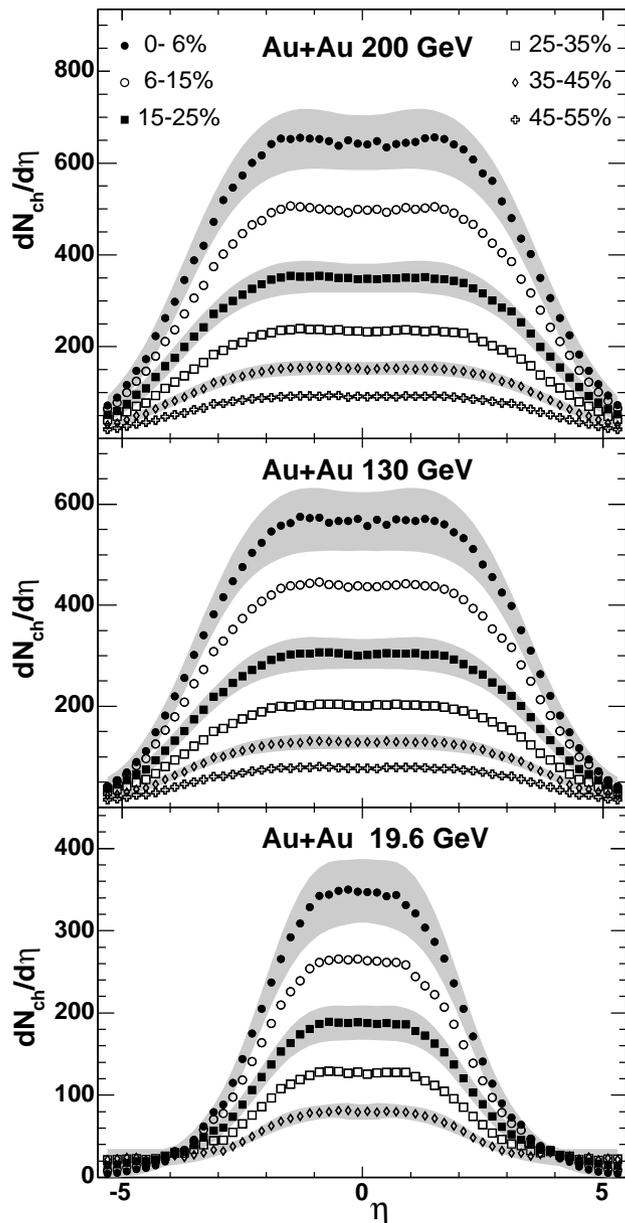

Energy dependence of the multiplicities: limiting fragmentation

Anna Staśto

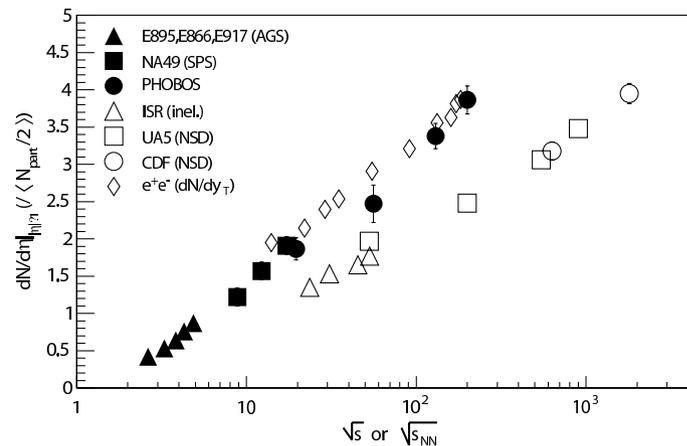
BNL

General features of multiplicities



- Growth with energy and centrality.
- Apparent flat region in the mid-pseudorapidity. In the rapidity space the distributions are however gaussian and no hint of plateau is seen.
- Growth with energy at mid-rapidity is mild:

$$\frac{2}{N_{\text{part}}} \frac{dN}{d\eta} \Big|_{\eta < 1} \sim \ln \sqrt{s}$$
 and no deviations from this behavior are observed.

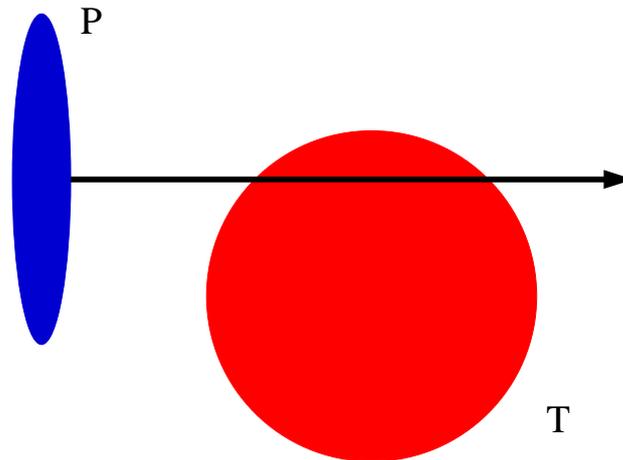


Hypothesis of limiting fragmentation

Benecke, Chou, Yang, Yen:

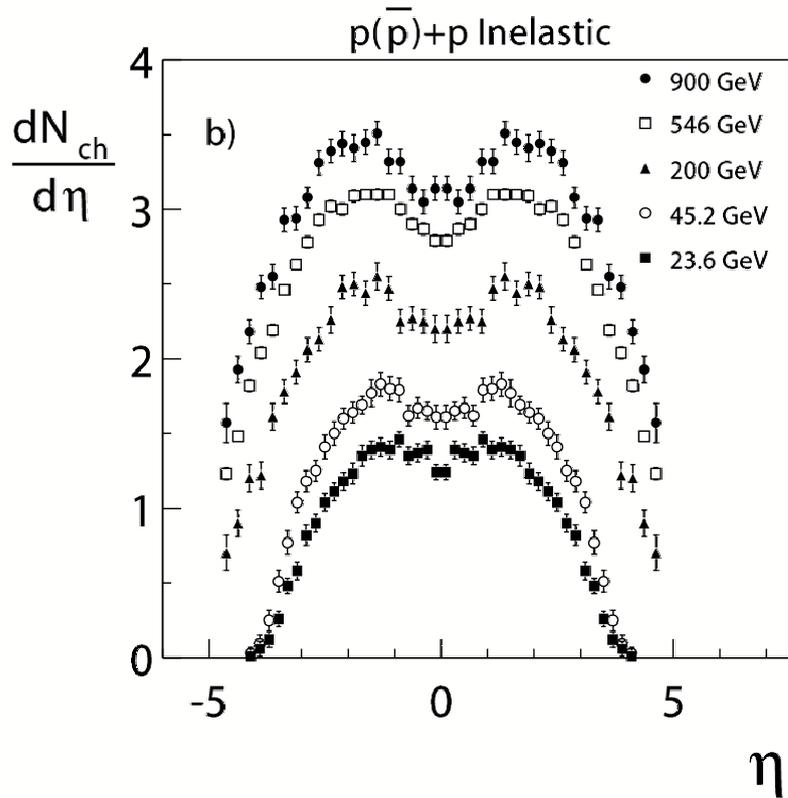
- For very high energy collisions in the lab system (target at rest) or a projectile system (projectile at rest) some of the outgoing particles approach *limiting distributions*.
- The limiting distributions represent the broken-up fragments of the target. The fragments of the projectile move with increasing velocity as $\sqrt{s} \rightarrow \infty$ (in the lab frame) and do not contribute to the limiting fragmentation. To study these fragments one has to go to the projectile system.
- In the laboratory frame the incoming particle is a Lorentz contracted system which passes through the target. The excitation of the target may cause a break up of the target.

Hypothesis of limiting fragmentation (contd.)

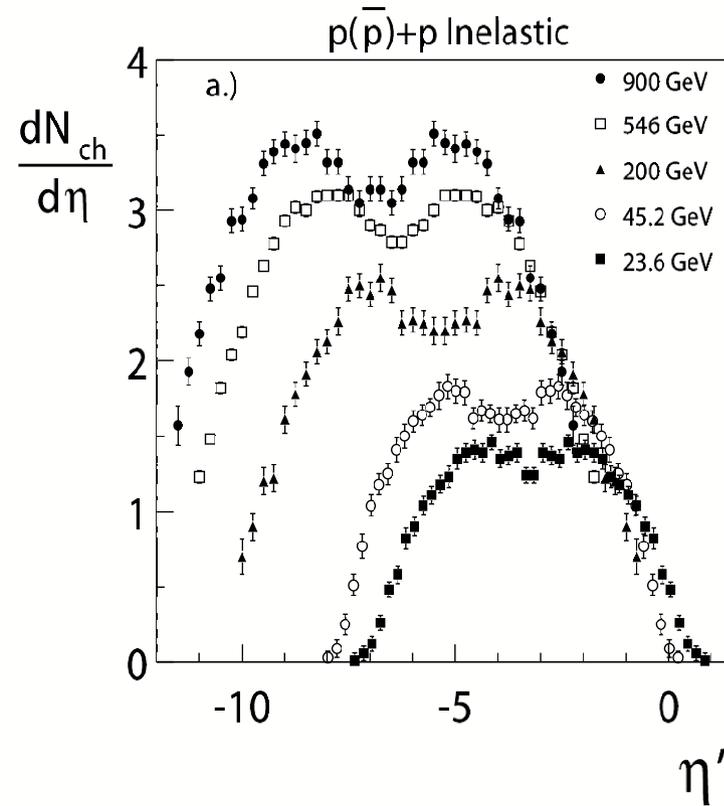


- *The constancy of the total cross section and of the elastic scattering cross section* suggests that the momentum and quantum-number transfer process between the projectile and the target does not appreciably change when the projectile is further and further compressed.
- The hypothesis of limiting fragmentation gives emphasis to the lab and projectile systems. In this it is very different from the statistical model. In the latter model model the two incoming particles collide and arrest each other in c.m. system the final product of the collision being emitted from this arrested amalgamation of the original particles.

proton-(anti)proton collisions



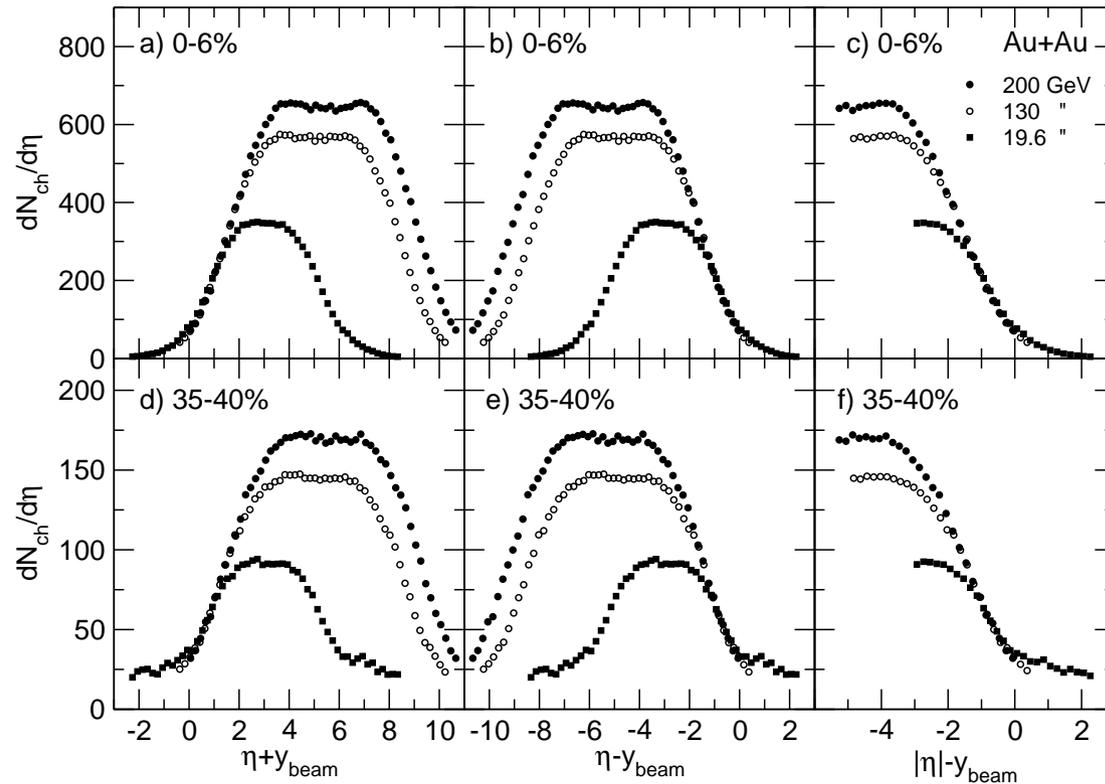
Pseudorapidity distribution



Shifted pseudorapidity distribution
in $\eta' \equiv \eta - Y_{beam}$

$$Y_{beam} = \ln \frac{\sqrt{s}}{m_p}$$

Nucleus-nucleus collisions

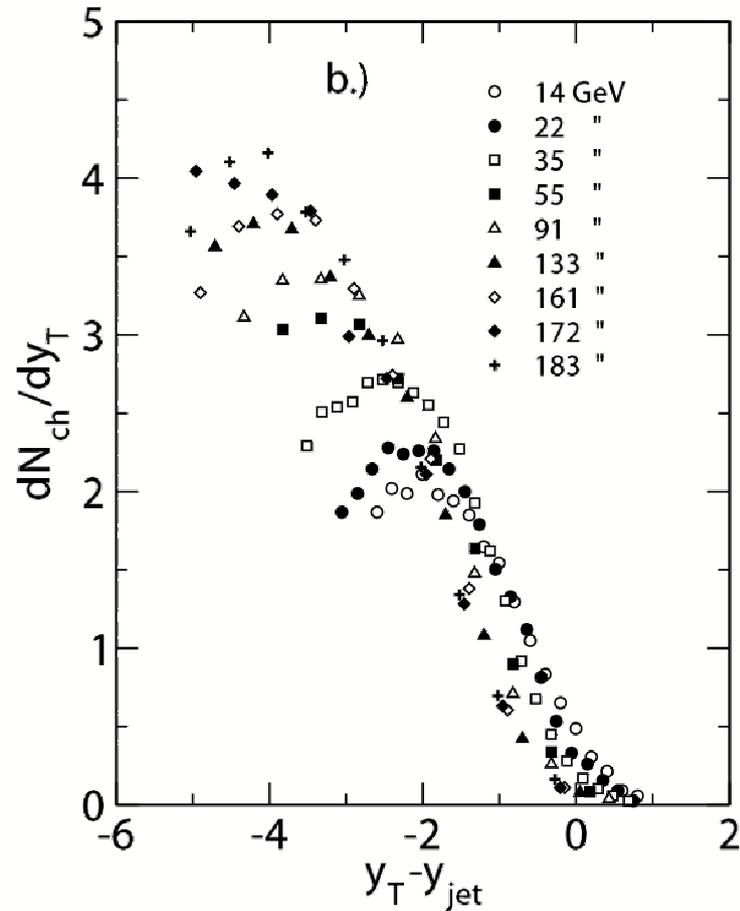


PHOBOS

Limiting fragmentation for both central and peripheral collisions.

e^+e^- annihilation

It works for e^+e^- too ...



Distribution vs $y - Y_{jet}$, the motion is described along the thrust axis.

Possible models

- (Hard) k_T factorization and gluon saturation in Color Glass Condensate, *Kharzeev, Levin, Nardi; Jalilian-Marian; Gelis, Venugopalan, A.S.*
- (Soft) Bremsstrahlung from color charges, *Białas, Jeżabek*
- (Soft) Color string model, *Braun, Pajares*
- ...

k_T factorization and gluon saturation

k_t factorization for gluon production at high energy $s \gg p_T$:

$$\frac{dN}{dyd^2p_T} = \frac{\alpha_s S_{AB}}{2\pi^4 C_F S_A S_B} \frac{1}{p_T^2} \int \frac{d^2k_T}{(2\pi)^2} \phi_A(x_1, k_T) \phi_B(x_2, |p_T - k_T|)$$

- ϕ_A, ϕ_B gluon distribution for the nuclei A,B respectively.
- $S_{A,B}$ total transverse area for nuclei.
- S_{AB} transverse area for an overlap region.
- p_T transverse momentum of the produced gluon.
- $x_1 = \frac{p_T}{m} e^{y-Y_{\text{beam}}}$, $x_2 = \frac{p_T}{m} e^{-y-Y_{\text{beam}}}$; longitudinal momentum fractions of the gluons probed in target and projectile.
- Already simplified formula, valid at zero impact parameter with uniform density (can do better by including impact parameter in $\phi_{A,B}$).

Khazzev, Levin, Nardi; Kovchegov, Tuchin; Szczurek ...

k_T factorization ...

Several points to mention about the formula:

- The formula contains mild divergence at small p_T , needs regularization.
- Functions ϕ are gluon distributions. In principle it is possible to include also quarks (*Szczurek*).
- Perturbative formula valid in principle at high p_T ; we know from experiment that the (integrated over p_T) rapidity distributions are dominated by low p_T particles.
- Functions $\phi(x, k_T)$ are *unintegrated* gluon distributions, in principle defined at small values of x (as well as the whole approach)
$$xg(x, Q^2) \sim \int^{Q^2} d^2k_T \phi(x, k_T)$$
- Extrapolation and suitable ansatz for ϕ at large x is needed.
- Experimentally measured hadrons, need to do a fragmentation from gluons (quarks) to pions, but the scales are very low \rightarrow would need nonperturbative fragmentation functions.

Unintegrated gluon distributions

- Several calculations available, mainly from the linear evolution equations (no saturation), see ex. *Szczurek*.
- Saturation ansatz: *Kharzeev, Levin, Nardi*
- ϕ from CGC *Albacete, Armesto, Salgado, Wiedemann; Gelis, Venugopalan, A.S.*:

$$\phi(x, k) \equiv Nk^2 \int drr J_0(kr) \text{Tr} \langle U(0) U^\dagger(r) \rangle_Y$$

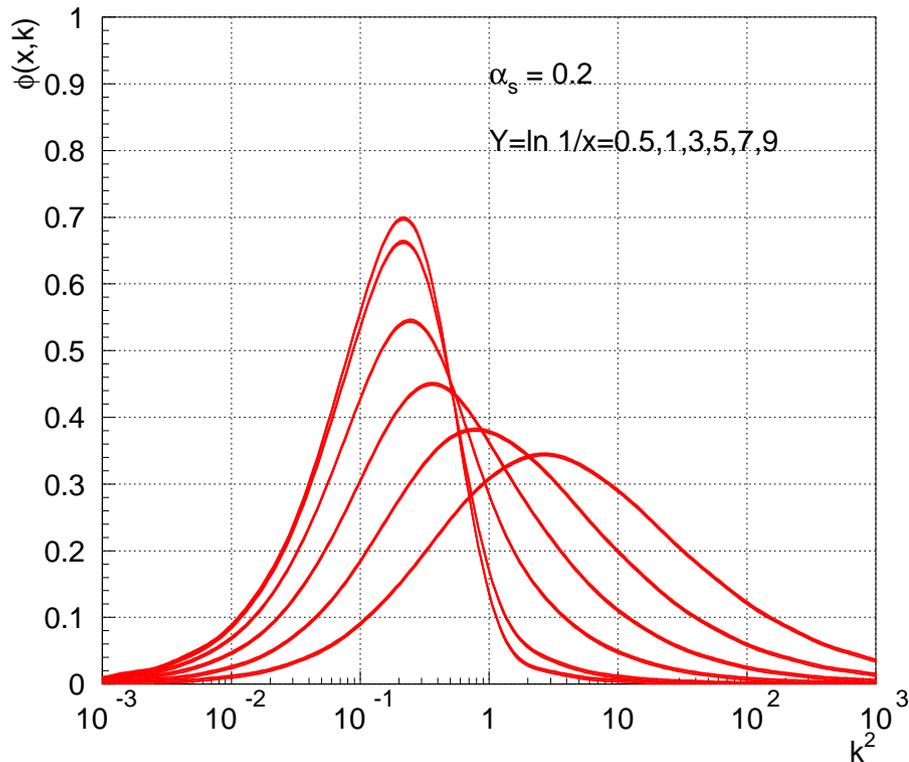
$\langle \dots \rangle$ CGC average over the color sources in the hadron and $Y \equiv \ln 1/x$.

- Define $\text{Tr} \langle U(0) U^\dagger(r) \rangle_Y \equiv N_c^2 (1 - T)^2$.
- Calculate T directly from CGC equations \rightarrow Balitsky-Kovchegov equation.

$$\frac{dT(r, Y)}{dY} = \bar{\alpha}_s (K \otimes T)(r, Y) - \bar{\alpha}_s (K \otimes T \otimes T)(r, Y)$$

where K is the BFKL kernel and r is coordinate conjugate to momentum k .

ϕ distribution from BK equation



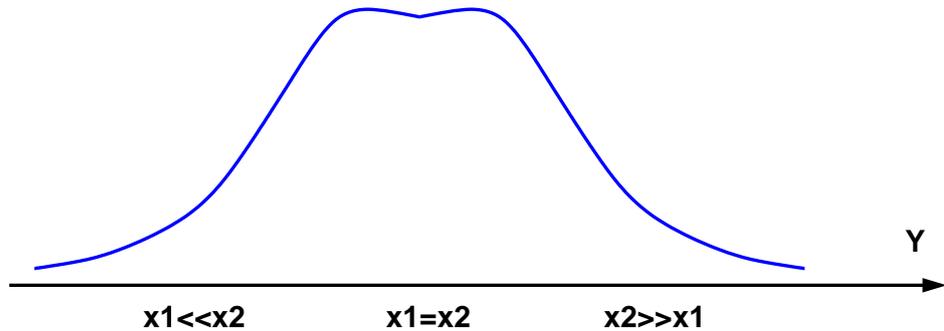
- Peak of the distribution is at the saturation scale $Q_s(Y)$.
- Soliton moves to higher k 's as Y increases.
- The area under the integral is constant, conserved during the evolution in x

$$\int \frac{d^2k}{k^2} \phi(x, k) = \text{Const.}$$

This is consequence of the definition of ϕ through the unitary matrices and the nonlinear evolution equation.

Qualitative analysis

Pseudo-rapidity distribution:



Recall:

$$x_1 = \frac{p_T}{m} e^{Y - Y_{beam}}$$

$$x_2 = \frac{p_T}{m} e^{-Y - Y_{beam}}$$

$$\frac{dN}{dy} \sim \int \frac{d^2 p_T}{p_T^2} \int d^2 k_T \phi_A(x_1, |k_T|) \phi_B(x_2, |p_T - k_T|)$$

- When $x_1 \sim x_2$: $Q_s^A(x_1) \sim Q_s^B(x_2) \rightarrow$ entanglement in momenta.
- When $x_1 \gg x_2$: $Q_s^A(x_1) \ll Q_s^B(x_2) \rightarrow$ separation in transverse momenta,

$$\frac{dN}{dy} \sim \int \frac{d^2 p_T}{p_T^2} \int d^2 k_T \phi_A(x_1, |k_T|) \phi_B(x_2, |p_T|)$$

Qualitative analysis

When $x_1 \gg x_2$ (or $x_2 \gg x_1$) we have separation of integrals in k_T space:

$$\frac{dN}{dy} \sim \int \frac{d^2 p_T}{p_T^2} \phi_B(x_2, |p_T|) \int d^2 k_T \phi_A(x_1, |k_T|)$$

- Integral over projectile density constant: $\int \frac{d^2 p_T}{p_T^2} \phi_B(x_2, |p_T|) = \text{const.}$
- Integral over target density:

$$\int^{Q_s(x_2)} d^2 k_T \phi_A(x_1, |k_T|) = x_1 g(x_1, Q_s(x_2))$$

Integrated parton density at large values of x_1 :

$$x_1 g(x_1, Q_s(x_2)) = x_1 g(x_1)$$

exhibits x_1 scaling.

Scaling in limiting fragmentation

$$\frac{dN}{dY} \simeq \mathcal{N} x_1 g(x_1) = \mathcal{F}(Y - Y_{beam}), x_1 \gg x_2$$

scaling with $Y - Y_{beam}$ (recall $x_1 \sim \exp(Y - Y_{beam})$).

For comparison with data:

- Need to model $\phi_A(x_1, k_T)$ at large x_1 .
- Since $x_1 g(x_1)$ should obey x_1 scaling

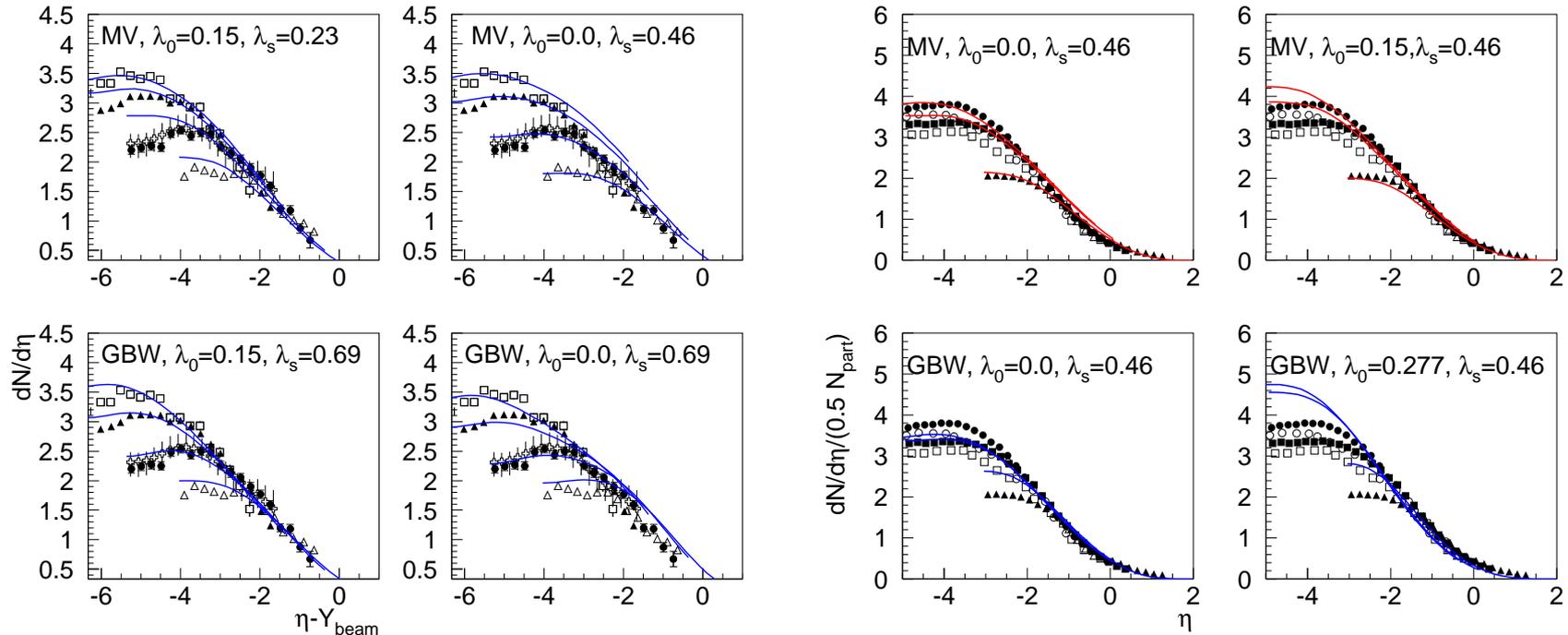
$$x_1 g(x_1) = x_1 g(x_1, Q_s^2) = \int^{Q_s^2} dk^2 \phi_A(x_1, k)$$

the distribution ϕ_A must be peaked at very low k_T and sharply fall for large k_T .

- $\phi_A(x_1, k_T)$ at large x_1 is the largest source of uncertainty when comparing with the data.

Proton-antiproton and AuAu(central) collisions

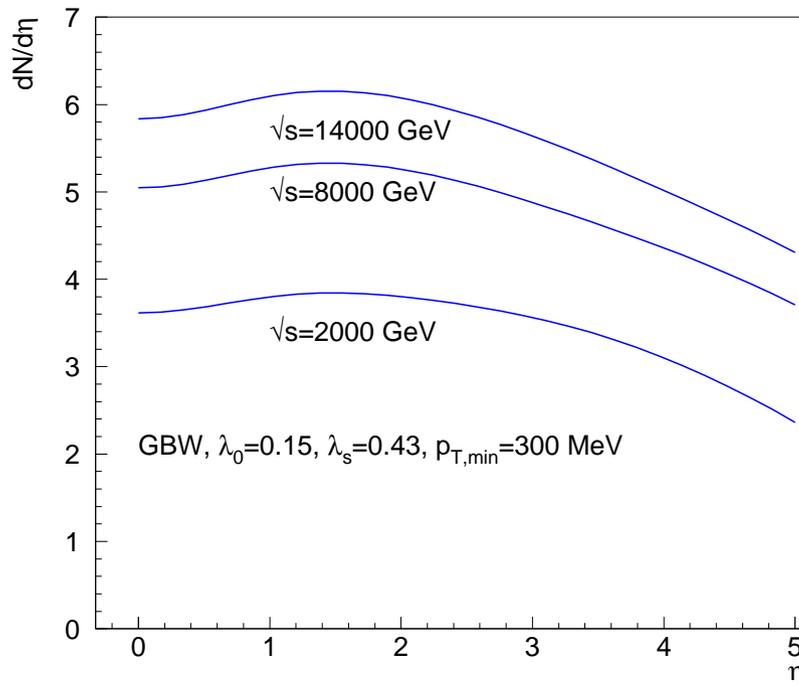
Gelis, Venugopalan, A.S.



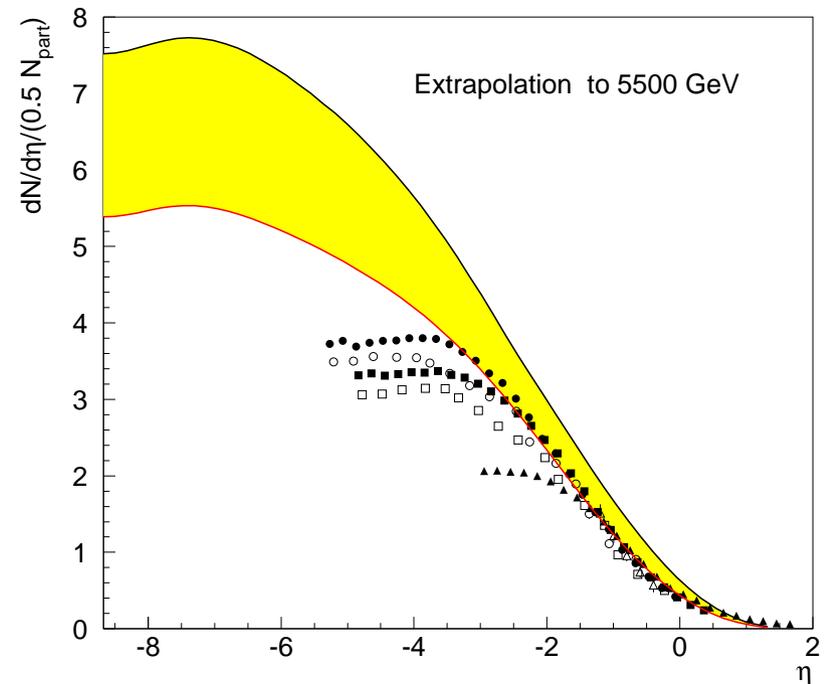
- Small violations of limiting fragmentation scaling due to the fact that in some models we do not have approximately scaling of $x_1 g(x_1)$.
- Additional uncertainties connected with $y \leftrightarrow \eta$ change and fragmentation functions.

Extrapolation to LHC

pp collisions:



AuAu central collisions:



Still there are many parameters: a lot of uncertainty in the predictions. Some models give violations of limiting fragmentation. For example MV input $\phi_A(x_1, k_T)$ at large x_1 has too large tail in k_T .

Limiting fragmentation scaling is related to x_1 scaling at large x_1 .

Bremsstrahlung from color sources

Białas, Jeżabek

- Particle production proceeds by number of color exchanges between two sets of partons, one from target and one from projectile.
- Color charges lead to the emission of particles by the bremsstrahlung process.
- In the fragmentation region the partons in the projectile are treated independently from the ones in the target.
- The original distribution of partons in the target is flat in the rapidity, which gives the linear increase of the multiplicity spectrum with Y .
- Increase of the multiplicity distribution stops when $Y = Y_0$, because the partons with life time longer than the exchange process can participate in the interaction.

Bremsstrahlung from color sources

Distribution of partons in rapidity is uniform:

$$dn(z_+) = b(1 - z_+)^{b-1} \frac{dz_+}{z_+}$$

Emission of particle clusters in the bremsstrahlung process:

$$dN(x_+) = a(1 - x_+)^{a-1} \frac{dx_+}{x_+}$$

The rapidity distribution of particles is then:

$$dN(x_+) = \lambda \int_{\max\{z_0, x_+\}} b(1 - z_+)^{b-1} \frac{dz_+}{z_+} \left[a \left(1 - \frac{x_+}{z_+} \right)^{a-1} \right]$$

If $x_+ > z_0$ (parton has to live long enough) we have

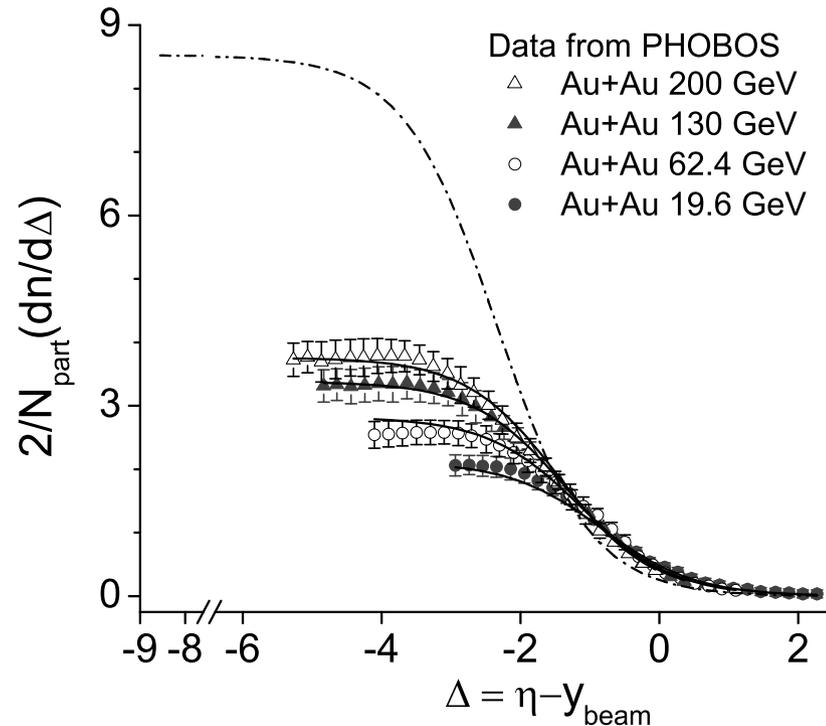
$$\frac{dN}{dy} = x_+ \frac{dN}{dx_+} = \lambda ab [C + \ln M/m + Y_{beam} - y]$$

Linear increase with increasing $Y_{beam} - y$.

Color string scenario

Braun, Pajares et al

- Color strings are stretched between the colliding partons, which then decay into observable particles.
- Taking into account transverse dimensions of the string leads to phenomenon of string fusion and percolation.
- Because of fusion multiplicities become significantly damped.



In case of gold - gold collisions the fusion of strings is essential in order to obtain the description of the experimental data.

What have we learned?

● Limiting fragmentation:

- Factorization of parton distributions in target and projectile at large rapidities.
- Then the multiplicity distribution is directly proportional to the parton density in the target (i.e. gluon and quark density at large x) which has to be nearly flat in rapidity and independent of the scales in the process.
- These models imply that the limiting fragmentation arises because the rapidity distribution of the produced particles are determined early in the scattering process, essentially by the form of the initial states.

● Applicability of CGC:

- In principle: string picture \leftrightarrow soft, CGC+Pomeron \leftrightarrow hard.
- CGC picture, (hard approach) describes multiplicities.
- Striking similarities: string fusion vs saturation in CGC.
- Qualitative and quantitative agreement of multiplicities (and average transverse momentum).