

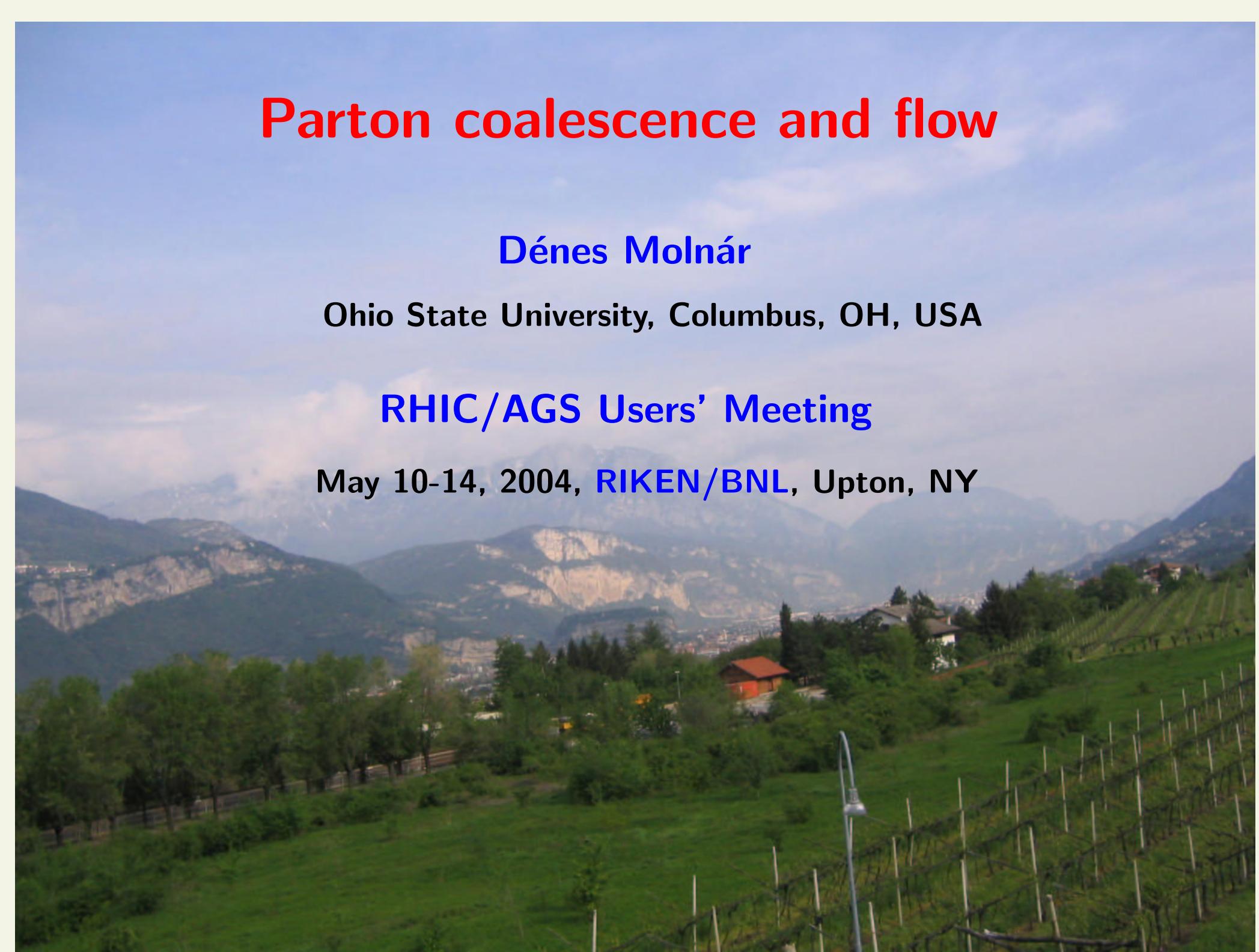
# Parton coalescence and flow

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# Outline

- **Introduction**

- motivation, what coalescence is
- simple parton coalescence formula

- **Spectra from coalescence (radial flow)**

- coalescence window at RHIC
- baryon enhancement

- **Azimuthal anisotropy**

- anisotropy amplification from coalescence
- elliptic flow scaling with quark number, mesons vs baryons
- charm hadron elliptic flow

- **Open issues and next steps**

- current and recent challenges
- coalescence dynamics

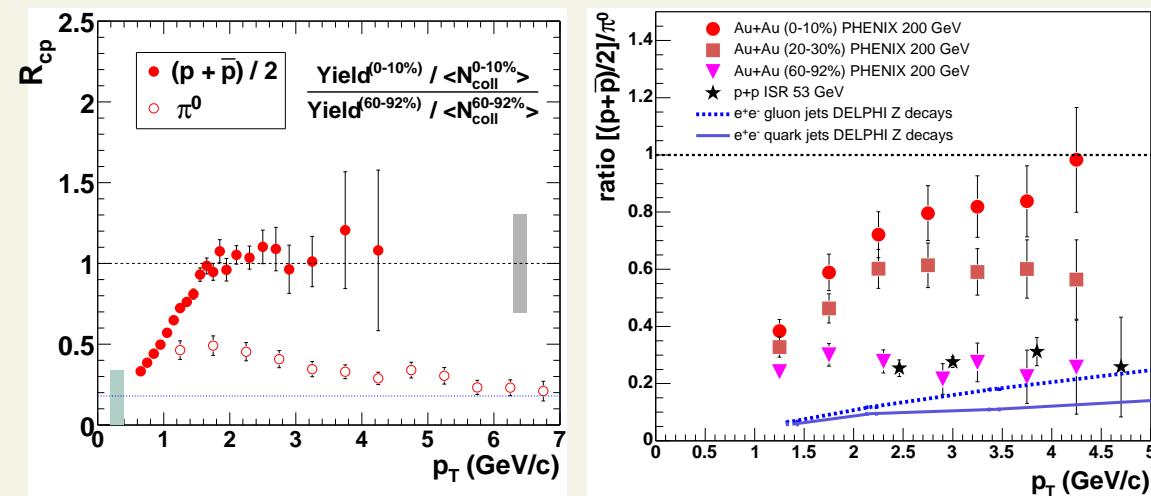
- **Summary**

# Introduction

# Two surprises at RHIC

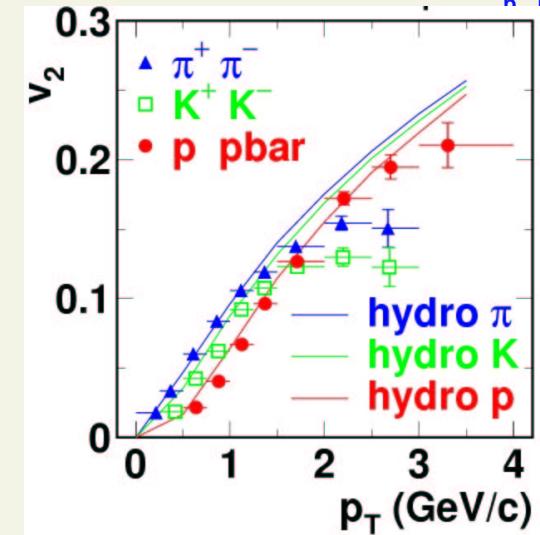
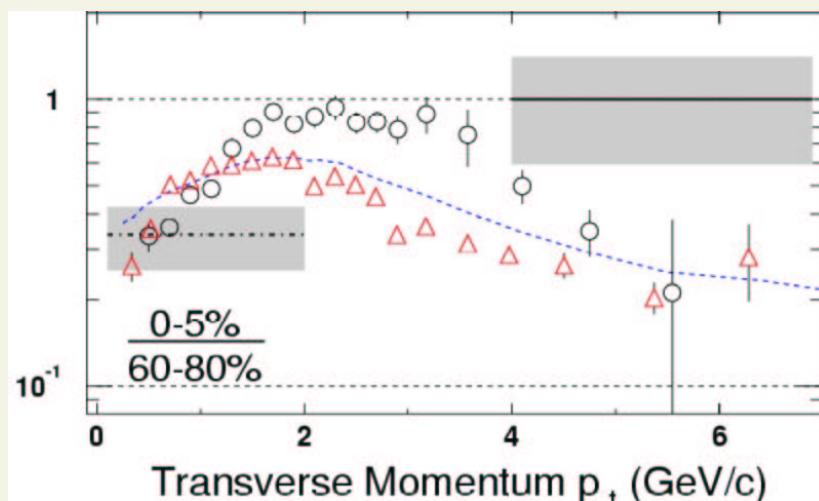
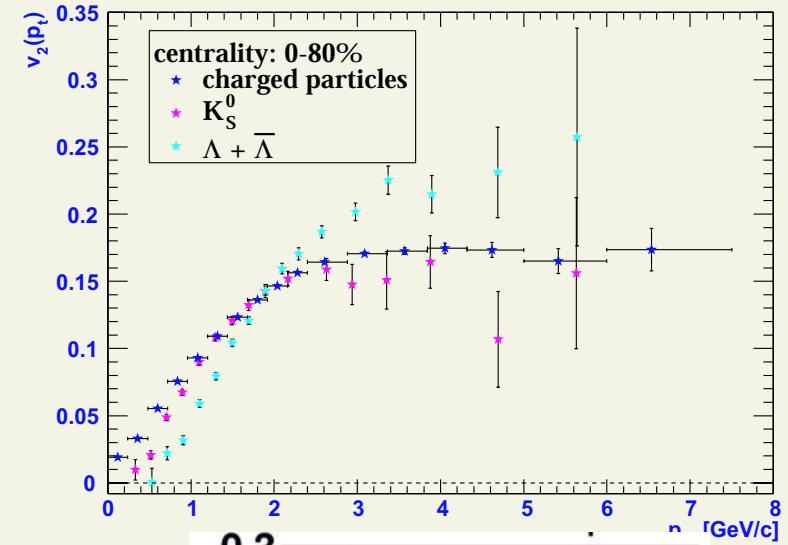
baryon **non-suppression**,  $p/\pi \sim 1$

d'Enterria [PHENIX], Sorensen [STAR]:



elliptic flow saturates **higher** for baryons

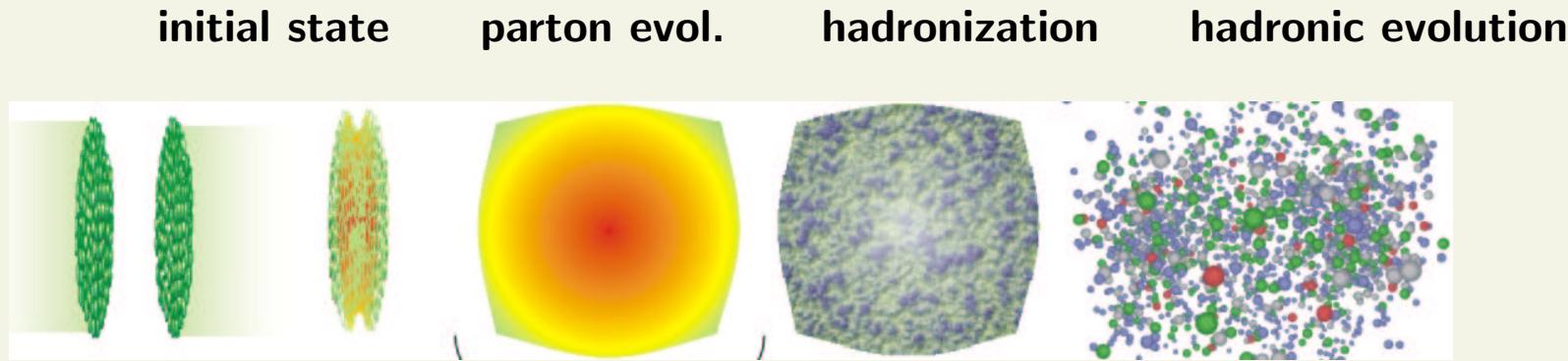
Sorensen [STAR], Esumi [PHENIX]:



→ parton coalescence provides an explanation ←

# Hadronization problem

- **heavy-ion collision stages**



- **hadronization:** **least understood part**
  - nonperturbative
  - confinement

**better knowledge is essential** in order to learn about partonic stage from observed hadrons

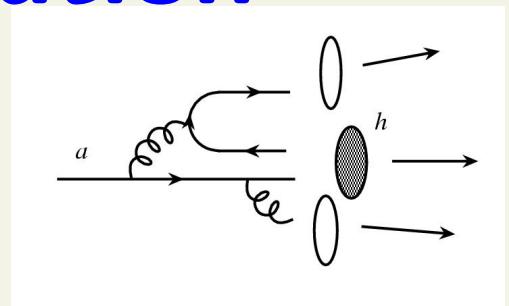
# Independent fragmentation

**factorized pQCD:**  $dN_h = f_i \otimes f_j \otimes d\sigma_{ij \rightarrow aX} \otimes D_{a \rightarrow h}$

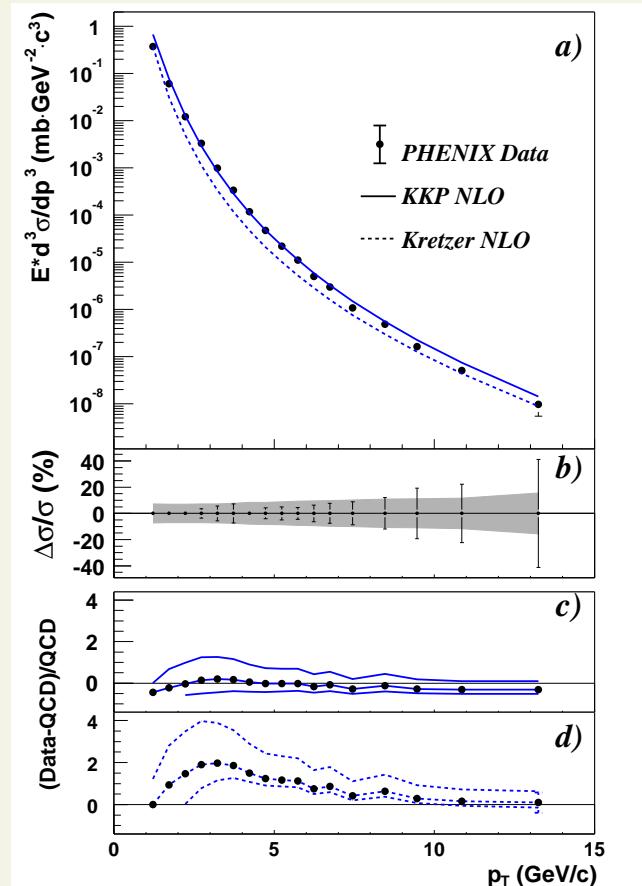
[applicable at large  $p_T$ ]

fragmentation function

- universal, measured (nonperturbative)



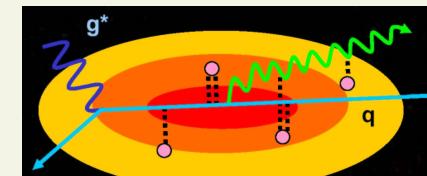
$p + p$ : works



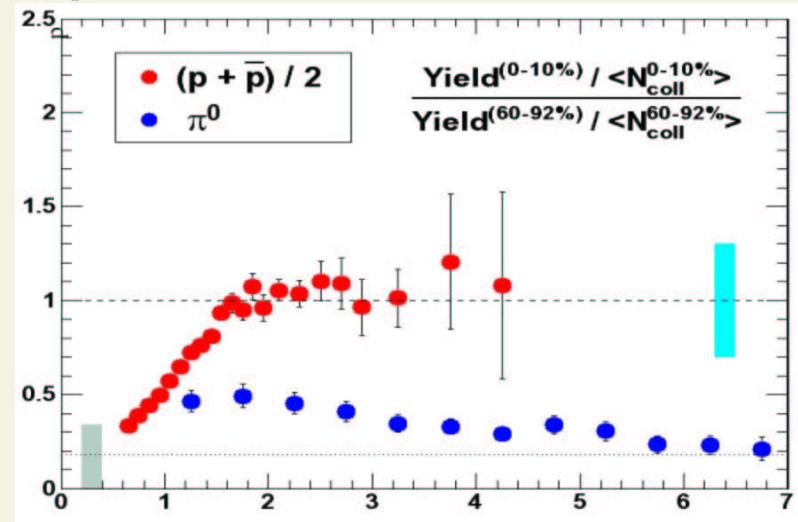
$A + A$ : works for pions, but fails for baryons

$$dN_h = f_{A,i} \otimes f_{A,j} \otimes d\sigma_{ij \rightarrow aX} \otimes E_{loss} \otimes D_{a \rightarrow h}$$

Wang, Gyulassy, Dokshitzer, Mueller, Levai,  
Vitev, Wiedemann, Guo, Salgado, Djordevic, ...



$R_{cp}$  from PHENIX: **baryon non-suppression**

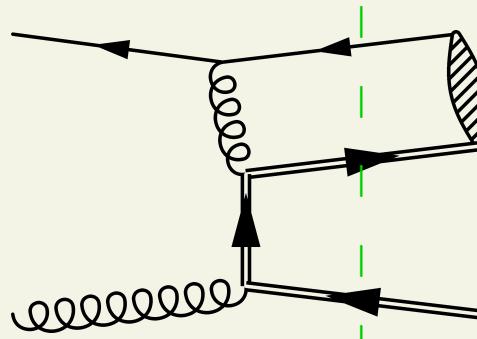


# Multi-parton hadronization

**leading particle effect:**  $\pi^-(u\bar{d}) + A \rightarrow D + X$  (E791 -  $E_{lab} = 500\text{GeV}$ )

$$\alpha[D^+(c\bar{d})] \equiv \frac{\sigma[D^+]-\sigma[D^-]}{\sigma[D^+]+\sigma[D^-]} \sim 0.2 - 0.7 \quad - \text{large asymmetry}$$

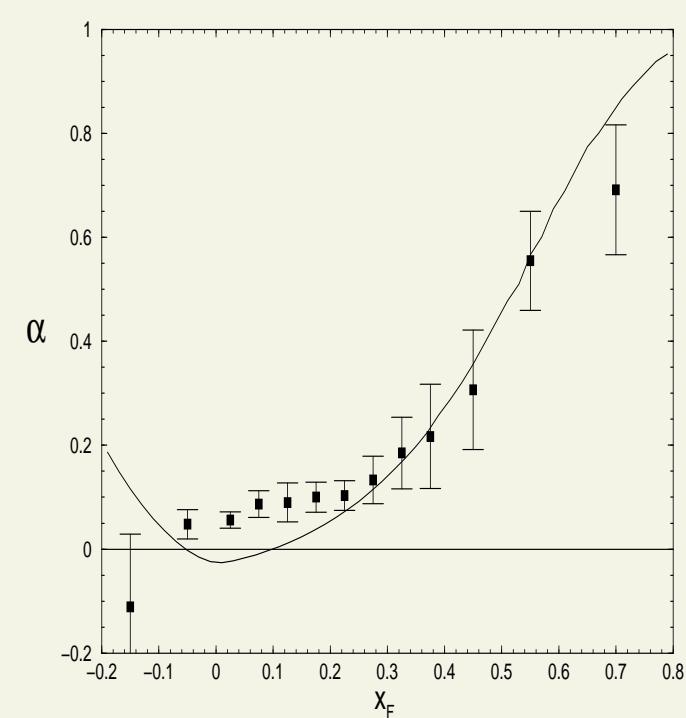
- LO pQCD:  $\alpha = 0$ , NLO still too small → power corrections large
- need  $\bar{d} + g \rightarrow (c\bar{d}) + \bar{c} \rightarrow D^+ + \bar{c}$  (+X) reproduces E791:



[Braaten & Jia, PRL89 ('02)]

$D^+$  favored because valence  $\bar{d}$  is already there

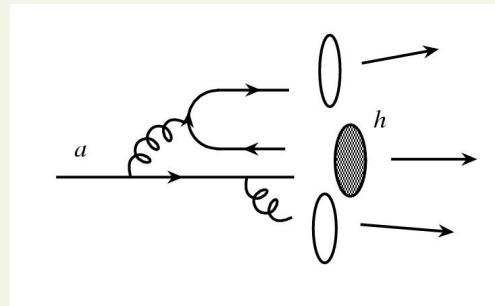
- beam-drag effect (natural in string models)



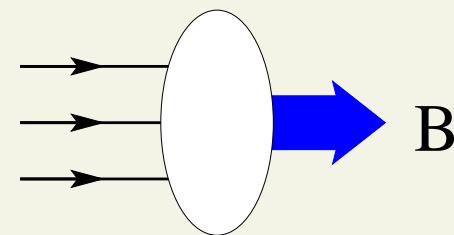
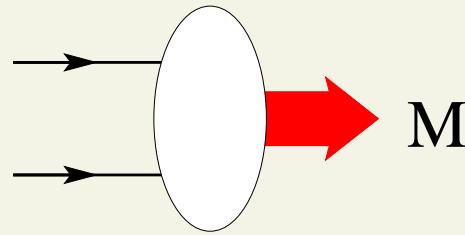
# Parton coalescence

Hwa, Yang, Biró, Zimányi, Lévai, Csizmadia, Ko, Lin, Voloshin, D.M., Greco, Fries, Müller, Nonaka, Bass, ...

- **heavy-ion collisions: large parton density  $\Rightarrow$  multi-parton processes probable besides jet fragmentation**



**additional hadronization channels via parton coalescence/recombination**



- consider lowest-order  $q\bar{q} \rightarrow M$ ,  $qqq \rightarrow B$  (**valence quarks only**)

# Applications

**hadron multiplicity:** Das & Hwa, PLB68 ('77)  
Biró et al, PLB347 ('95) - ALCOR  
Csizmadia & Lévai, JPG28 ('02) - MICOR

**baryon/meson ratio:** Hwa & Yang, PRC65 ('02)  
Greco, Ko, Levai, PRL90 ('03); PRC68 ('03)  
Fries, Müller, Nonaka, Bass, PRL90 ('03); PRC68 ('03)  
Hwa, Yang, PRC67 ('03)  
Fries, Müller, Nonaka, Bass, JPG30 ('04)  
Hwa & Yang, nucl-th/0401001

**resonances:** Nonaka, Müller, Asakawa, Bass, Fries, PRC69 ('04),  
Zimányi & Lévai, nucl-th/0404060

**elliptic flow:** Ko & Lin, PRL89 ('02)  
Voloshin, NPA715 ('02)  
D.M. & Voloshin, PRL91 ('03)  
Nonaka, Fries, Bass, PLB583 ('04)  
D.M., JPG30 ('04)  
Greco, Ko, nucl-th/0404020  
D.M., nucl-th/0404065

**charm hadron elliptic flow:** Lin & D.M., PRC68 ('03)  
Greco, Ko, Rapp, nucl-th/0312100

# Formalism

# Traditional coalescence

- **original problem:**  $n + p \rightarrow d$

Butler & Pearson, PR129 ('63)

Schwarzschild & Zupancic, PR129 ('63)

Sato & Yazaki, PLB98 ('81)

Gyulassy, Frankel & Remler, NPA402 ('86)

Dover, Heinz, Schnedermann & Zimányi PRC44 ('91)

Nagle, Kumar, Kusnezov, Sorge & Mattiello, PRC53 ('96)

Kahana, Kahana, Pang, Baltz, Dover, Schnedermann & Schlagel, PRC54 ('96)

Scheibl & Heinz, PRC59 ('99)

...

# Simple coalescence formula

- **basic equations:**  $q\bar{q} \rightarrow \text{meson}$ ,  $qqq$  (or  $\bar{q}\bar{q}\bar{q}$ )  $\rightarrow \text{baryon}$

$$\frac{dN_M(\vec{p})}{d^3p} = g_M \int \left( \prod_{i=1,2} \cancel{d^3x_i} d^3p_i \right) \cancel{W_M}(x_1 - x_2, \vec{p}_1 - \vec{p}_2) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$
$$\frac{dN_B(\vec{p})}{d^3p} = g_B \int \left( \prod_{i=1,2,3} \cancel{d^3x_i} d^3p_i \right) \cancel{W_B}(x_{12}, x_{13}, \vec{p}_{12}, \vec{p}_{13}) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) f_\gamma(\vec{p}_3, x_3) \delta^3(\vec{p} - \sum \vec{p}_i)$$

hadron yield      space-time      wave-fn.      quark distributions

**assumes:**

- weak binding
- no 2-body or 3-body correlations
- rare process - otherwise violates unitarity (need low phasespace density)
- 3D hypersurface (e.g., equal time - sudden approximation)

comes from the QM projection theorem:  $P_\phi(t) = |\langle \phi | \Psi(t) \rangle|^2$

- need  $f_q$ ,  $f_{\bar{q}}$ , hypersurface, and hadron wave functions

# Applying the formulas

- $f_q$ ,  $f_{\bar{q}}$ , and hypersurface can in principle be obtained from a dynamical calculation
- however, most common approach:

educated guess for distributions, with a few parameters fitted to a subset of data  
[analogous to: hydro  $\leftrightarrow$  blastwave; hydro, transport HBT  $\leftrightarrow$  HBT source parameterizations]

- thermal constituent spectra + radial flow
- uniform temperature & density, fixed proper time

yield from fragmentation included additively

$$dN_h = dN_h^{coal}(f_q \otimes f_{\bar{q}} \otimes W_h, \text{hypersurface}) + dN_h^{frag}(d\sigma \otimes E_{loss} \otimes \text{FFs})$$

- useful simplifications:

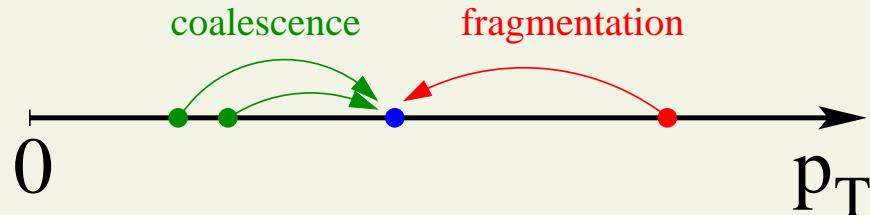
slowly varying spatial distributions: -  $W_M(\Delta \vec{p}, \Delta \vec{x}) \rightarrow \tilde{W}_M(\Delta \vec{p}) \delta^3(\Delta \vec{x}), \dots$

if also narrow wave fn: -  $W_M(\Delta \vec{p}, \Delta \vec{x}) \rightarrow (2\pi)^3 \delta^3(\Delta \vec{p}) \delta^3(\Delta \vec{x}), \dots$

# Spectra (radial flow)

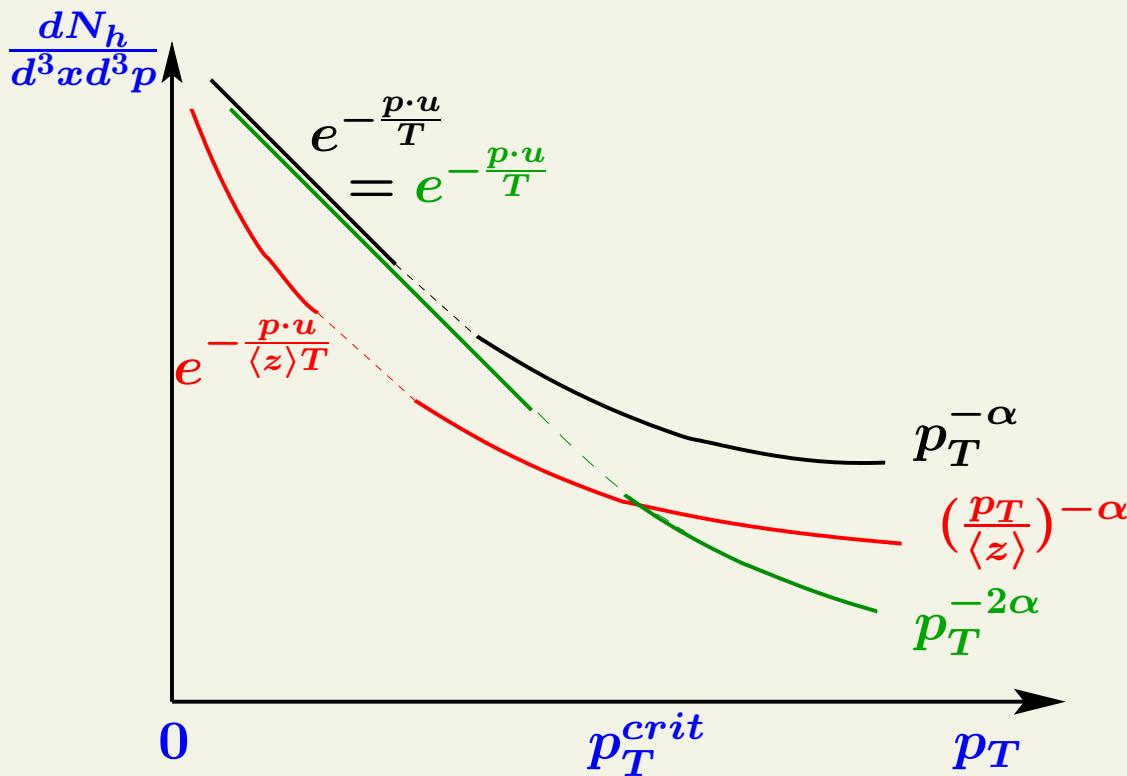
# Coalescence window

Coalescence competes with fragmentation and wins below  $p_T < p_T^{crit}$



**coal:**  $p_T \rightarrow \approx n p_T, n = 2, 3$

**frag:**  $p_T \rightarrow z p_T, z < 1$



- for exponential, coal wins
- for power-law, frag wins



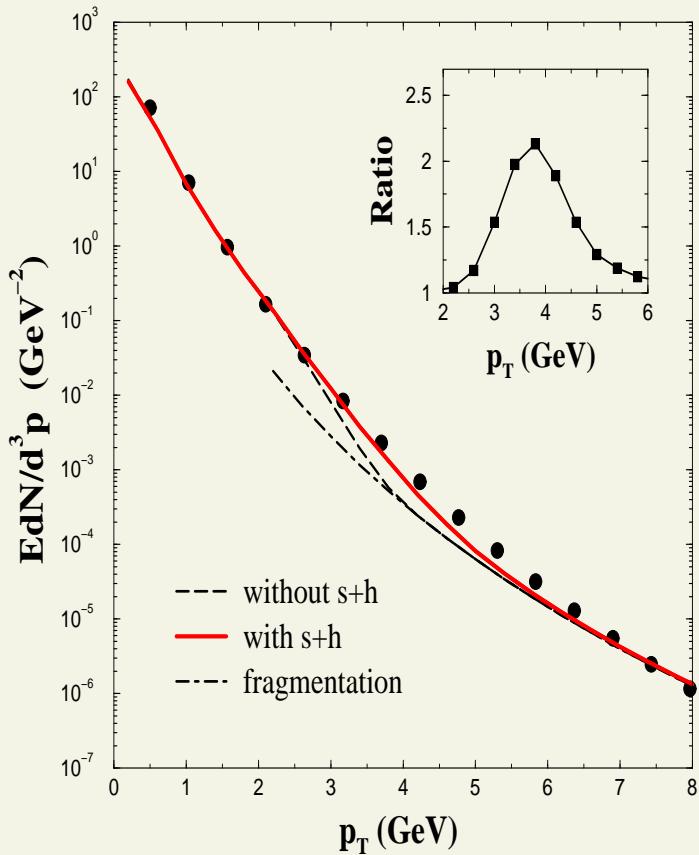
coalescence yield drops  
steeper than fragmentation  
yield

$p_T^{crit}$ : decreases with incr. centrality

# Parametrized soft component

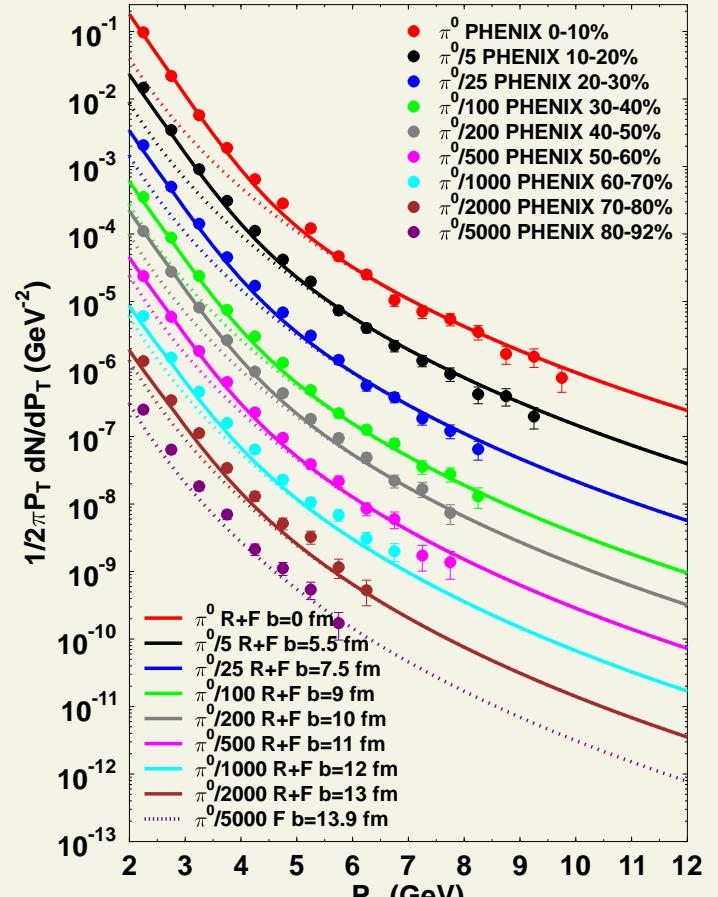
(!) a quark-antiquark “plasma”, thermalized at  $T \approx T_c$ , w/ strong radial flow works

Greco, Ko, Levai ('03):  $T_q = 170$  MeV,  $v_T^{max} = 0.5$



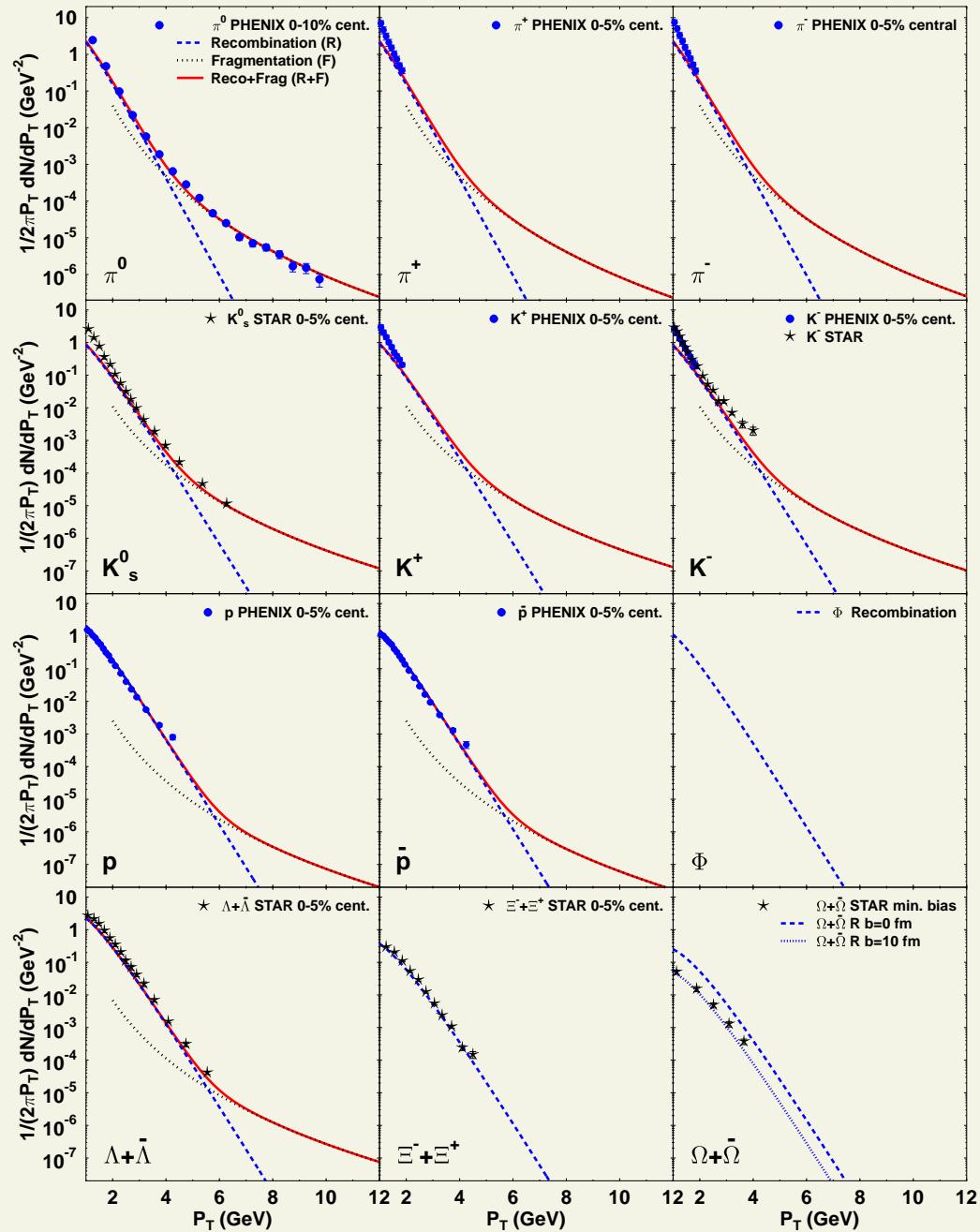
$[A_T = 225 \text{ fm}^2, \tau = 4 \text{ fm}, L/\lambda = 3.5]$

Fries et al ('03):  $T_q = 175$  MeV,  $v_T^{max} = 0.55$



$[A_T = 155 \text{ fm}^2, \tau = 5 \text{ fm}, \lambda = 0.67]$

- coal. dominates meson production out to  $p_T \sim 4$  GeV ( $\sim 6$  GeV for baryons)



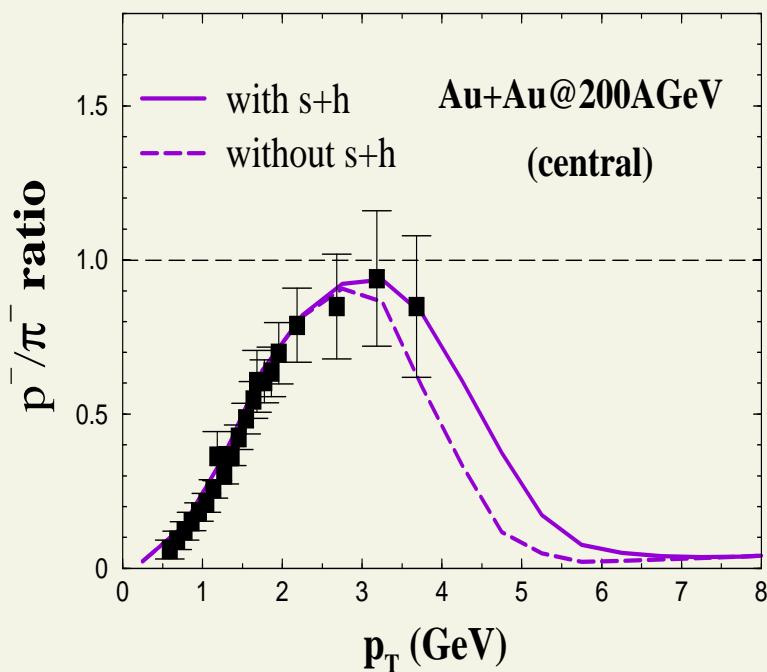
30-50% typical accuracy

# Baryon enhancement

Thermal  $p_\perp < \sim 2$  GeV partons contribute, via recombination, out to  $2 \times \sim 2$  GeV for mesons, while  $3 \times \sim 2$  GeV for baryons

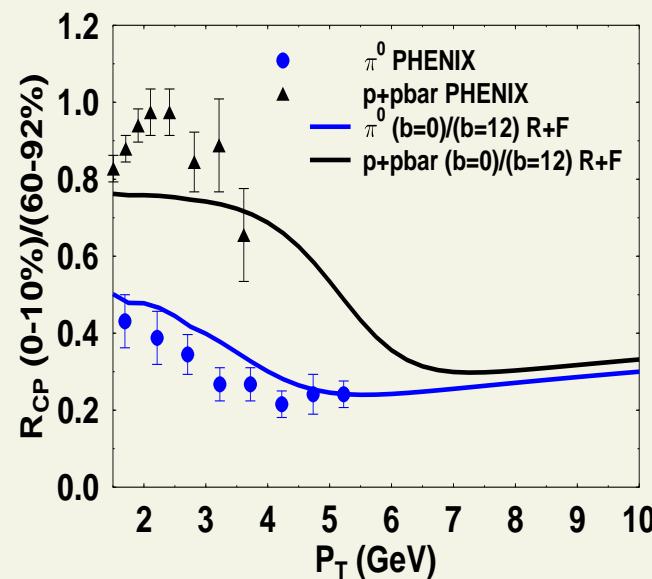
$\Rightarrow$  thermal tails extend into the intermediate  $p_T$  regime  $\rightarrow$  fixes  $p/\pi$

Greco, Ko, Levai ('03):

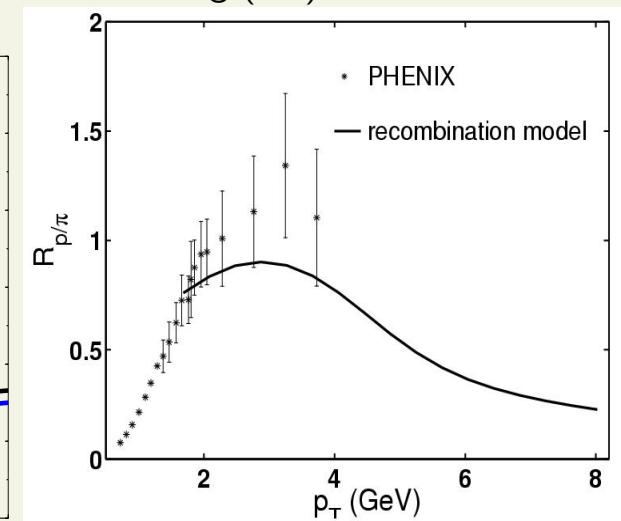


$[A_T = 225 \text{ fm}^2, \tau = 4 \text{ fm}, L/\lambda = 3.5]$

Fries et al ('03):



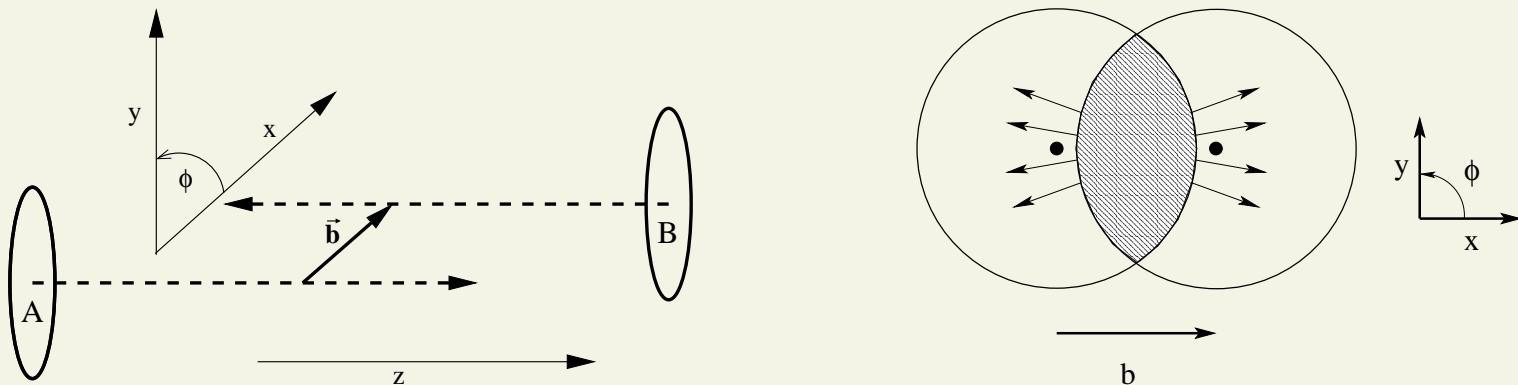
Hwa & Yang ('04):



# Azimuthal anisotropy

# Azimuthal anisotropy

- momentum-space **anisotropy** of particle production in A+A collisions



$$\frac{dN}{d\phi dX} \equiv \frac{1}{2\pi} \frac{dN}{dX} [1 + 2 \sum_{n=1} v_n(X) \cos(n\phi)] \rightarrow \begin{aligned} \text{directed flow: } & v_1(X) \equiv \langle \cos \phi \rangle_X \\ \text{elliptic flow: } & v_2(X) \equiv \langle \cos 2\phi \rangle_X \end{aligned}$$

...

$X$  : event and particle selection, e.g., centrality, transverse momentum

# Anisotropy amplification

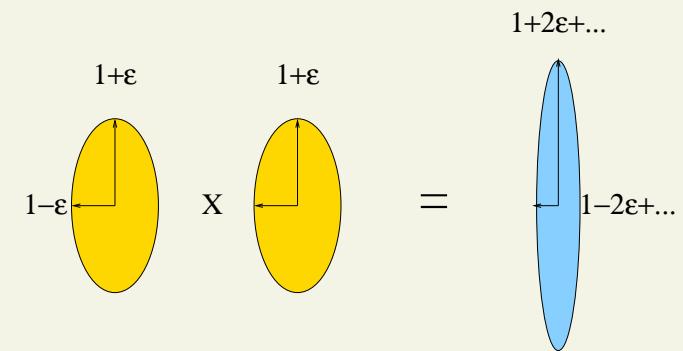
[Voloshin & D.M. ('03)]

**narrow wave fn.:**  $(2 \times \frac{\vec{p}}{2}) \rightarrow \vec{p}, \quad (3 \times \frac{\vec{p}}{3}) \rightarrow \vec{p}$

$$\frac{dN_M}{d\phi} \propto \left( \frac{dN_q}{d\phi} \right)^2, \quad \frac{dN_B}{d\phi} \propto \left( \frac{dN_q}{d\phi} \right)^3$$

$$v_k^M(p_\perp) = v_k^a\left(\frac{p_\perp}{2}\right) + v_k^{\bar{b}}\left(\frac{p_\perp}{2}\right) + \mathcal{O}(v^3), \quad k = 1, 2, \dots$$

$$v_k^B(p_\perp) = v_k^a\left(\frac{p_\perp}{3}\right) + v_k^b\left(\frac{p_\perp}{3}\right) + v_k^c\left(\frac{p_\perp}{3}\right) + \mathcal{O}(v^3)$$



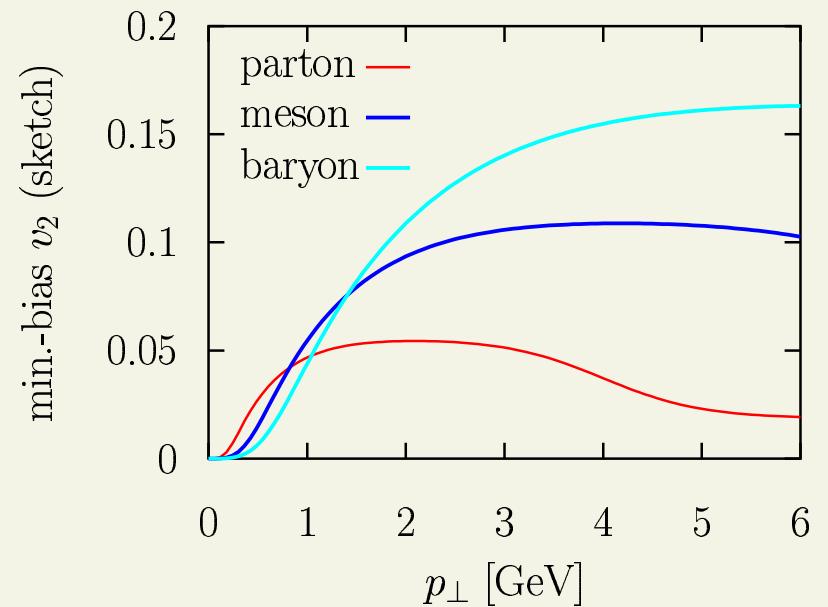
⇒ **hadron  $v_2$  amplified at high  $p_\perp$**

if all quarks have same  $v_2$ :

3× for baryons    }    50% larger  $v_2$   
2× for mesons    }

“  $v_2^h(p_\perp) \approx n \times v_2^q(p_\perp/n)$  ”

→ 5× for pentaquark, 6× for deuteron



- if  $v_2$  depends on quark flavor, further differentiation by flavor content

# Determination of parton $v_2$

Via measuring  $v_2(p_\perp)$  for various hadrons, one can extract quark flows AND test consistency of coalescence model

Consider, e.g.,  $v_2^q \neq v_2^s$  (light vs. strange)

$$\begin{array}{lll}
 v_2^\pi(p_\perp) & \approx & 2v_2^q\left(\frac{p_\perp}{2}\right) \\
 v_2^K(p_\perp) & \approx & v_2^q\left(\frac{p_\perp}{2}\right) + v_2^s\left(\frac{p_\perp}{2}\right) \\
 v_2^\phi(p_\perp) & \approx & 2v_2^s\left(\frac{p_\perp}{2}\right) \\
 \\ 
 v_2^p(p_\perp) & \approx & 3v_2^q\left(\frac{p_\perp}{3}\right) \\
 v_2^{\Lambda,\Sigma}(p_\perp) & \approx & 2v_2^q\left(\frac{p_\perp}{3}\right) + v_2^s\left(\frac{p_\perp}{3}\right) \\
 v_2^{\Xi}(p_\perp) & \approx & v_2^q\left(\frac{p_\perp}{3}\right) + 2v_2^s\left(\frac{p_\perp}{3}\right) \\
 v_2^\Omega(p_\perp) & \approx & 3v_2^s\left(\frac{p_\perp}{3}\right)
 \end{array}$$

2 unknowns, 7 equations  $\Rightarrow$  e.g.,

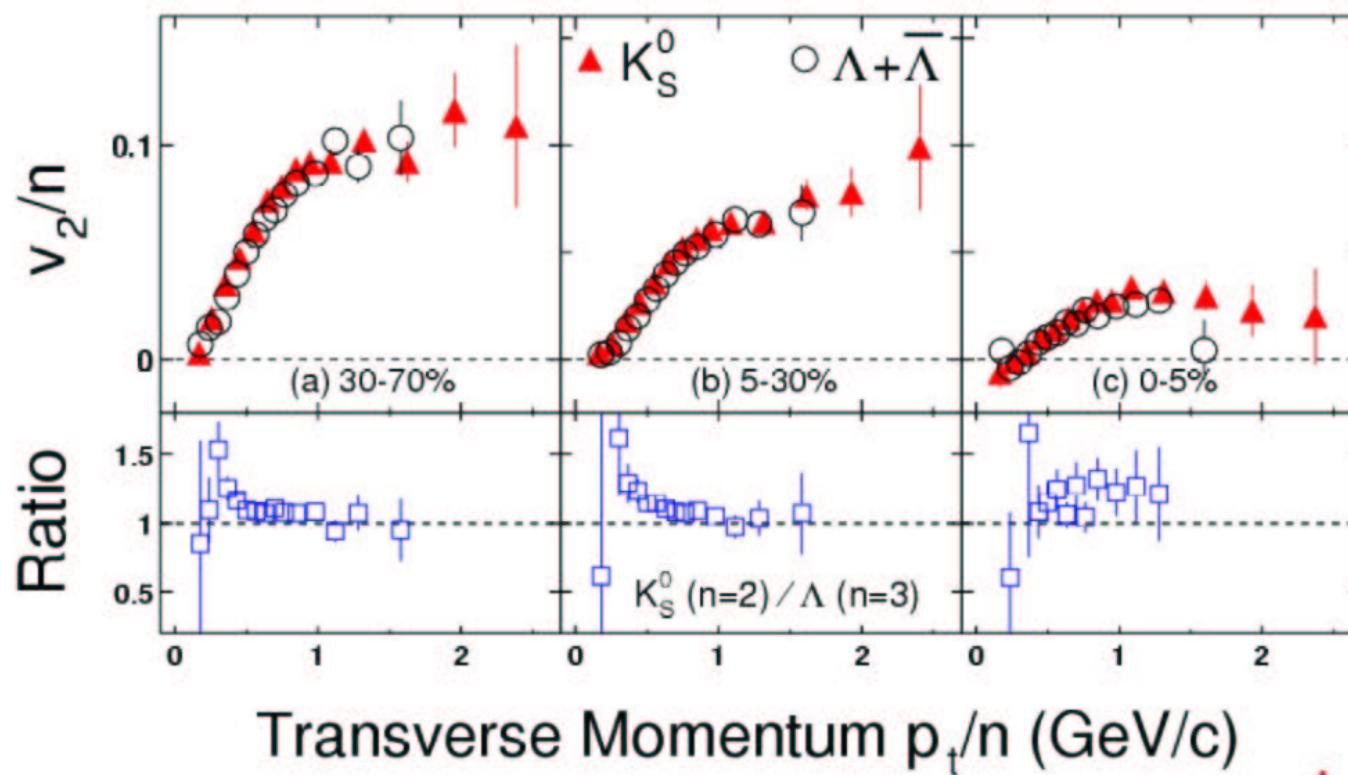
$$\begin{aligned}
 v_2^q(p_\perp) &= v_2^\pi(2p_\perp)/2 = v_2^p(3p_\perp)/3 = v_2^\Lambda(3p_\perp) - 2v_2^p(3p_\perp)/3 \\
 v_2^s(p_\perp) &= v_2^\phi(2p_\perp)/2 = v_2^\Omega(3p_\perp)/3 = v_2^K(2p_\perp) - v_2^\pi(2p_\perp)/2
 \end{aligned}$$

**Simplest  $v_2^q = v_2^s$  case:**  $v_2^{hadron}(np_\perp)/n$  universal fn. ( $n = 2, 3$  for meson,baryon)

# Experimental test of $v_2$ scaling

first good news - STAR, SQM2003:  $K_0^S, \Lambda$

This **parton coalescence** rescaling seems to work for each of our centrality intervals

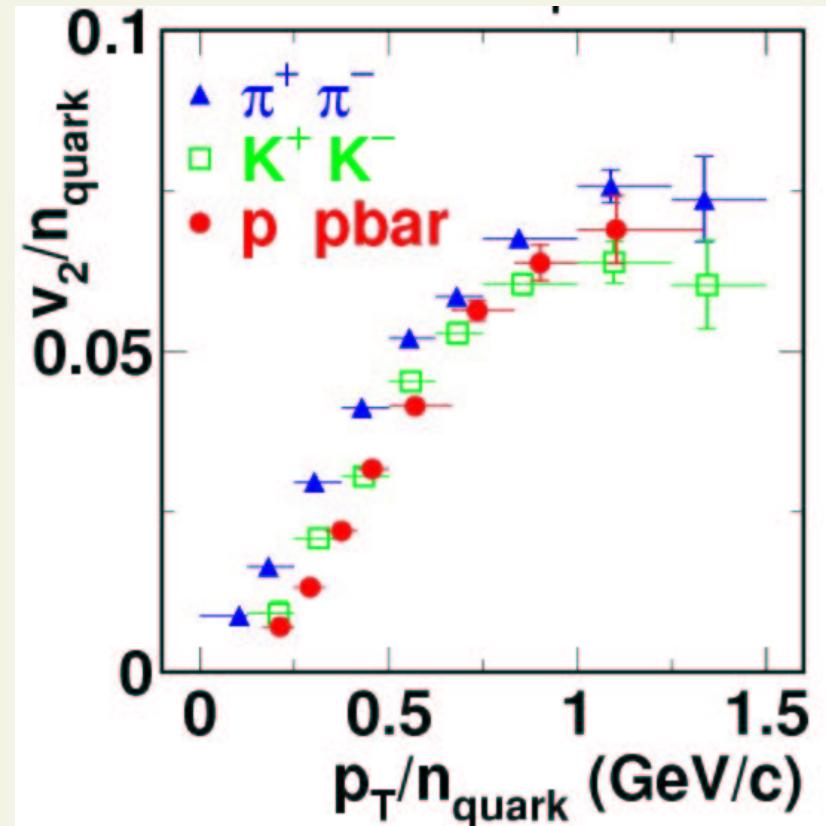
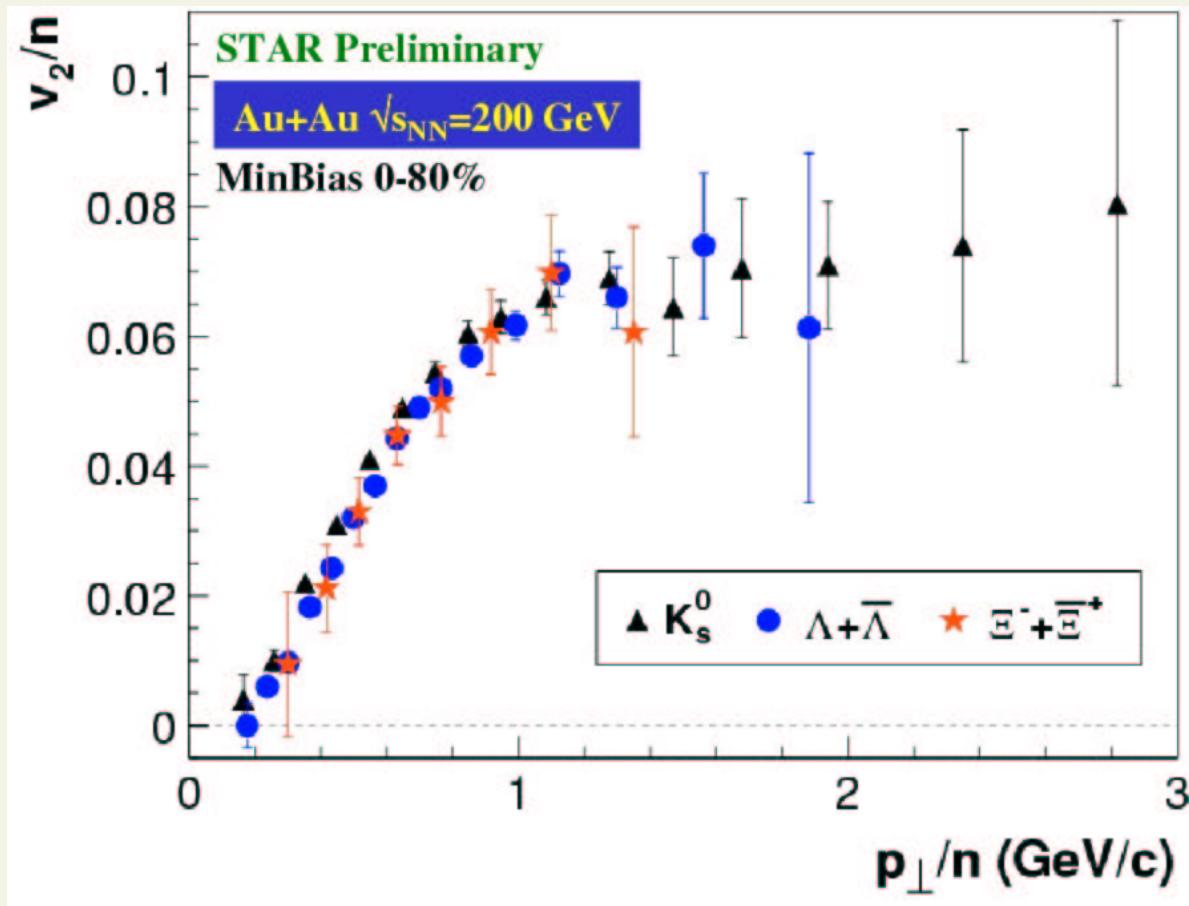


Paul Sorensen



Castillo [STAR], HIC03:  $\Xi$  flow

PHENIX, PRL91 ('03):  $\pi, K, p$

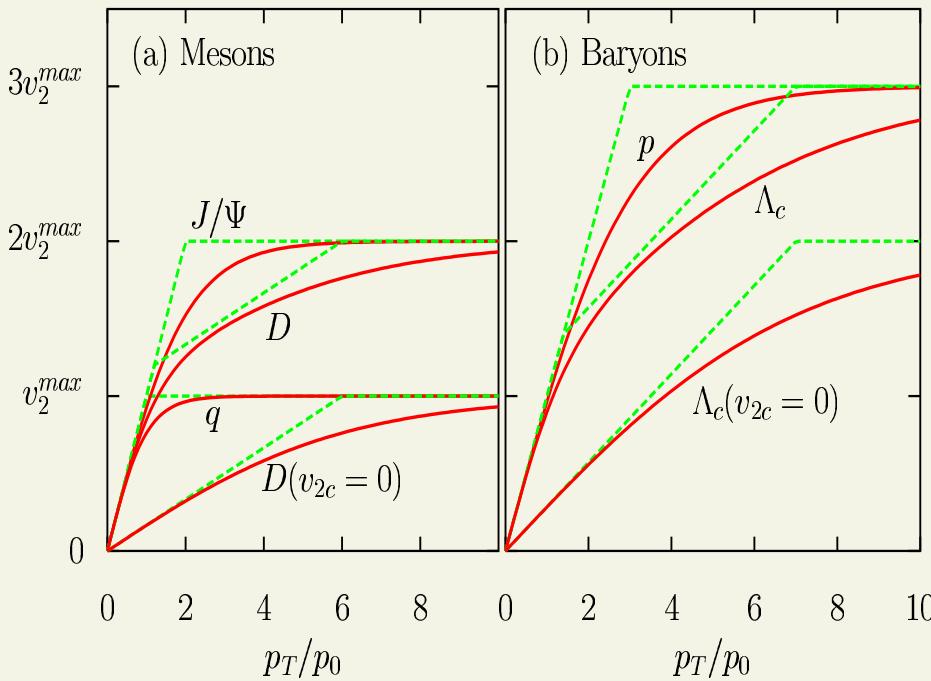


- coalescence predictions confirmed for  $\pi, K, K_0, p, \Lambda, \Xi$  - also for  $\Omega, \phi$  (poor statistics)
  - pions are little off the curve, likely due to resonance decays [Greco & Ko]
- surprisingly, RHIC data indicate  $v_2^q \approx v_2^s$

# Elliptic flow for charm

## generic expectations:

Lin & D.M. ('03)

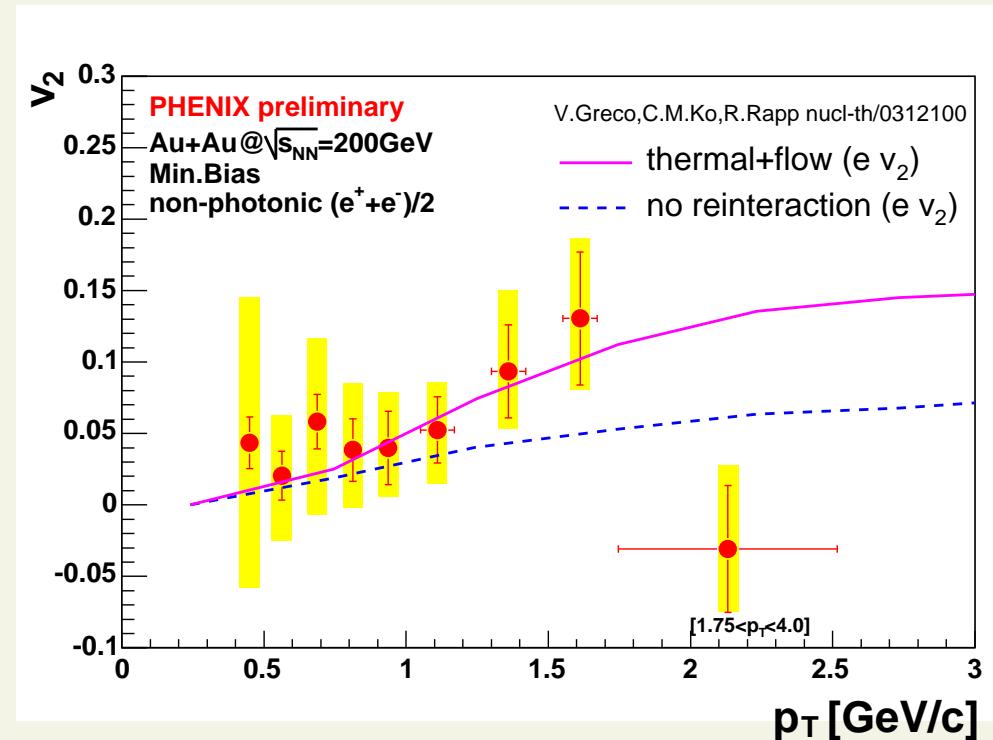


simpistic (linear & flat); more realistic (based on parton cascade MPC)

- $v_2(p_\perp)$  rises slower, saturates later for asymmetric systems ( $D$ ,  $D_s$ ,  $\Lambda_c$ )
  - heavy quark carries most of the hadron momentum (momentum proportional to constituent mass)
- nonzero  $v_2$  for  $D$ ,  $D_s$ ,  $\Lambda_c$  even for zero charm  $v_2$  (no thermalization)

## decay electron $v_2$ : $D \rightarrow K \nu e$ , $D \rightarrow K^* \nu e$

Kaneta, nucl-ex/0404014; Greco & Ko, nucl-th/0312100:



e's from hadron decays and  $\gamma$ -conversion subtracted  
 $\equiv$  "non-photonic"

- need better data to discriminate between scenarios with or without charm flow

# **Open issues and next steps**

# A few recent issues

- **entropy:**  $2 \rightarrow 1, 3 \rightarrow 1 ??$ 
  - globally, problem is cured by resonance decays and larger  $s/n$  for massive particles
  - locally, the issue is still interesting (but less of a worry)
- **binding energy:** only relevant for Goldstones  $\pi$  &  $K$ 
  - likely a small correction because resonance channels  $K^* \rightarrow K; \rho, \omega, \Delta, \dots \rightarrow \pi$  dominate production
- **extend to low- $p_T$  regime:** simple formula not applicable (unitarity)
  - does coal w/ unitarity still reduce to statistical models/hydrodynamics?
- **constituents vs partons:** so far - two, mostly independent components
  - Hwa & Yang: hadrons made of constituents, which in turn consist of partons limited to momentum space → need extension to 6D phasespace
- **hadron correlations:** coal dilutes jetlike correlations??
  - not necessarily: constituent correlations possible  $\Rightarrow$  map uniquely to hadron correlations
  - a correlation source: parton showers [Hwa & Yang, nucl-th/0401001]

# Coalescence dynamics - puzzle

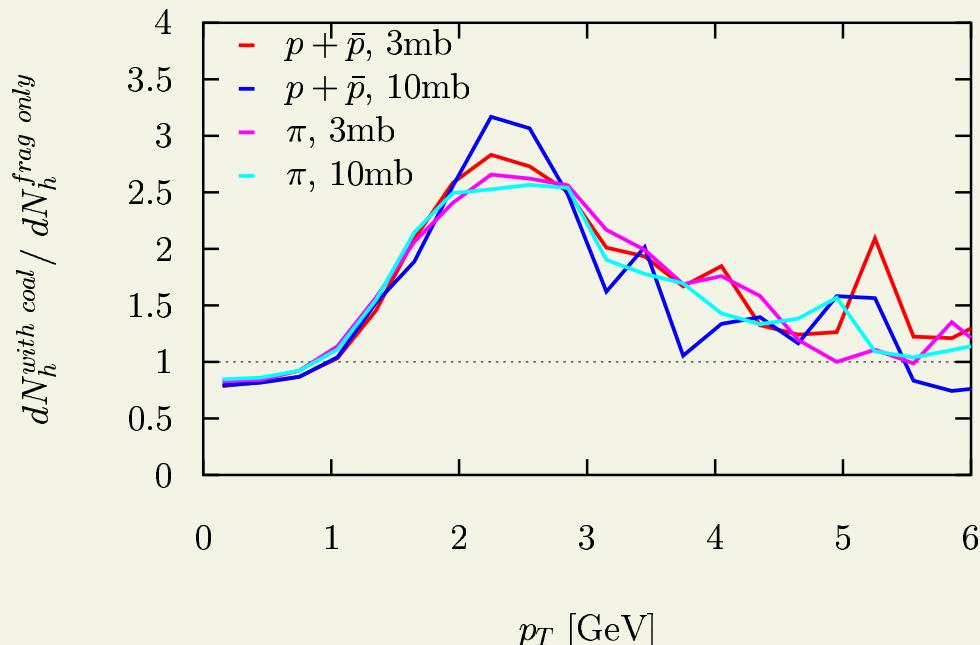
parton transport + coalescence (e.g., MPC, AMPT, VNI/BMS)

hadronization:

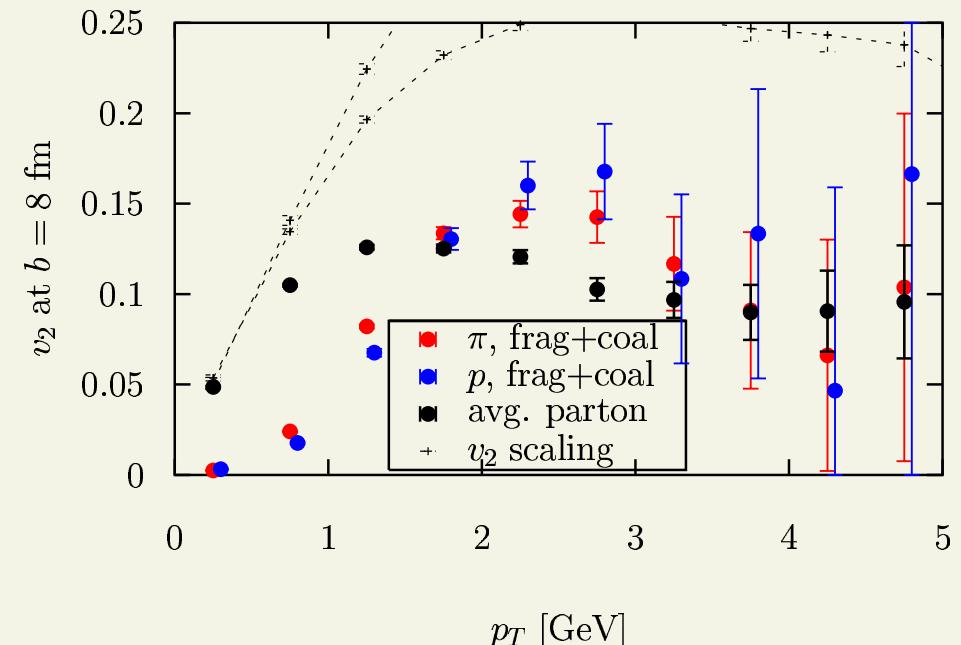
- partons nearby in phasespace coalesce, those without a partner fragment
- weakly-bound hadrons

Au+Au @ 200 GeV,  $b=8$  fm - hadron enhancement

D.M. ('04):



elliptic flow



- **2 – 3× enhancement over fragmentation for  $1.5 < p_T < 4$  GeV**

**but same for protons as pions  $\Rightarrow p/\pi$  stays low**  
 **$\rightarrow$  baryons have more constituents  $\Rightarrow$  are more “fragile”**

- **flow amplification negated**
  - $\rightarrow$  25-30% frag contribution, reduces flow
  - $\rightarrow$  **dynamical correlations**,  $f(\vec{x}, \vec{p}) \neq F(\vec{x})g(\vec{p})$
  - $\rightarrow$  imperfect scaling
- **baryon-meson splitting disappeared**

# Summary

- hadronization via quark coalescence is important at RHIC, dominates hadroproduction at intermediate  $1.5 < p_\perp < 4 - 6$  GeV
- both the observed large baryon/meson ratios and elliptic flow scaling with quark number (baryon flow enhancement) follow from the simple coalescence formulas - predictions for  $\phi$ ,  $\Omega$ , and charm hadron flow are yet to be tested
  - ⇒ strong indication for quark degrees of freedom at hadronization for narrow wave functions, elliptic flow of quarks can be extracted from that of hadrons via simple relations
- spacetime dynamics ( $x-p$  correlations, “diffuse” freezeout) can significantly influence both the flow scaling and baryon/meson ratio enhancement
  - for weakly-bound hadrons, baryon production is reduced because baryons are more fragile than mesons (except for the “sudden” hadronization scenario)
  - ⇒ further studies required to find a dynamical approach that preserves features of the simple models