Even a small viscosity matters

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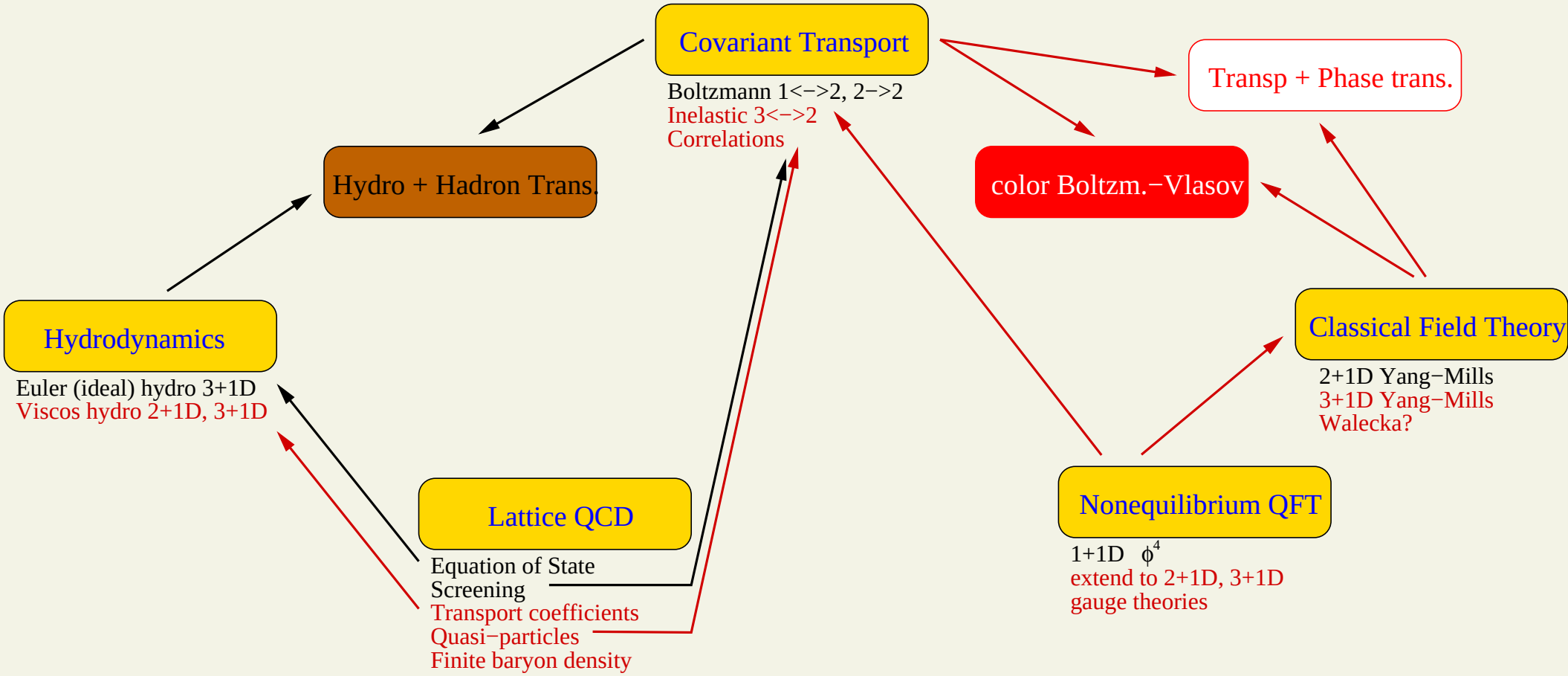
Dynamical frameworks



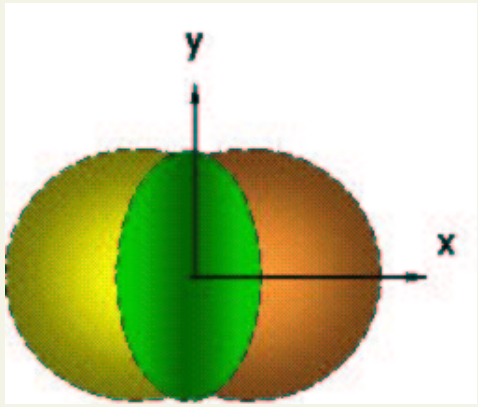
lots of progress - but still a long way to go

Observations at RHIC

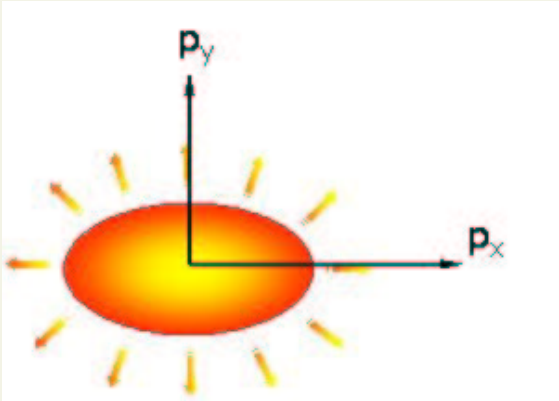
- **strong attenuation of energetic particles** **"jet quenching"**
- **remarkable "elliptic flow"** (v_2) **collectivity**

spatial anisotropy $\longrightarrow$ **final azimuthal momentum anisotropy**

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



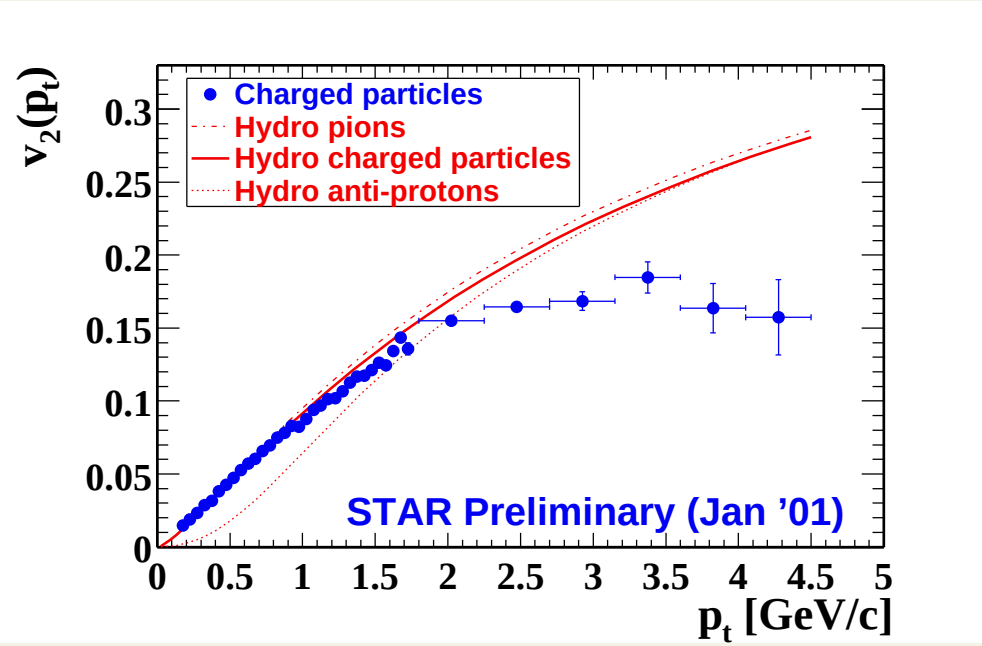
$$\longrightarrow \quad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$



RHIC - large $2:1$ anisotropy

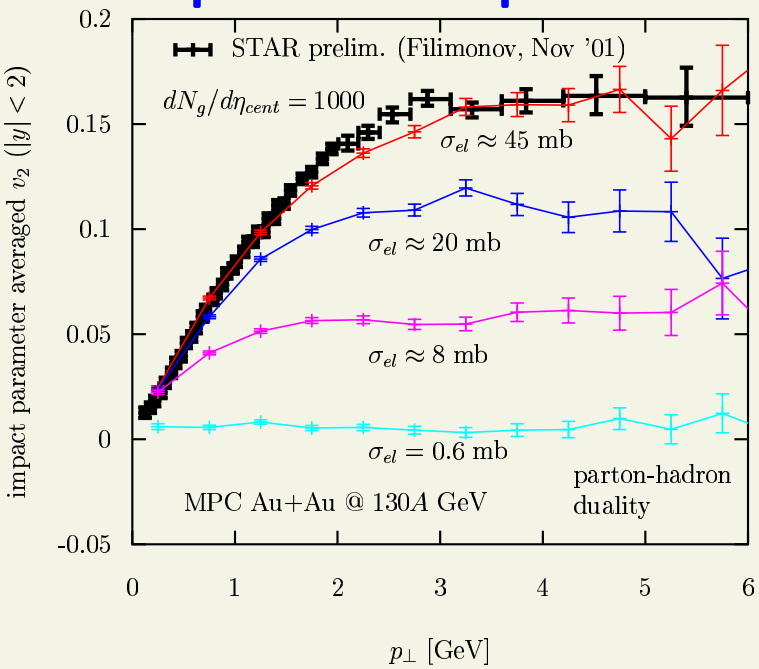
Kolb, Heinz et al

ideal hydrodynamics



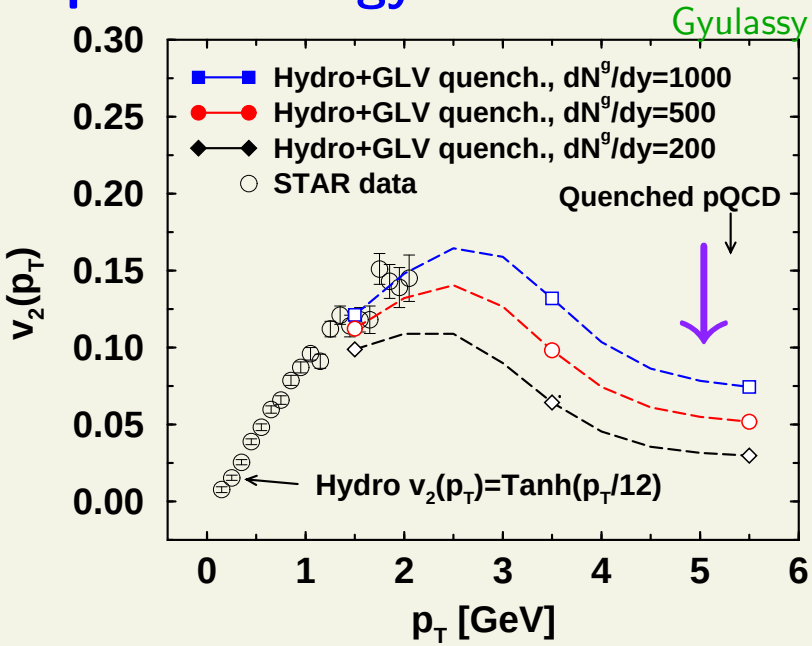
DM & Gyulassy

covariant parton transport



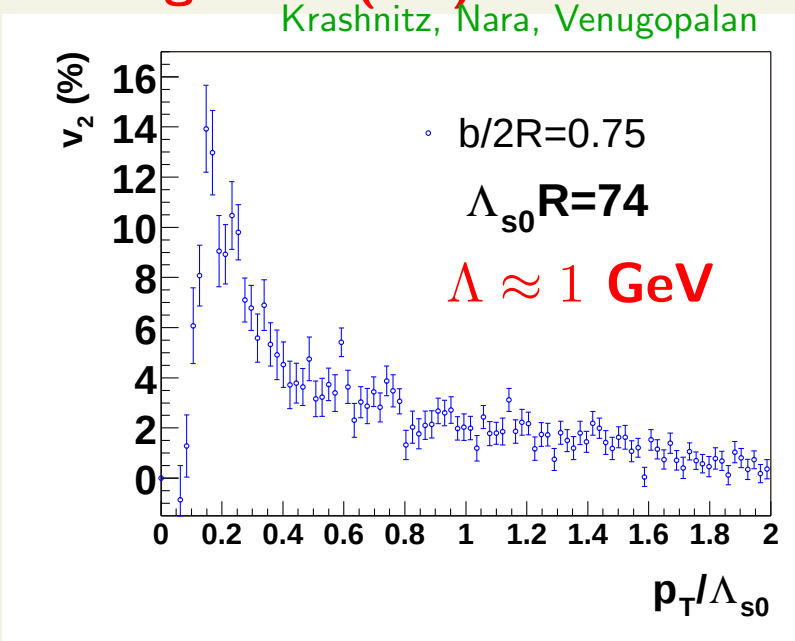
parton energy loss...

Gyulassy & Vitev



classical Yang-Mills (2D)...

Krashnitz, Nara, Venugopalan



Parton transport theory

Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, ...

Covariant, nonequilibrium approach - formulated in terms of local rates.

Transport eqn: $f_i(\vec{x},\vec{p},t)$ **- quark/gluon phase space distributions**

$$p^\mu \partial_\mu f_i(\vec{x},\vec{p},t) = \overbrace{S_i(\vec{x},\vec{p},t)}^{\text{source}} + \overbrace{C_i^{el.}[f](\vec{x},\vec{p},t)}^{2\to 2\ (\mathrm{ZPC},\mathrm{GCP},\ldots)} + \overbrace{C_i^{inel.}[f](\vec{x},\vec{p},t)}^{2\leftrightarrow 3\ (\mathrm{MPC},\mathrm{Xu-Greiner})} + \ldots$$

algorithms: OSCAR code repository @ http://nt3.phys.columbia.edu/OSCAR

HERE: utilize MPC model/algorithm DM, NPA 697 ('02)

Example: Molnar's Parton Cascade

Elementary processes: elastic $2 \to 2$ **processes** + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \to q'\bar{q}'$ + $ggg \leftrightarrow gg$

Equation for $f^i(x,\vec{p})$**:** $\quad i = \{g, d, \bar{d}, u, \bar{u}, \ldots\}$

$2 \to 2$

$$
\begin{aligned}
p_1^\mu \partial_\mu \tilde{f}^i(x,\vec{p}_1) \quad = \quad & \frac{\pi^4}{2} \sum_{jkl} \iiint\limits_{2\ 3\ 4} \left(\tilde{f}^k_3 \tilde{f}^l_4 - \tilde{f}^i_1 \tilde{f}^j_2 \right) \left| \bar{\mathcal{M}}^{i+j\to k+l}_{12\to 34} \right|^2 \delta^4(12-34) \\
& + \frac{\pi^4}{12} \iiiint\limits_{2\ 3\ 4\ 5} \left(\frac{\tilde{f}^i_3 \tilde{f}^i_4 \tilde{f}^i_5}{g_i} - \tilde{f}^i_1 \tilde{f}^i_2 \right) \left| \bar{\mathcal{M}}^{i+i\to i+i+i}_{12\to 345} \right|^2 \delta^4(12-345) \\
& + \frac{\pi^4}{8} \iiiint\limits_{2\ 3\ 4\ 5} \left(\tilde{f}^i_4 \tilde{f}^i_5 - \frac{\tilde{f}^i_1 \tilde{f}^i_2 \tilde{f}^i_3}{g_i} \right) \left| \bar{\mathcal{M}}^{i+i\to i+i+i}_{45\to 123} \right|^2 \delta^4(123-45) \\
& + \tilde{S}^i(x,\vec{p}_1)
\end{aligned}
$$

$2 \leftrightarrow 3$

$3 \leftrightarrow 2$

$\leftarrow$ **initial conditions**

with shorthands:

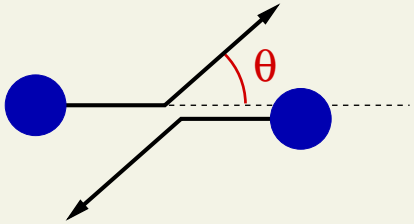
$$
\tilde{f}^q_i \equiv (2\pi)^3 f_q(x,\vec{p}_i), \quad \int\limits_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \quad \delta^4(p_1+p_2-p_3-p_4) \equiv \delta^4(12-34)
$$

mean free path: characterizes local conditions

$$\lambda(x) \equiv \frac{1}{\textbf{cross section} \times \textbf{density(x)}} \qquad \begin{cases} \lambda = 0 & - \text{ideal hydrodynamics} \\ \lambda = \infty & - \text{free streaming} \end{cases}$$

transport opacity: time-integrated, spatially averaged [DM & Gyulassy NPA 697 ('02)]

$$\chi \equiv \langle n_{coll} \rangle \langle \sin^2 \theta_{CM} \rangle \quad \sim \# \text{ of collisions per parton} \times \text{mom. transfer efficiency}$$



$$\chi = \int dz\, \rho(z)\, \sigma_{transp} = \int dz\, \frac{1}{\lambda_{tr}(z)}$$

near equilibrium: related to transport coefficients (viscosity, diffusion constants)

e.g., $\eta \sim \frac{T}{\sigma_{tr}}$

Connection to hydro

- **energy-momentum tensor:** $T^{\mu\nu}(x) = \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(x,\vec{p})$

  charge current: $N_c^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu c_i f_i(x,\vec{p})$

  entropy current: $S^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu f_i(x,\vec{p}) \left\{1 - \ln[f_i(x,\vec{p})h^3]\right\}$

  conservation laws: $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu N_c^\mu = 0$

  dissipation: $\partial_\mu S^\mu \geq 0 \quad \Rightarrow$ **entropy production, in general**

- **Ideal fluid approximation** $\lambda \to 0$: **local equilibrium** $f = (2\pi)^{-3} \exp[p_\mu u^\mu(x)/T(x)]$

$$T_{id}^{\mu\nu} = (e+p)u^\mu u^\nu - p\, g^{\mu\nu}$$

$$\partial_\mu S^\mu = 0 \quad \Rightarrow \text{entropy conserved}$$

- **Navier-Stokes** $\lambda \ll length\ \&\ time\ scales$: **near equilibrium - slowly varying**

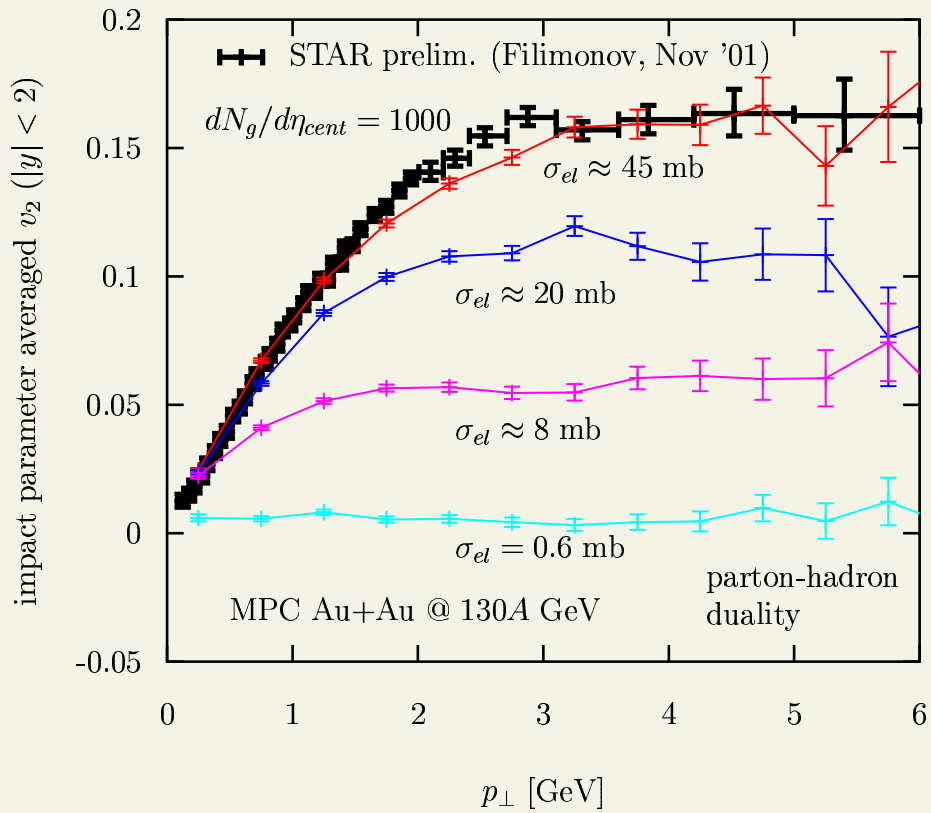
$n_B = 0$ $\qquad$ $\eta \propto T/\sigma_{tr}$

$$T_{NS}^{\mu\nu} = T_{id}^{\mu\nu} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \zeta \Delta^{\mu\nu}\nabla_\alpha u^\alpha \quad \to \text{shear \& bulk viscosity}$$

$$\partial_\mu S_{NS}^\mu = \tfrac{\eta}{2T}(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha)^2 + \tfrac{\zeta}{T}(\nabla_\mu u^\mu)^2 \quad \to \text{entropy production}$$

Parton transport and RHIC

DM & Gyulassy, NPA 697 ('02): $v_2(p_T, \chi)$



parton transport model MPC

$2 \to 2$ **only**

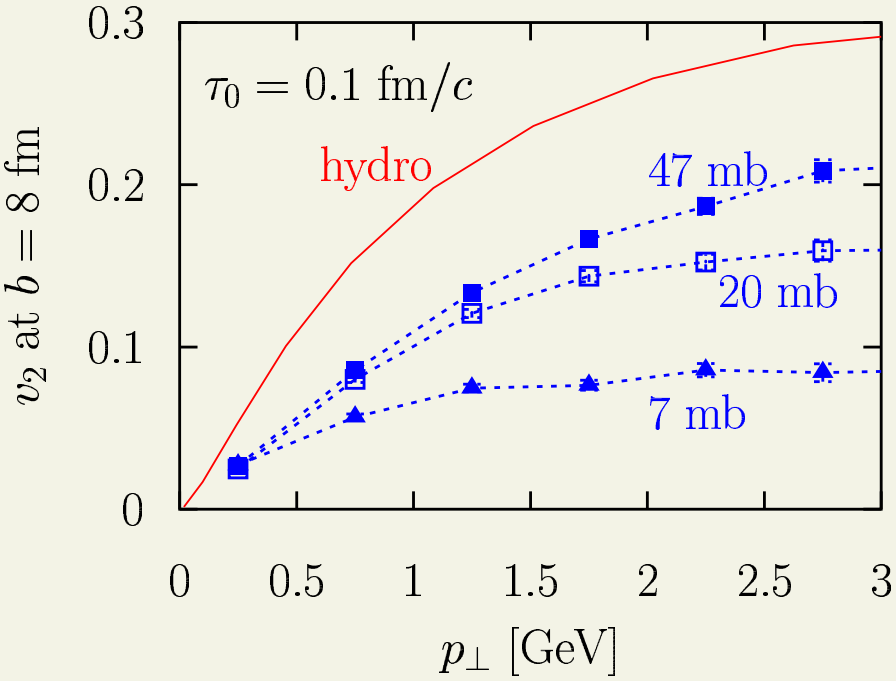
Au+Au @ 130 GeV, $b = 8$ fm

- minijet initconds
- 1 parton $\longrightarrow$ 1 π hadronization

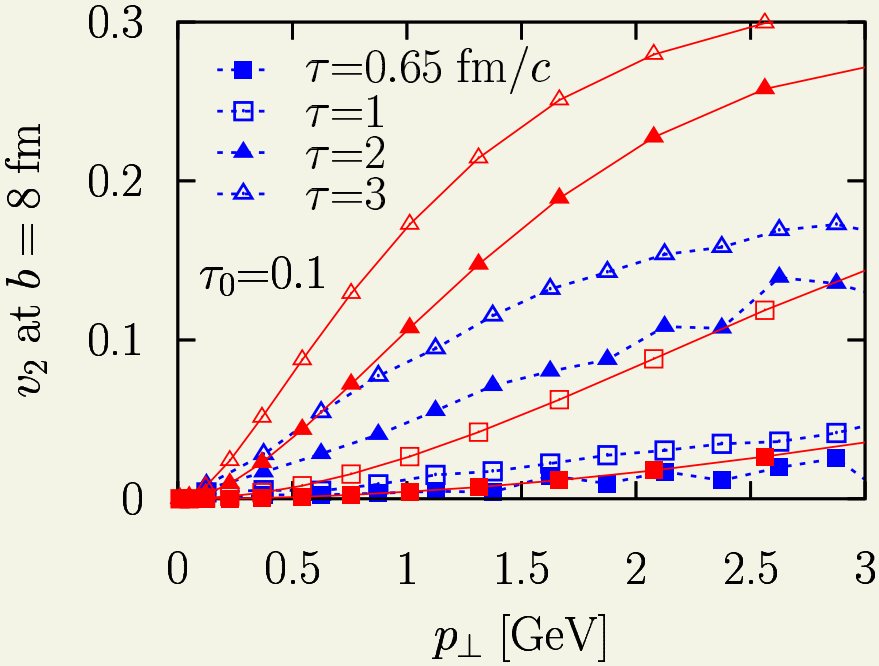
$15\times$ **perturbative rates needed**

No, still not ideal fluid

DM & Huovinen, PRL94 ('05): **final** $v_2(p_T)$



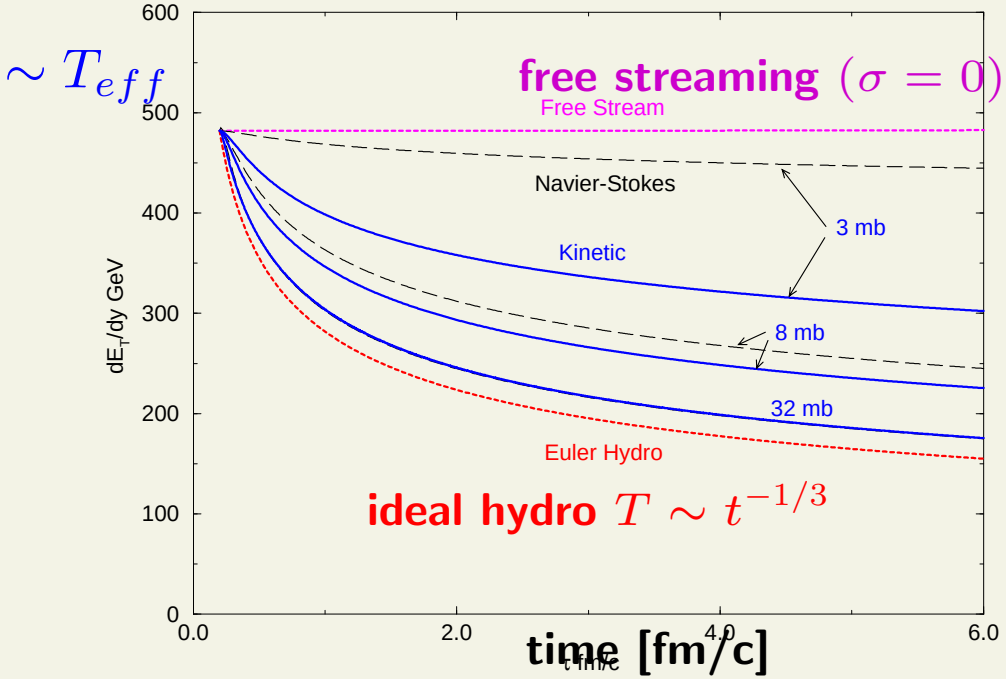
$v_2(\tau, p_T)$



→ **dense, strongly-interacting system, but still dissipative**

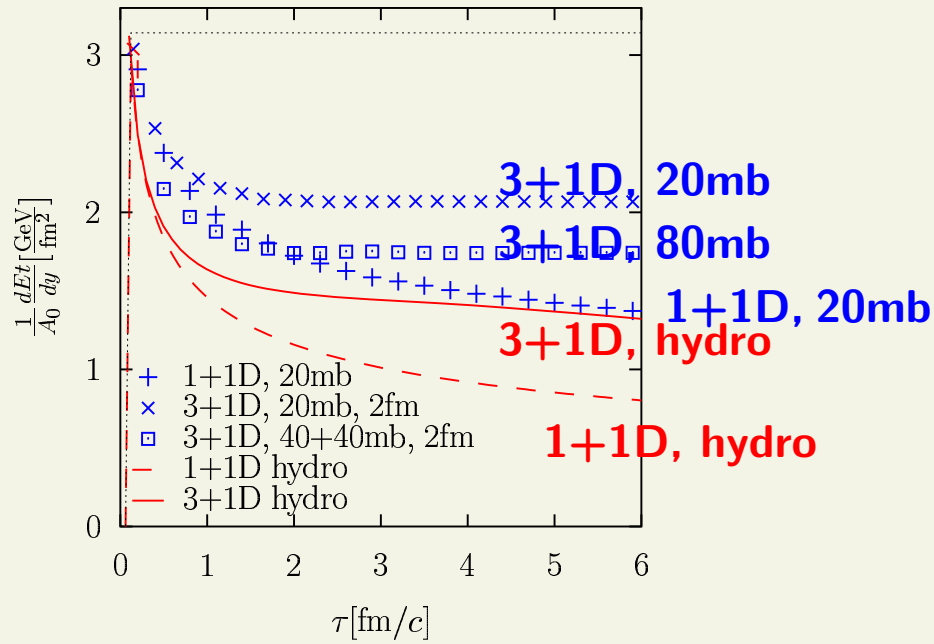
strong gradients → even a (very) small viscosity matters

Gyulassy, Pang, Zhang ('97): 1+1D kinetic

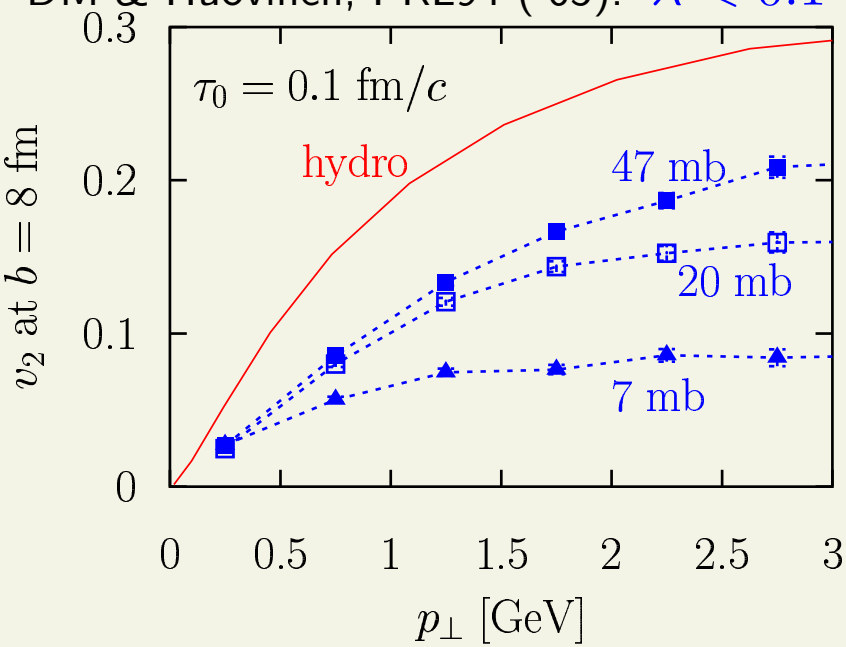


DM & Gyulassy ('00): 3+1D kinetic theory

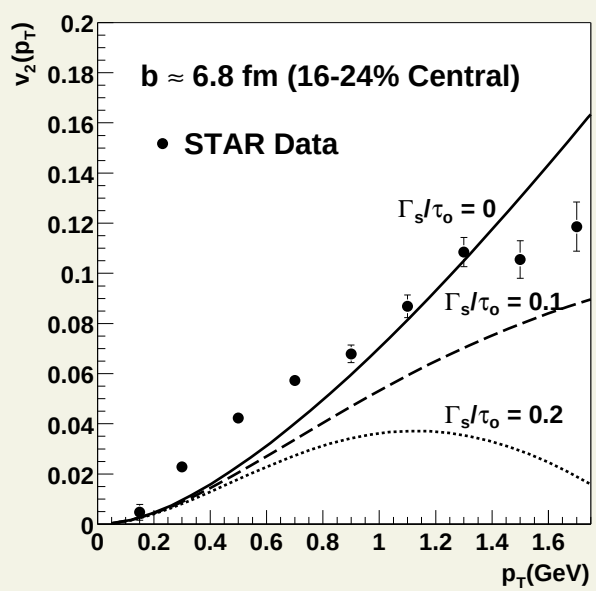
MPC vs hydro (1+1D and 3+1D)



DM & Huovinen, PRL94 ('05): $\lambda < 0.1$ **fm**



Teaney ('04): $\Gamma_s \equiv 4(\eta/s)/3T$



← **minimal** Γ_s

"Minimal" viscosity

$\sigma_{gg} \sim 50$ **mb** $\Rightarrow \lambda_{MFP}$ **is very short**

quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

$\Rightarrow$ **kinetic theory:** $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz '85

viscosity: $\eta = s\frac{\lambda T}{5}$ $\Rightarrow$ **minimal viscosity** $\eta/s \geq 1/15$

"universal" conjecture: $\eta/s \geq 1/4\pi$ Son et al ('02), ('04) - $\mathcal{N} = 4$ SYM

very short MFP at RHIC $\rightarrow$ **most ideal fluid possible? ("perfect fluid")**

Viscous hydro (Navier-Stokes)

zero baryon density limit: $\partial_\mu T^{\mu\nu} = 0$ **with**

$$T^{\mu\nu} = (\varepsilon+p)u^\mu u^\nu - pg^{\mu\nu} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha, \qquad \Delta^{\mu\nu}=g^{\mu\nu}-u^\mu u^\nu \quad \nabla^\mu=\Delta^{\mu\nu}\partial_\nu$$

η, ζ - **shear and bulk viscosities**

sound attenuation: $\omega(k) = c_s k - \frac{i}{2}\frac{1}{\varepsilon+p}\left(\zeta + \frac{4}{3}\eta\right)k^2 = c_s k - \frac{i}{2}k^2\Gamma_s \qquad (c_s^2 \equiv \frac{\partial\varepsilon}{\partial p})$

slower cooling in 1D Hubble flow: $\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon+p}{\tau}\left(1 - \frac{\Gamma_s}{\tau}\right)$ Gyulassy & Danielewicz '85

in heavy ion collisions, large gradients $\Rightarrow$ even a small viscosity can matter

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I think that there is a moral to this story, namely that it is more important to have beauty in one's equations that to have them fit experiment.

— P.A.M. Dirac

It is advances in theory, more than anything else, that

Millennium Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven Prize Problems. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each. During the **Millennium Meeting** held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled The Importance of Mathematics, aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

One hundred years earlier, on August 8, 1900, David Hilbert delivered his famous lecture about open mathematical problems at the second International Congress of Mathematicians in Paris. This influenced our decision to announce the millennium problems as the central theme of a Paris meeting.

The **rules** for the award of the prize have the endorsement of the CMI Scientific Advisory Board and the approval of the Directors. The members of these boards have the responsibility to preserve the nature, the integrity, and the spirit of this prize.

Paris, May 24, 2000

Please send inquiries regarding the Millennium Prize Problems to **prize.problems@claymath.org**.

Birch and Swinnerton-Dyer Conjecture
Hodge Conjecture
Navier-Stokes Equations
P vs NP
Poincaré Conjecture
Riemann Hypothesis
Yang-Mills Theory

Rules
Millennium Meeting Videos

DM @ RHIC/AGS Users' Mtg (June 2005):

estimate effect of "minimal" viscosity $\eta/s \equiv \kappa = 1/(4\pi)$: DM '05

for 1D Bjorken expansion, ideal gas $\epsilon = 3p \propto T^4$, $\partial_i u_j \neq 0$ ($i, j \in \{0, z\}$)

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + p}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2} \approx \frac{4}{3}\kappa\frac{\epsilon + p}{T\tau^2}$$

$$\Rightarrow \frac{T(\tau)}{T_0} = \left(\frac{\tau_0}{\tau}\right)^{1/3}\left[1 + \frac{2\kappa}{3\tau_0 T_0}\left(1 - \left(\frac{\tau_0}{\tau}\right)^{2/3}\right)\right]$$

entropy production already relevant during QGP phase at RHIC

$$\frac{\tau s}{\tau_0 s_0} = \frac{\tau T^3}{\tau_0 T_0^3} \approx 1.1 - 1.2$$

($\tau_0 = 0.6$ fm/c, $T_0 \sim 300$ MeV, $\tau_{QGP} \sim 3 - 5$ fm/c, $\kappa = 1/4\pi$)

even more in realistic case (not uniform 1D, also mixed phase)

challenge: naive Navier-Stokes formulation is severely acausal

causal formulation: viscosities AND microscopic relaxation times Israel & Stewart

$$e.g. \quad \dot{\pi}_{ij}(x) = -\frac{\pi_{ij}(x) - \pi_{ij}^{naive}(x)}{\tau_{rel}}$$

$\Rightarrow$ **these could be inferred from heavy-ion data**

2+1D algorithms are being developed ... Muronga, Teaney, Chaudhuri, Heinz ...

heavy-ion collisions: even viscous version breaks down eventually
$\longrightarrow$ **decoupling problem**

$\longrightarrow$ **opaque transport gives in fact Navier-Stokes + self-consistent decoupling**

η/s for transport

"minimal" viscosity - corresponds to $\lambda_{tr} \approx 1/(3T_{eff}) \approx 0.1$ **fm**

estimate from χ**: assuming 1D Bjorken expansion**

$$\chi = \int dz\, \rho(z)\sigma_{tr} \sim \int d\tau \rho_0 \frac{\tau_0}{\tau}\sigma = \frac{\tau_0}{\lambda_{tr}(\tau_0)} \ln \frac{L}{\tau_0}$$

for $b = 8$ **fm @ RHIC, transport gives** $\chi \approx 20$

$\Rightarrow \lambda_{tr}(\tau_0) \sim 1.5 - 2 \times 10^{-2}$ **fm (!)**

estimate from average density:

$$\lambda_{tr}(\tau_0) = \frac{\tau_0 A_T}{\sigma_{tr} dN/d\eta} \sim 1 - 2 \times 10^{-2} \text{ fm}$$

note: nonuniform - where most action is, it is even smaller

$\Rightarrow \sigma_{gg} = 50$ **mb is already better than best-case scenario**