GLAUBER MODEL:

FOUNDATION, APPLICATIONS, AND UNCERTAINTIES

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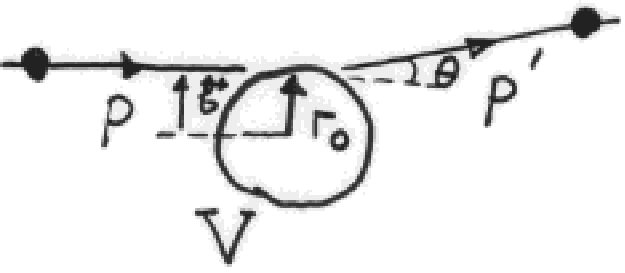
1. Eikonal approximation in quantum mechanic. and field theory
2. Glauber-Gribov multiple scattering theory
3. Application to nuclear collisions
4. Approximations and uncertainties

1. Eikonal approximation

consider the scattering amplitude



$$f(s,t) = \frac{1}{2ip} \sum_\ell (2\ell+1)\left[e^{2i\delta_\ell} - 1\right] P_\ell(\cos\theta)$$

at high p, the number of partial waves is large: $n_\ell \sim p \cdot r_0 \gg 1$

introduce impact parameter b:

$$pb = \ell + 1/2$$

At large ℓ, small θ

$$P_\ell(\cos\theta) \simeq \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i(2\ell+1)\sin\frac{\theta}{2}\cos\varphi} =$$

$$= \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i\vec{q}\vec{b}} \; ; \qquad \text{we used } (2\ell+1)\sin\tfrac{\theta}{2}\cos\varphi = 2p\sin\tfrac{\theta}{2}\,\frac{\ell+1/2}{p}\cdot$$

$$\cdot\cos\varphi = \vec{q}\cdot\vec{b}$$

$$\Rightarrow \quad \boxed{f(s,t) \simeq \frac{ip}{2\pi}\int d^2\vec{b}\, e^{i\vec{q}\vec{b}}\left[1 - e^{i\chi(s,b)}\right]} \qquad \vec{q} \simeq \vec{q}_\perp, \; t \simeq -\vec{q}_\perp^{\,2}$$

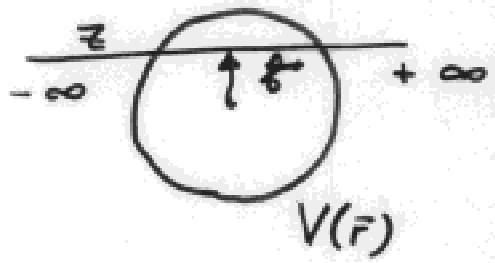
$$\chi(s,b) \equiv 2\delta(s,b)$$

$\sum_\ell \to \int d\vec{b}$

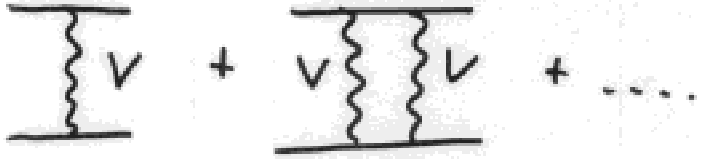
"EIKONAL APPROXIMATION"

For potential scattering,

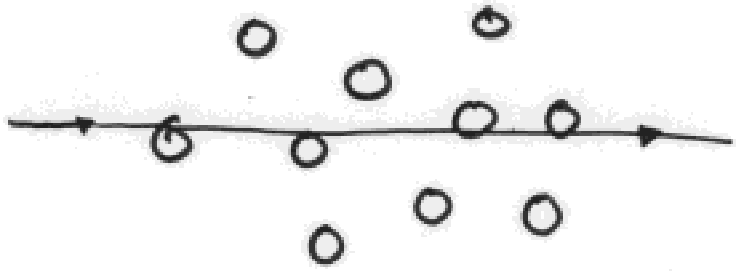
phase shift $$\chi(b) = -\frac{1}{v}\int_{-\infty}^{+\infty} V(r)\,dz$$



$$f(s,t) = \frac{ip}{2\pi}\int d^2b\, e^{i\bar{q}\bar{b}}\left[1 - e^{-\frac{i}{v}\int V(r)dz}\right]$$

expand exponential => 

- Consider scattering on a composite system:



assume the scatterings on different nucleons are independent; then

$$\chi(\bar{b}, \bar{r}_1, \bar{r}_2, \ldots \bar{r}_A) = \sum_{j=1}^{A} \chi_j(\bar{b} - \bar{s}_j)$$

$$F_{fi}^{A}(t) = \frac{ip}{2\pi}\int d^2b\, e^{i\bar{q}\bar{b}} \langle f | 1 - \prod_{j=1}^{A} e^{i\chi_j(\bar{b}-\bar{s}_j)} | i\rangle =$$

$$= \frac{ip}{2\pi}\int d^2b\, e^{i\bar{q}\bar{b}} \langle f | 1 - \prod_{j=1}^{A}[1 - \gamma_j(\bar{b}-\bar{s}_j)] | i\rangle$$

where $\gamma(b) = \frac{1}{2\pi i p}\int d^2q\, e^{-i\bar{q}\bar{b}} f(\bar{q})$ is the elementary scattering amplitude

- Physical meaning ?

i) consider purely imaginary $f(\vec{q}) \Rightarrow \gamma_j(\vec{b}-\vec{s}_j)$ is probability of absorptio

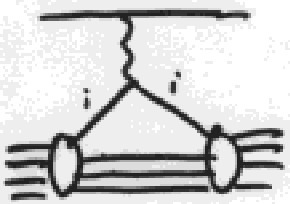
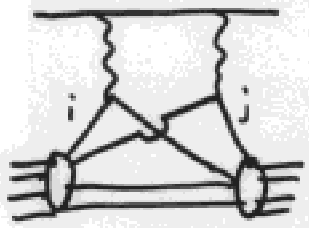
$$F_{fi}(t) = \frac{ip}{2\pi}\int d^2b\, e^{i\vec{q}\vec{b}} \langle f| 1 - \prod_{j=1}^{A}\{1-\gamma_j(\vec{b}-\vec{s}_j)\}|i\rangle$$

$\prod_{j=1}^{A}\{1-\gamma_j(\vec{b}-\vec{s}_j)\}$: Prob. of not being absorbed by the nucl ... by any of the nucleo

$1 - \prod_{j=1}^{A}\{1-\gamma_j(\vec{b}-\vec{s}_j)\}$: Probability of particle absorption by any of the nucleons

ii) $$\Gamma(\vec{b}, \vec{r}_1, \ldots \vec{r}_A) \equiv 1 - \prod_{j=1}^{A}\{1-\gamma_j(\vec{b}-\vec{s}_j)\} =$$

$$= \sum_{i=1}^{A}\gamma_i(\vec{b}-\vec{s}_i) + \sum_{i>j}\gamma_j(\vec{b}-\vec{s}_j)\cdot\gamma_i(\vec{b}-\vec{s}_i) + \ldots$$

$\sum$  + $\sum_{i>j}$  + ...

single scattering

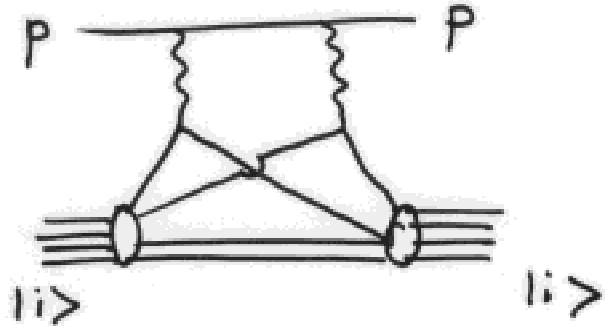
double scattering

- Note: coherent sum $\Rightarrow$ interference

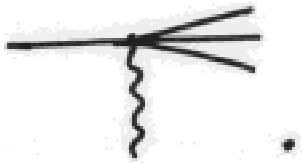
2. Gribov's inelastic shadowing

consider the contribution of double scattering to forward scattering amplitude:

A)



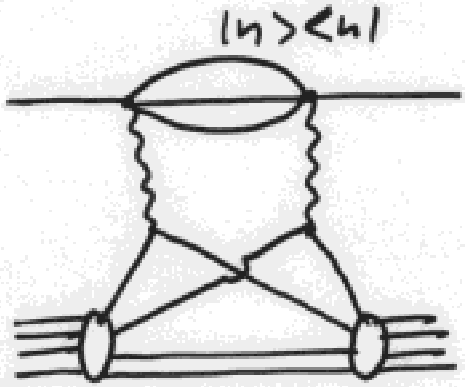
If the particle is composite (like all hadrons), it can be excited by the interaction:



Therefore, the following diagram will contribute, and interfere with A):

B)

$\sum_n$



The excitation is accompanied by the longitudinal momentum transfer $\Delta p_L \simeq \frac{M_f^2 - M_i^2}{2P}$; diagram B) is important when $\Delta p_L \cdot R_A \lesssim 1$

Formation time & formation zone in Quantum Mechanics

Time-dependent approach

$$\Delta E \, \Delta t \gtrsim 1$$

$$\tau_f \simeq \frac{1}{M_f - M_i}$$

$$z_c \equiv \Delta z = \frac{P_i}{\bar{M}} \tau_f$$

$$\bar{M} = \frac{M_f + M_i}{2}$$

$$z_c \simeq \frac{2P_i}{M_f^2 - M_i^2}$$

Time-independent approach

$$\Delta p \, \Delta x \gtrsim 1$$

$$\Delta p_L = \sqrt{E^2 - M_i^2} - \sqrt{E^2 - M_f^2} \simeq$$

$$\simeq \frac{M_f^2 - M_i^2}{2P_i}$$

$$z_c \equiv \Delta z \simeq \frac{1}{\Delta p_L} \simeq$$

$$\simeq \frac{2P_i}{M_f^2 - M_i^2}$$

When the initial momentum P_i is large, the process develops in the Lab at large longitudinal distances

Application to nuclear collisions

- Inelastic AB cross section

$$\sigma_{AB}^{in} = \int d^2b \overline{(1 - P_0(b))}$$

interaction probability

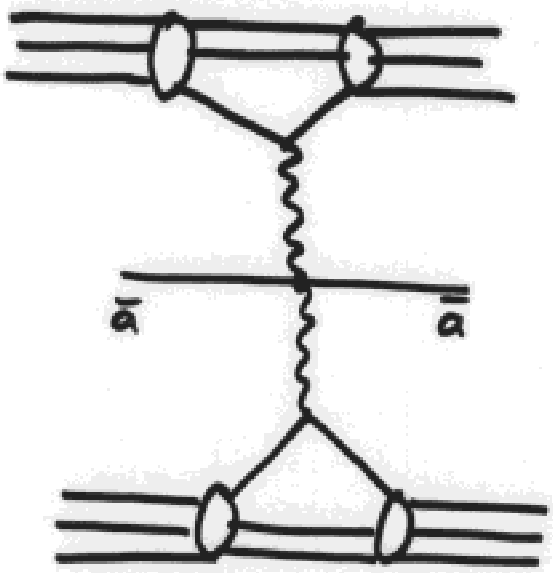
probability of no interaction

$$P_0(b) = \left(1 - \sigma_{NN}^{in} T_{AB}(b)\right)^{AB}$$

$$T_{AB}(b) = \int d^2s \, T_A(\vec{s}) \, T_B(\vec{b} - \vec{s})$$

"nuclear overlap function"

- Inclusive spectra



AGK theorem:

$$E \frac{d^3\sigma_{AB}^{q}}{d^3p} = T_{AB}(b) \, E \frac{d^3\sigma_{NN}^{q}}{d^3p}$$

integrate over b :

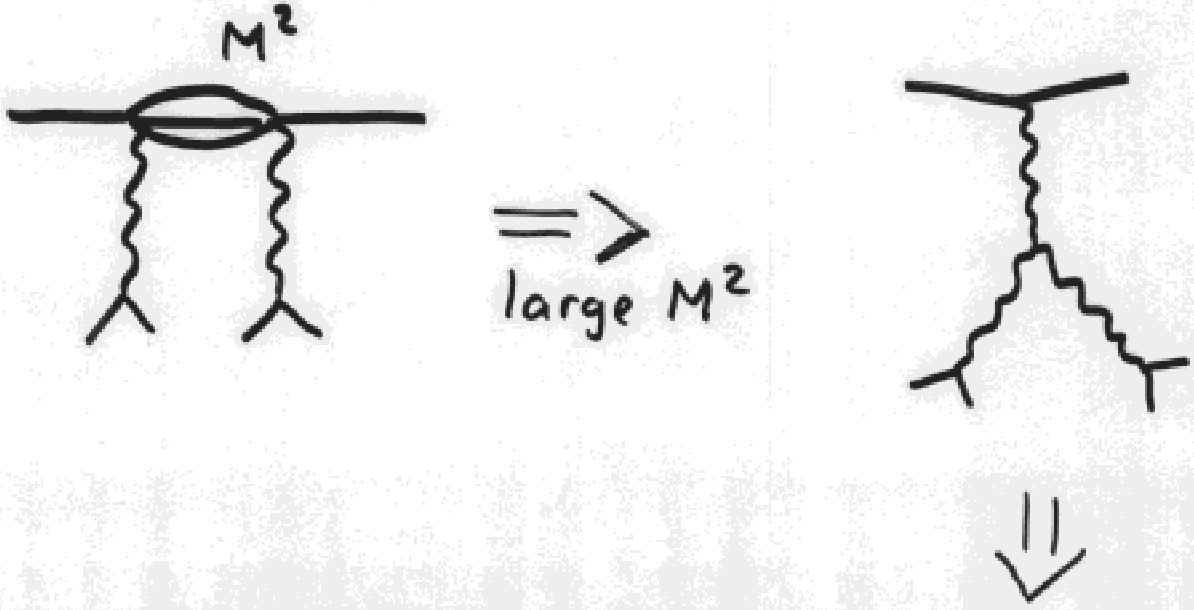
$$E\frac{d^3\sigma^*_{AB}}{d^3p} = AB\ E\frac{d^3\sigma^*_{NN}}{d^3p}$$

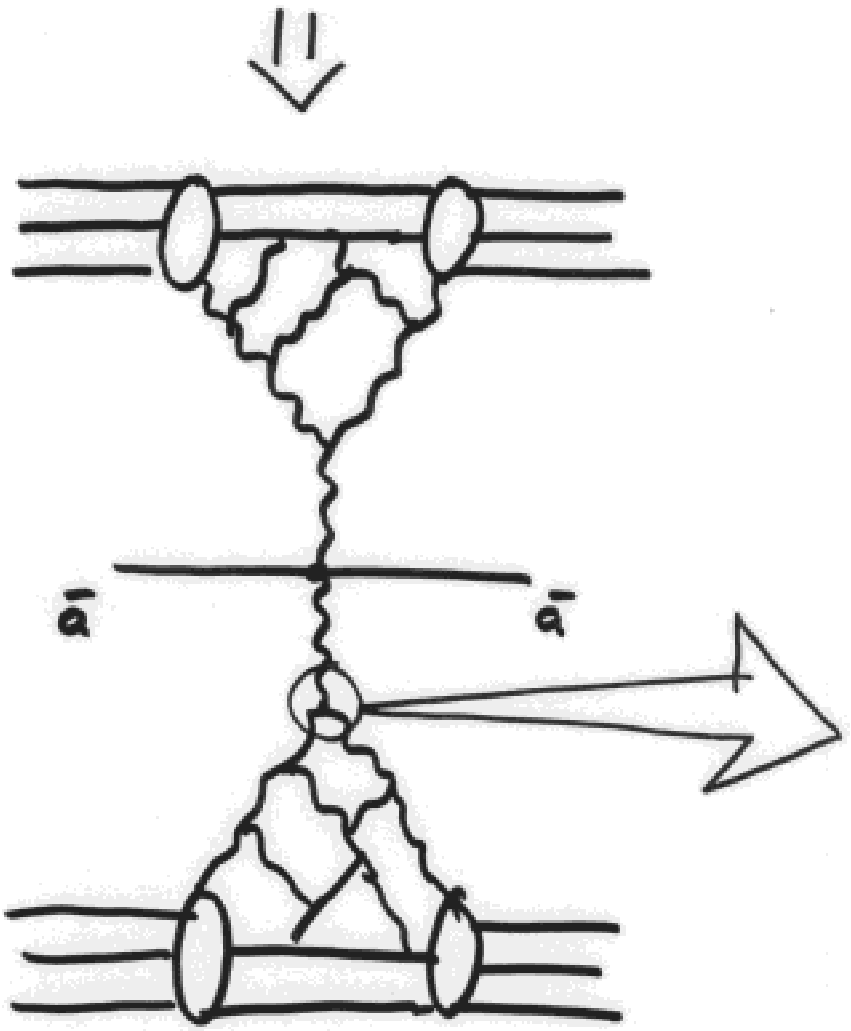
$$\frac{dn_{ch}}{d\eta} = AB\ \frac{1}{\sigma^{in}_{AB}}\ \frac{d\sigma^{ch}_{NN}}{d\eta}$$

=> multiplicity is proportional to the number of collisions

Is this always true ?

No. The theorem does not apply to the diagrams of this type:





interactions of gluon ladders

dominant at high energy, large A

("gluon saturation", "color glass condensate"...)

In the saturation regime,

$$\frac{dn}{d\eta} \sim N_{part} \cdot \underbrace{xG(x, Q_s^2)}_{\updownarrow} \sim 1/\alpha_s(Q_s^2)$$

gluon structure function

frozen α_s at small Q_s^2 => wounded nucleon model

Let x be the fraction of multiplicity coming from "hard" processes in pp collisio

Then

$$\frac{dn}{d\eta} = (1-x)\, n_{pp} \frac{\langle N_{part} \rangle}{2} + x\, n_{pp} \langle N_{coll} \rangle$$

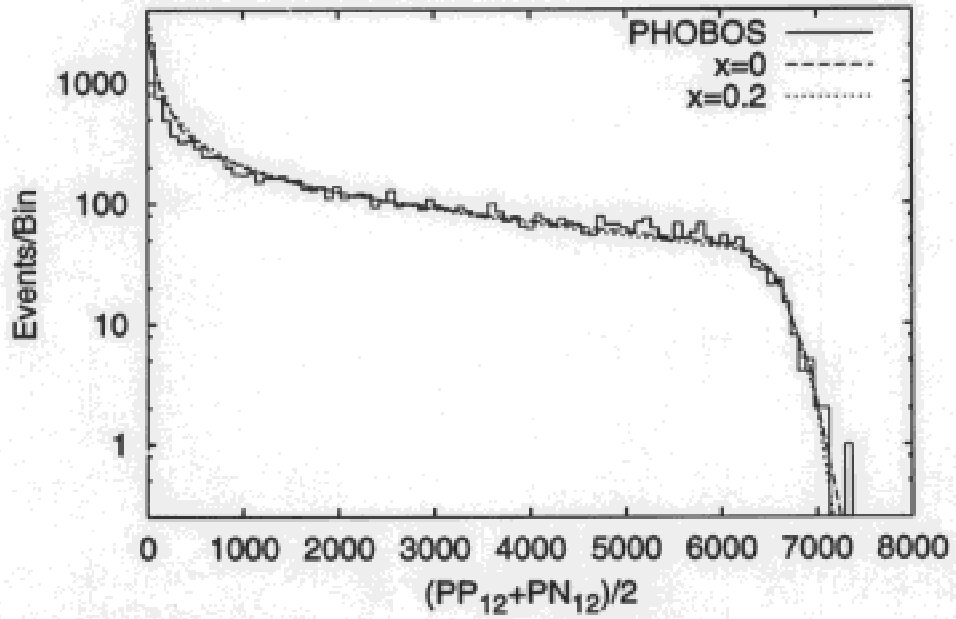
The χ section

$$\frac{1}{\sigma} \frac{d\sigma}{dn} = \int d^2b\, P(n;b)\,(1 - P_0(b))$$

is <u>in</u>sensitive to x

DK, M. Nardi (based on KLNS)

$\sigma_{in} = 7.04 \pm 0.05$ bn



← $3 < |\eta| < 4.$

pseudo-rapidity

PHOBOS, hep-ex/0007036 PRL '00

The average number of wounded nucleons in an AB collision at impact parameter b

$$N_w^{AB}(b) = A\int d^2s\, T_A(\bar{s})\left\{1-\left[1-\sigma_N T_B(\bar{s}-\bar{b})\right]^B\right\} + B\int d^2s\, T_B(\bar{s}-\bar{b})\left\{1-\left[1-\sigma_N T_A(\bar{s})\right]^A\right\}$$

$$T_A(\bar{s}) = \int dz\, \rho_A(z,\bar{s})$$

Normalization:

$$\int d^2b\, N_w^{AB}(b) = A\int d^2s\left\{1-\left[1-\sigma_N T_B(\bar{s})\right]^B\right\} + B\underbrace{\int d^2s\left\{1-\left[1-\sigma_N T_A(\bar{s})\right]^A\right\}}_{\text{inelastic } pA \text{ cross section}},$$

since $\int d^2b\, T(\bar{s}-\bar{b}) = 1$

$$\Rightarrow \underline{\int d^2b\, N_w^{AB}(b) = A\sigma_B + B\sigma_A = \sigma_{AB}\cdot \bar{N}_w}$$

Dispersion in the number of wounded nucleons

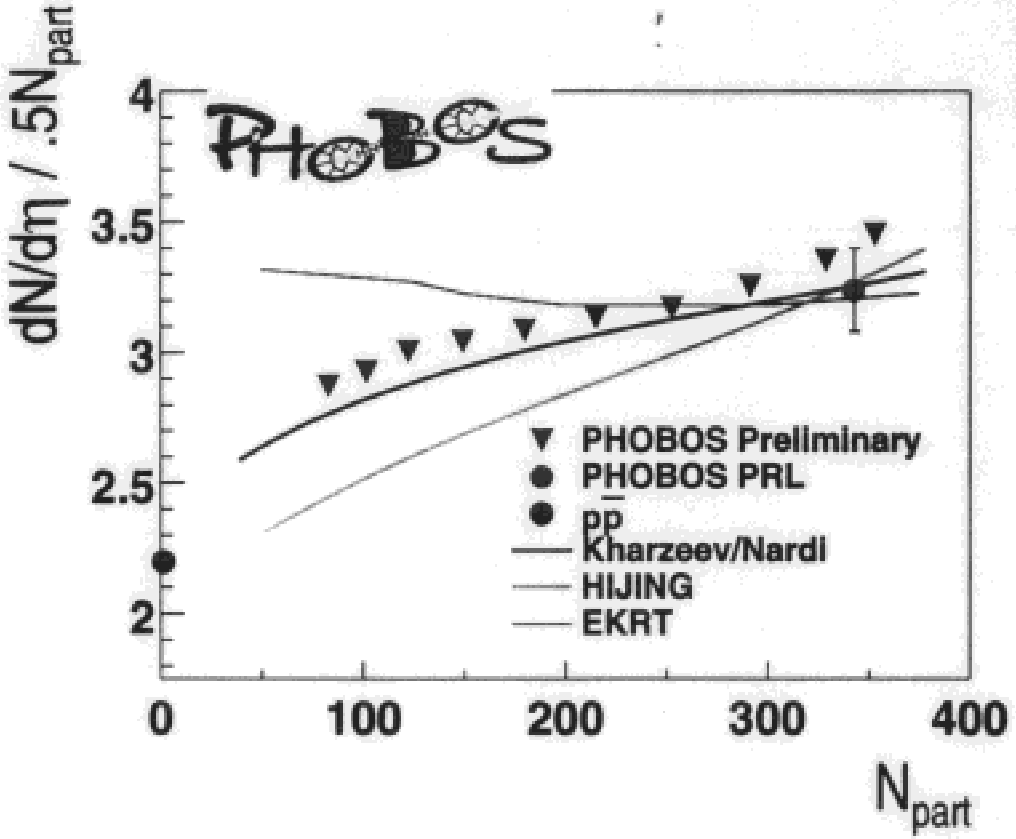
$$D_{AB}^2 = a\, N_w^{AB}(b)$$

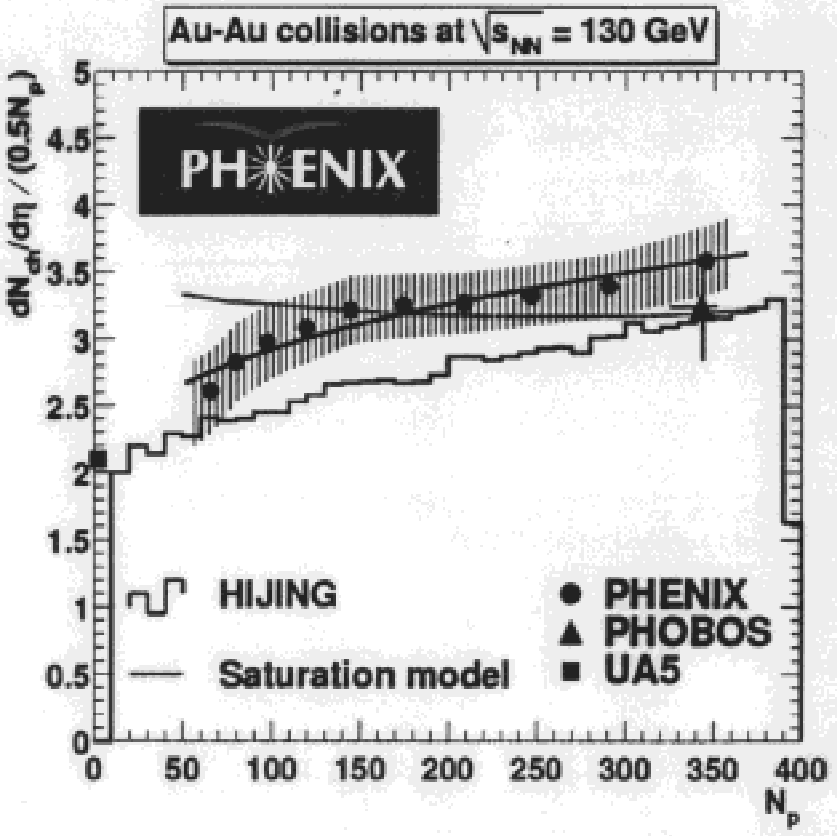
$\Rightarrow$ E_T-b correlation function

$$P_{AB}(E_T,b) = \frac{1}{\sqrt{2\pi q^2 a\, N_w^{AB}(b)}}\exp\left\{-\frac{\left[E_T - qN_w^{AB}(b)\right]^2}{2q^2 a\, N_w^{AB}(b)}\right\}$$

$$\int dE_T\, P_{AB}(E_T,b) = 1$$

dN/dη vs Centrality at η=0





data: PHOBOS Coll., nucl-ex/0106006

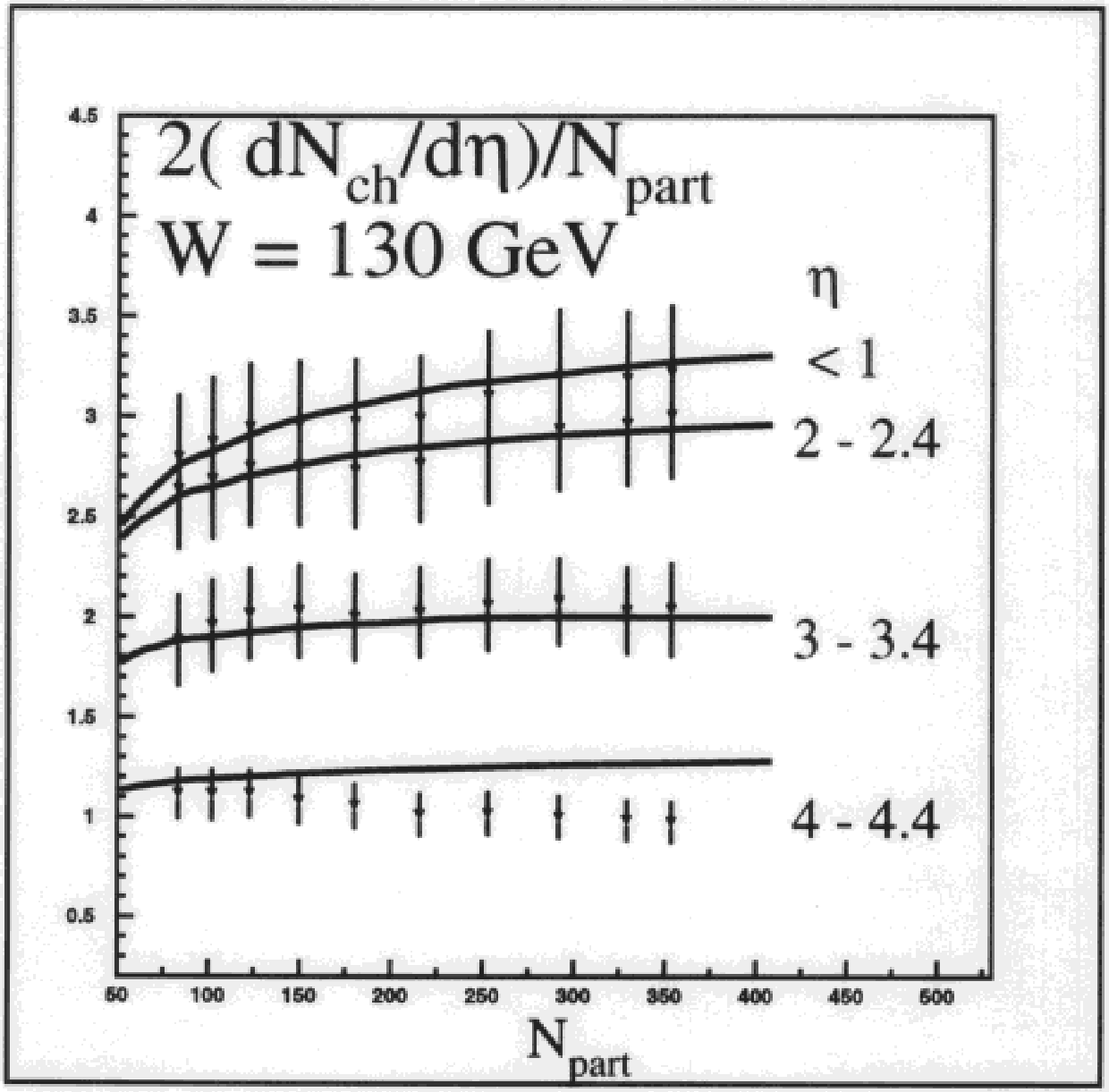


Figure 2:

D.K., E. Levin

to appear

$\left.\frac{dN_{ch}}{d\eta}\right|_{\eta=0} (\sqrt{s} = 200\,GeV) \simeq 620$

($\simeq$ 10% increase)

data: PHOBOS Coll., nucl-ex/0106006

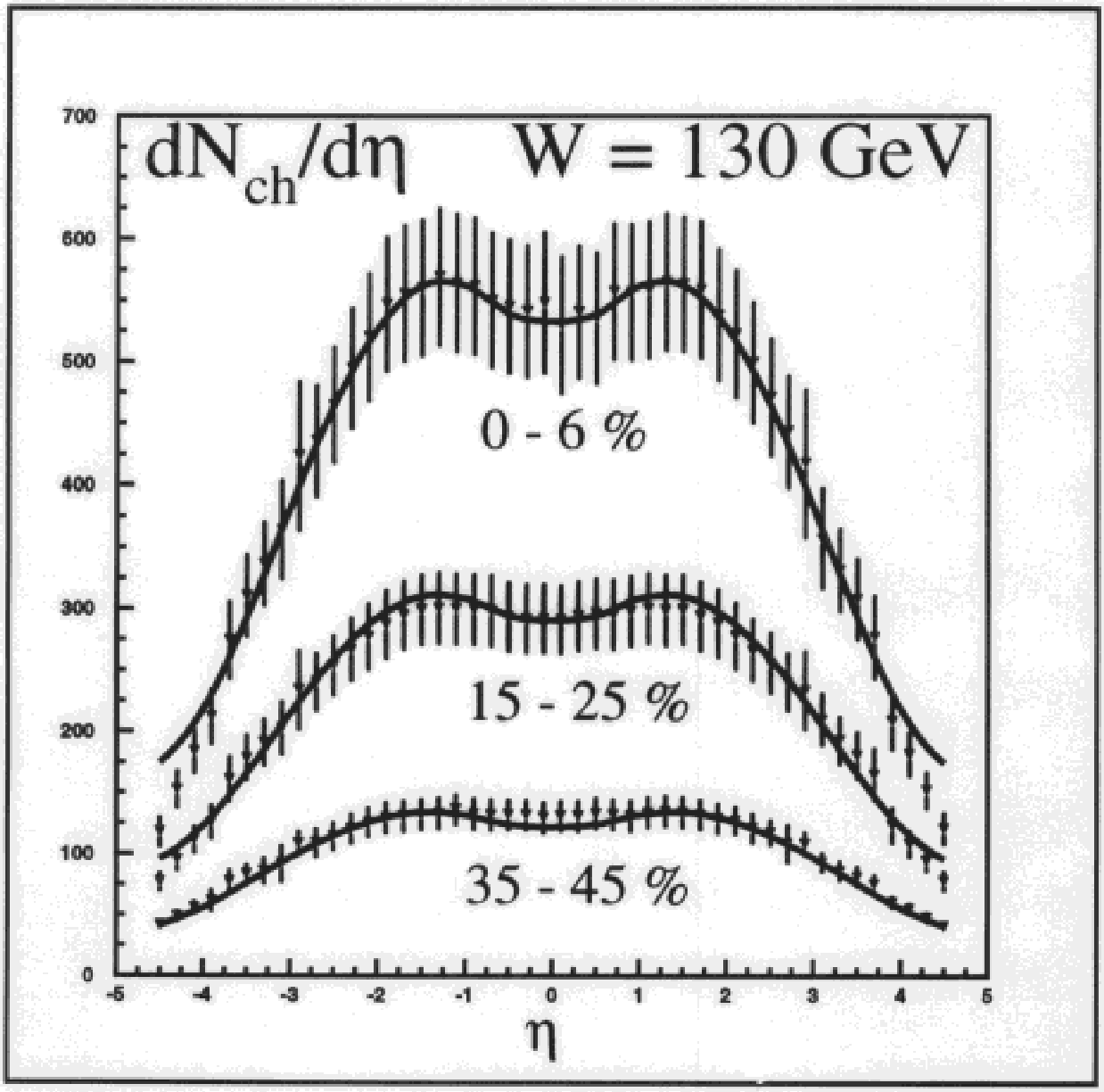
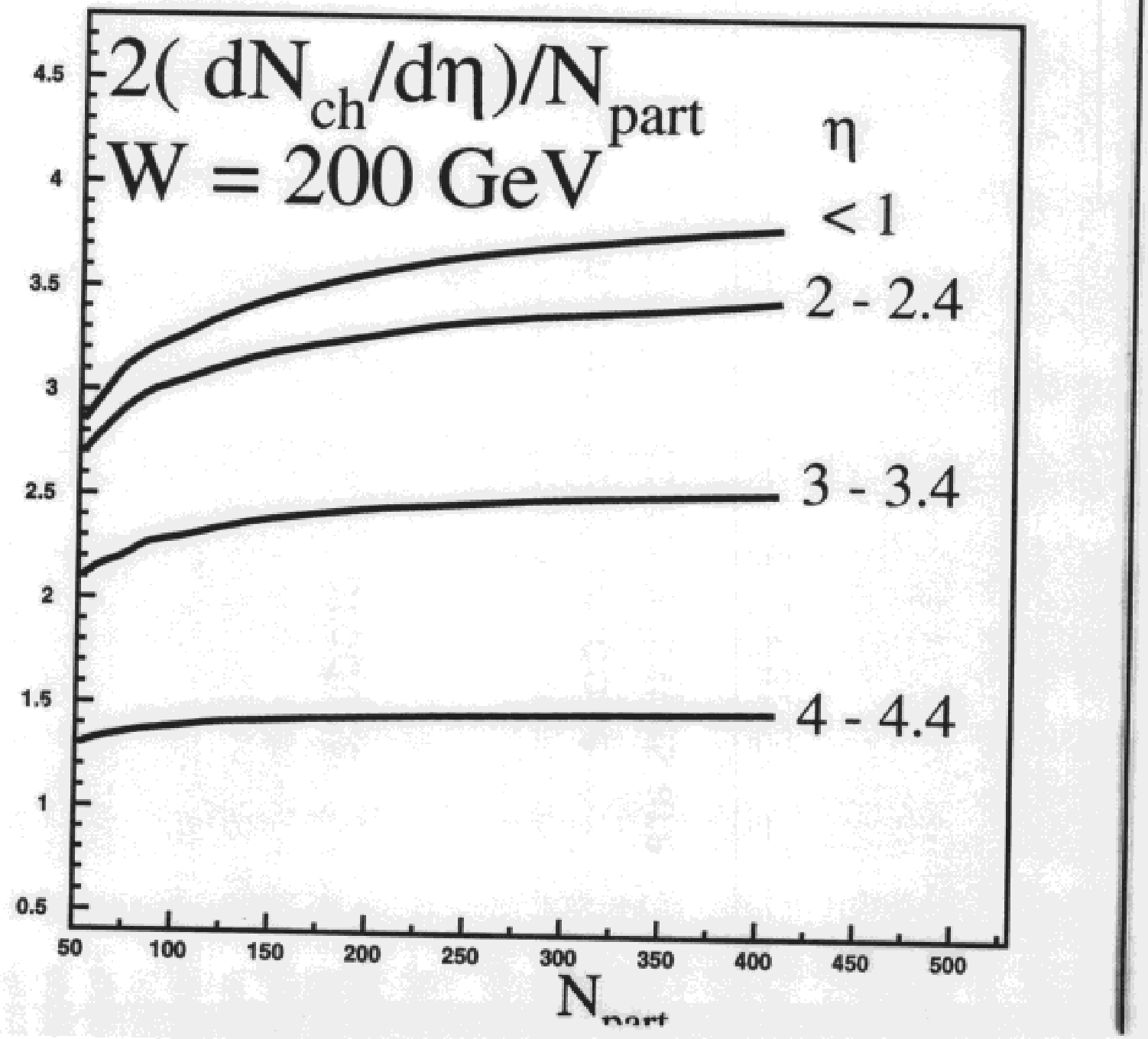
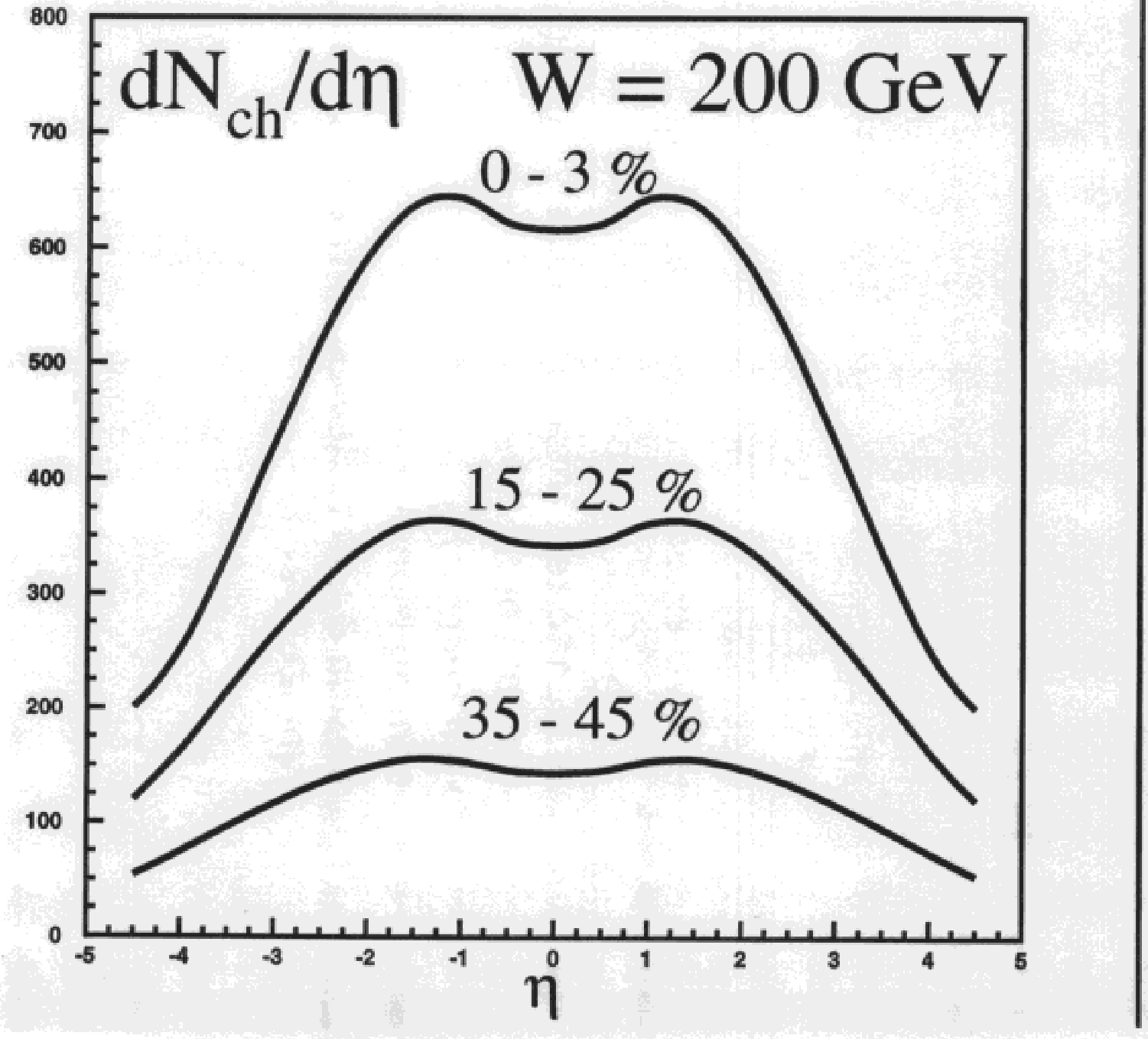


Figure 3:

D. K., E. Levin
to appear


2(dN_ch/dη)/N_part
W = 200 GeV
η
< 1
2 - 2.4
3 - 3.4
4 - 4.4
4.5
4
3.5
3
2.5
2
1.5
1
0.5
50
100
150
200
250
300
350
400
450
500
N_part


dN_ch/dη
W = 200 GeV
0 - 3 %
15 - 25 %
35 - 45 %
800
700
600
500
400
300
200
100
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
η

Approximations and uncertainties

• Eikonal	OK	
• Purely imaginary amplitudes	?	$\rho = \frac{\mathrm{Re}\,F}{\mathrm{Im}\,F} \sim 0.2$? measure at RHIC
• Coulomb - nuclear interference	OK	→ talk by A. Baltz affected by ρ
• Small (compared to nuclear size) range of interaction	??	$R^2 = 2R_p^2 + \alpha'_p \ln\left(\frac{s}{s_0}\right)$ range grows with energy, approximation gets worse
• Nuclear matter distribution = charge distribution (as measured in eA)	?	neutron "halo"?
• NN correlations	OK	can be systematically included
Inelastic shadowing = gluon saturation	???	PHYSICS