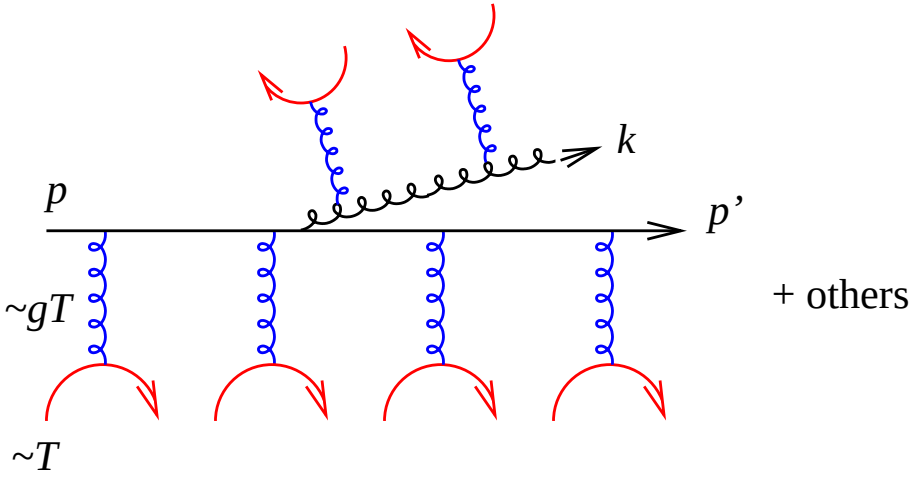
Guangyou Qin, S. Turbide, C. Gale, G. D. Moore and S. Jeon,

Department of Physics
McGill University

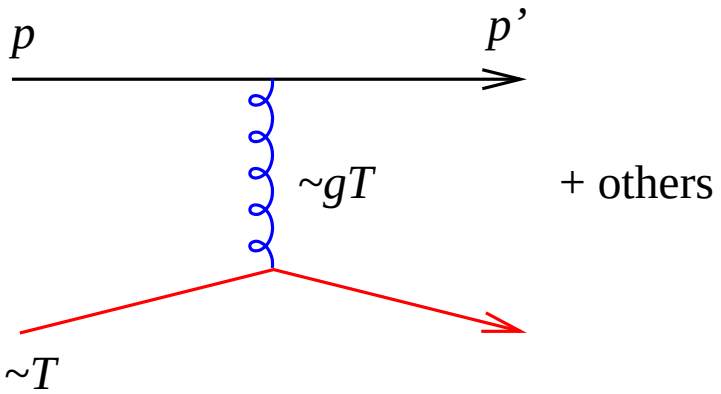
Dec. 11, 2007, BNL

The Issue

- QGP makes jets lose energy
  - Radiational (Inelastic)



  - Elastic



- Can we use this to characterize QGP?

The Issue

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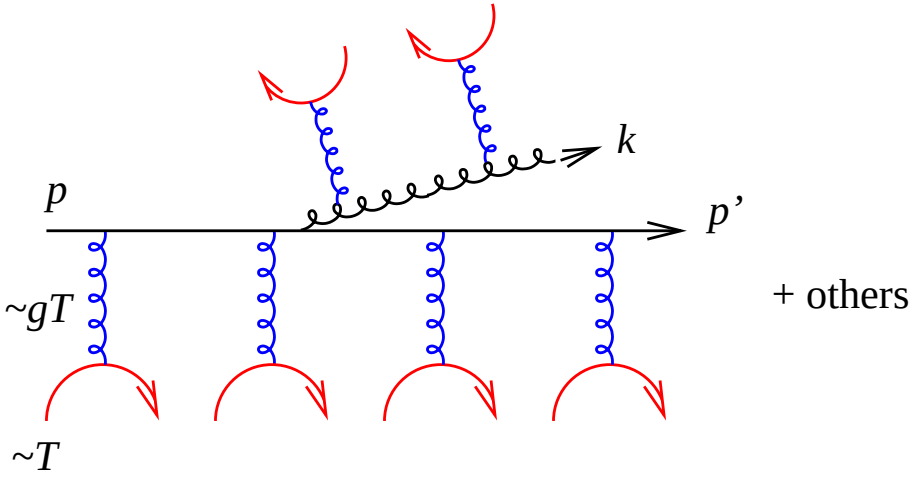

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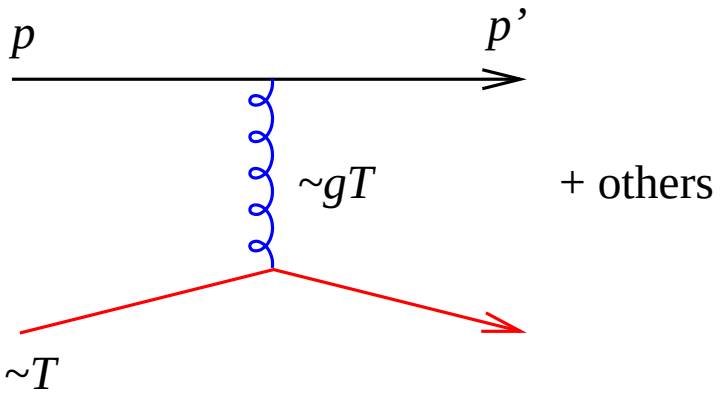


- Can we use this to characterize QGP?

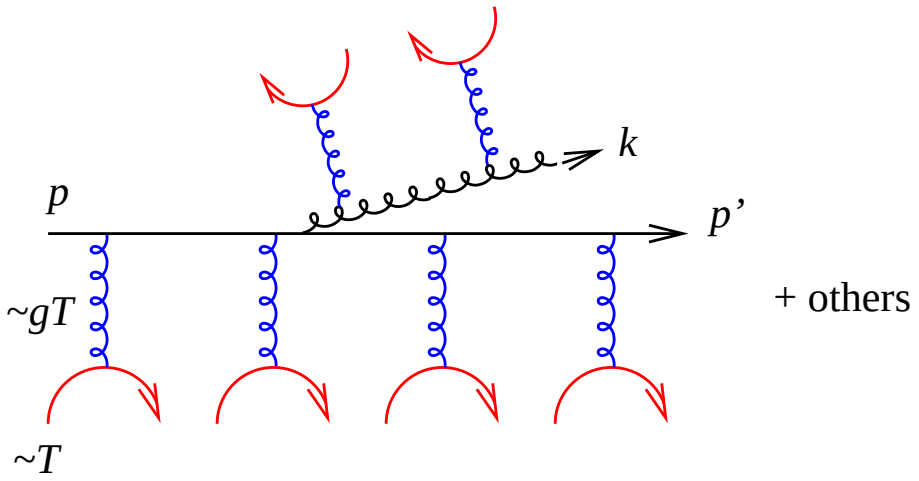
Big Picture

- $\sum\!\!\!\!\!\!\int \left(f_{a/A} \otimes f_{b/A} \otimes \frac{d\sigma_{ab\to cd}}{dx} \right) \otimes$ (E-loss module) $\otimes D_{\rm frag}$
- Parton-parton scattering: $\left(f_{a/A} \otimes f_{b/A} \otimes \frac{d\sigma_{ab\to cd}}{dx} \right)$
- $D_{\rm frag}$: As in vacuum but with reduced energy.
- Energy loss module – Three separate pieces
  - Energy change rate: $\frac{d\Gamma}{dtdk}(\epsilon, k; T)$
  - Evolution:

$$\frac{dP(\epsilon,t)}{dt} = \int dk \frac{d\Gamma}{dtdk} P(\epsilon + k, t) - \int dk \frac{d\Gamma}{dtdk} P(\epsilon, t)$$

  - $T(\mathbf{x}, t), u^\mu(\mathbf{x}, t)$: Must be obtained independently.
- Still schematic. There are theoretical and conceptual problems to further consider.

Rough Idea - Radiational (following BDMPS)



- Point here: Radiated gluon also undergoes multiple scatterings.
- Bethe-Heitler Spectrum (low ω)

$$\omega \frac{dI}{d\omega} \approx \frac{\alpha_s N_c}{\pi}$$

- Medium dependence comes through a scattering length scale l

$$\omega \frac{dI}{d\omega dz} \approx \frac{1}{l} \frac{\alpha_s N_c}{\pi}$$

Rough Idea - Radiational (following BDMPS)

- If all scatterings are incoherent

$$l = l_{\rm mfp} = 1/\rho\sigma \quad \text{and} \quad \omega\frac{dI}{d\omega dz} \approx \frac{\alpha_s N_c}{\pi l_{\rm mfp}}$$

- Coherence matters when multiple scatterings are needed to get $O(1)$ phase change: $1 \sim l_{\rm coh}\omega\left\langle\theta^2\right\rangle$
  Both the radiated gluon and the original parton undergo random walk:

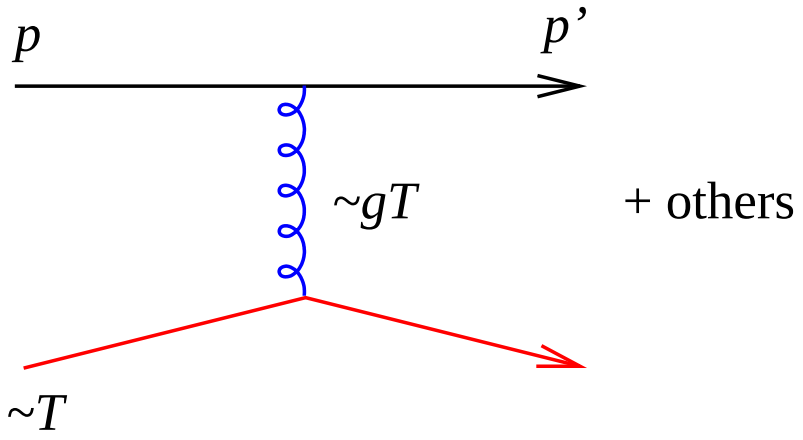
$$\left\langle\theta^2\right\rangle \approx N_{\rm coh}\left(\bar{\theta}_g^2 + \bar{\theta}_q^2\right) \approx \frac{l_{\rm coh}}{l_{\rm mfp}}\left(\frac{\mu^2}{\omega^2} + \frac{\mu^2}{E^2}\right)$$

  or

$$l_{\rm coh} = l_{\rm mfp}\sqrt{\frac{\omega}{E_{\rm LPM}}}\sqrt{\frac{E^2}{E^2+\omega^2}} \quad \text{and} \quad \omega\frac{dI}{d\omega dz} \approx \frac{\alpha_s N_c}{\pi l_{\rm mfp}}\sqrt{\frac{E_{\rm LPM}}{\omega}}$$

  for $\omega > E_{\rm LPM} \equiv \mu^2 l_{\rm mfp} \sim T$.

Rough Idea - Collisional (Following Bjorken)



- Energy loss per unit length

$$\frac{dE}{dz} \approx \int d^3k\, \rho(k) \int dq^2\, J\omega \frac{d\sigma^{\rm el}}{dq^2}$$

where

- $\rho(k)$: density, $J\omega \approx q^2/2k$: flux factor
- Elastic cross-section (Coulombic) $\frac{d\sigma}{dq^2} \approx C_R \frac{2\pi\alpha_s^2}{(q^2)^2}$

Rough Idea - Collisional

- with thermal ρ, this yields

$$\left(\frac{dE}{dz}\right)_{\text{coll}} \propto \alpha_s^2 T^2 \ln(E/\alpha_s T)$$

- Compare:

$$\left(\frac{dE}{dz}\right)_{\text{rad}} \propto \alpha_s^2 T \sqrt{TE}$$

- NOTE: We actually need $\omega dI/d\omega dz$ in place of dE/dz.

Rough Idea - Putting them together

- Fokker-Planck Eqn.

$$\frac{dP(E,t)}{dt} = \int_{-\infty}^{\infty} d\omega \left[P(E+\omega,t)\frac{d\Gamma(E+\omega,\omega)}{d\omega dt} - P(E,t)\frac{d\Gamma(E,\omega)}{d\omega dt} \right]$$

- $\Gamma = \Gamma_{\rm el} + \Gamma_{\rm inel}$
- Inelastic part is solved as it is.
- Elastic part – Soft exchange dominated:

$$\frac{d\Gamma}{d\omega dt} = (1+n_B(\omega_0))\frac{1}{|\omega_0|}\frac{dE}{dt}\delta(\omega-\omega_0) + n_B(|\omega_0|)\frac{1}{|\omega_0|}\frac{dE}{dt}\delta(\omega+\omega_0)$$

This gets the first moment (Drag) and the second moment (Diffusion) right for small ω_0 (Mustafa and Thoma).

Rough Idea - R_{AA}

Use BDMPS expression for the quenching factor for $1/p^n$ ($E = p$) with large n but with the energy range extended to $\omega < 0$:

$$R_{AA}(p) \approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^t dt' \, (d\Gamma/d\omega dt)(1 - e^{-\omega n/p})\right)$$

For the radiation rate, use simple estimates

$$\frac{d\Gamma}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{\lambda} \quad \text{for} \quad 0 < \omega < \lambda\mu^2$$

$$\frac{d\Gamma}{d\omega dt} \approx \frac{\alpha}{\pi\omega} N_c \sqrt{\frac{\mu^2}{\lambda\omega}} \quad \text{for} \quad \lambda\mu^2 < \omega < \lambda\mu^2(L/\lambda)^2$$

$$\frac{d\Gamma}{d\omega dt} \approx \frac{\alpha}{\pi|\omega|}\frac{N_c}{\lambda} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$

Rough Idea - R_{AA}

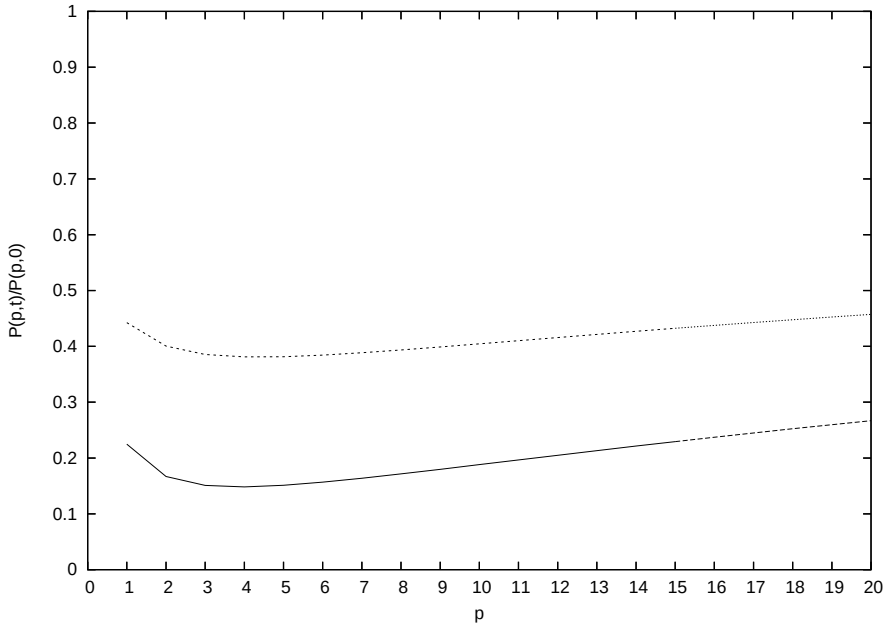
For elastic energy loss,

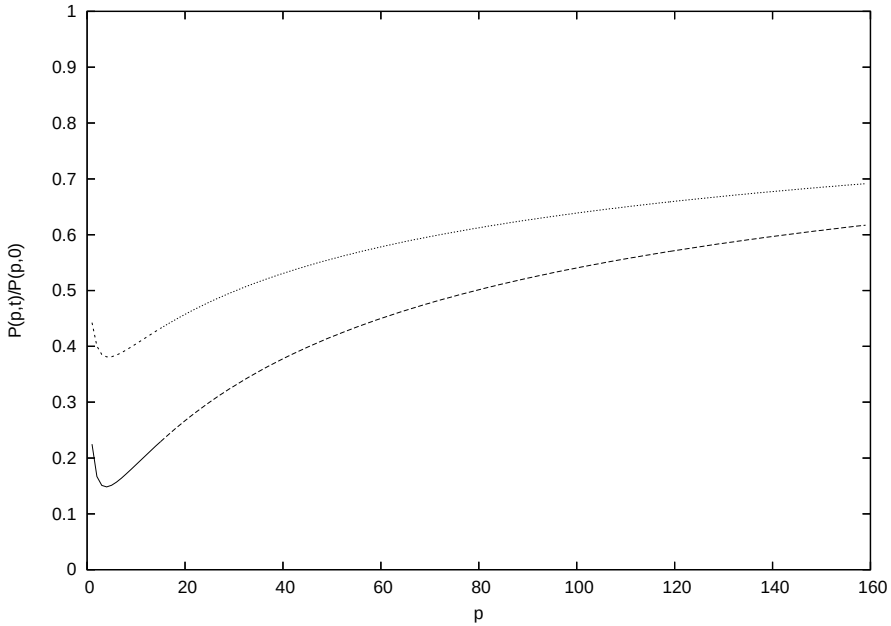
$$
\begin{aligned}
R_{AA}^{\mathrm{el}} &\approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^t dt' \, (d\Gamma_{\mathrm{el}}/d\omega dt)(1 - e^{-\omega n/p})\right) \\
&\approx \exp\left(-t\left(\frac{dE}{dt}\frac{K(\omega_0)}{|\omega_0|}\right)\right) \\
&\approx \exp\left(-t\left(\frac{dE}{dt}\right)\left(\frac{n}{p}\right)\left(1 - \frac{nT}{p}\right)\right)
\end{aligned}
$$

valid for $p > nT$ and we used

$$
\begin{aligned}
K(\omega_0) &= (1 + n_B(|\omega_0|))(1 - e^{-|\omega_0|n/p}) + n_B(|\omega_0|)(1 - e^{|\omega_0|n/p}) \\
&\approx |\omega_0|\left(\frac{n}{p}\right)\left(1 - \frac{nT}{p}\right) \quad \text{for small } \omega_0
\end{aligned}
$$

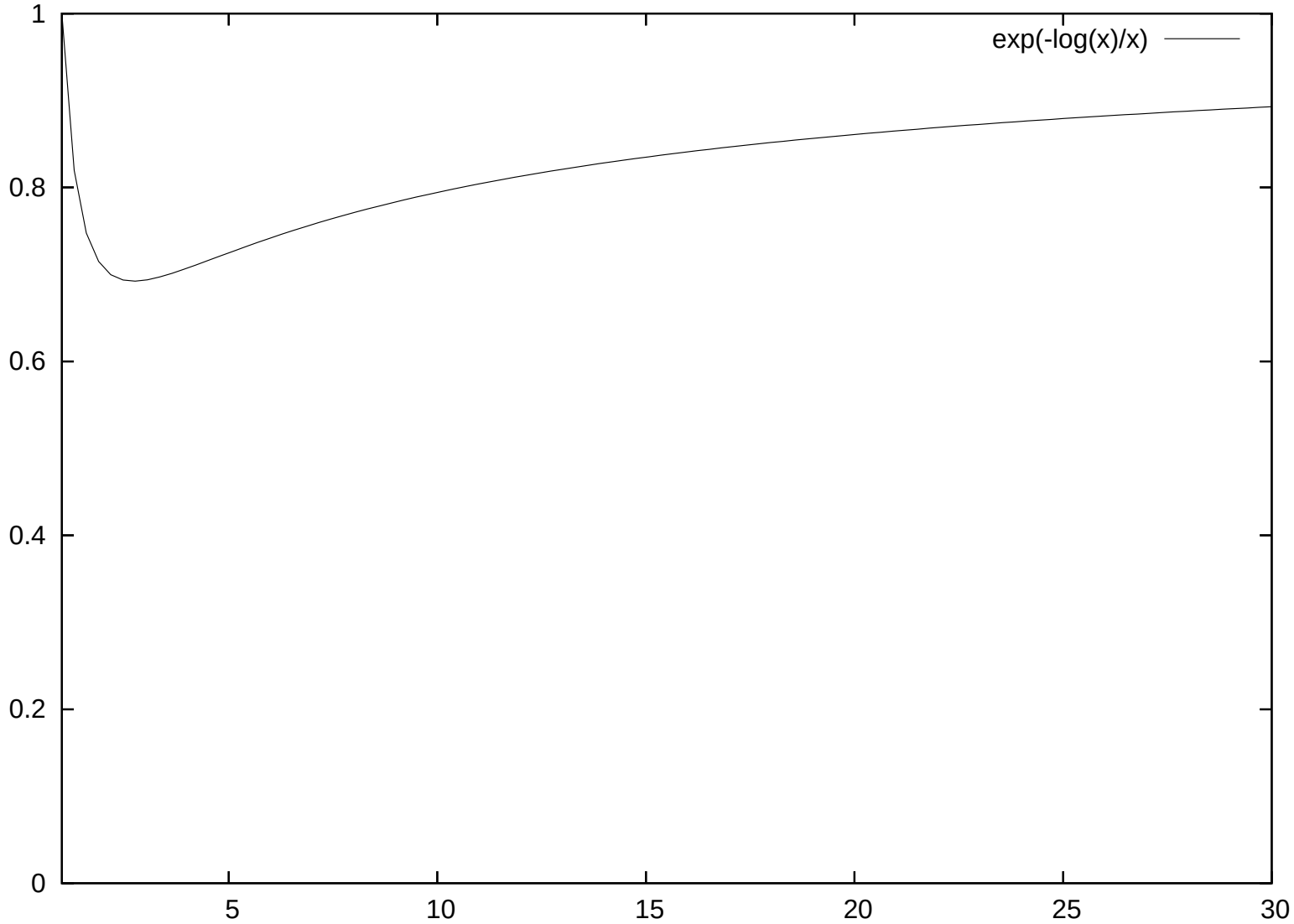
Rough Idea - R_{AA}





- Upper line: Without elastic
- Lower line: With elastic
- Features are roughly produced in both cases.

Why is it always flat?



- $R \sim \exp(-C \ln(p/p_0)/p)$ just not that sensitive to p

More on the evolution:

Poisson ansatz (BDMS, GLV, Wiedemann, Salgago, ...)
The expression

$$P(p,t) = \int d\epsilon\, D(\epsilon,p,t)\, P_0(p+\epsilon)$$

where

$$D(\epsilon,p,t) = e^{-\int d\omega \frac{dI}{d\omega}(p,\omega,t)} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \frac{dI}{d\omega_i}(p,\omega_i,t) \right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i \right)$$

solves

$$\frac{dP(p,t)}{dt} = \int d\omega \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(p,\omega,t) P(p+\omega,t) - P(p,t) \int d\omega \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(p,\omega,t)$$

with

$$\frac{dI}{d\omega}(p,\omega,t) = \int_{t_0}^{t} dt' \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(p,\omega,t')$$

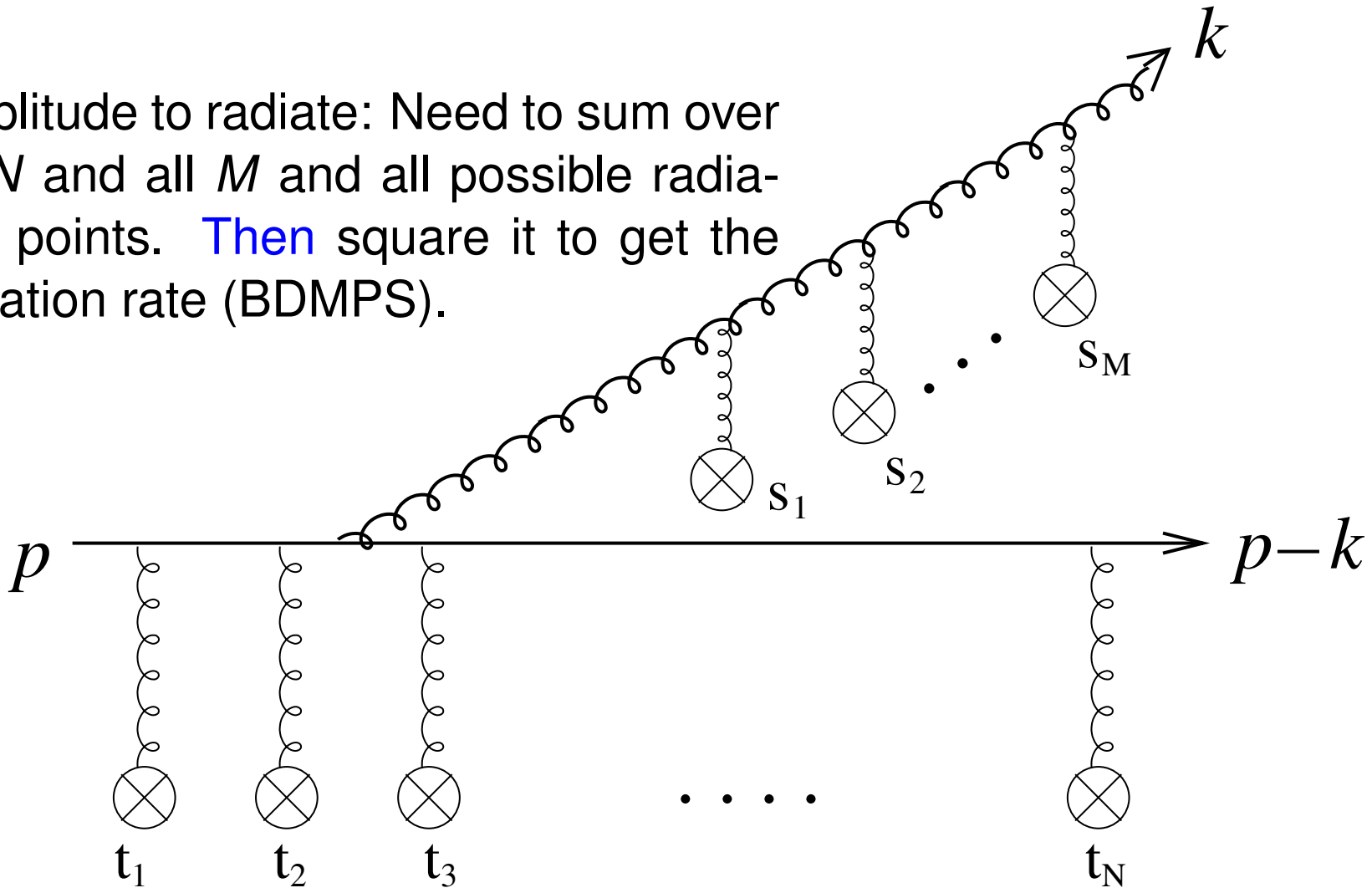
We solve:

$$\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} + 2P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt},$$

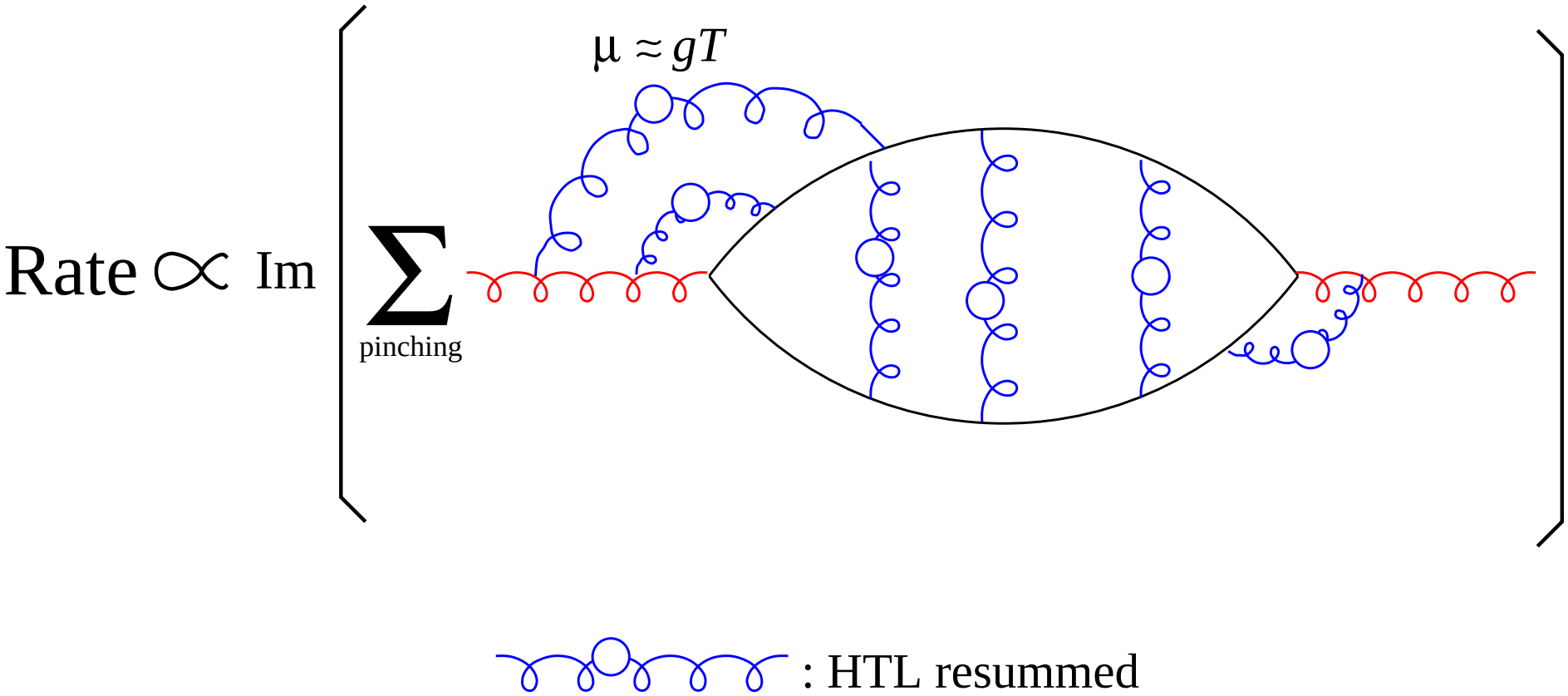
$$\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dkdt} + P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} - P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt} + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(k-p/2)\right)$$

Gluon Radiation

Amplitude to radiate: Need to sum over all N and all M and all possible radiation points. Then square it to get the radiation rate (BDMPS).



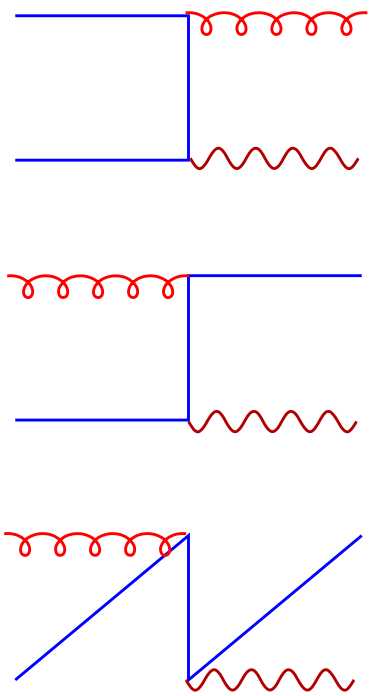
Diagrams



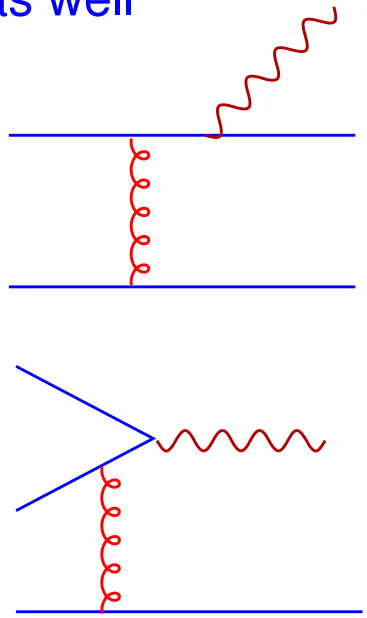
Why is this so hard?

- Collinear enhancement in photon & gluon radiations
  Aurenche, Gelis, Kobes and Zaraket, PRD58:085003,1998, Arnold, Moore and Yaffe (AMY), JHEP 0206:030,2002; JHEP 0112:009,2001; JHEP 0111:057,2001

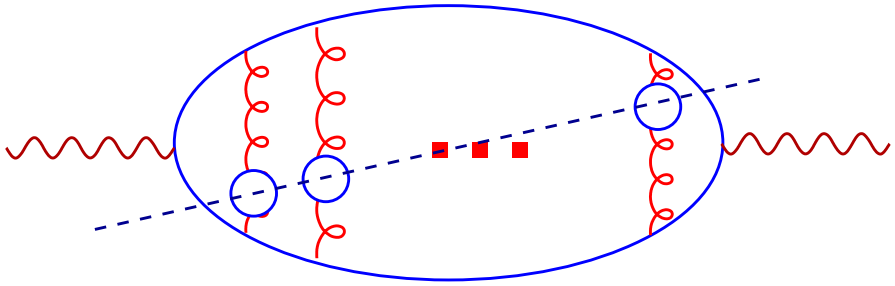
Leading order



Collinear enhancement makes these leading order as well

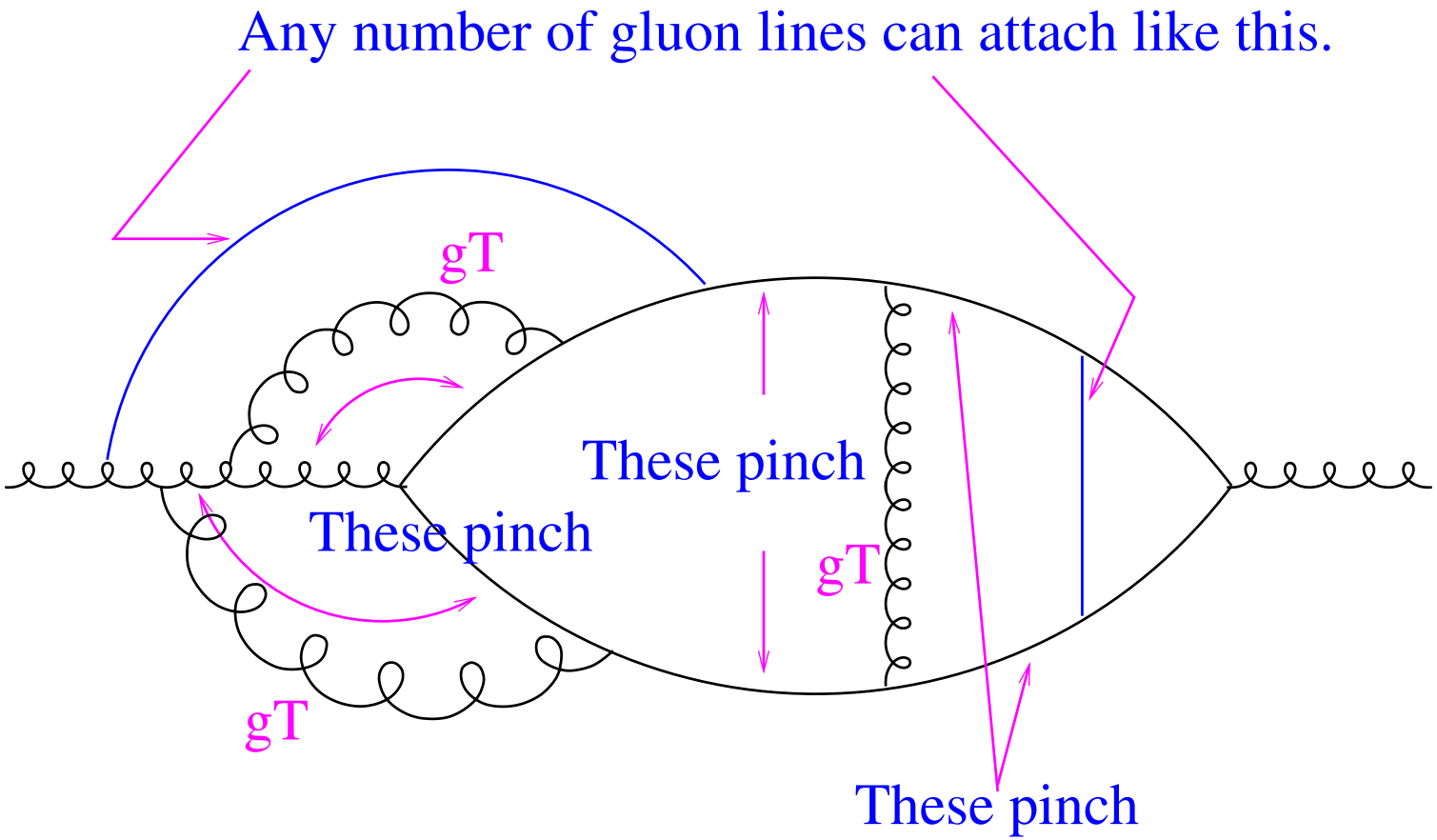


Need to resum all these, too (AMY)



○ : Hard Thermal Loop

Gluon ladder diagrams



Adding one more rung = O(1).
Need to resum.

SD equation for the vertex

Equation for the vertex **F**

$$
\begin{aligned}
2\mathbf{h} &= i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g^2\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^2}C(\mathbf{q}_\perp)\times \\
&\quad\times\Big\{(C_s - C_{\rm A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-k\,\mathbf{q}_\perp)] \\
&\qquad +(C_{\rm A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}+p\,\mathbf{q}_\perp)] \\
&\qquad +(C_{\rm A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-(p-k)\,\mathbf{q}_\perp)]\Big\}, \\
\delta E(\mathbf{h},p,k) &= \frac{\mathbf{h}^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p}\,.
\end{aligned}
$$

- m^2: Medium induced thermal masses.

Gluon Radiation Rate

Rate using **F**

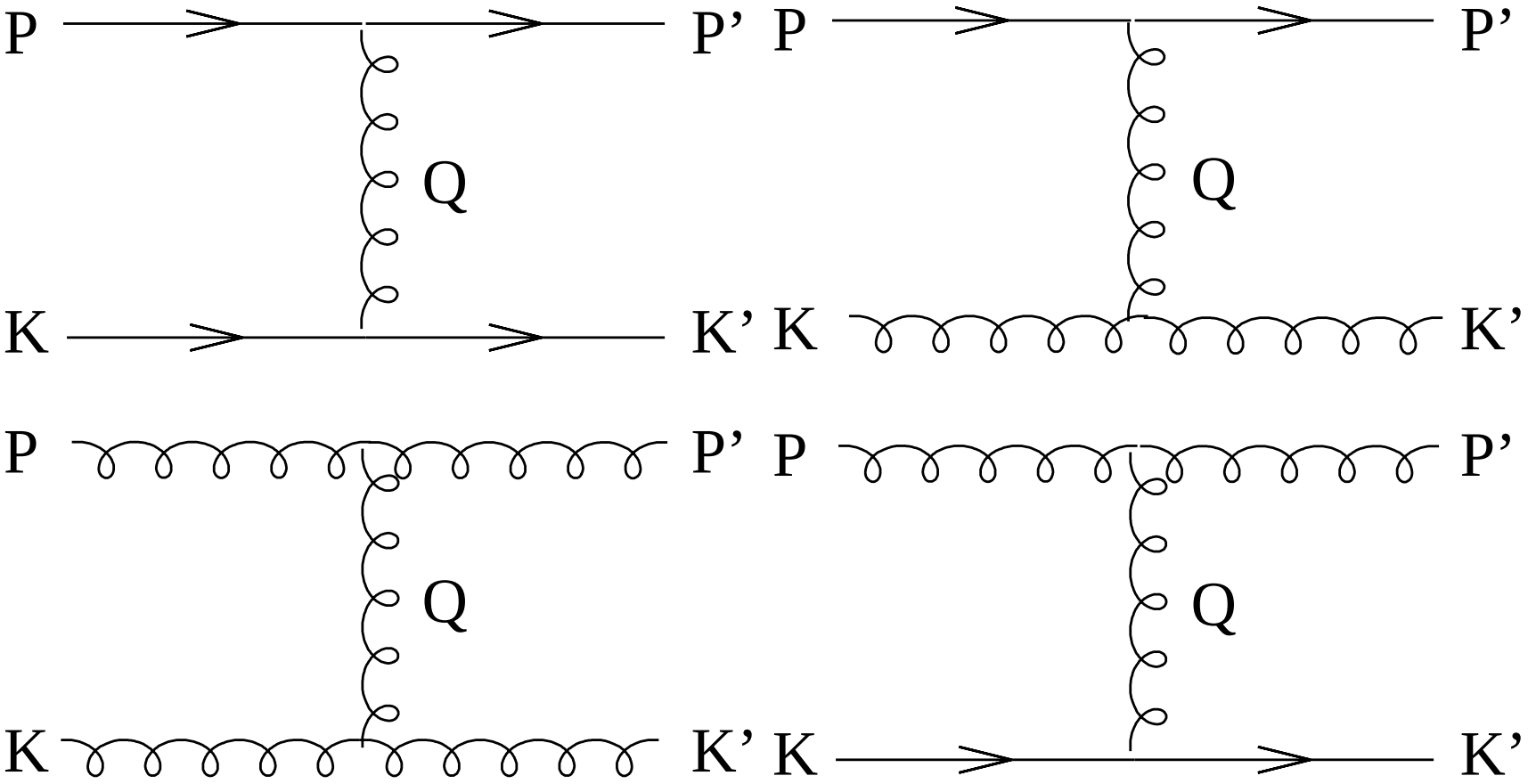
$$\frac{d\Gamma_g(p,k)}{dkdt} = \frac{C_s g_s^2}{16\pi p^7}\frac{1}{1\pm e^{-k/T}}\frac{1}{1\pm e^{-(p-k)/T}}\times$$
$$\times\left\{\begin{array}{cc} \frac{1+(1-x)^2}{x^3(1-x)^2} & q\to qg \\ N_f\frac{x^2+(1-x)^2}{x^2(1-x)^2} & g\to qq \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g\to gg \end{array}\right\}$$
$$\times\int\frac{d^2\mathbf{h}}{(2\pi)^2}2\mathbf{h}\cdot\mathrm{Re}\,\mathbf{F}(\mathbf{h},p,k)\,,$$

where $x\equiv k/p$ is the momentum fraction in the gluon (or either quark, for the case $g\to qq$). $\mathbf{h}\equiv\mathbf{p}\times\mathbf{k}$: 2-D vector. $O(gT^2)$

- Correctly incorporates *both* the BH limit and the LPM limit.

Elastic scattering rate

Coulombic *t*-channel dominates



Elastic scattering rate

Kinetic theory expression

$$\begin{aligned}\frac{dE}{dt} &= \frac{g_k}{2E}\int_{k,k',p'} \delta^4(p+k-p'-k')\,(E-E')|M|^2\, f(E_k)[1 \pm f(E'_k)] \\ &= C_r \pi\, \alpha_s^2 T^2 \left[\ln(ET/m_g^2) + D_r\right]\end{aligned}$$

where C_r and D_r are channel dependent $O(1)$ constants.

Approximation:

$$\frac{d\Gamma}{d\omega dt} \approx (1+n_B(\omega_0))\frac{1}{|\omega_0|}\frac{dE}{dt}\delta(\omega-\omega_0) + n_B(|\omega_0|)\frac{1}{|\omega_0|}\frac{dE}{dt}\delta(\omega+\omega_0)$$

Gets the first moment and the second moment right.

We solve:

$$\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} + 2P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt},$$

$$\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dkdt} + P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} - P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt} + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(k-p/2)\right)$$

Are we being sensible?

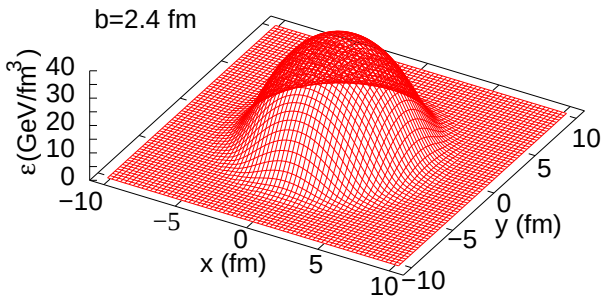
- For the rate, we correctly handle:
  - Thermal dispersion corrections for all species in the plasma
  - Real moving plasma particles to scatter from, rather than idealized static scatterers
  - LPM effect–transition between BH and strong LPM handled in smooth and correct way. NOTE that the LPM effect is often numerically not that large.
  - Stimulated emission and absorption processes are included
- However the rate is determined in a fixed-temperature, extensive medium $\Longrightarrow$ Fokker-Planck valid for $l_{\rm coh} \ll T/(dT/dx)$
- Which is more important? Multiple soft radiation (AMY does that correctly) or rare hard radiation (AMY is incomplete for this)?
  - For jet energy loss, multiple soft radiation is more important because the initial spectrum is steeply falling.

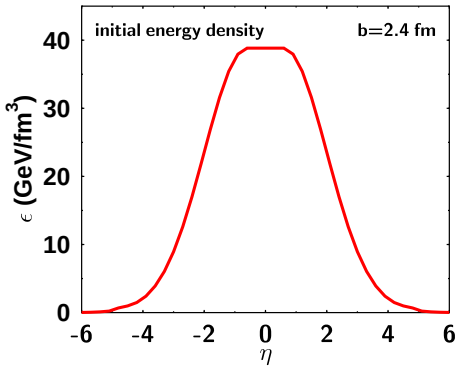
Thermal vs. Static medium

- Medium = static scattering centers – Gyulassy, Wang, BDMPS, GLV, Wiedemann, Kovner, Salgado, ...
  - Medium is characterized by $\hat{q} = \mu^2/\lambda$ and the color representation of the scatterers.
  - Vacuum radiation ('energy loss') can be included.
  - Uses Poisson ansatz to do real time calc.
- Dynamic thermal medium – AMY, Jeon, Moore, Turbide, Gale, ...
  - Thermal quarks and gluons
  - Use kinetic theory to do real time calc - A bit easier to include medium expansion
- In both cases, medium expansion must be independently modeled.

(3+1)-D relativistic hydrodynamics (Nonaka & Bass)

- Based on conservation laws: $\partial_\mu T^{\mu\nu} = 0, \partial_\mu j^\mu = 0$.
- For ideal fluid, $T^{\mu\nu} = (\epsilon + p)U^\mu U^\nu - pg^{\mu\nu}, j^\mu = n_B U^\mu$.
- EOS: Bag model + Hadron with extended volume
- Initial conditions: $\epsilon(x, y, \eta) = \epsilon_{\max} W(x, y; b) H(\eta)$, $n_B(x, y, \eta) = n_{B\max} W(x, y; b) H(\eta)$





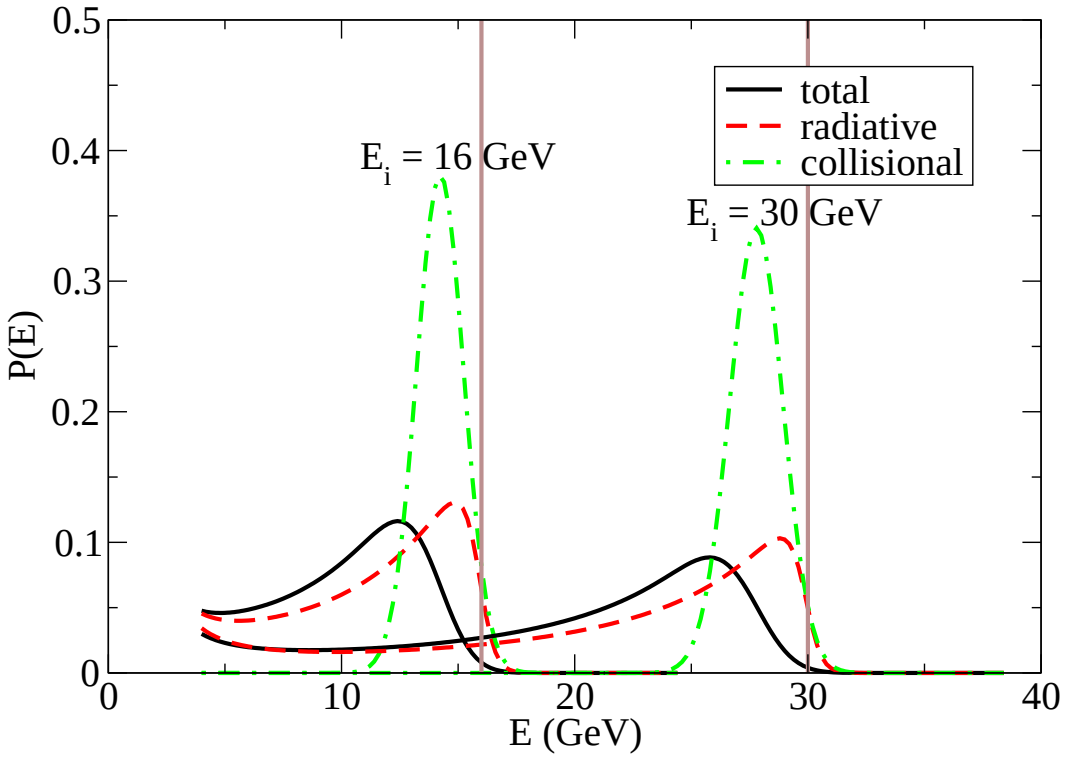
- Particle spectra: Cooper-Frye Formula

$$E\frac{dN_i}{d^3p} = \int_\Sigma p \cdot d\sigma \frac{g_i}{(2\pi)^3} \frac{1}{\exp\left[(p \cdot U - \mu_i)/T_{\rm f}\right] \pm 1}$$

Nonaka and Bass, Phys.Rev.C75:014902,2007

Evolution of a single jet at RHIC

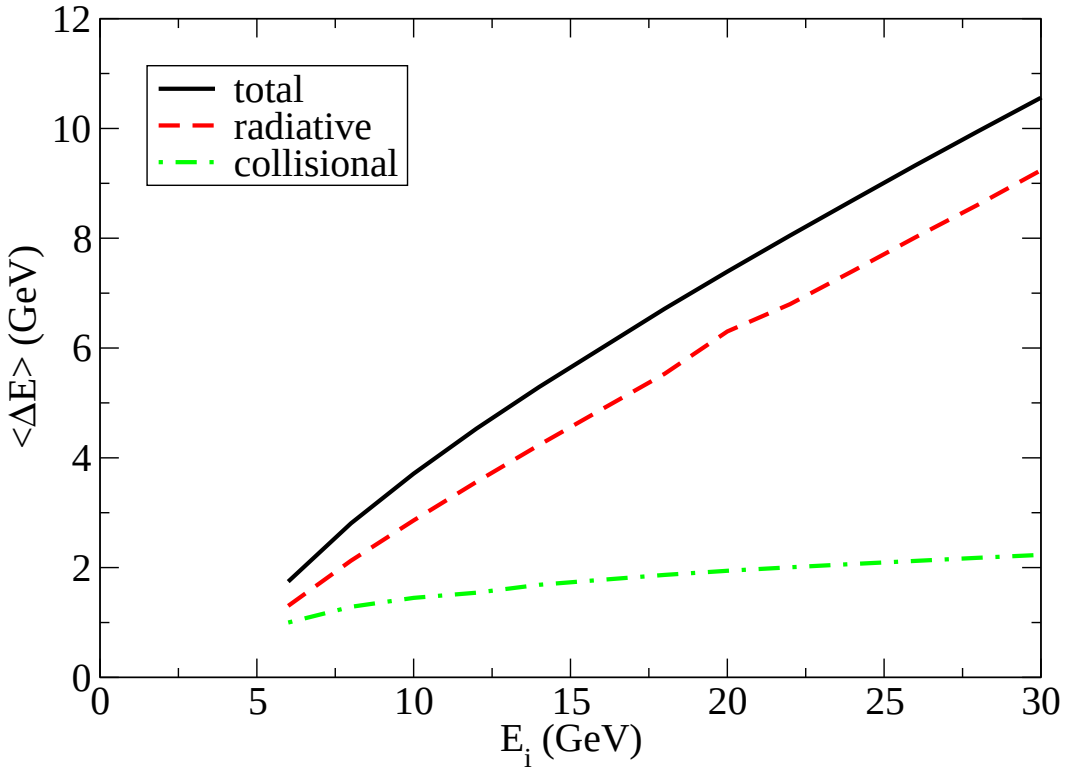
- The final momentum distribution $P(E, t_f)$ of a single quark jet after passing through RHIC medium ($b = 2.4$ fm)



- Medium described by (3+1)D ideal hydrodynamics.
- The jet starts at the center and propagates in plane.
- Jet energy loss turned off in hadronic phase.

Averaged energy loss of a single jet at RHIC

- The averaged energy loss of a quark jet after passing through RHIC medium ($b = 2.4$ fm)



- Averaged energy: $\langle E \rangle = \int dE E P(E) / \int dE P(E)$

Qin *et al.*, arXiv:0710.0605 [hep-ph], PRC, in press, arXiv:0705.2575 [hep-ph]

Pion Production

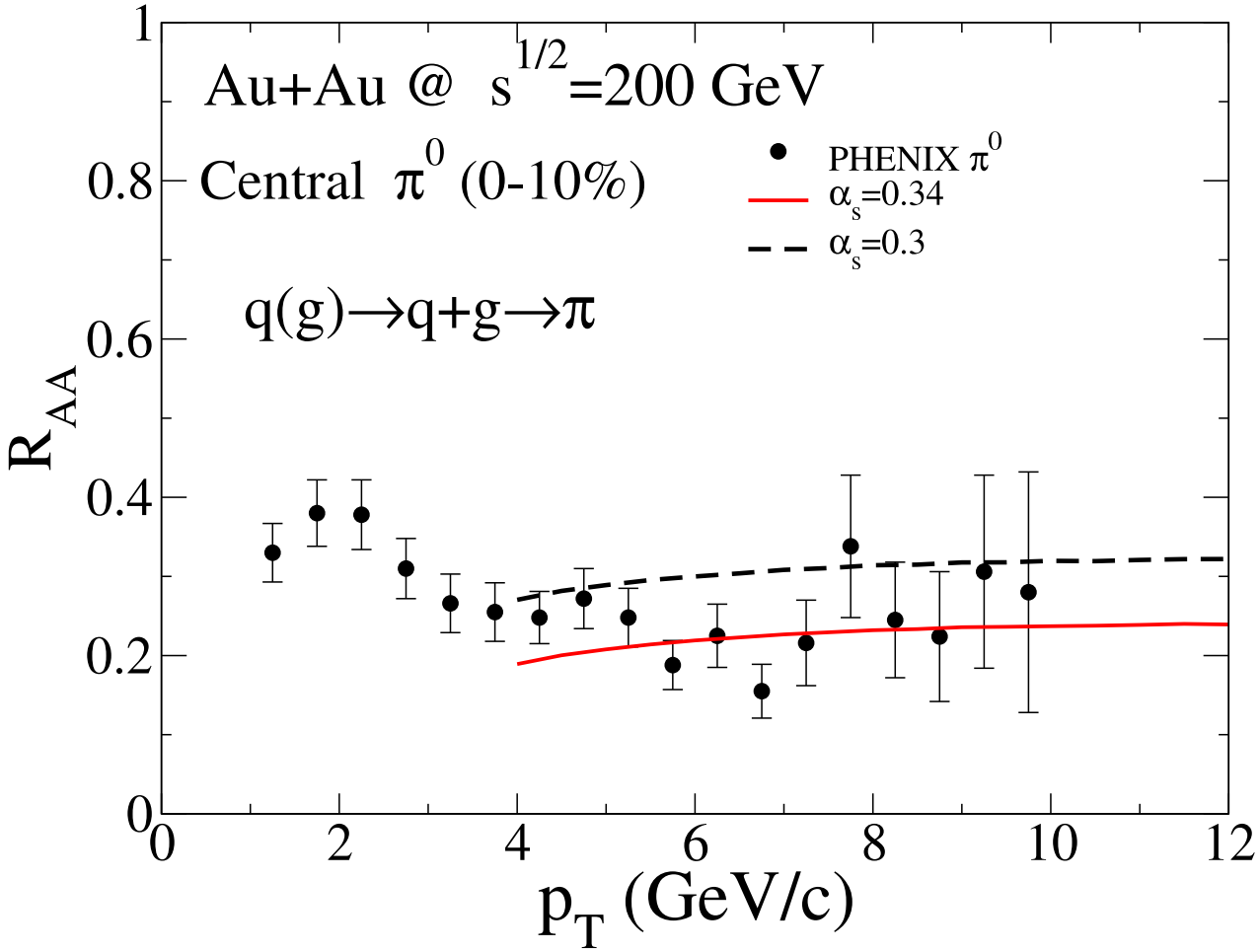
$$\frac{dN_{AA}}{dyd^2\mathbf{p}_T} = \frac{\langle N_{\rm coll}\rangle}{\sigma_{in}} \sum_{a,b,c,d} \int dx_a dx_b\, g_A(x_a,Q) g_A(x_b,Q) \times K_{\rm jet} \frac{d\sigma_{a+b\to c+d}}{dt} \frac{\tilde{D}_{\pi^0/c}(z,Q)}{\pi z}$$

with $\tilde{D}_{\pi^0/c}(z,Q) = \int d^2 r_\perp\, \mathcal{P}(\mathbf{r}_\perp) \tilde{D}_{\pi^0/c}(z,Q,\mathbf{r}_\perp,\mathbf{n})$, and $\tilde{D}_{\pi^0/c}(z,Q,\mathbf{r}_\perp,\mathbf{n}) =$

$$\int dp_f \frac{z'}{z} \left(P_{qq/c}(p_f;p_i;\Delta t)\, D_{\pi^0/c}(z',Q) + P_{g/c}(p_f;p_i;\Delta t)\, D_{\pi^0/c}(z',Q) \right)$$

with $z = p_T/p_i$ and $z' = p_T/p_f$.

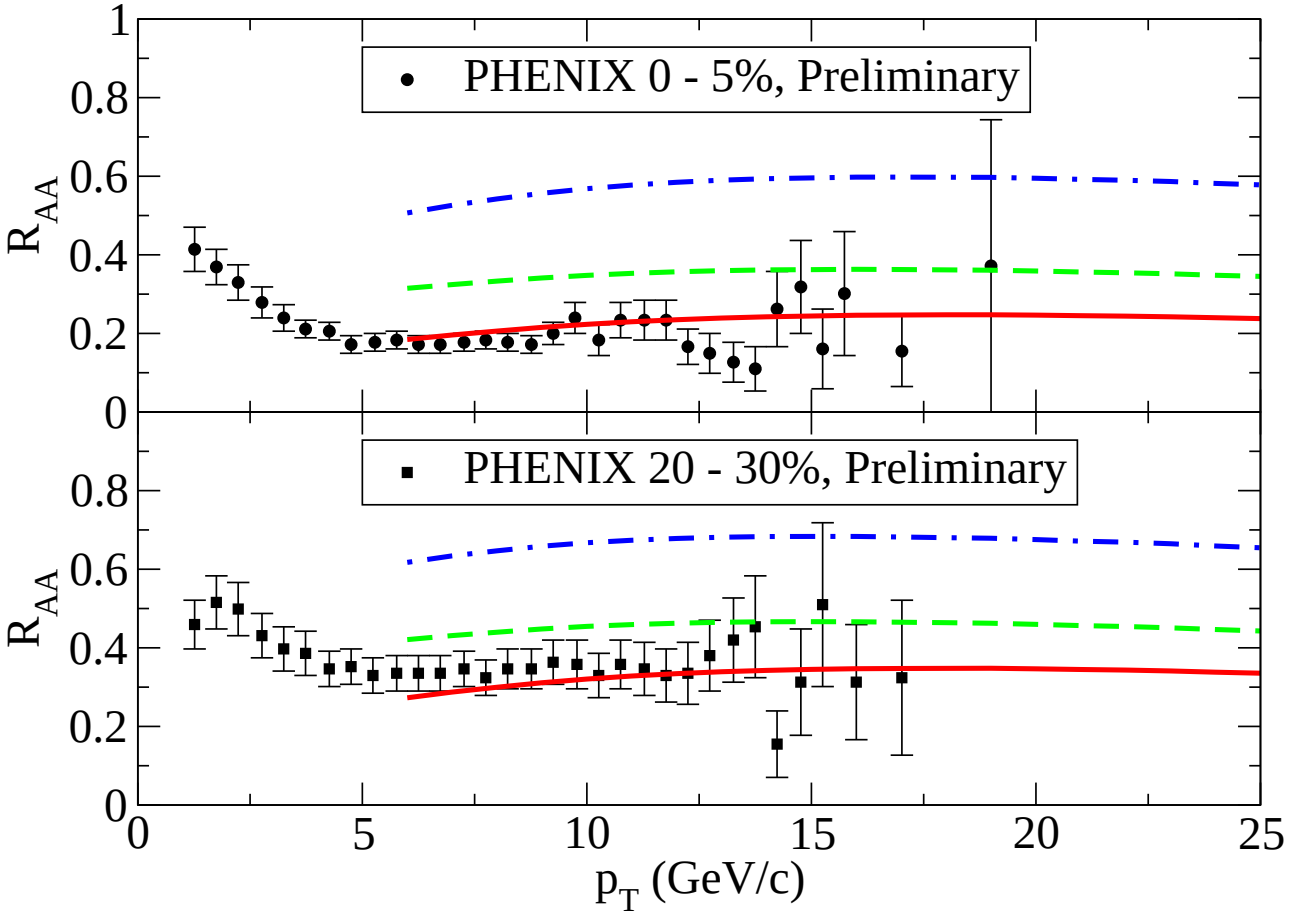
R_{AA} at RHIC - π^0 - Radiation only



$T_i = 370$ MeV, $dN/dy = 1260$. 1-D Bjorken expansion.

Best $\alpha_s = 0.33$ S.Turbide, C.Gale, S.J. and G.Moore, PRC72:014906,2005

R_{AA} at RHIC – π^0 - Full

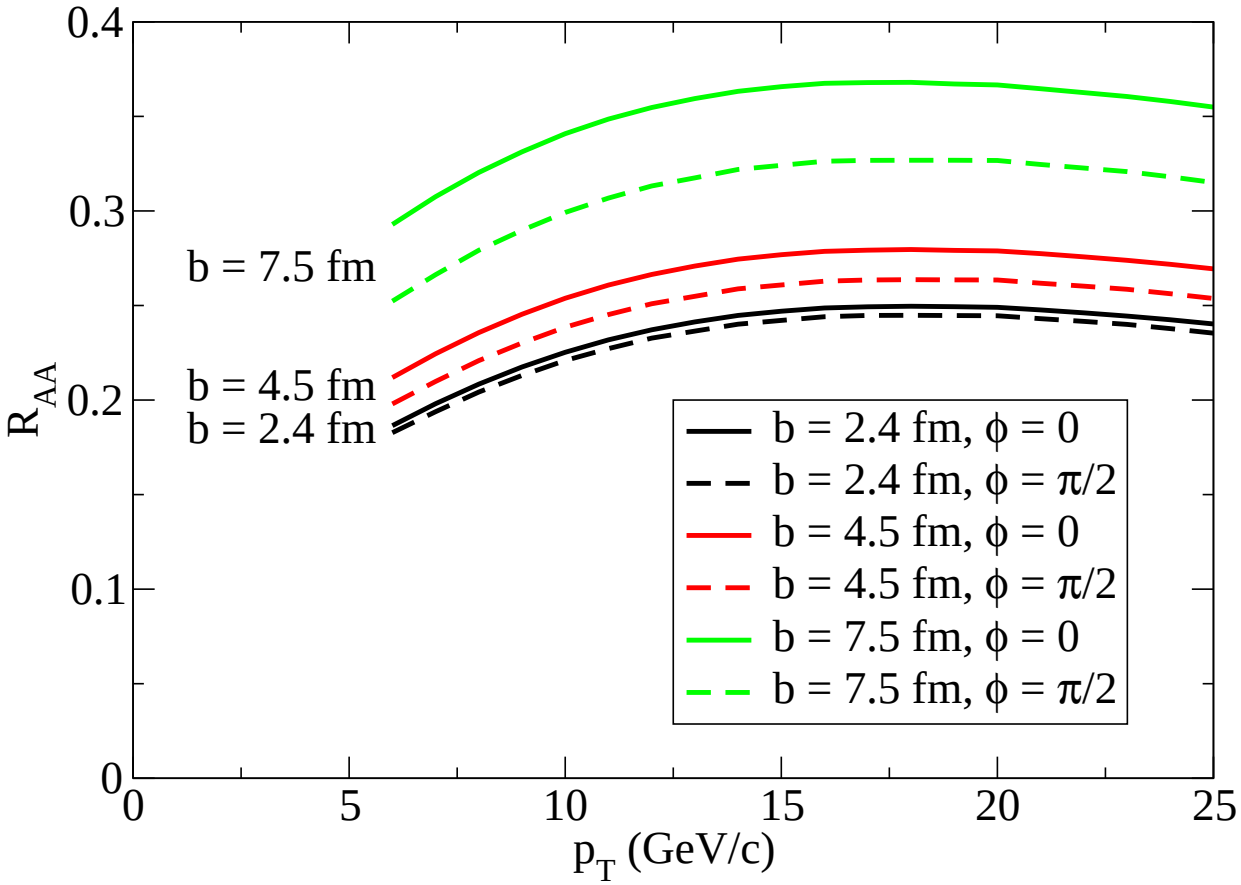


- total, rad, coll
- Strong coupling α_s tuned from 0.33 to 0.27

Qin *et al.*, arXiv:0710.0605 [hep-ph], PRC, in press, arXiv:0705.2575 [hep-ph]

R_{AA} vs. reaction plane at RHIC

- R_{AA} for π^0 in plane and out of plane at RHIC (different b)

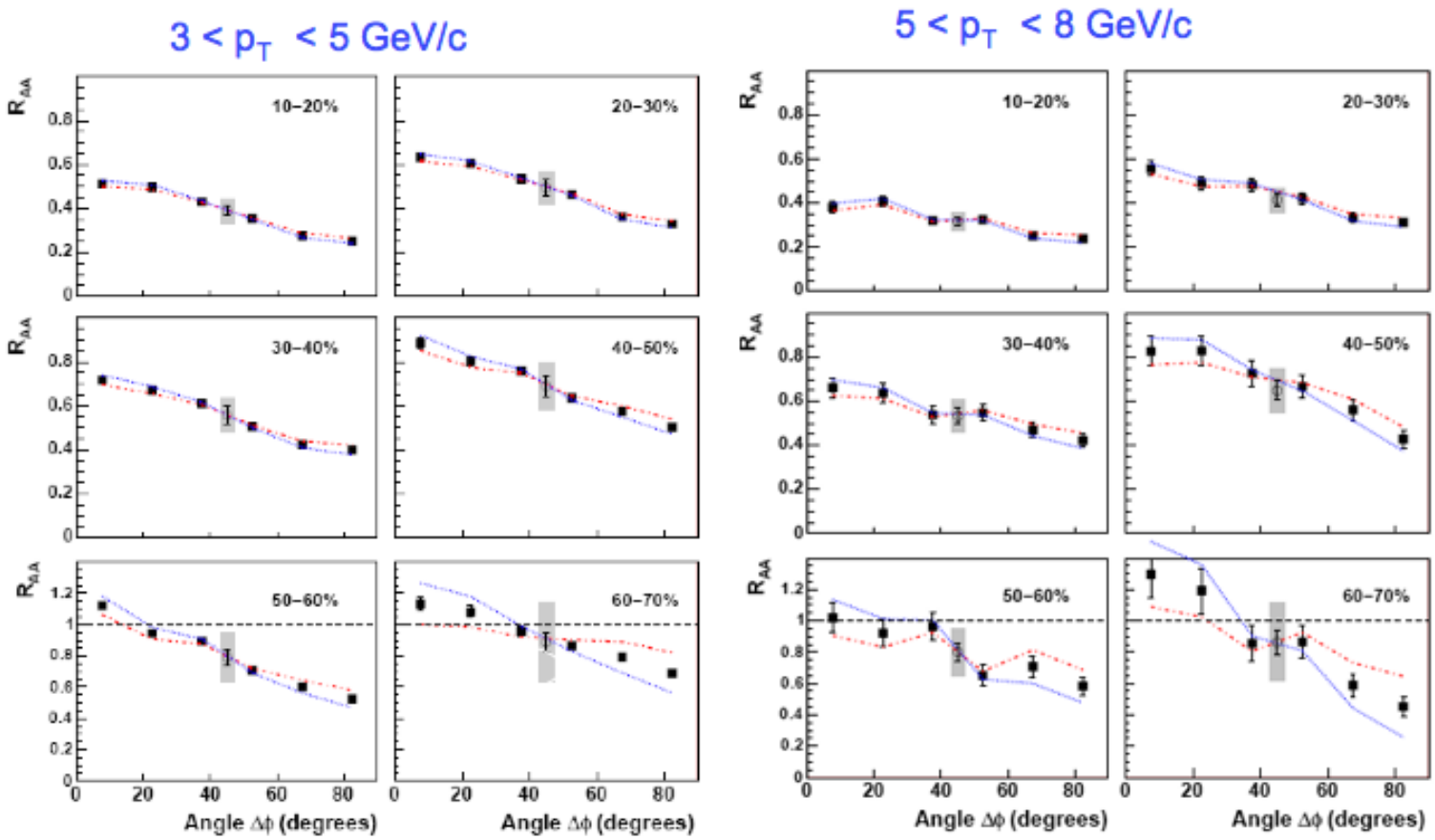


- Jets propagating out of plane ($\phi = \pi/2$) are suppressed more than in plane ($\phi = 0$)

PHENIX data from QM06 (Pantuev's talk)

The results:

PHENIX Run2, nucl-ex/0611007, submitted PRC



-R_{AA} in plane and out of plane changes by factor ~2

- For peripheral bins no suppression in plane, while a factor ~2 out of plane 13

Summary

- Calculated Jet Quenching with radiational *and* collisional energy loss
- Dynamic medium - 3+1 D Hydro Evolution included
- Best we can do using perturbative results (We keep track of radiated gluons as well)
- R_{AA} is still flat
- $R_{AA}(\phi)$ needs more work
- Radiational part plays bigger role, but collisional part not negligible.
- Best fit α_s reduced by 10 %
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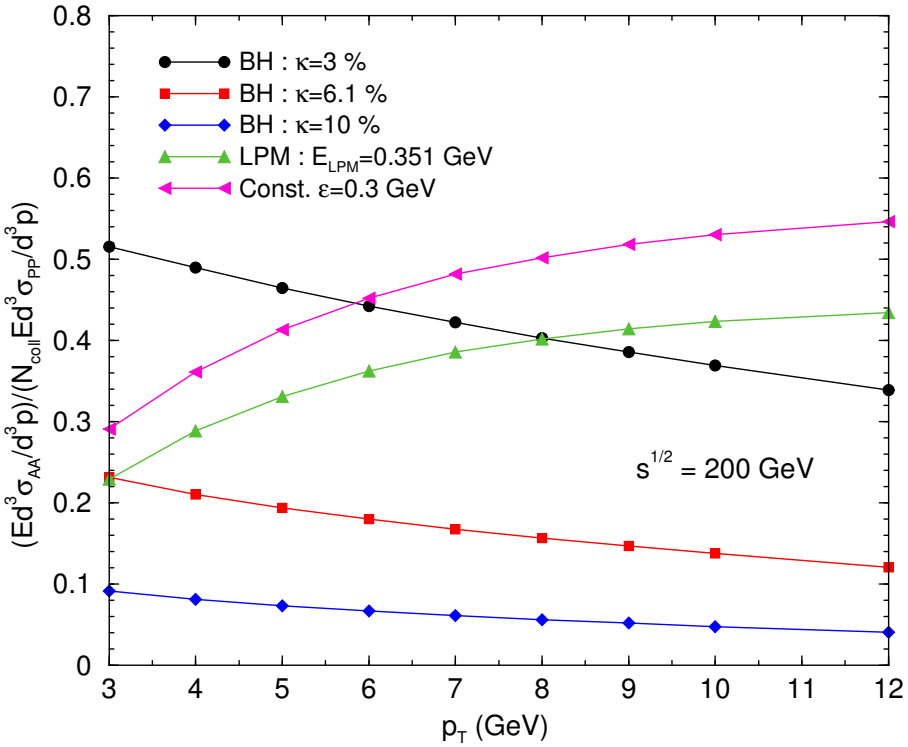
Came a long way from here...



"High" p_t Spectra

PHOBOS - $\frac{dN}{dp_t^2} \sim$ # part.'s NOT # coll.'s ?

PHENIX: $R_{AA} \approx$ const. p_t: 4 → 8 GeV

$\left.\frac{dN}{dp_t^2}\right|_{NLO} \bar{c} \quad \frac{\Delta E}{E} \sim .07/\text{scatt.'s}$

WRONG! Jeon, Jalilian-Marian, Sarcevic

Right: $\Delta E \sim \sqrt{E}$ Baier, Dokshitzer, Mueller, Schiff

QM 02, Jeon, Jalilian-Marian, Sarcevic. Rob's summary.