Consequences of the MC Glauber Initial State: Correlations & Fluctuations

Peter Steinberg
Brookhaven National Laboratory

Workshop on Viscous Hydrodynamics and Transport Models
April 29, 2008

Acknowledgements

Glauber Modeling in High-Energy Nuclear Collisions

Michael L. Miller,[1] Klaus Reygers,[2] Stephen J. Sanders,[3] and Peter Steinberg[4]

online at annualreviews.org

Ann.Rev.Nucl.Part.Sci.57:205-243,2007

The PHOBOS+ Glauber Working Group:
Mark Baker, Constantin Loizides, Steve Manly, PAS, Uli Heinz

Thanks to:
W. Busza, G. Roland, B. Alver, G. Veres

Acknowledgements

Phys. Rev. C **77**, 014906 (2008)

The Importance of Correlations and Fluctuations on the Initial Source Eccentricity in High-Energy Nucleus–Nucleus Collisions

B.Alver[4], B.B.Back[1], M.D.Baker[2], M.Ballintijn[4], D.S.Barton[2], R.R.Betts[7], R.Bindel[8], W.Busza[4], V.Chetluru[7], E.García[7], T.Gburek[3], J.Hamblen[9], U.Heinz[6], D.J.Hofman[7], R.S.Hollis[7], A.Iordanova[7], W.Li[4], C.Loizides[4], S.Manly[9], A.C.Mignerey[8], R.Nouicer[2], A.Olszewski[3], C.Reed[4], C.Roland[4], G.Roland[4], J.Sagerer[7], P.Steinberg[2], G.S.F.Stephans[4], M.B.Tonjes[8], A.Trzupek[3], G.J.van Nieuwenhuizen[4], S.S.Vaurynovich[4], R.Verdier[4], G.I.Veres[4], P.Walters[9], E.Wenger[4], B.Wosiek[3], K.Woźniak[3], B.Wysłouch[4]

[1] Physics Division, Argonne National Laboratory, Argonne, IL 60439-4843, USA
[2] Physics and C-A Departments, Brookhaven National Laboratory, Upton, NY 11973-5000, USA
[3] Institute of Nuclear Physics, Kraków, Poland
[4] Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, MA 02139-4307, USA
[5] Department of Physics, National Central University, Chung-Li, Taiwan
[6] Department of Physics, Ohio State University, Columbus, OH 43210, USA
[7] Department of Physics, University of Illinois at Chicago, Chicago, IL 60607-7059, USA
[8] Department of Chemistry, University of Maryland, College Park, MD 20742, USA
[9] Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA
(Dated: April 27, 2008, Revision: 1.66)

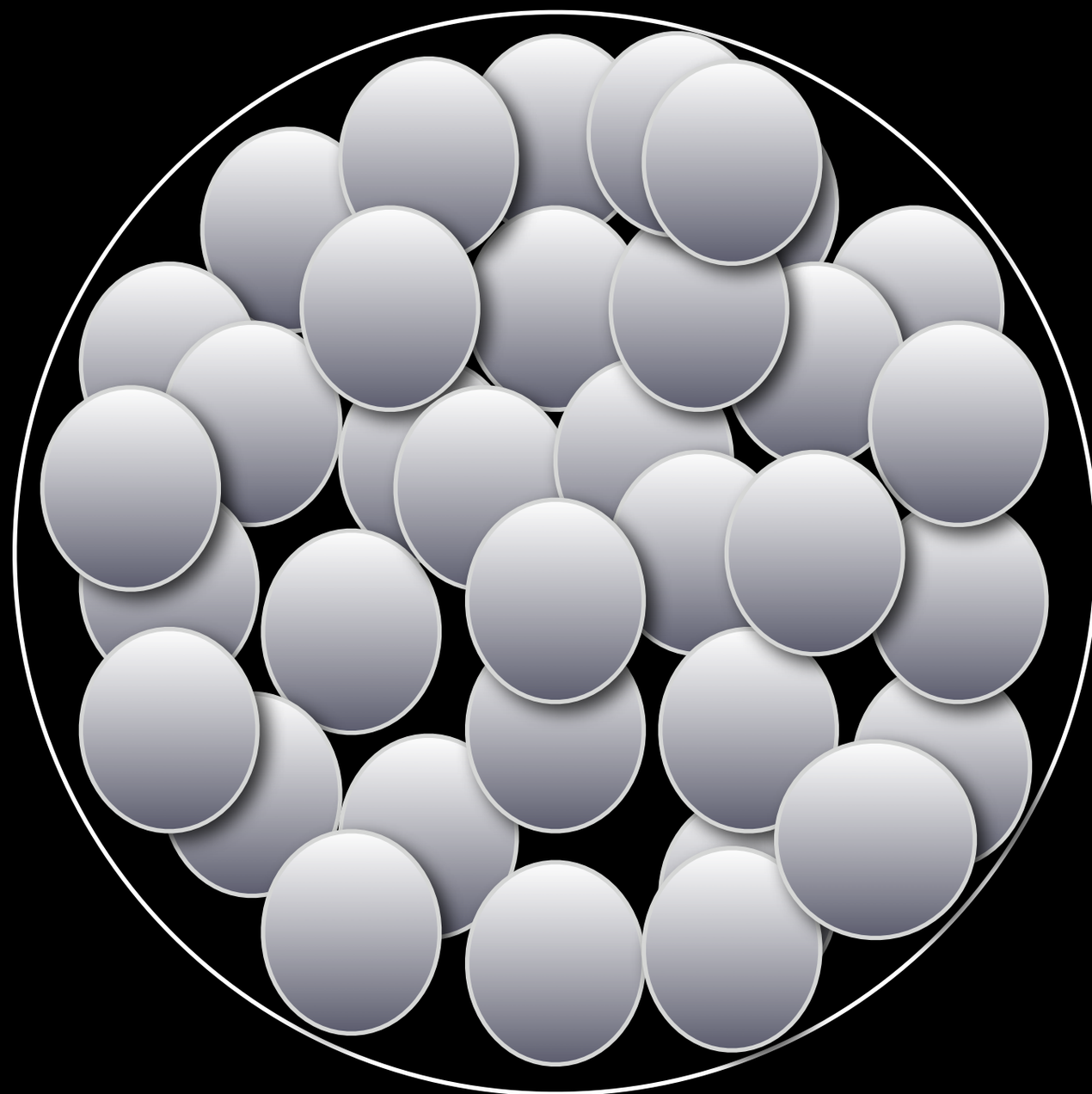
In relativistic heavy-ion collisions, anisotropic collective flow is driven, event by event, by the initial eccentricity of the matter created in the nuclear overlap zone. Interpretation of the anisotropic flow data thus requires a detailed understanding of the effective initial source eccentricity of the event sample. In this paper, we investigate various ways of defining this effective eccentricity using the Monte Carlo Glauber (MCG) approach. In particular, we examine the participant eccentricity, which quantifies the eccentricity of the initial source shape by the major axes of the ellipse formed by the interaction points of the participating nucleons. We show that reasonable variation of the density parameters in the Glauber calculation, as well as variations in how matter production is modeled, do not significantly modify the already established behavior of the participant eccentricity as a function of collision centrality. Focusing on event-by-event fluctuations and correlations of the distributions of participating nucleons we demonstrate that, depending on the achieved event-plane resolution, fluctuations in the elliptic flow magnitude v_2 lead to most measurements being sensitive to the root-mean-square, rather than the mean of the v_2 distribution. Neglecting correlations among participants, we derive analytical expressions for the participant eccentricity cumulants as a function of the number of participating nucleons, $N_{\rm part}$, keeping non-negligible contributions up

Two Kinds of Glauber Models

Where is the matter? (Participant Eccentricity)

Correlations & Fluctuations

What is a Nucleus?



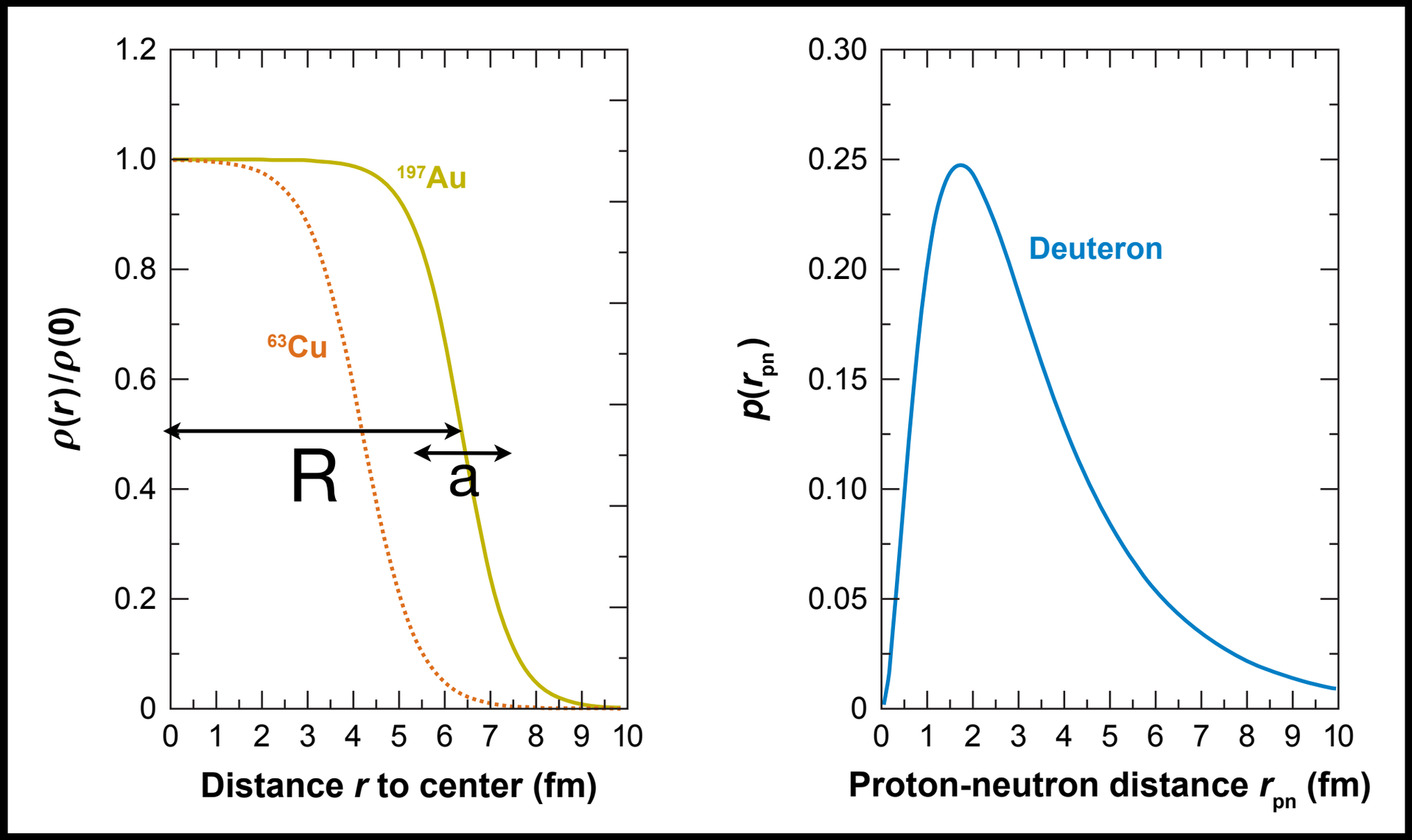
A bound state of nucleons, with positions chosen according to the Fermi distribution

What is a Nucleus?

An average density distribution of nucleon positions

Ann.Rev.Nucl.Part.Sci.57:205-243,2007

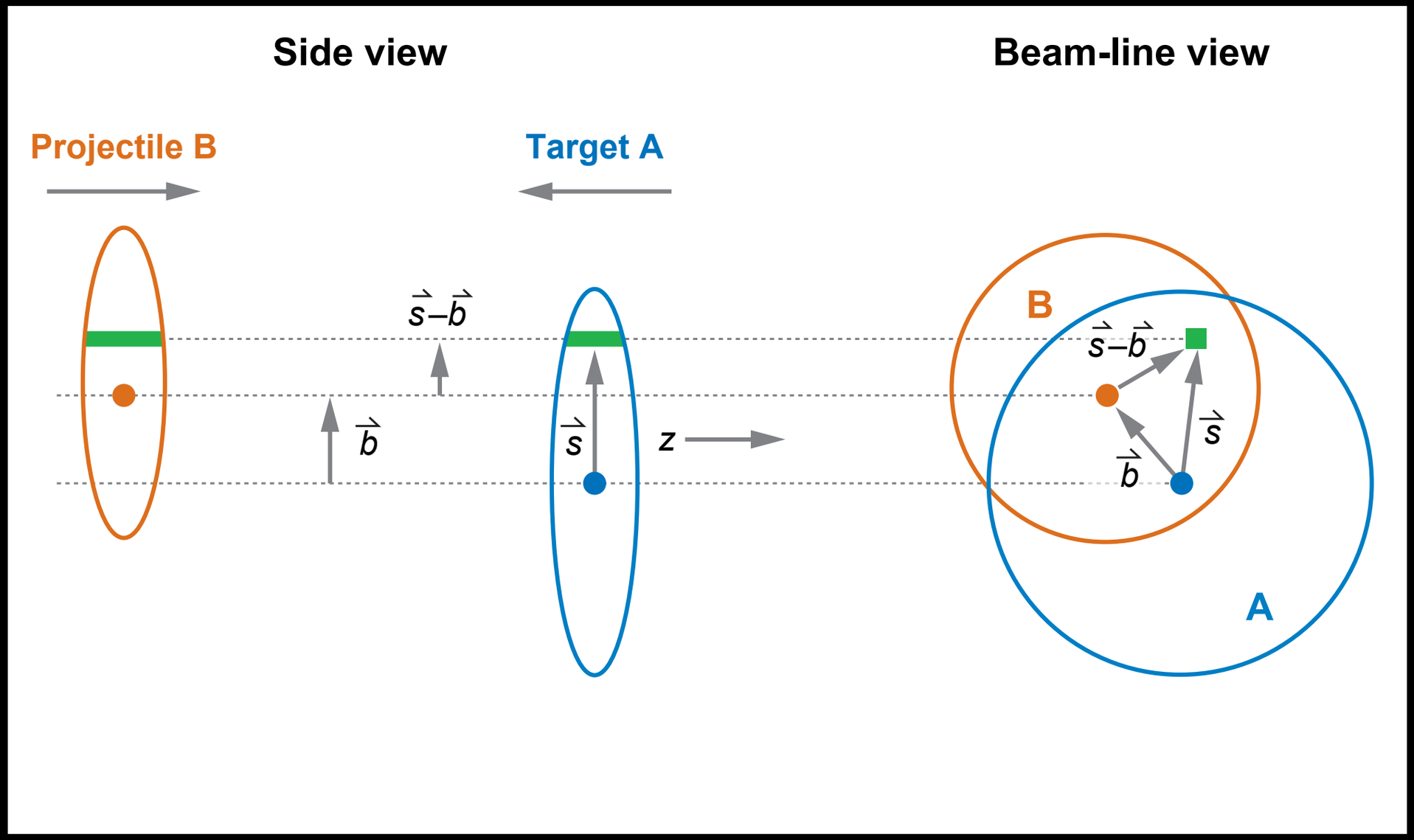
Nuclear Distributions



Distributed according to a Fermi distribution
(or Hulthen, for d+Au)

Ann.Rev.Nucl.Part.Sci.57:205-243,2007

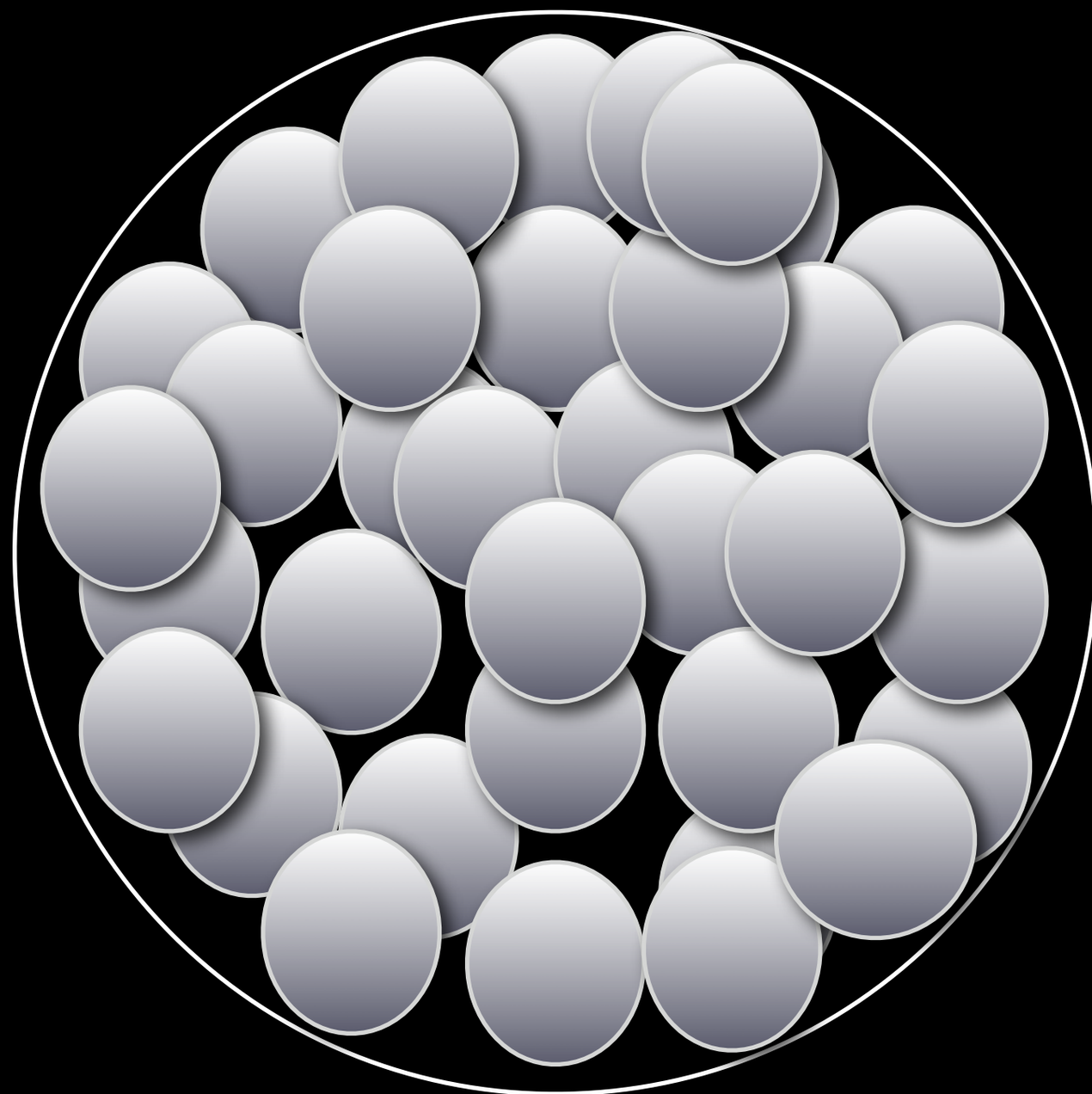
Optical Limit Approach



$$\sigma_{AB} = \int d^2b \left\{1 - [1 - \sigma_{inel}^{NN} T_{AB}(b)]^{AB}\right\}$$

everything based on smooth, averaged densities

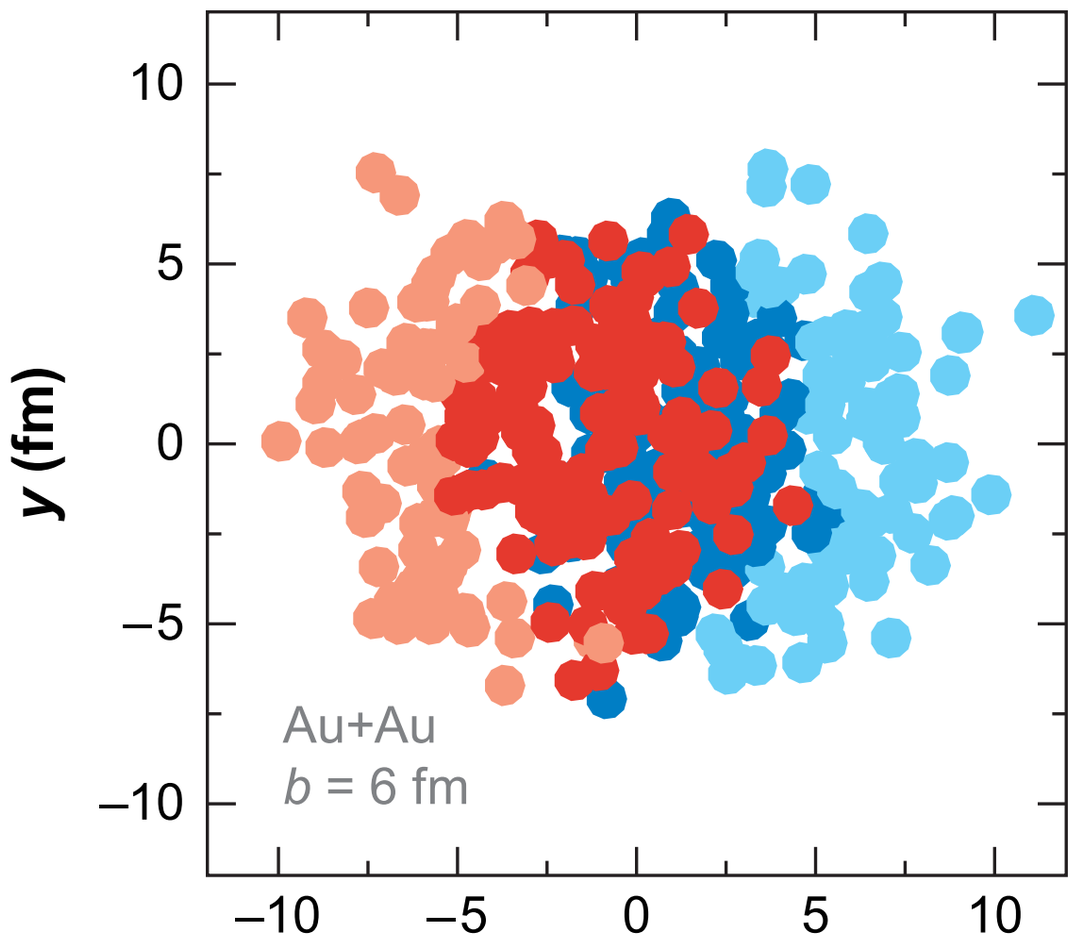
Take it from the top

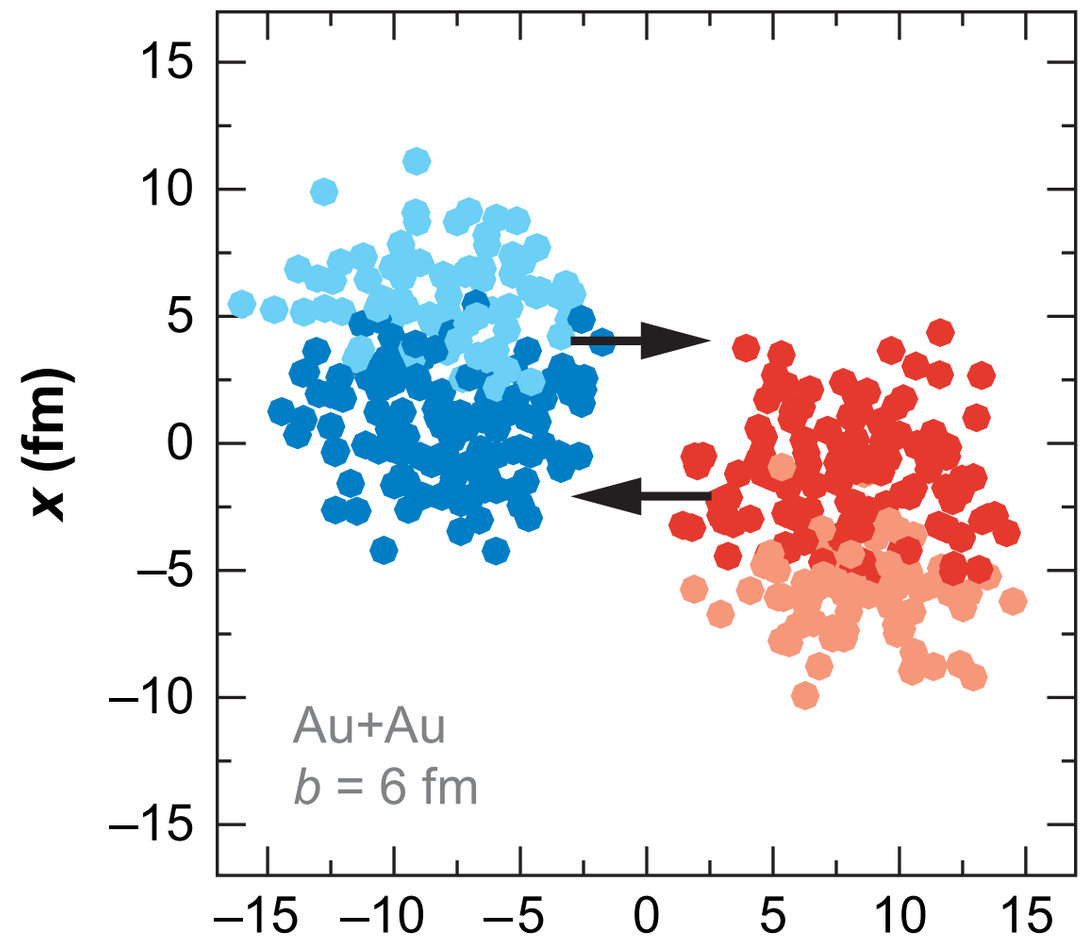


A bound state of nucleons, with positions chosen according to the Fermi distribution

Ann.Rev.Nucl.Part.Sci.57:205-243,2007

Glauber Monte Carlo



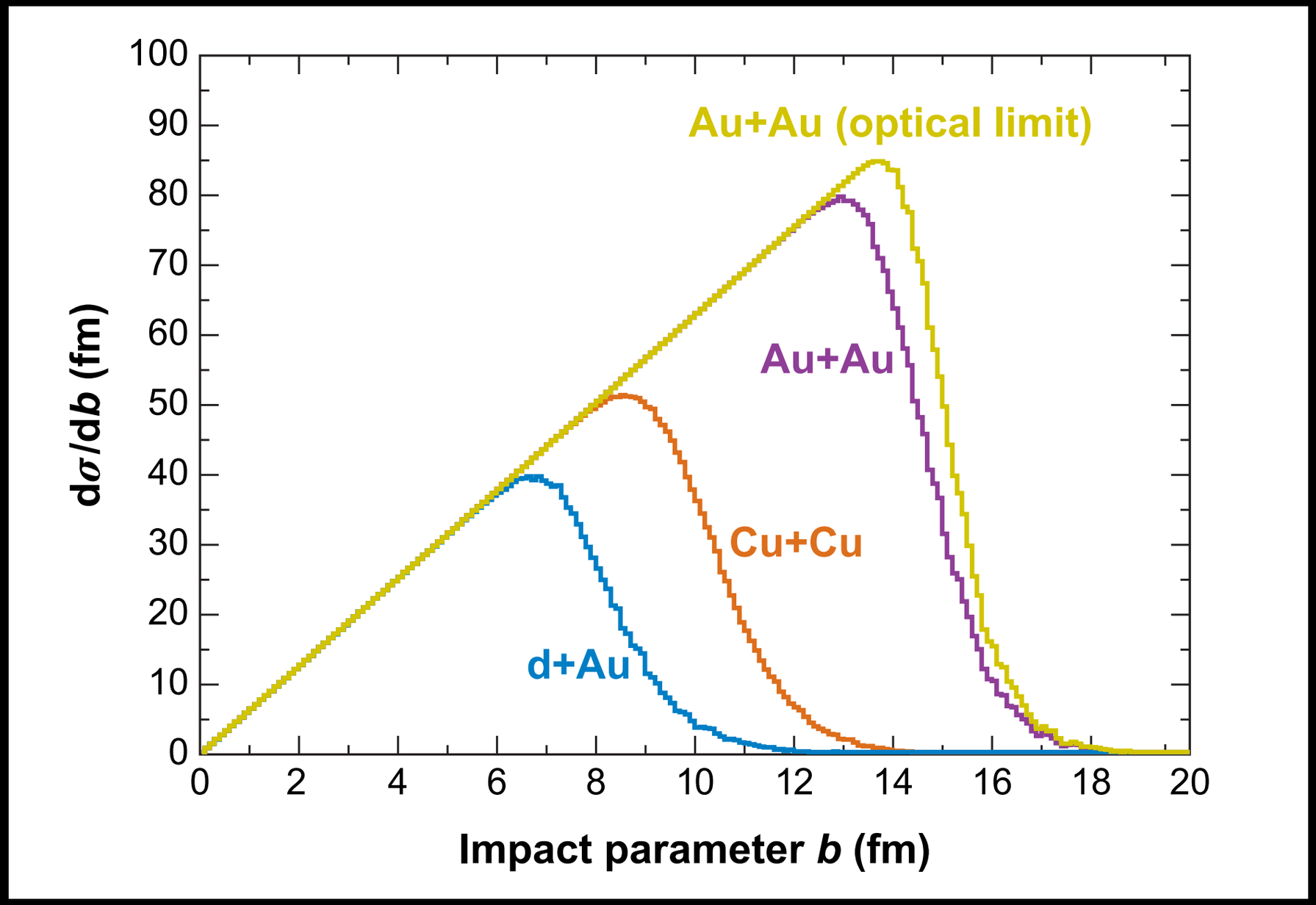


$$\sigma_{inel}^{AB} = \int d^2b \int d^2s_1^A \cdots d^2s_A^A d^2s_1^B \cdots d^2s_B^B \times$$
$$\hat{T}_A(\mathbf{s_1^A}) \cdots \hat{T}_A(\mathbf{s_A^A}) \hat{T}_B(\mathbf{s_1^B}) \cdots \hat{T}_B(\mathbf{s_B^B}) \times$$
$$\left\{ 1 - \prod_{j=1}^{B} \prod_{i=1}^{A} [1 - \hat{\sigma}(\mathbf{b} - \mathbf{s_i^A} + \mathbf{s_j^B})] \right\}$$

A complicated 800-dimensional integral, but more intuitive!

Ann.Rev.Nucl.Part.Sci.57:205-243,2007

Effect on Total Cross Section



Total cross section systematically <u>larger</u> in optical approach

"Eclipsing"

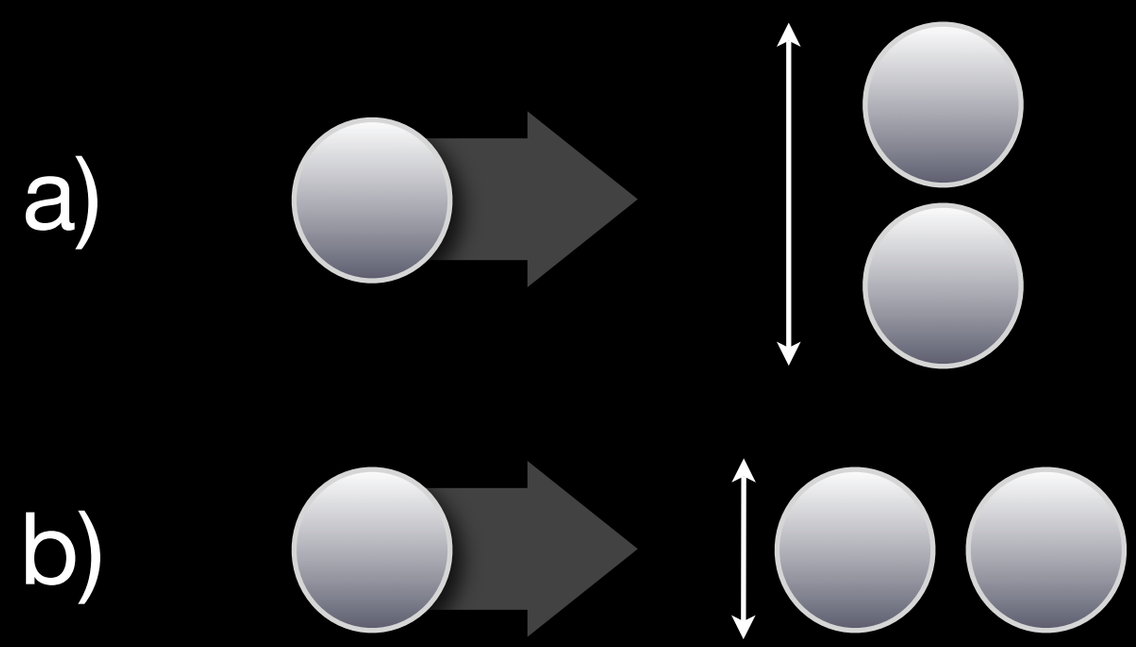
PHYSICAL REVIEW VOLUME 100, NUMBER 1 OCTOBER 1, 1955

Cross Sections in Deuterium at High Energies

R. J. GLAUBER
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received May 27, 1955)

Recent measurements of nucleon attenuation at 1.4 Bev (where $\lambda\!\!\!^{-}=0.1\times10^{-13}$ cm) seem, on the contrary, to reveal a substantial lack of additivity of the neutron and proton cross sections, in deuterium.[1,2] Measurements with incident protons and incident neutrons both indicate that the deuteron cross section is less than the sum of the free-particle cross sections. The measured differences, although obviously subject to uncertainty, amount to 9 mb and 6 mb respectively, values to be compared with $\sigma(n,p)=42$ mb and $\sigma(p,p)=48$ mb.

Some simple considerations may be of help in indicating the nature of the effect. At these energies the attenuation of the incident amplitude by incoherent processes such as meson production may be schematically represented as due to a certain amount of absorption of the incident wave by the nucleons. Since the incident wavelengths in these cases are evidently much smaller than the ranges of interaction, the nucleons may be thought of as casting fairly well-defined shadows. It is then clear that absorption or scattering by either nucleon is reduced when it enters the shadow of the other. Astronomers have long been familiar with a time-reversed analog of this effect; the decrease in luminosity of binary star systems during eclipses.



$$\sigma_a > \sigma_b$$

Optical vs. Glauber

- **Not a purely academic question**

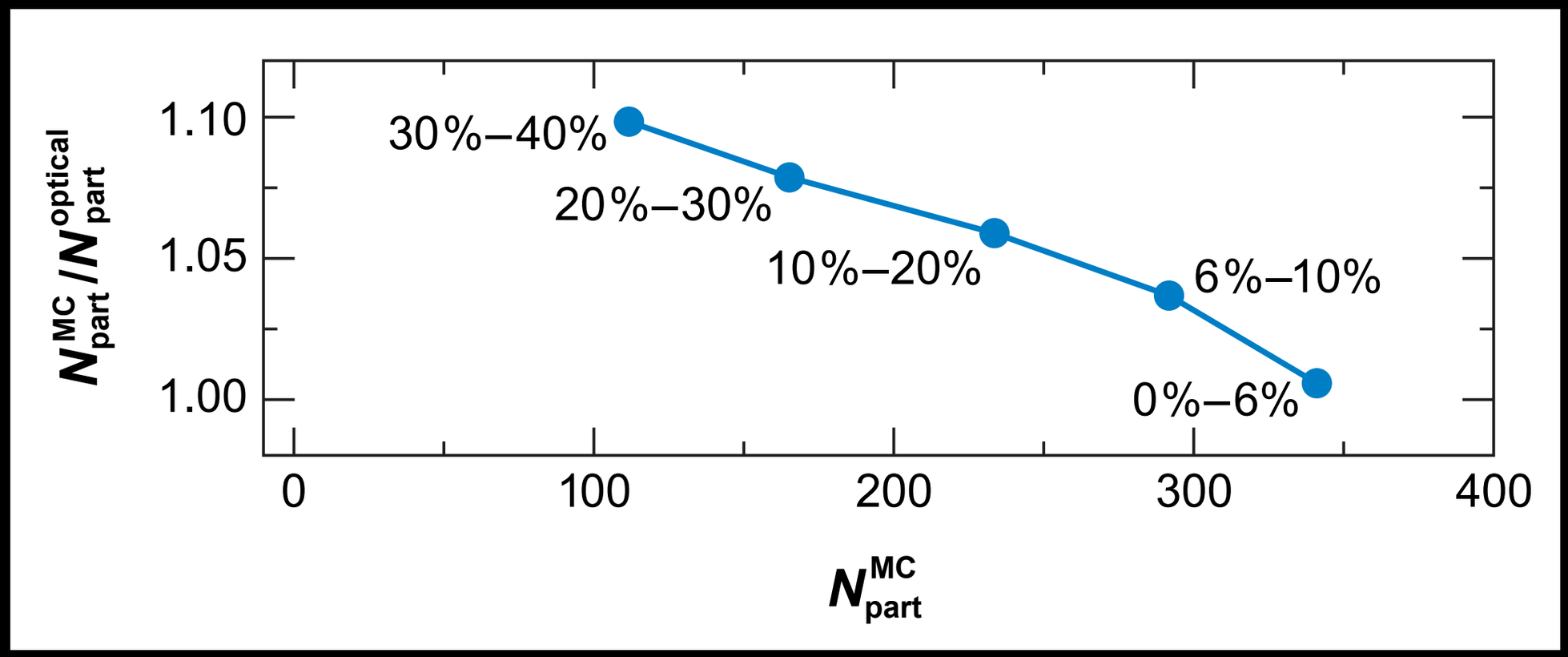
- **Nuclear configuration can shift event to event, which is how we do physics at RHIC (e.g. v_2)**

  we average measurements, not vice versa!

- **Our techniques for estimating the geometry should accomodate this**

Ann.Rev.Nucl.Part.Sci.57:205-243,2007

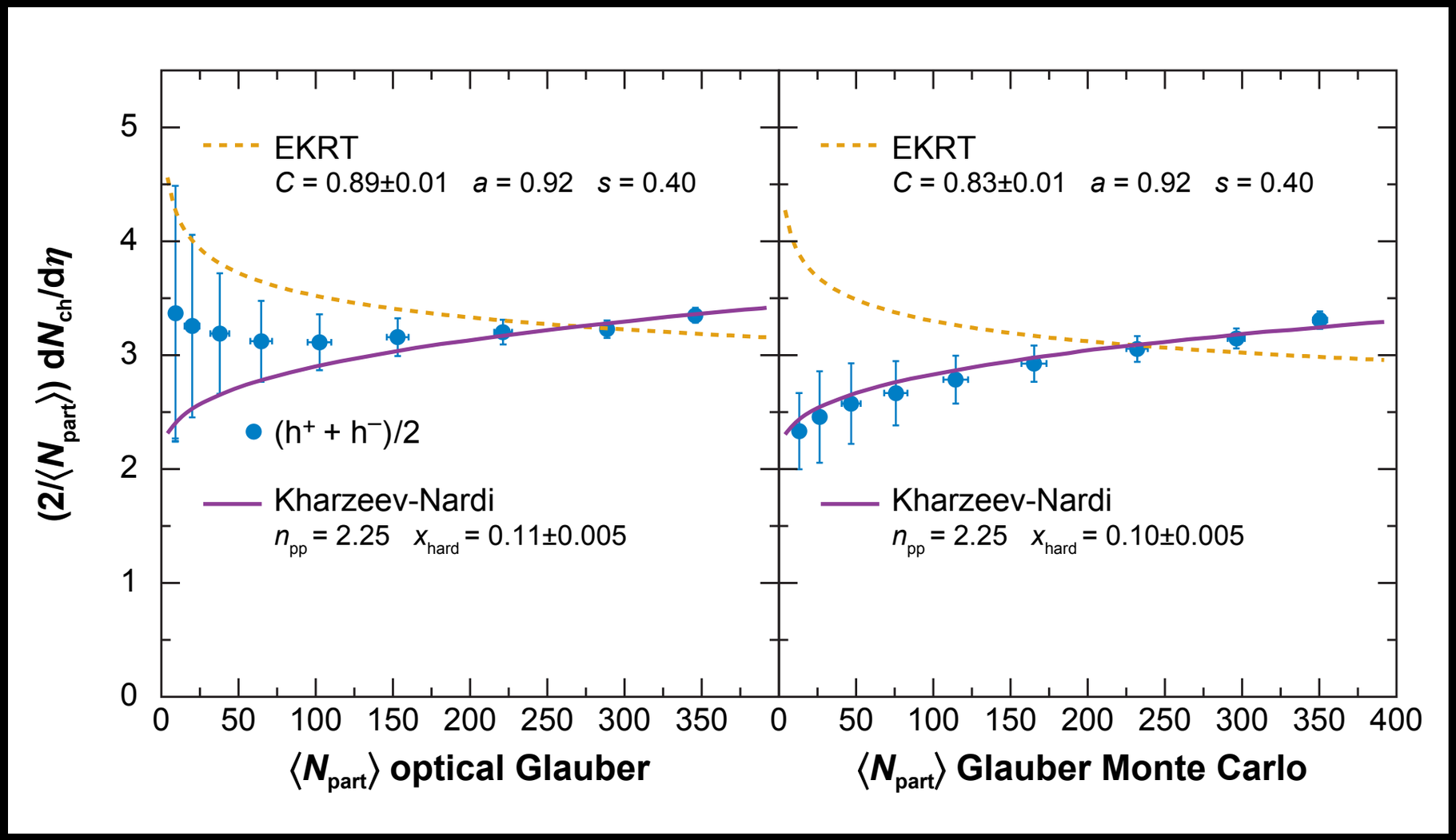
Optical vs. MC



Generically, ignoring fluctuations leads to <u>underestimating</u> N_{part} in peripheral events

Ann.Rev.Nucl.Part.Sci.57:205-243,2007

Effect on Observables



Interpretation of data can be changed by using different (i.e. wrong) Glauber approach

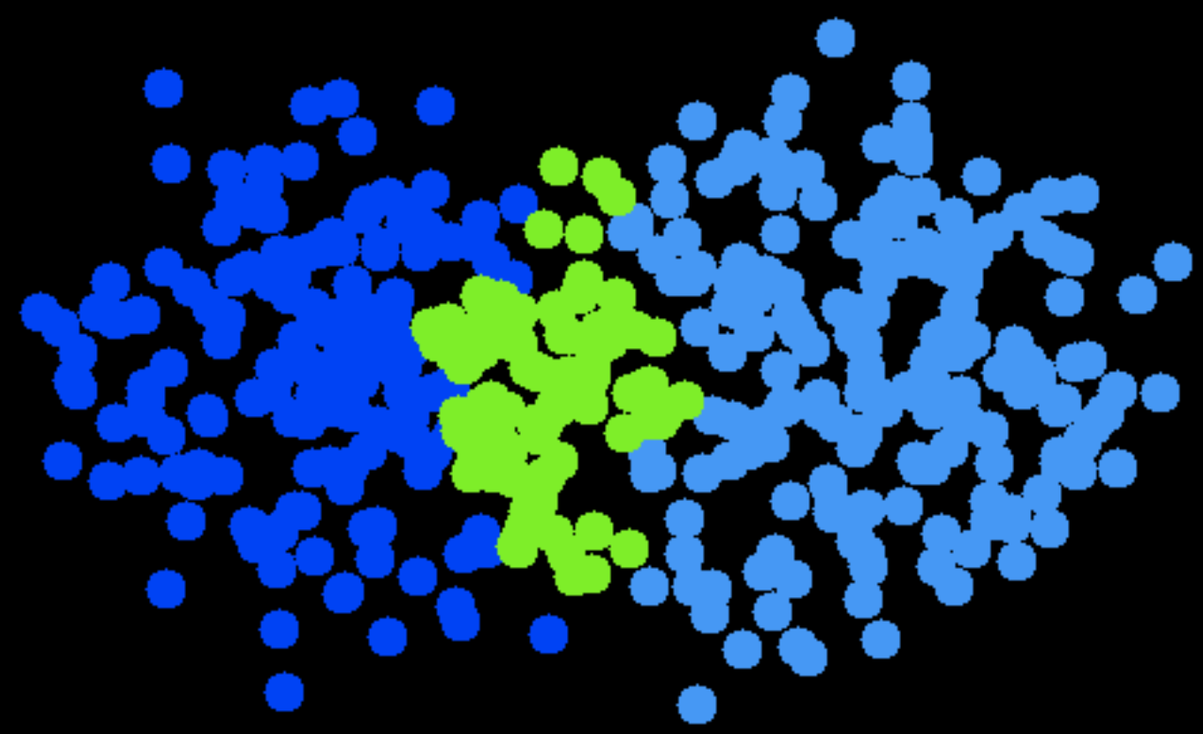
Role of Glauber @ Early Times

- **The inelastic cross section shows that Glauber matters as to whether anything happens at all!**
  - Do CGC-shadowed calculations give σ_{tot}?

- **It can also give us a hint as to how and where matter was produced**

SLP Approach

Sudden Localized Participant Approach

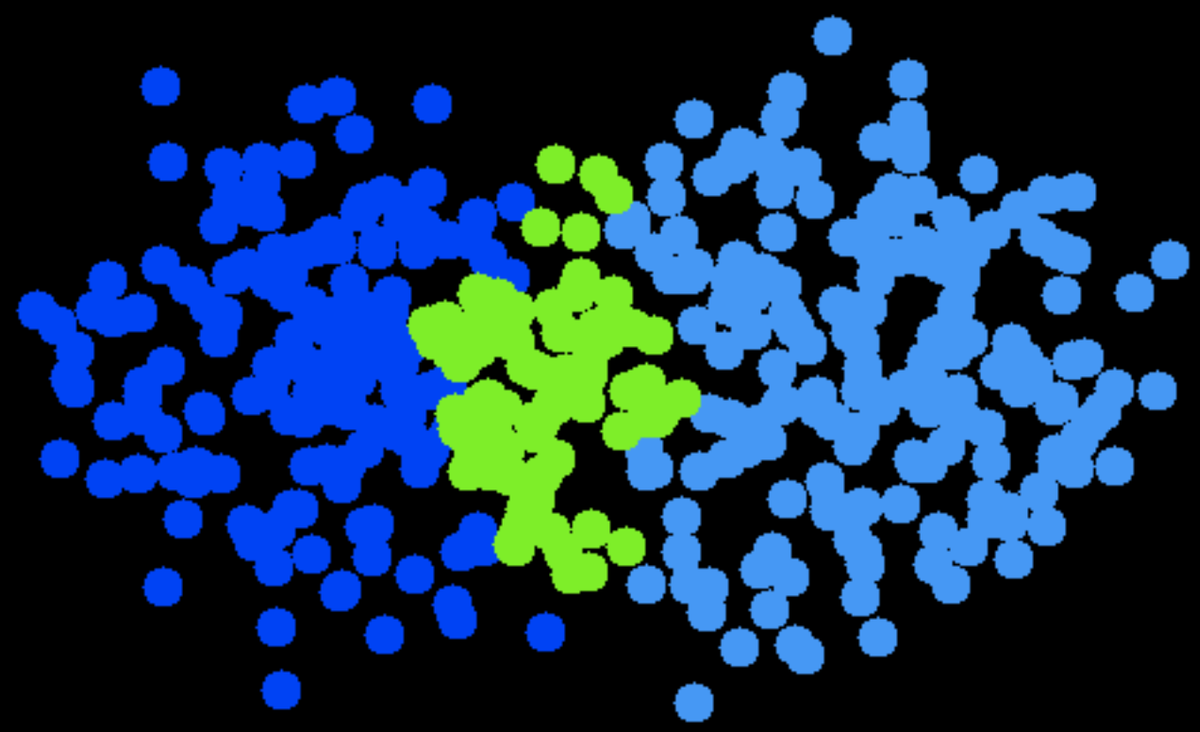


If total entropy is linear with N_{part},
let us also assume that the matter is created
<u>where</u> the soft interactions occur.

If it thermalizes <u>suddenly</u>, then this is the initial state
for hydrodynamic evolution

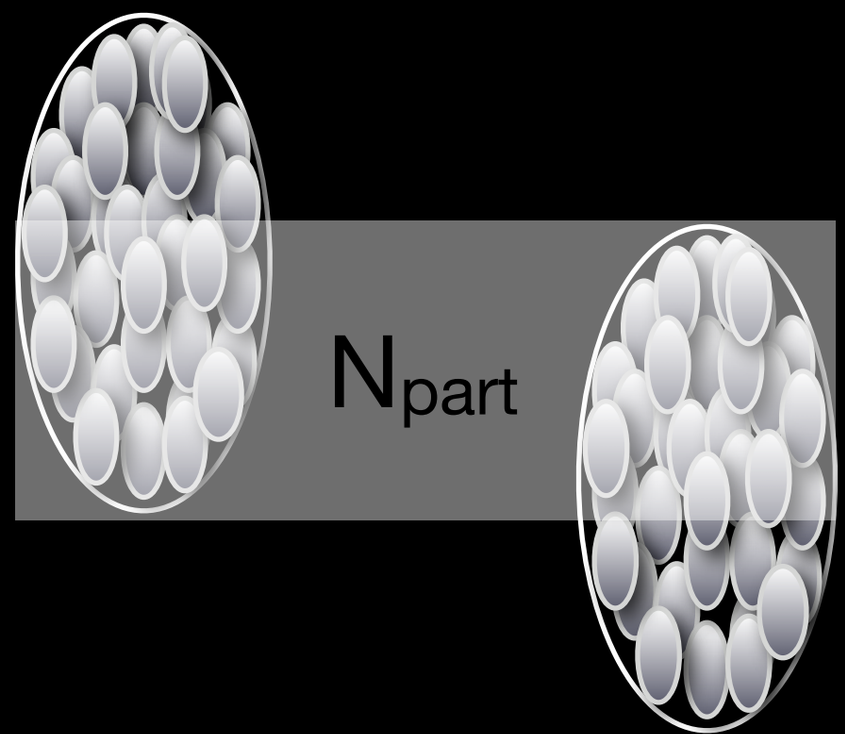
SPLAT

Sources are Participants, Localized At Thermalization



Ann.Rev.Nucl.Part.Sci.57:205-243,2007

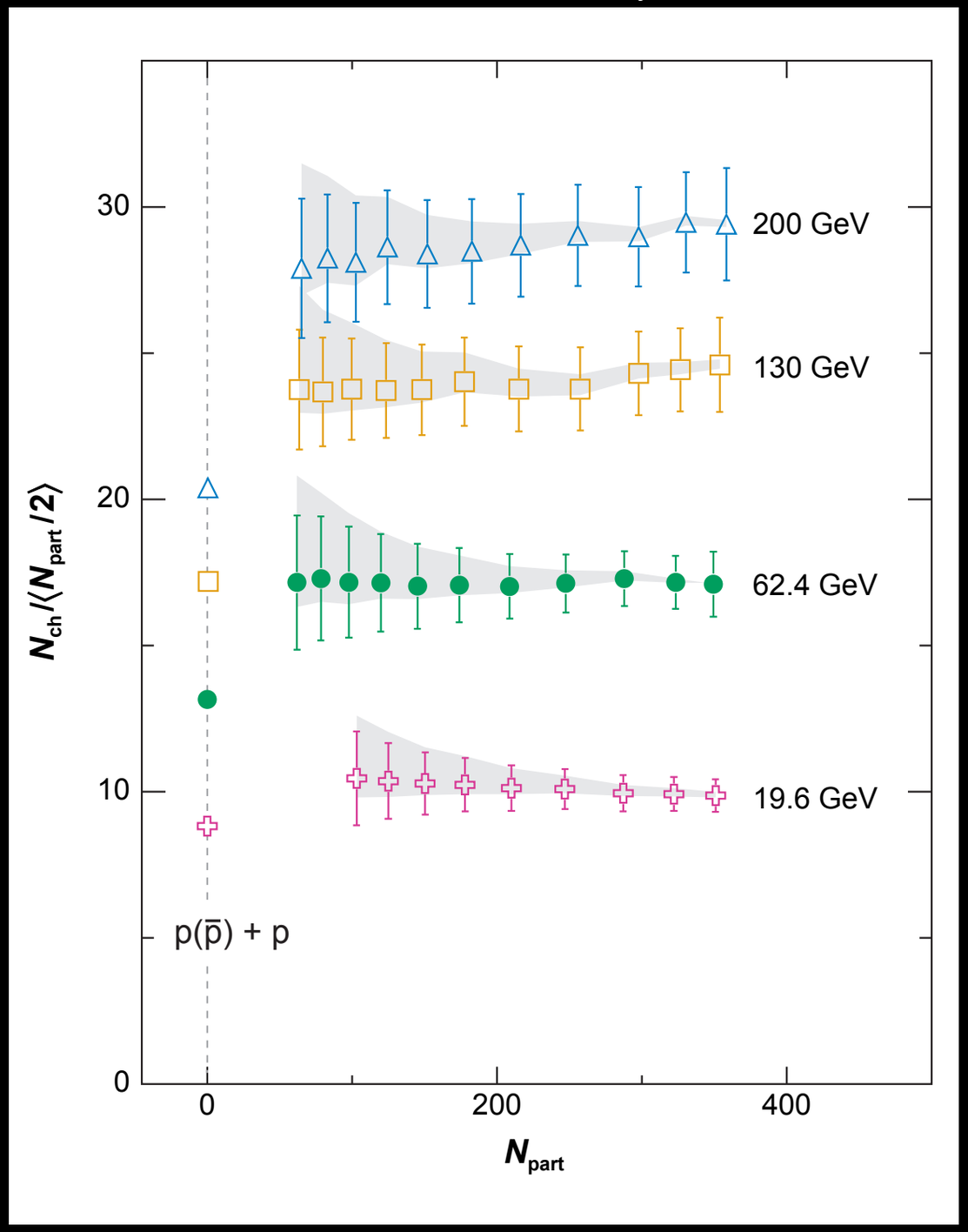
Total Multiplicity



Total produced entropy scales <u>linearly</u> with N_{part}

No information on where matter was created

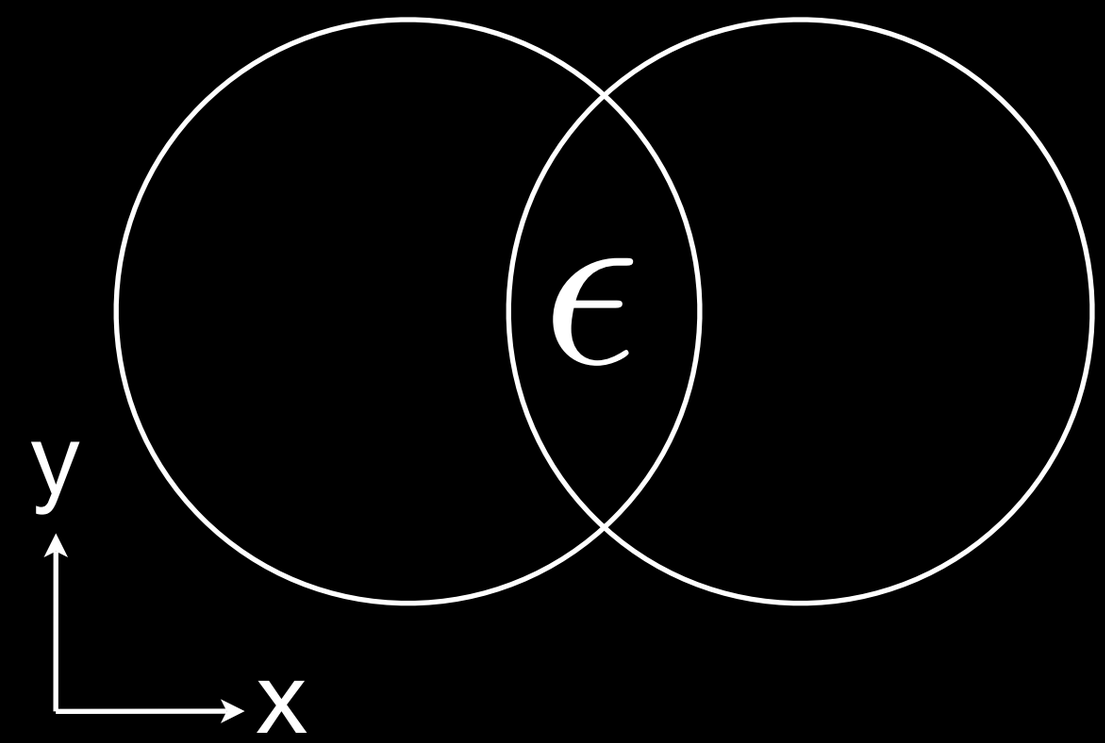
Phys.Rev.C74:021902,2006



Eccentricity

Overlap zone where matter thermalizes has a particular "shape" vs. impact parameter

$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$

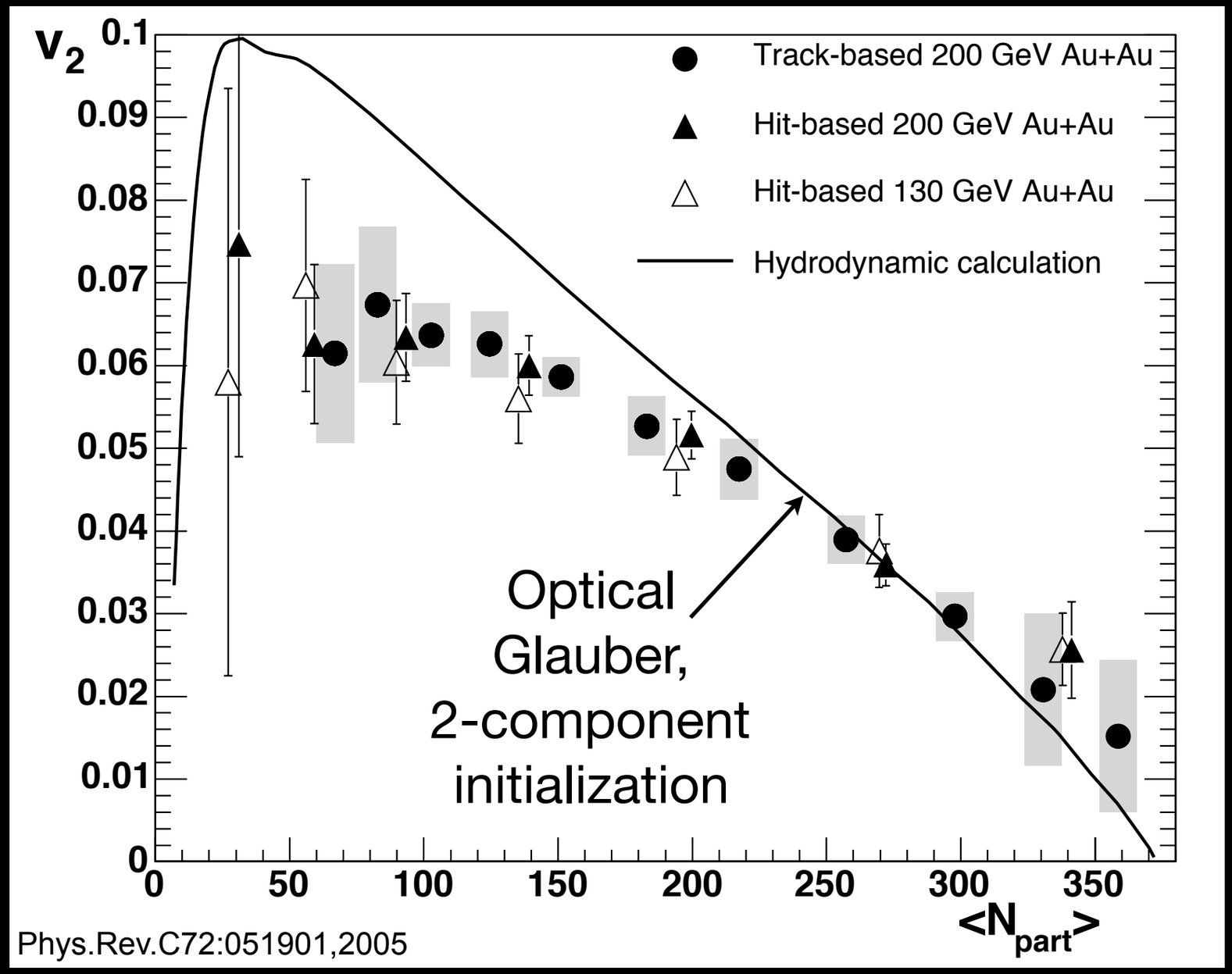


Generically, hydro predicts complete transfer of spatial anisotropy into momentum anisotropy!

$$v_2 \propto \epsilon$$

Hydro is sensitive to where the matter was (and not what!)

Hydro @ RHIC



hydro scales

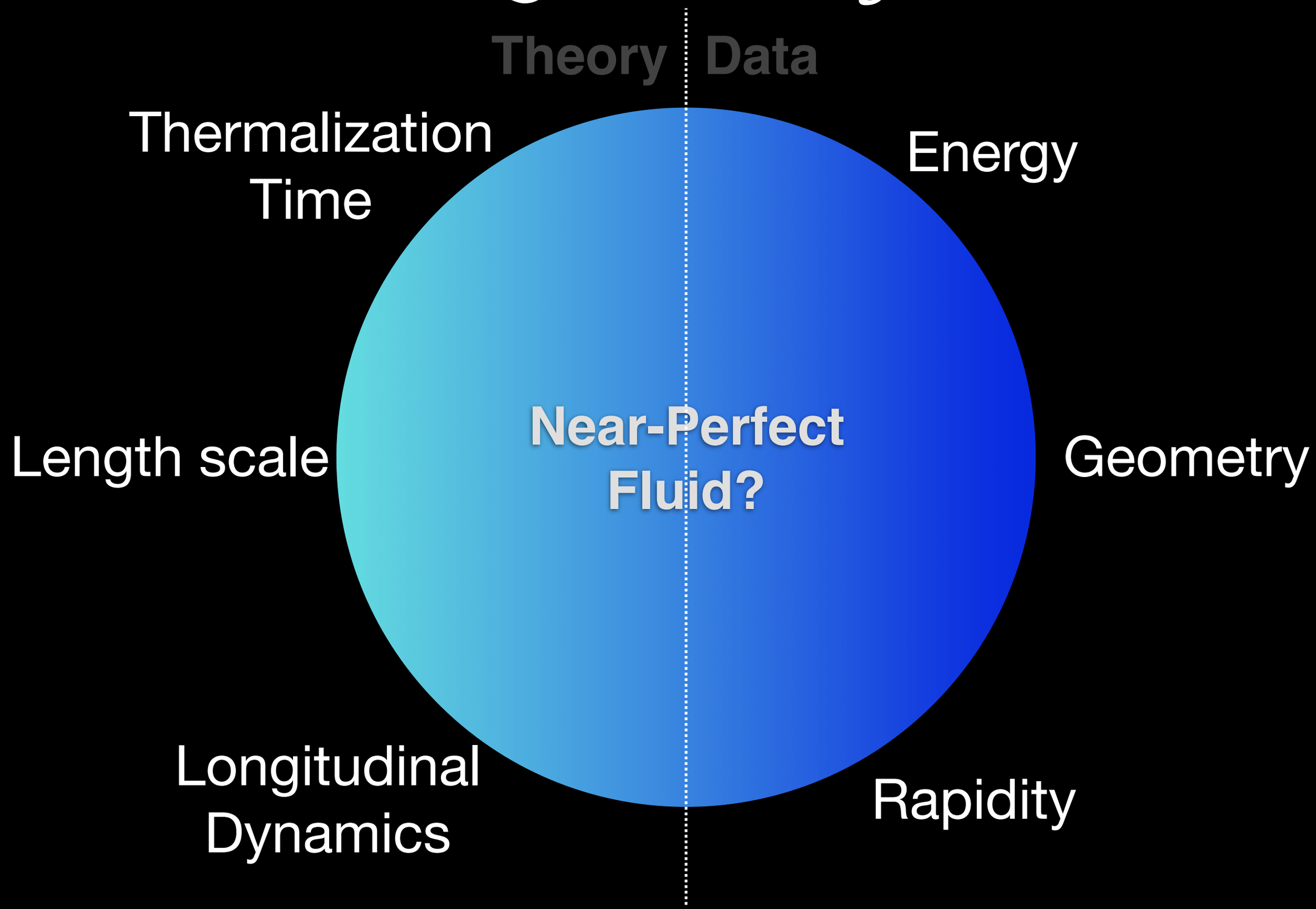
$$\tau_0 \sim 0.6 fm/c$$
$$\epsilon \sim 30\ GeV/fm^3$$

$$\Longleftrightarrow$$

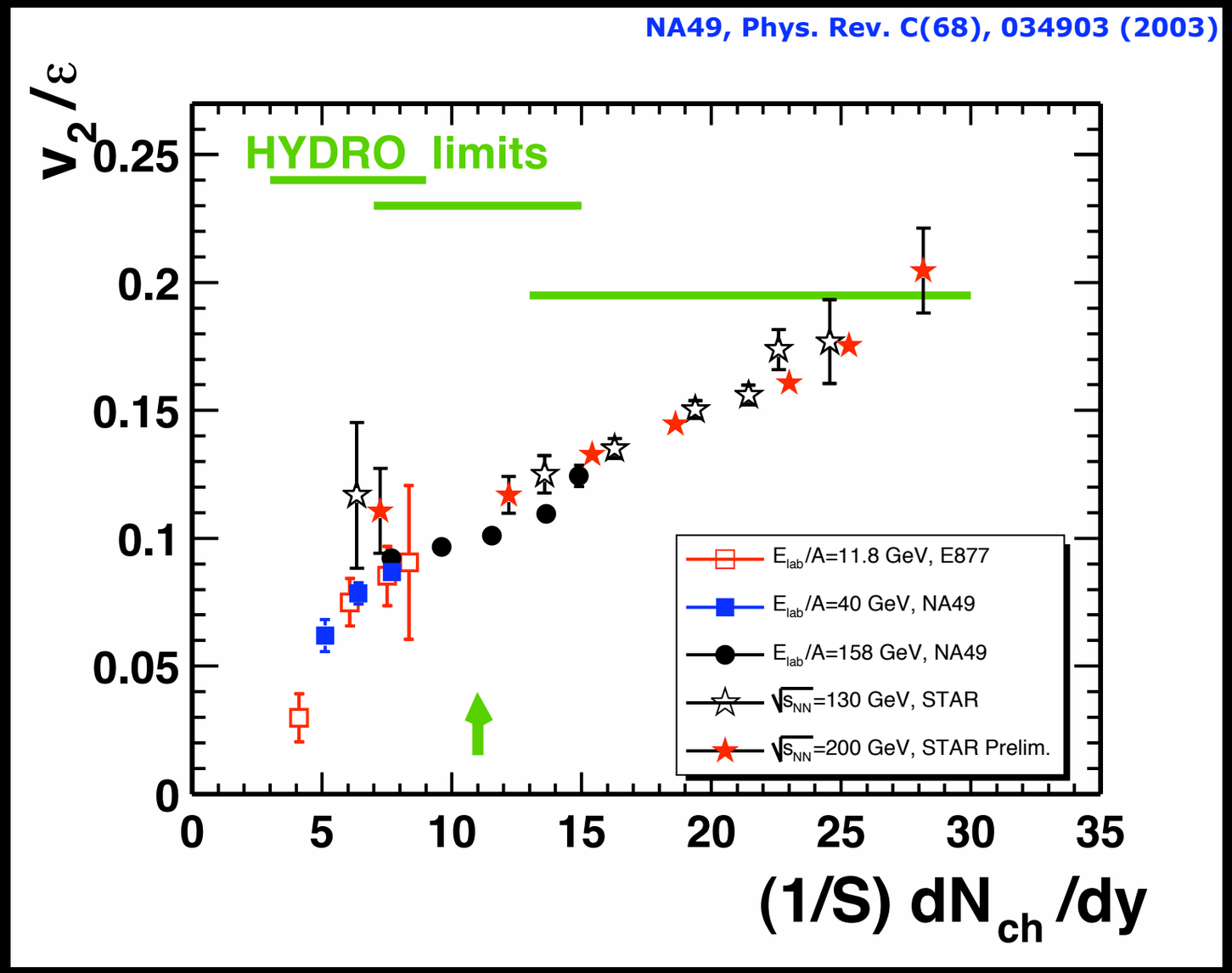
$$\tau_0 \sim 1\ fm/c$$
$$\epsilon \sim 500\ MeV/fm^3$$

hadronic scales

The Edge of Liquidity



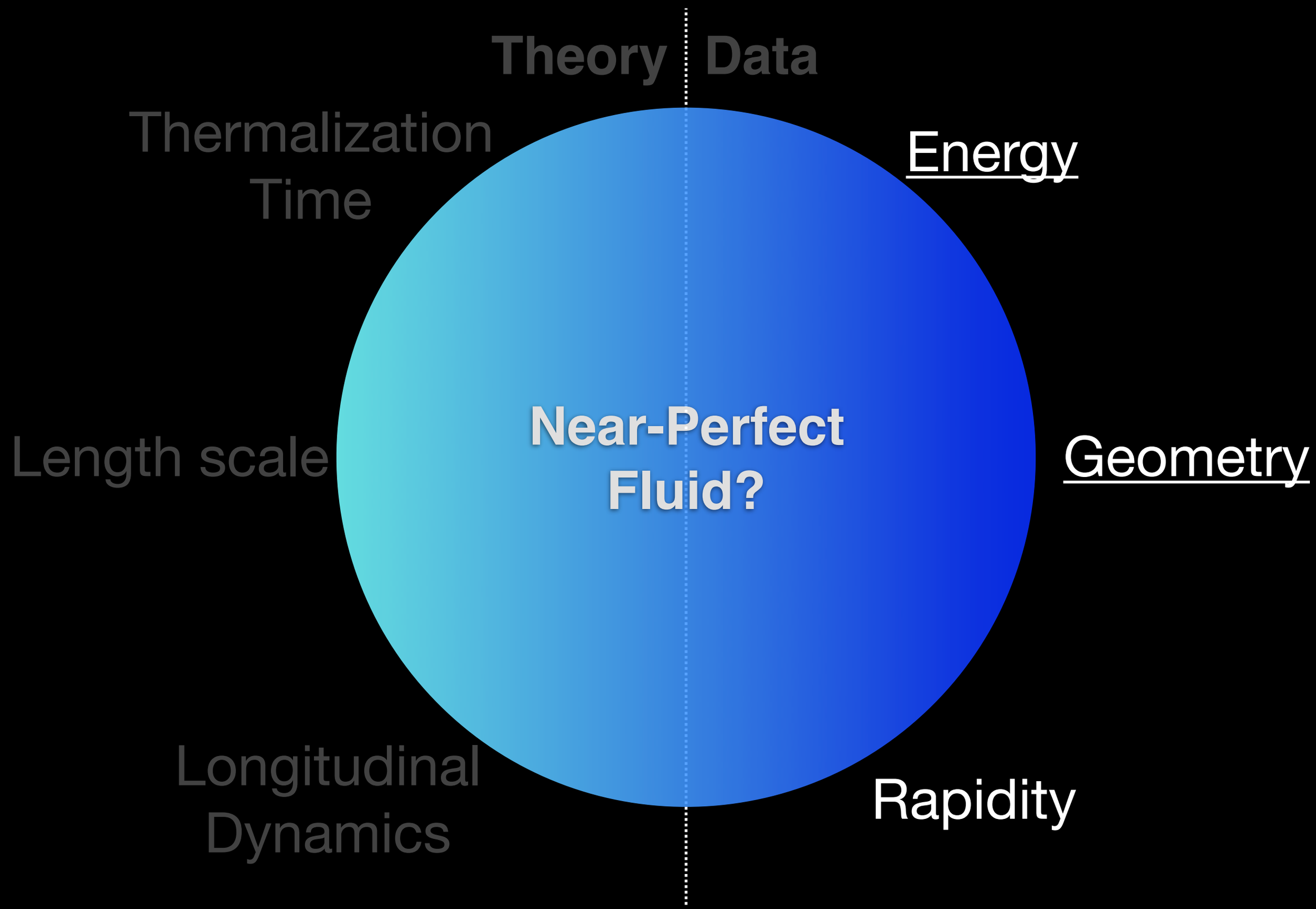
"Scaling Behavior"



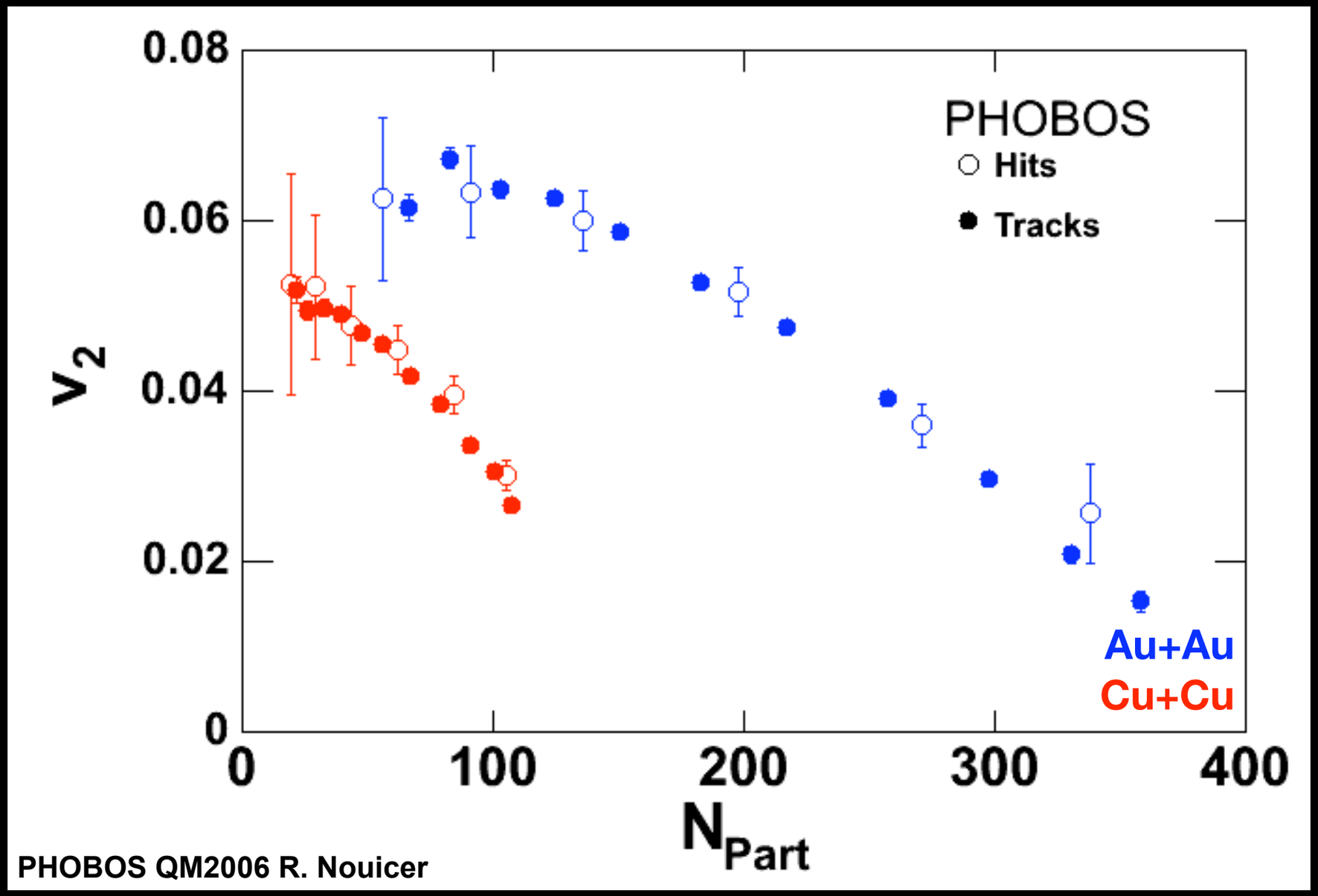
"hydro limit"?

"pressure" $\frac{v_2}{\epsilon}$ is a simple function of $\frac{dN/dy}{S}$ "transverse density"

Is this hydrodynamic equilibration, or just the approach to it? In any case, it seems to be universal

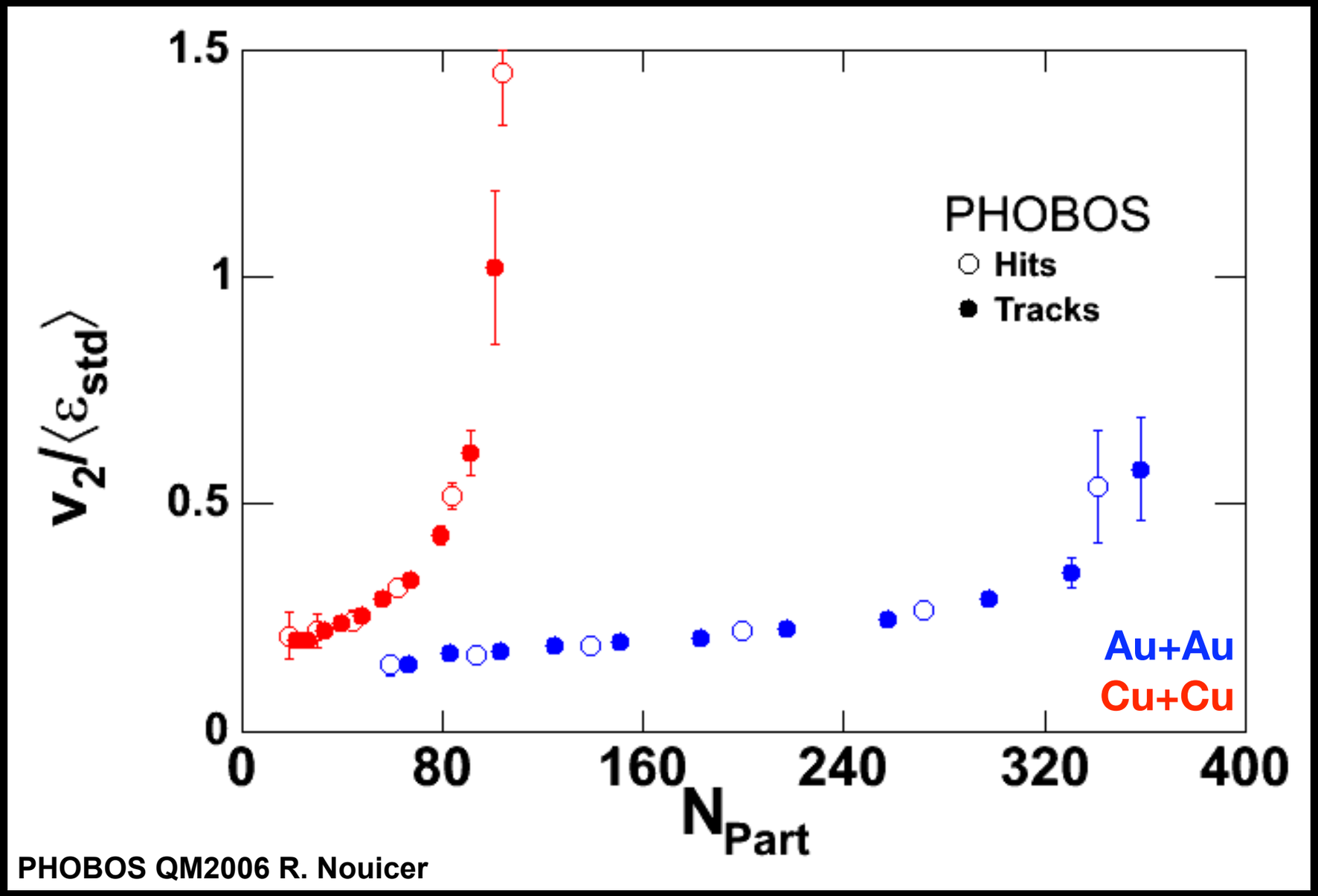


Does v_2 follow ε ?



v_2 does not go to zero when eccentricity should (b~0)

Something wrong...

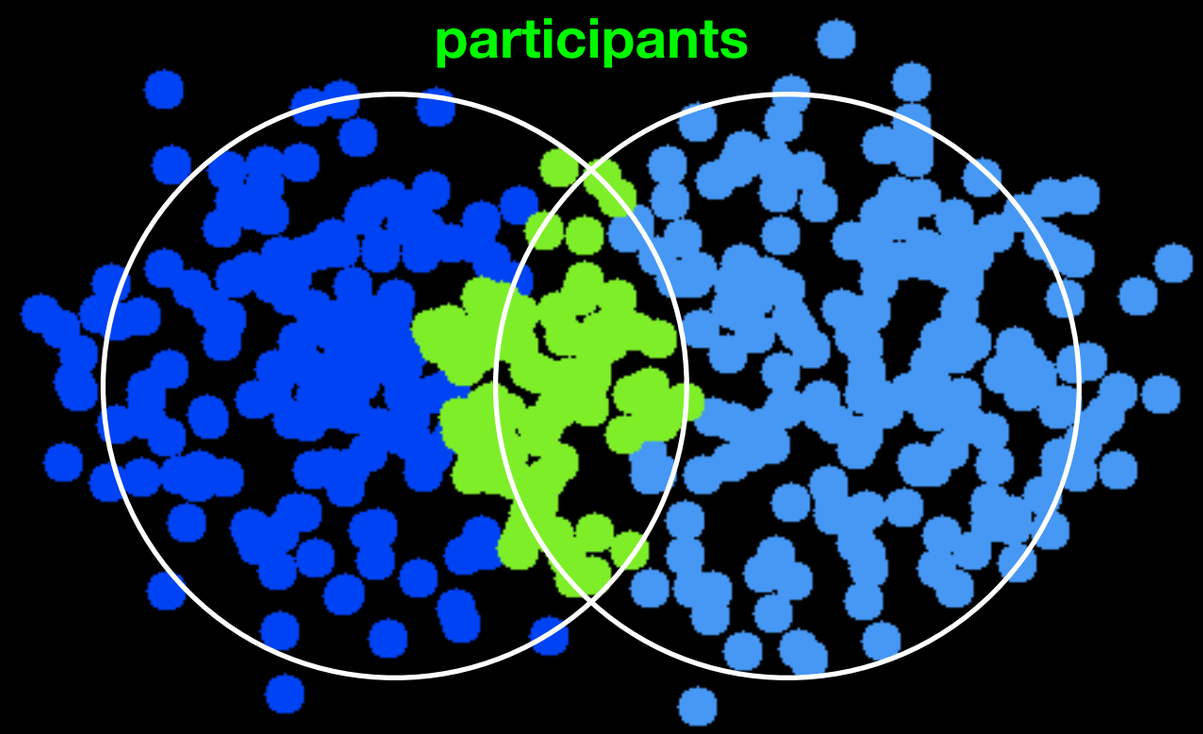


PHOBOS QM2006 R. Nouicer

Eccentricity Fluctuations

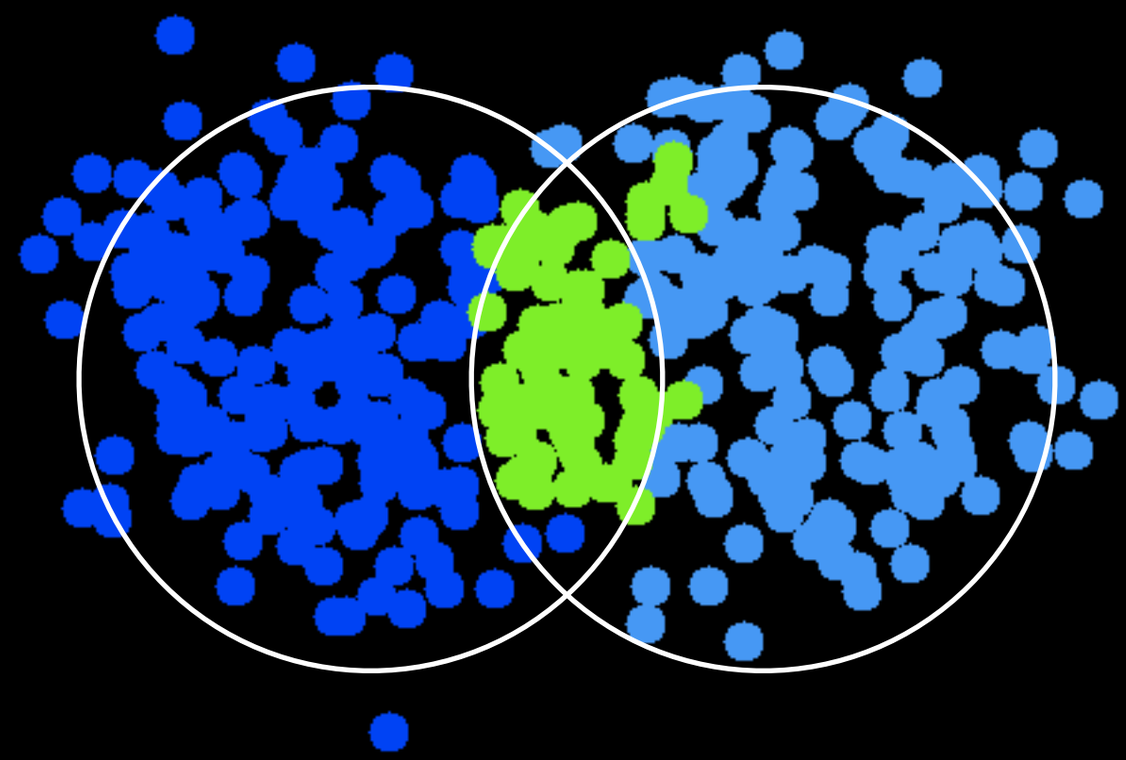
Optical limit

Glauber Monte Carlo

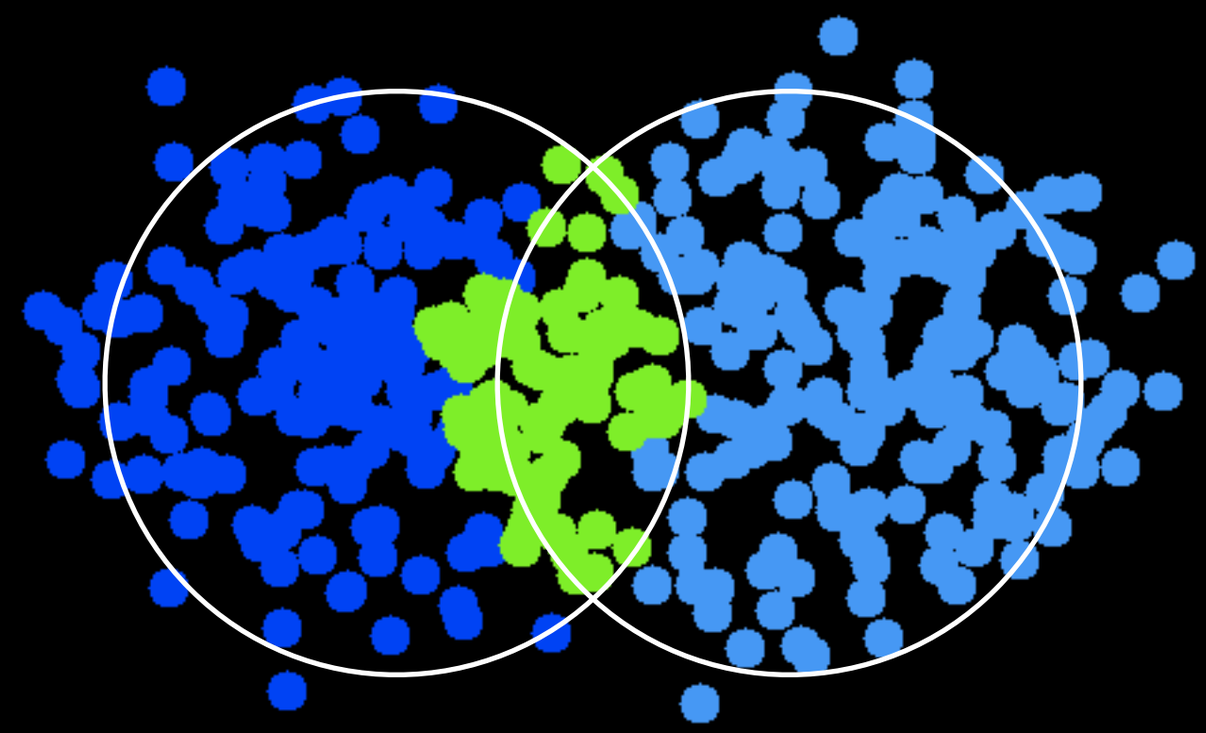


We know nuclei are made of nucleons,
Why “insist” that an <u>average</u> density
matters for flow measurements?

Au+Au



Au+Au

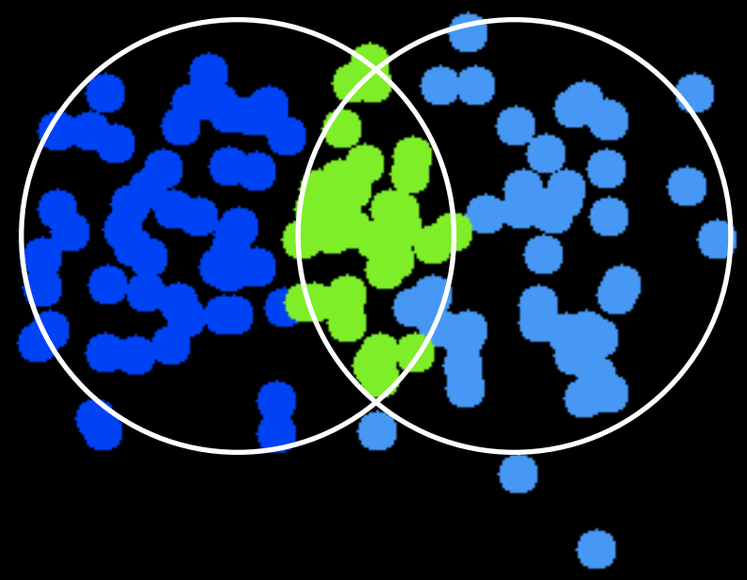


Participants trace out overlap zone, but include

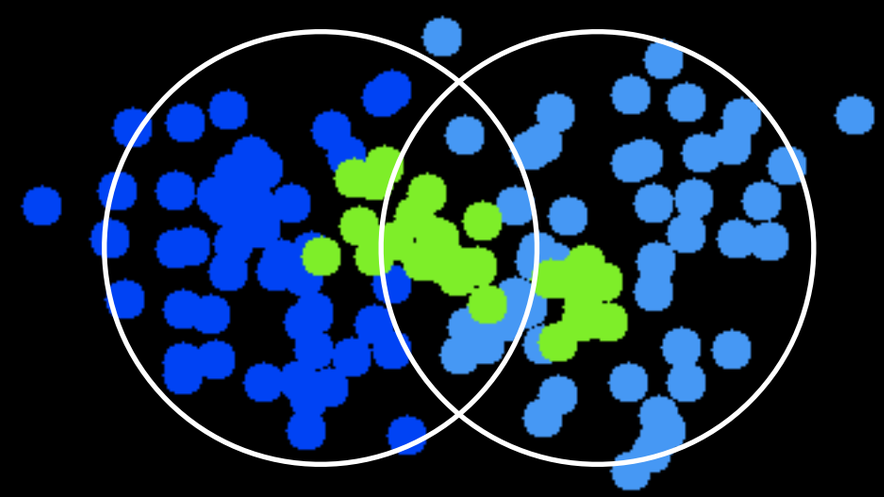
1. Fluctuations (finite number per event)
2. <u>Correlations</u> (it takes two to tango...)

(NB: these are snapshots of nucleon configurations, not stable nuclear states!)

Cu+Cu

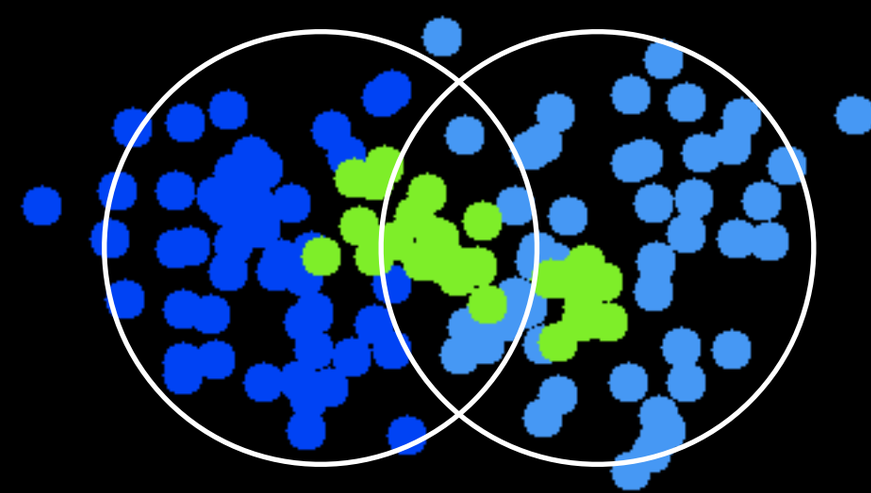


Cu+Cu



Fluctuations can seriously deviate from nominal overlap zone for small numbers of nucleons

Cu+Cu

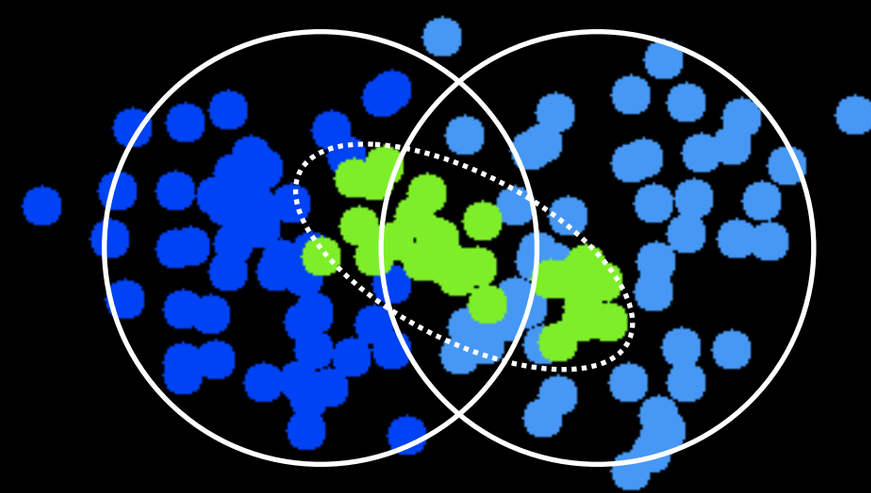


$$\epsilon_{RP} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$

similar to “Standard eccentricity” $\epsilon_{std} = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$

Cu+Cu

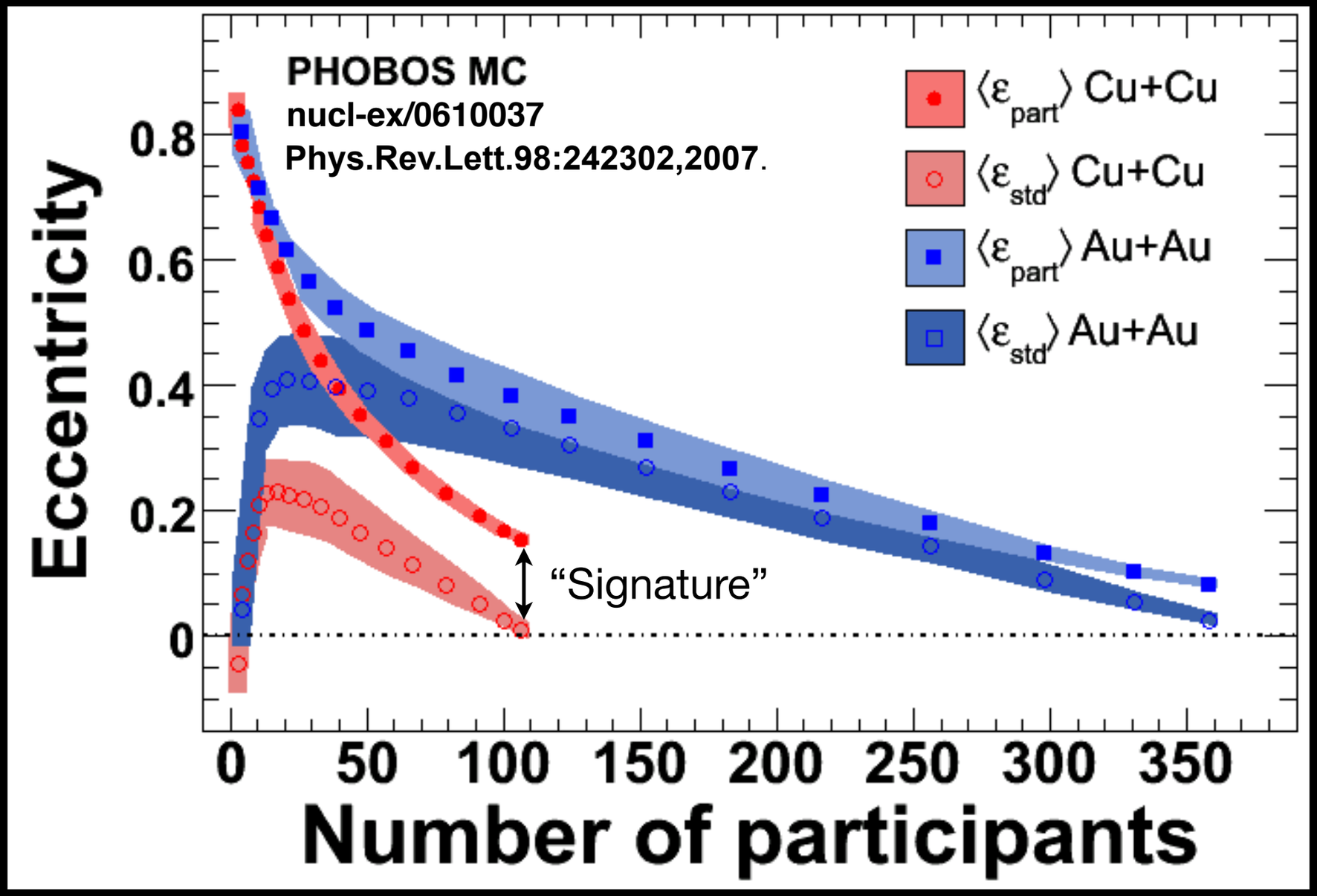
Principal axes make sense if v_2 depends on shape of produced matter (in SLP), not the reaction plane



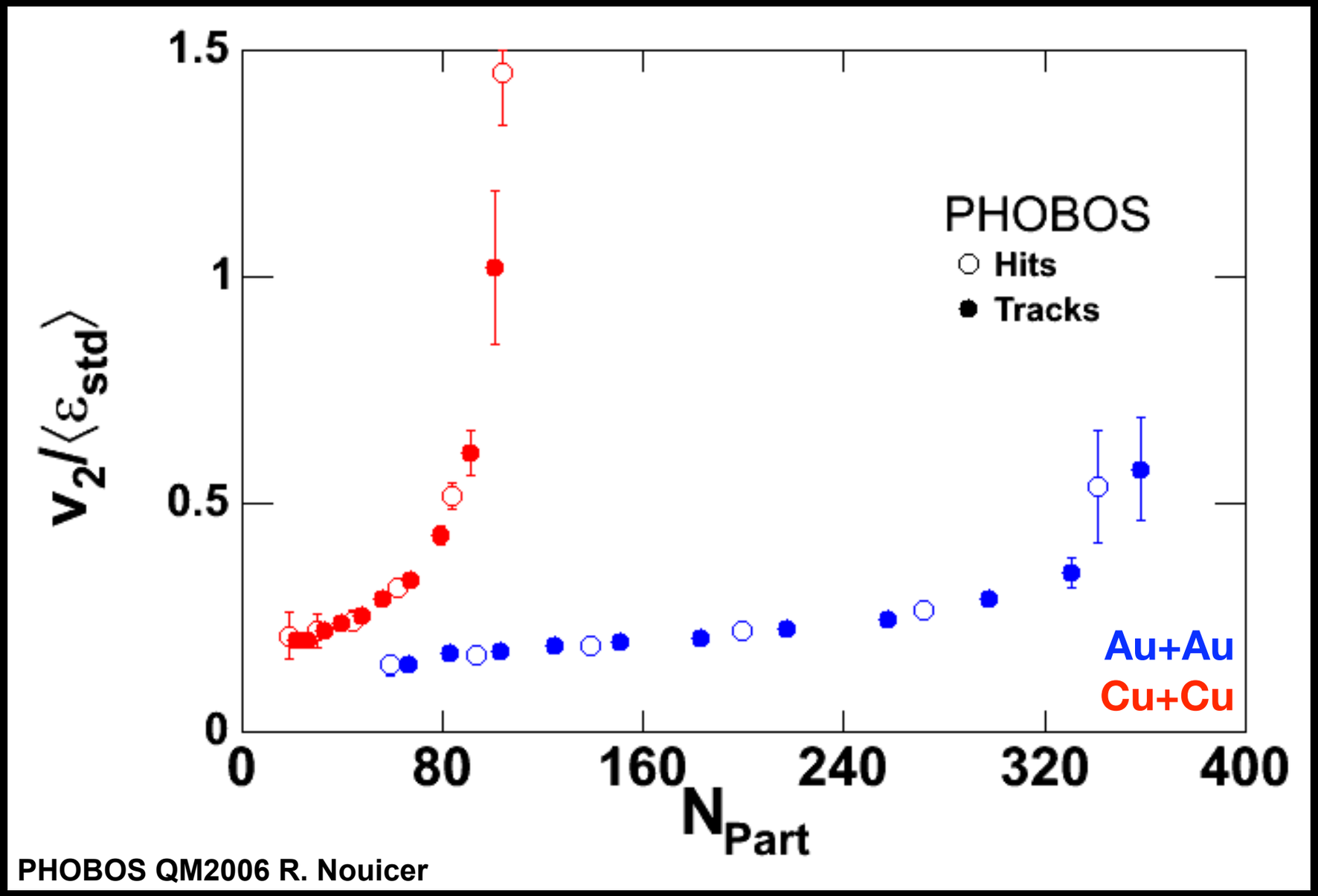
$$\epsilon_{part} = \frac{\sigma_y'^2 - \sigma_x'^2}{\sigma_y'^2 + \sigma_x'^2} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy}^2)^2}}{\sigma_y^2 + \sigma_x^2}$$

"Participant eccentricity"

Participant vs. Standard

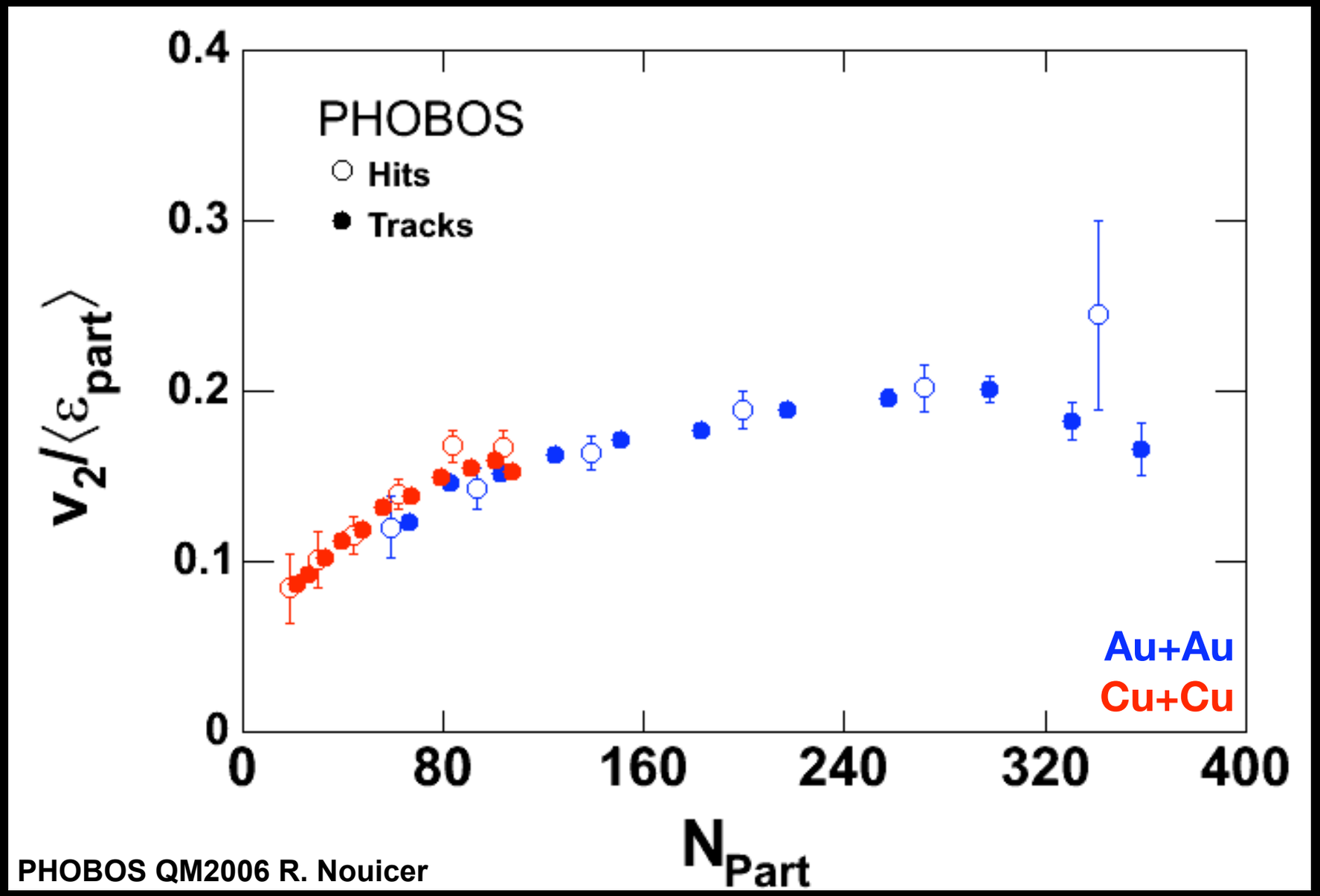


Something wrong...



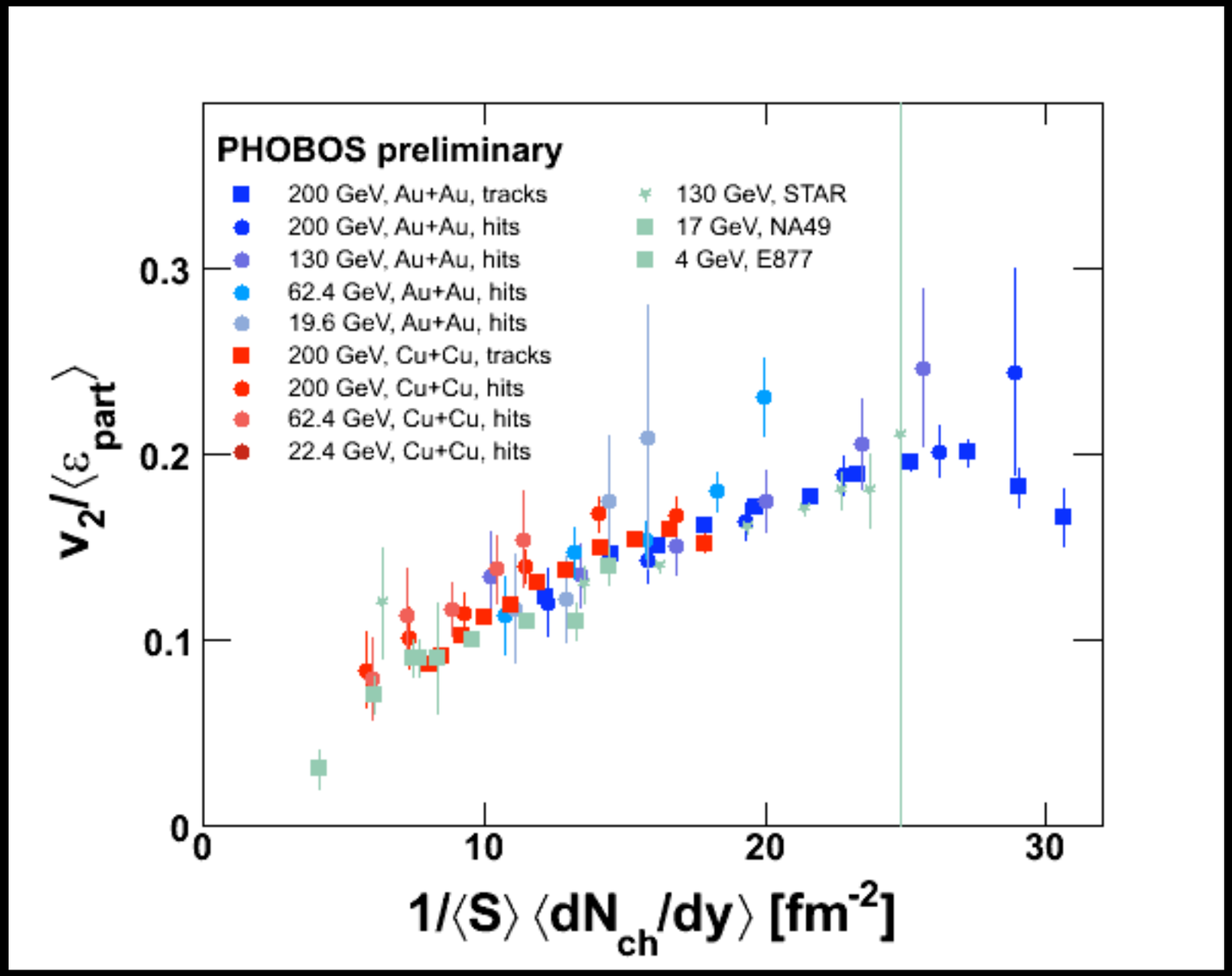
PHOBOS QM2006 R. Nouicer

...leads to scaling



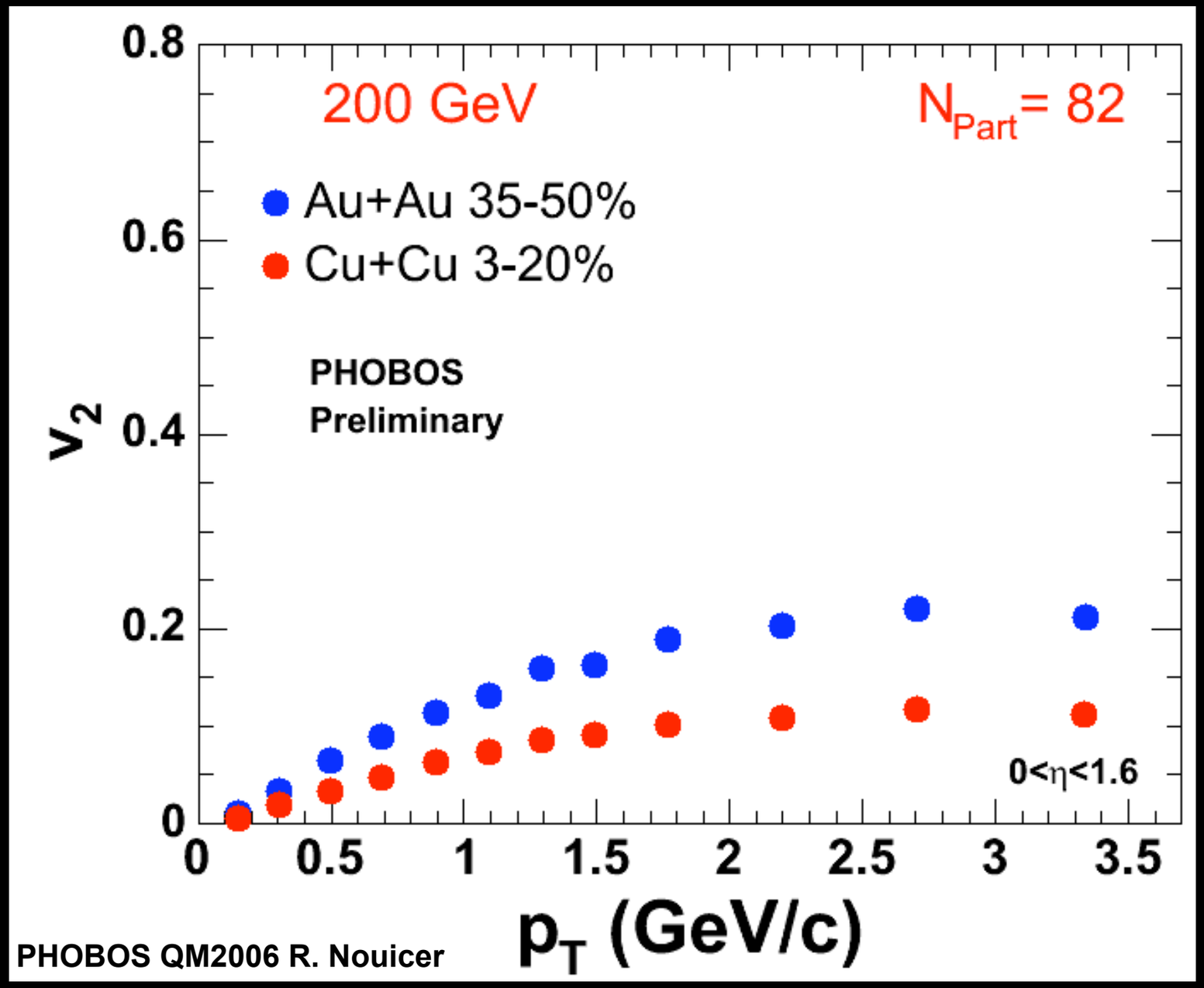
vs. Areal Density

PHOBOS QM2006



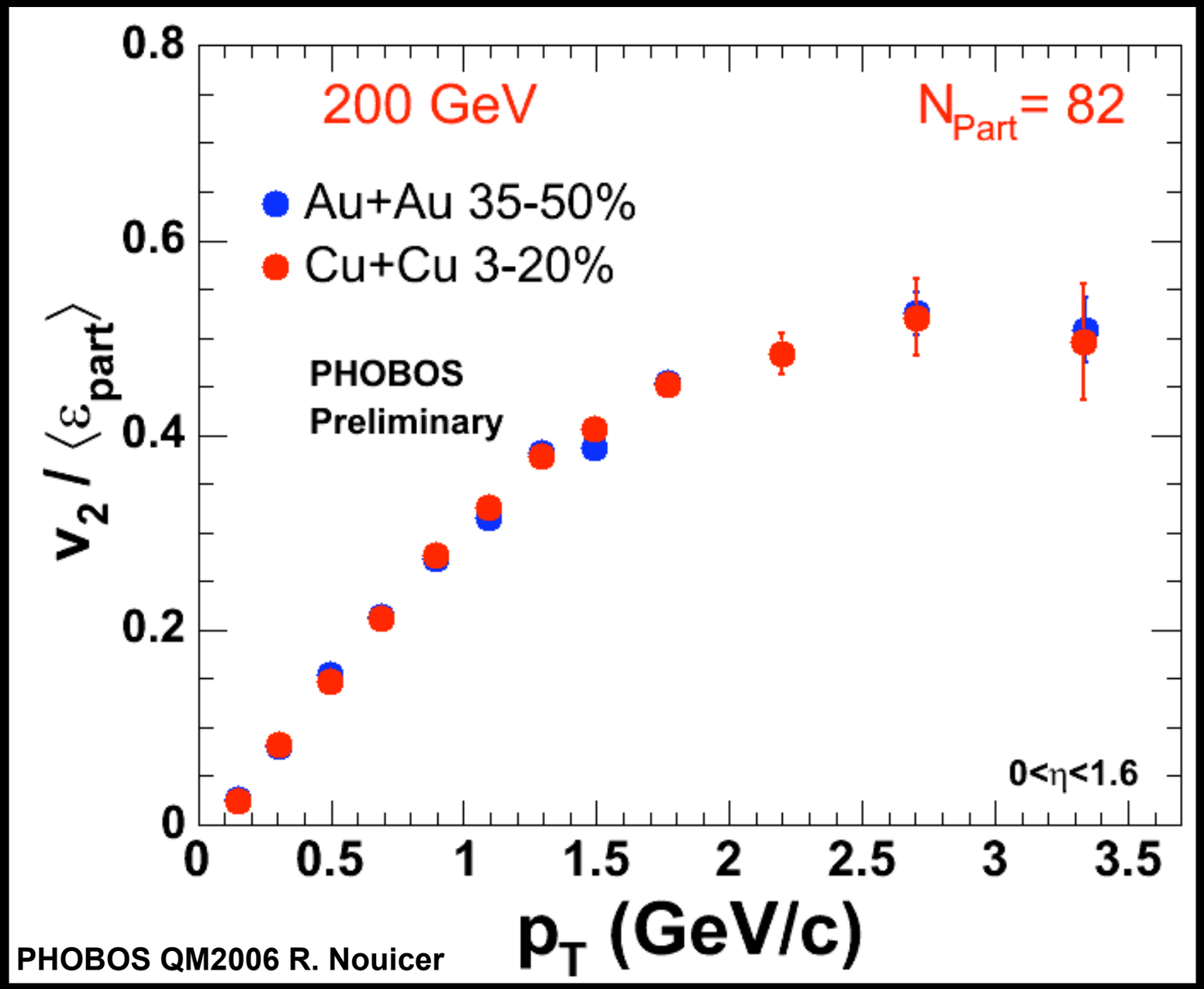
statistical errors only

Transverse Momentum



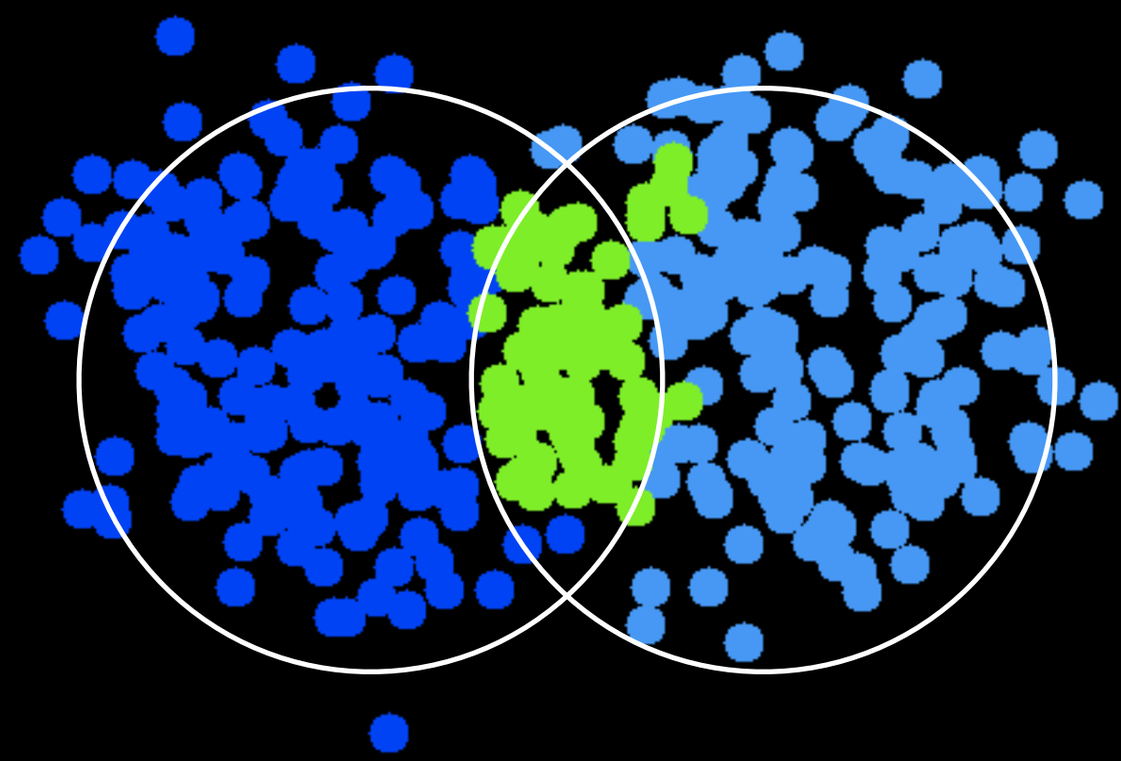
Choose two bins with same N_{part} (~same density)

Transverse Momentum



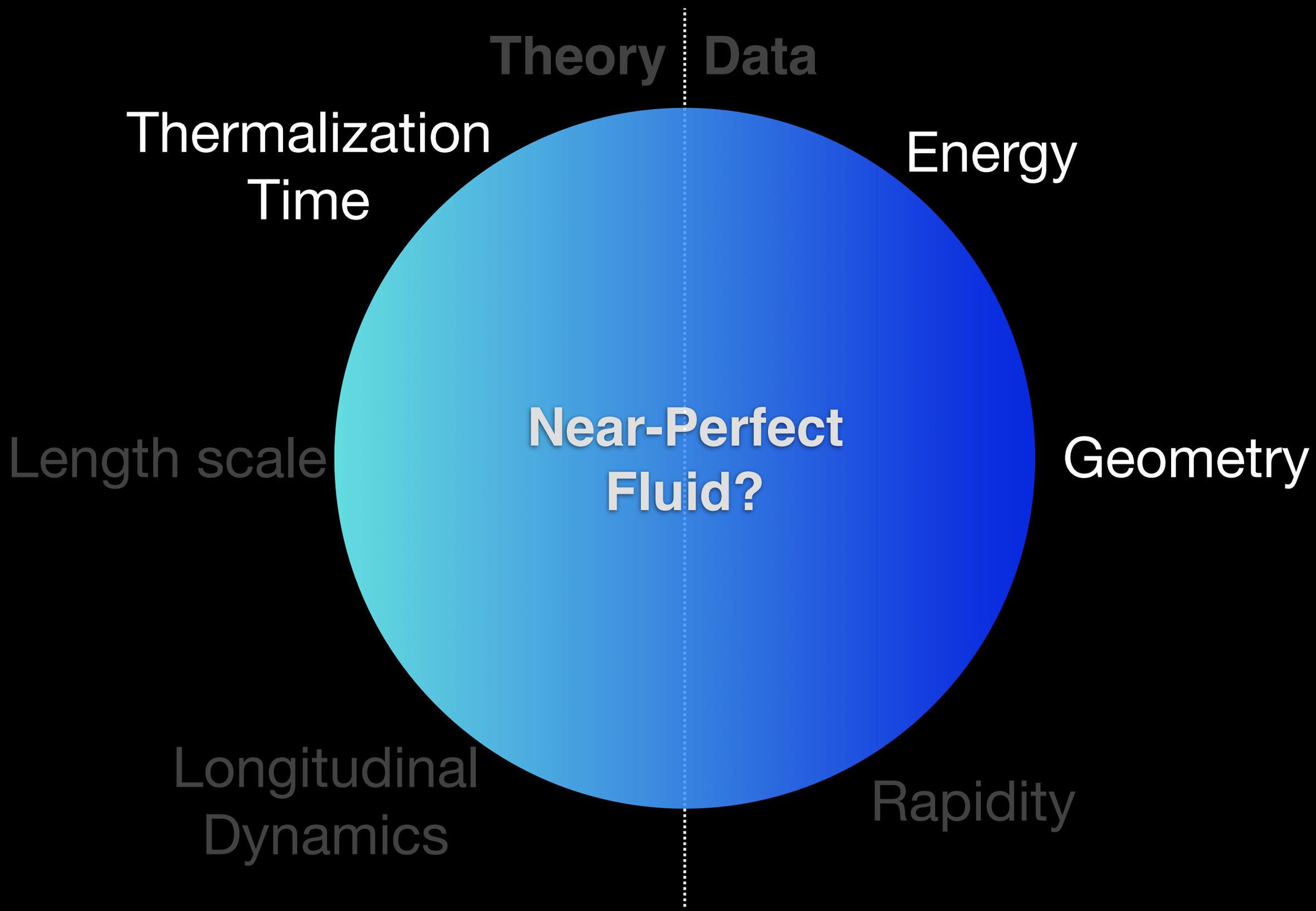
Unity of geometry, system, energy, p_T
<u>at same N_{part}</u>

"Freeze-in"



Configuration established <u>early</u> and <u>preserved</u>:
substantial viscosity would
generate new entropy due to inhomogeneities
under different geometric conditions

Data suggests geometry is “frozen in” immediately (SLP)



Digging Deeper

Initial success of participant eccentricity raised many questions:

1. Robustness to nuclear density variations
2. Robustness to particle production model
3. Role of “correlations”

Phys. Rev. C **77**, 014906 (2008)

Glauber Parameters

Cu+Cu **Au+Au**

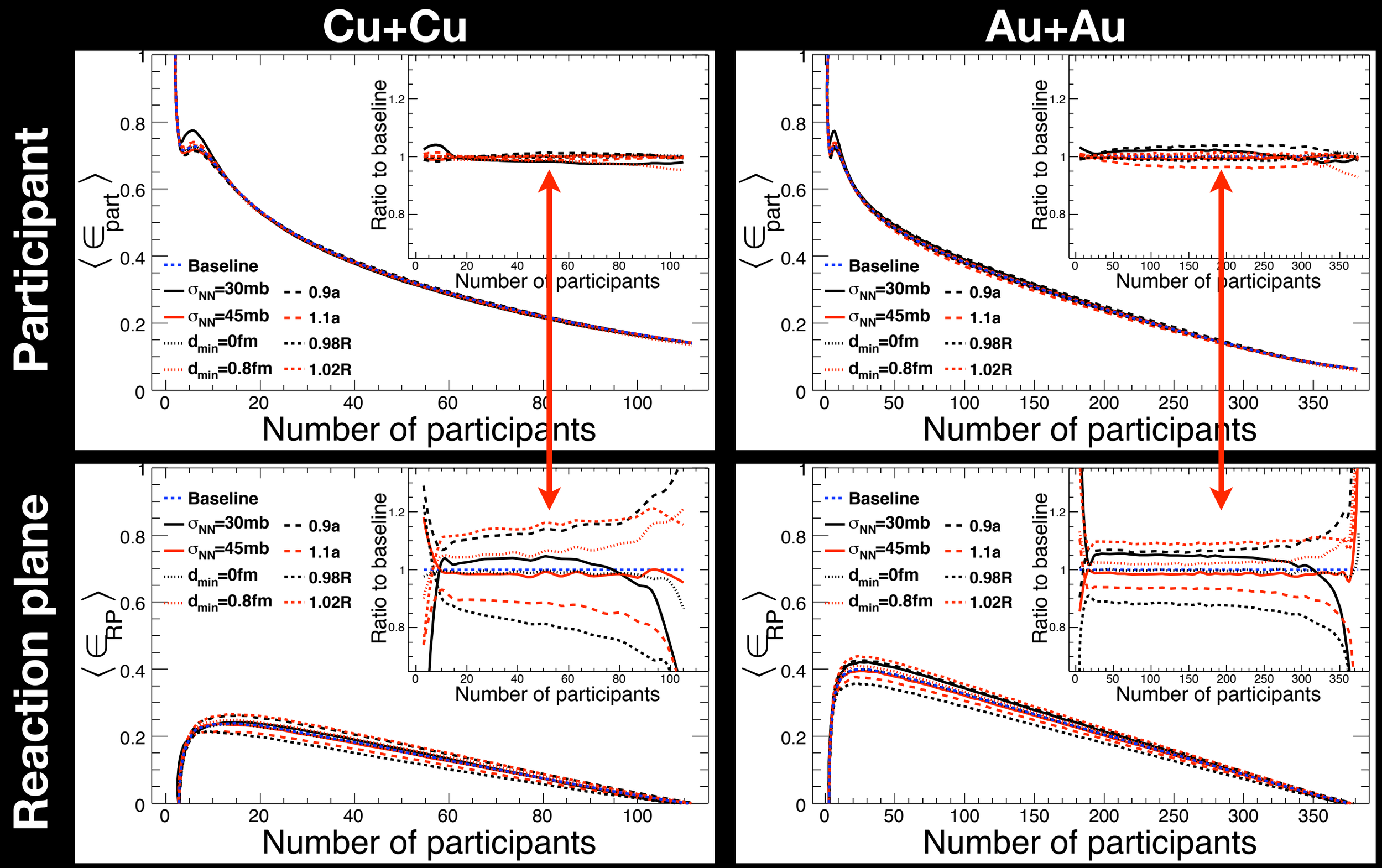
Participant

Reaction plane

Nuclear radius (R)
Skin depth (a)
NN cross section (σ_{NN})
Minimal distance (d)

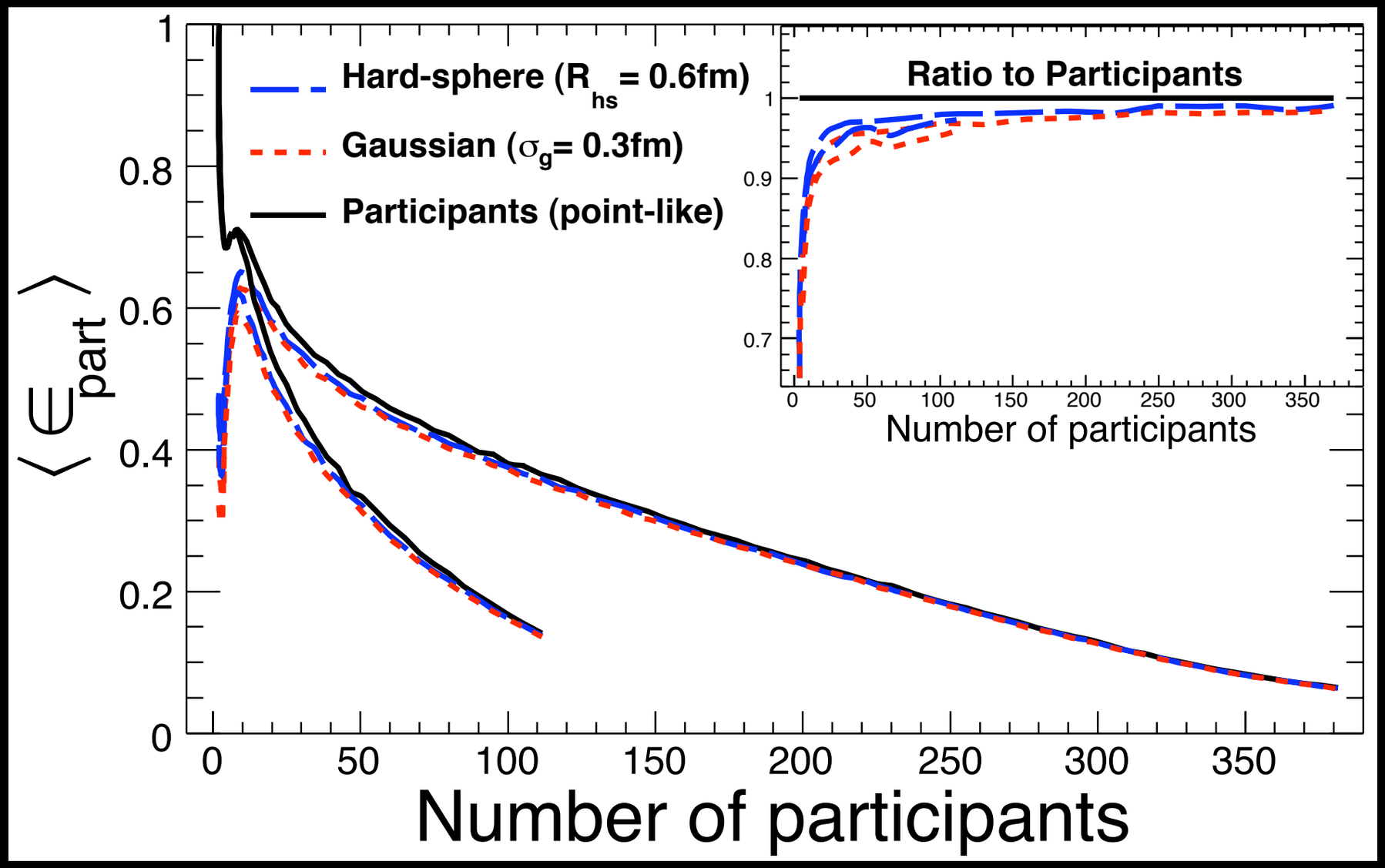
Phys. Rev. C **77**, 014906 (2008)

Glauber Parameters



Phys. Rev. C **77**, 014906 (2008)

Matter Deposition

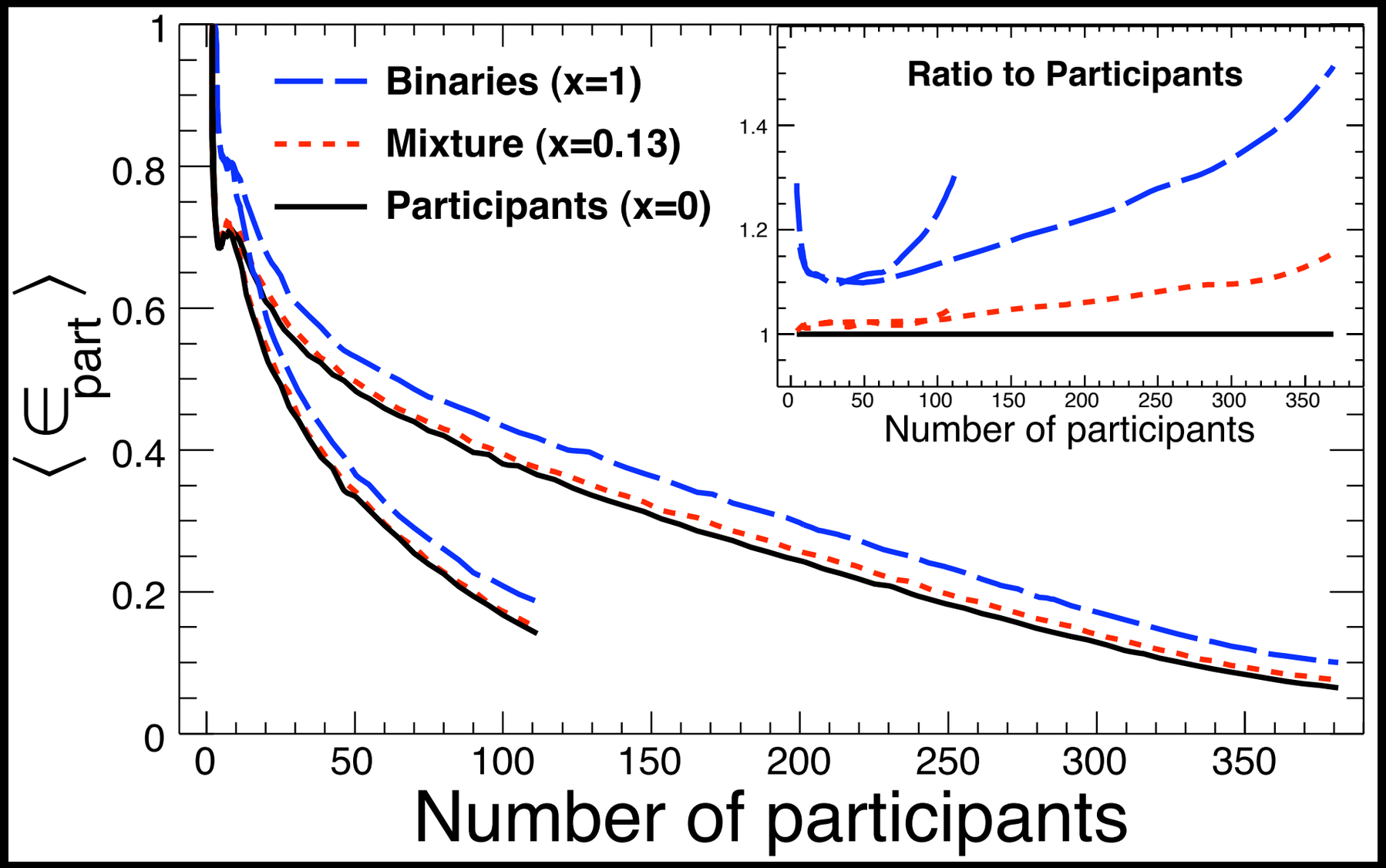


Explored two implementations of “smearing” matter
Hard sphere and Gaussian

Smearing is a 5% effect at most between peripheral & central

Phys. Rev. C **77**, 014906 (2008)

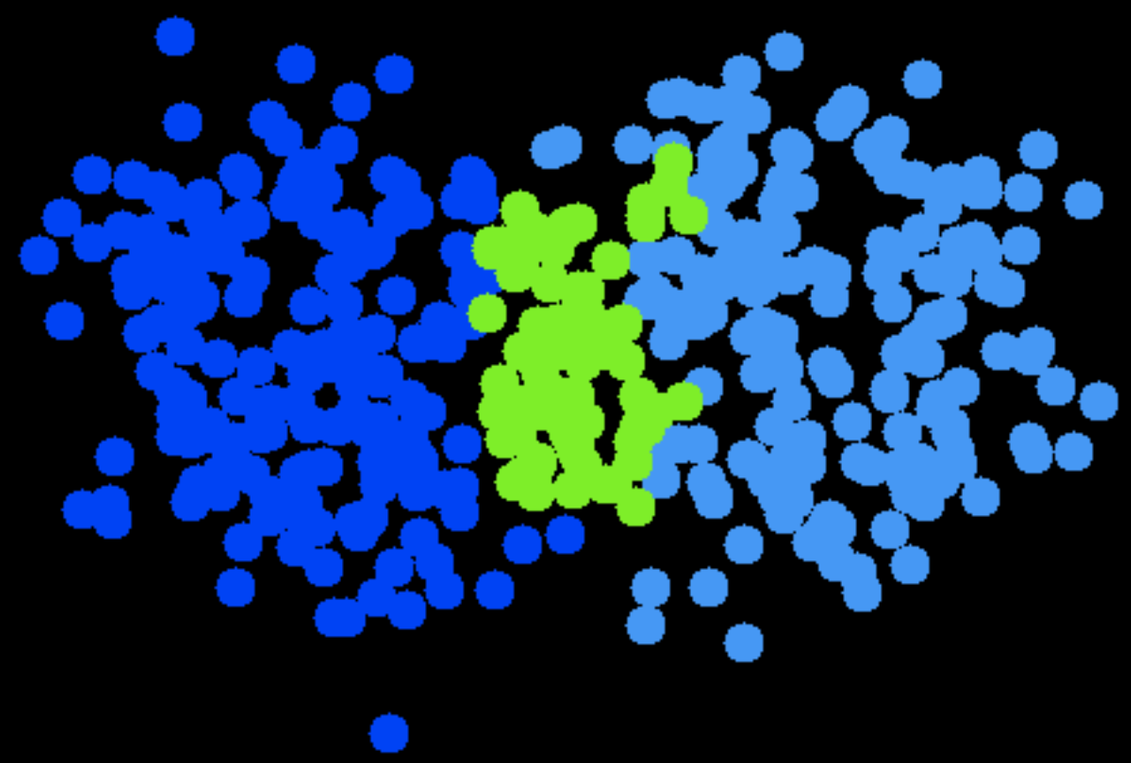
Participants vs. Collisions



"Two component model" $$\frac{dN}{d\eta} = n_{pp}\left\{(1-x)\frac{N_{part}}{2} + xN_{coll}\right\}$$

At most a 10% effect between peripheral & central

Correlations & Fluctuations



- **Something only accessible with MC Glauber**
  - Effect of fluctuations on v_2 estimation
  - Corrected eccentricity scaling of v_2
  - Cumulants and correlations of nucleons

Cumulants of ε

- **Basic intuition:**
  - if v_2 **α ε** , and v_2 is estimated by correlating n particles, then one is really measuring a higher moment of v_2 and thus the eccentricity

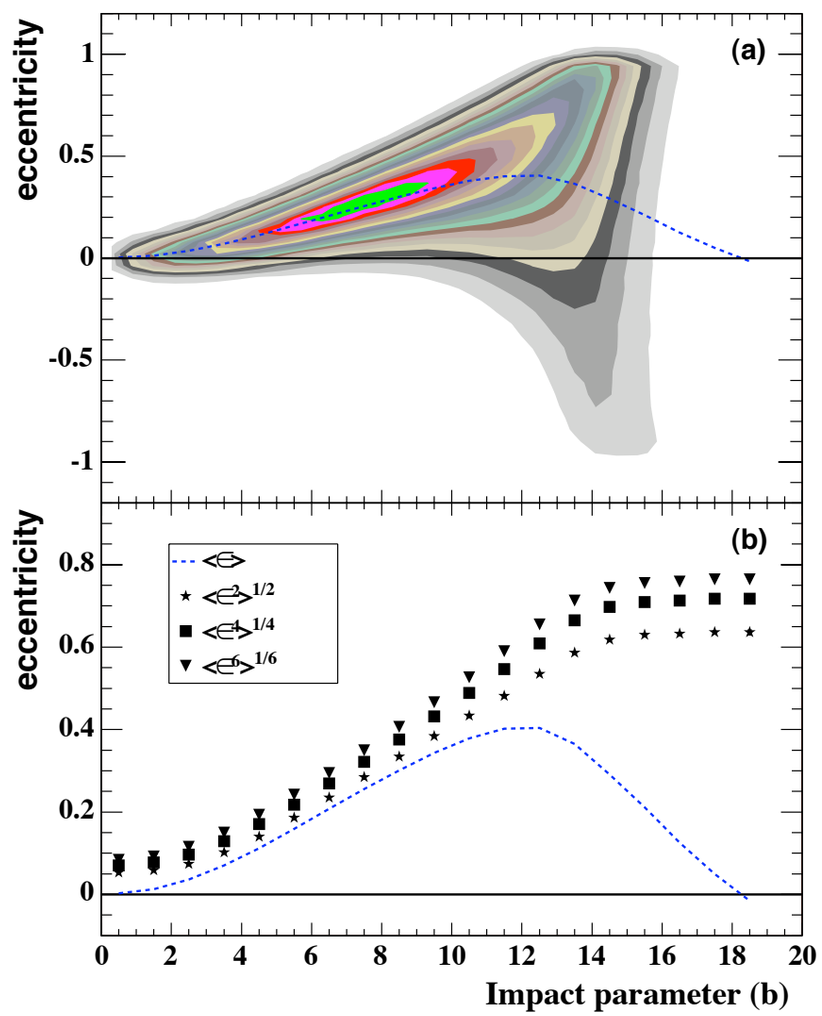
$$v_2\{n\} \sim \langle \epsilon^n \rangle^{1/n}$$

- **Cumulants are more natural**

$$\epsilon\{2\}^2 \equiv \langle \epsilon^2 \rangle \qquad \epsilon\{4\}^4 \equiv 2\langle \epsilon^2 \rangle^2 - \langle \epsilon^4 \rangle$$

Order of ε cumulant should match order of v_2 cumulant

Miller & Snellings



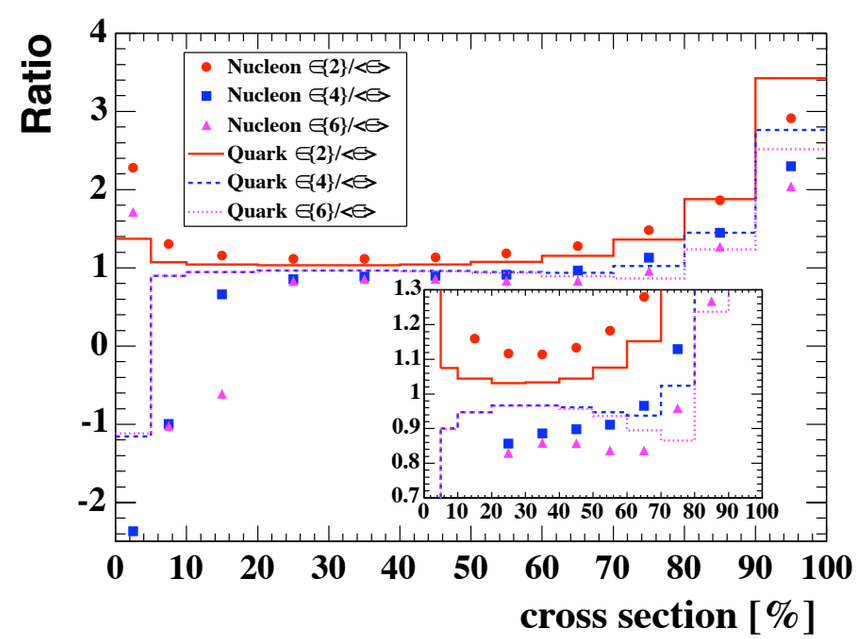


FIG. 3: Eccentricity cumulants calculated for Au+Au collisions in a Monte Carlo Glauber model compared to $\langle\epsilon\rangle$ as a function of centrality (from left to right is from central to peripheral collisions, respectively). The symbols are the Monte Carlo Glauber results using nucleons, the lines are for constituent quarks (see text).

culation of ϵ and therefore the determination of the cor-

Interestingly, moments even of ε_{std} track each other, except for $\langle\varepsilon_{std}\rangle$ in peripheral & central

nucl-ex/0312008, submitted to PRC

Bhalerao & Ollitrault

Bhalerao & Ollitrault suggested that while 2-particle cumulant is systematically higher, 4-particle cumulant goes back to "standard" eccentricity

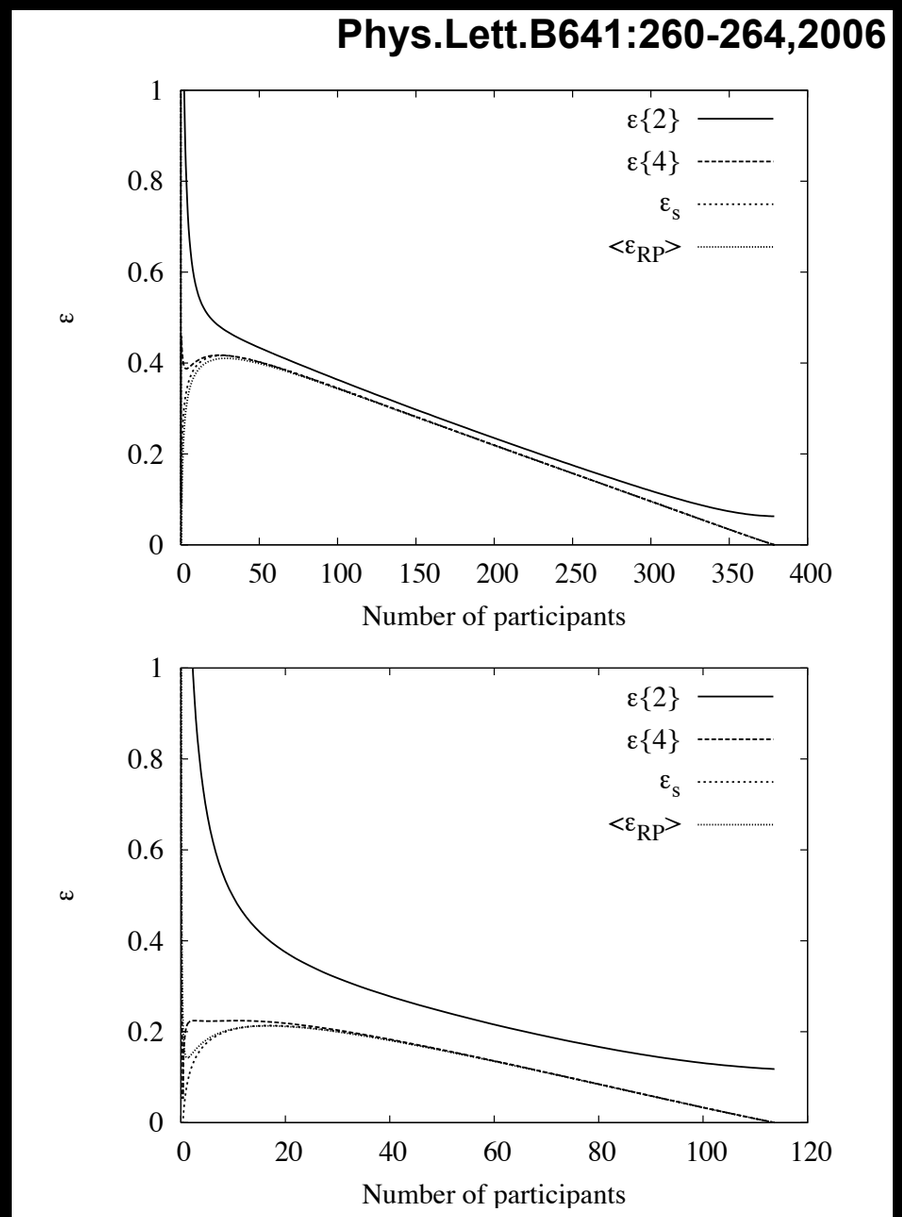


FIG. 2: Standard eccentricity ε_s, participant eccentricities $\varepsilon\{2\}$ and $\varepsilon\{4\}$, and reaction-plane eccentricity $\langle\varepsilon_{RP}\rangle$, vs the number of participant nucleons for a Au-Au collision (top) and a Cu-Cu collision (bottom).

Phys. Rev. C **77**, 014906 (2008)

Fluctuations and v_2

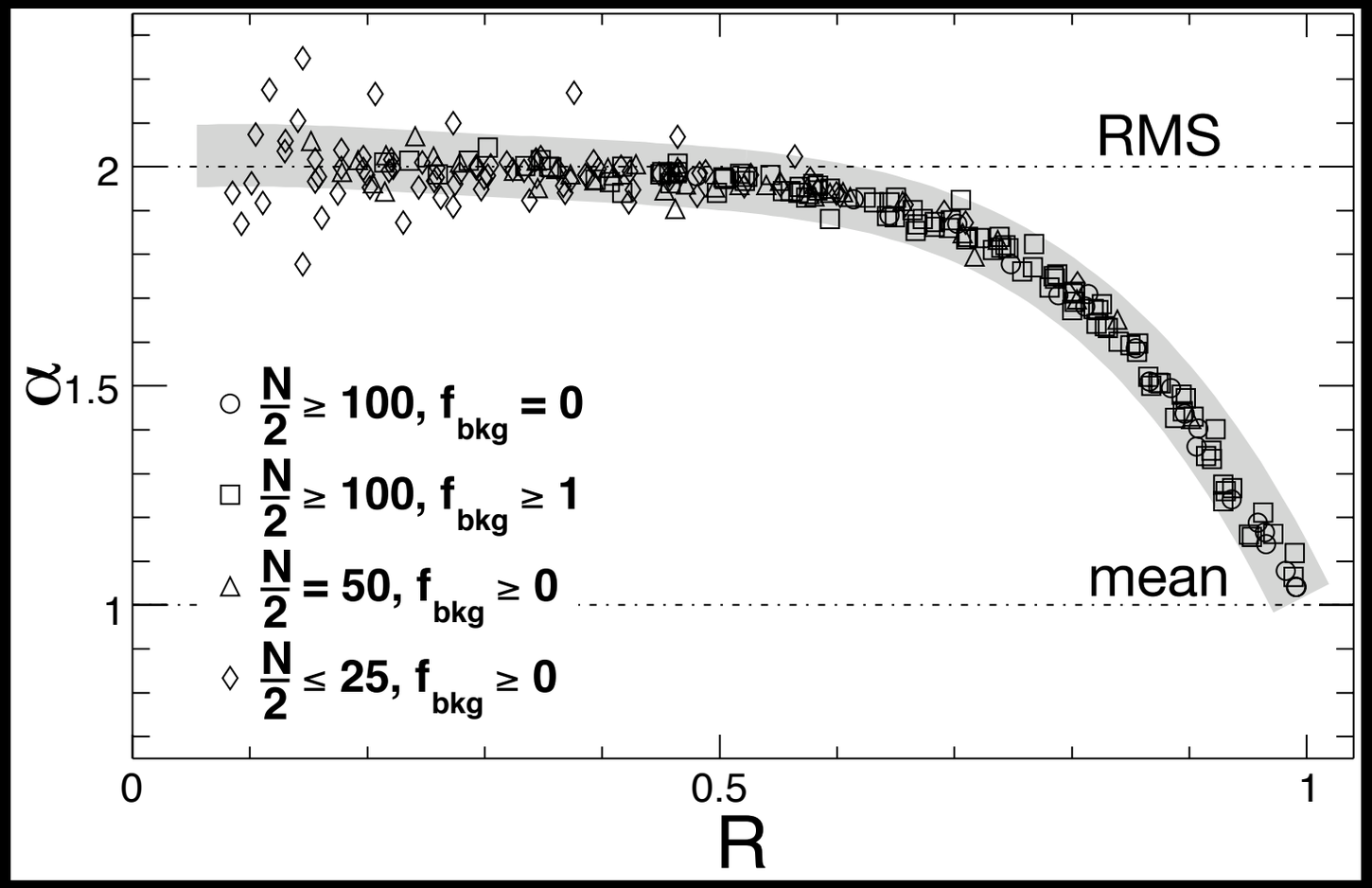
Does PHOBOS reaction plane method measure <u>mean</u> or <u>RMS</u> of v_2?

Simulations, with N particles and background fraction f_{bkg} let us extract:

$$\langle\langle v_2^\alpha \rangle\rangle^{1/\alpha} = v_2\{EP\}$$

vs.

$$R = \sqrt{\langle \cos(2\psi_1 - 2\psi_2) \rangle}$$



Typical PHOBOS resolutions are 0.1-0.5 → α=2

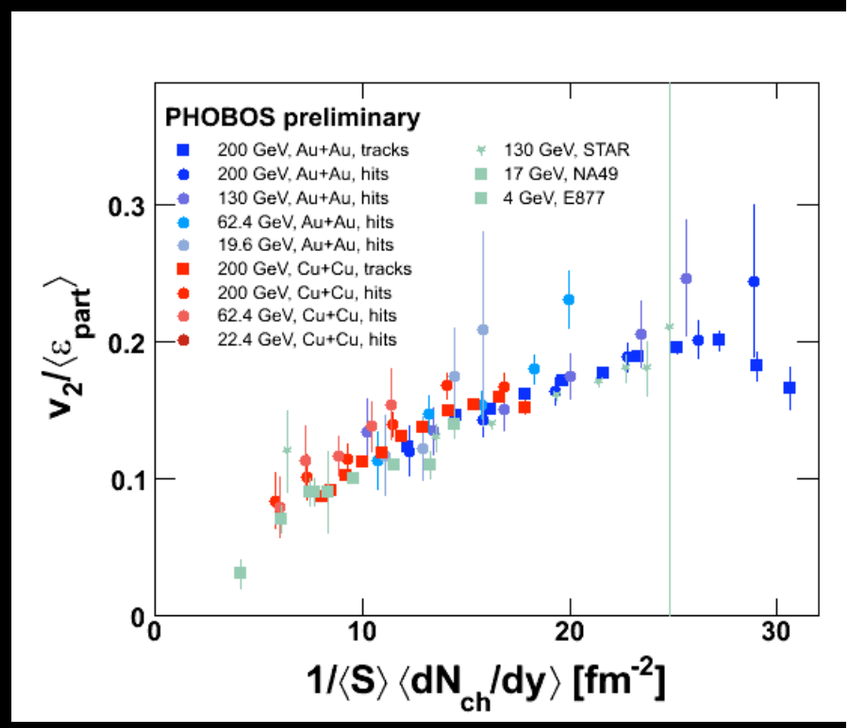
Typical v_2 fluctuations:

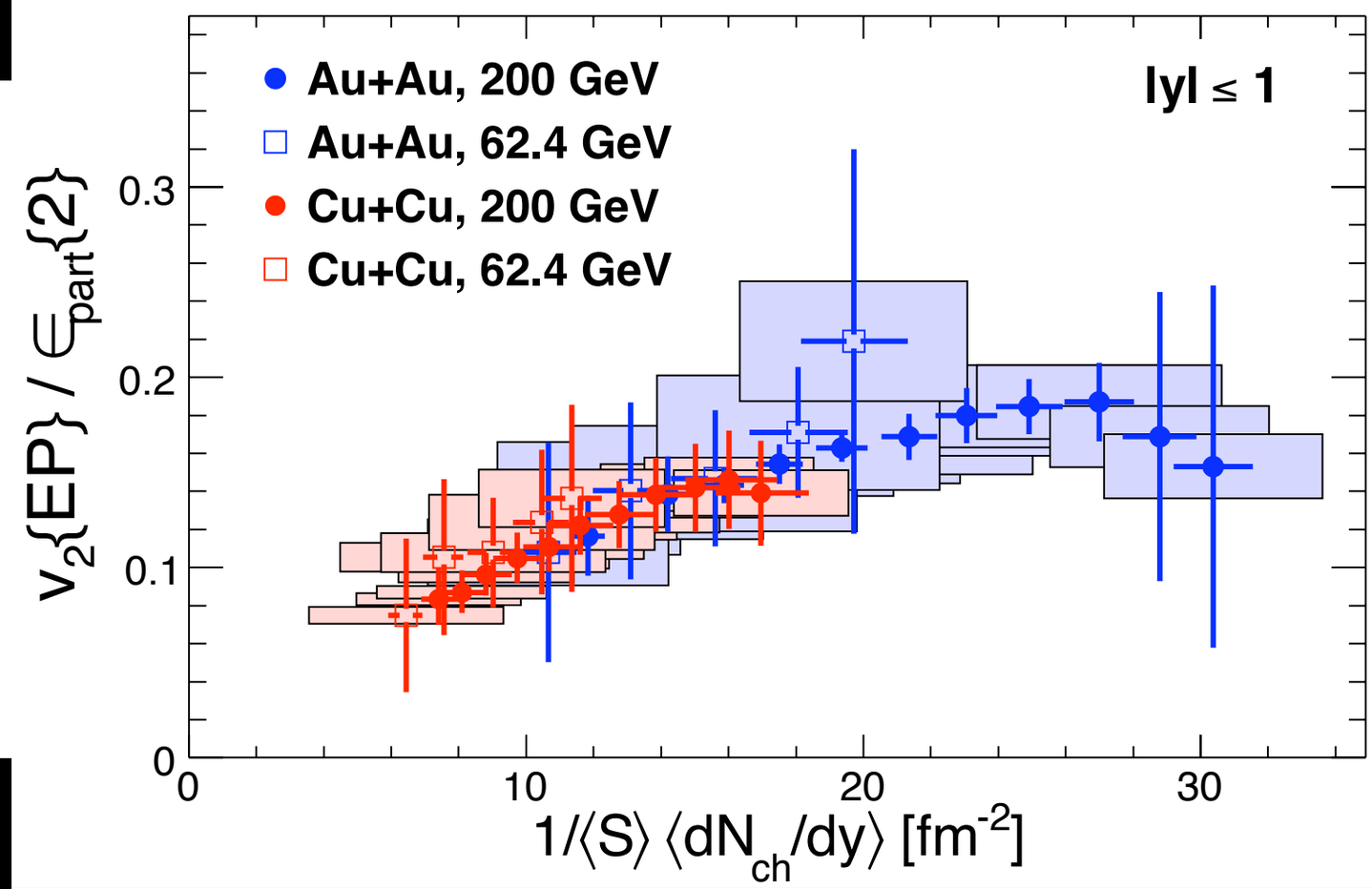
$$\sigma_{v_2}/v_2 = 0.4 \rightarrow v_2\{2\} = 1.08 v_2$$

(upper limit)

Phys. Rev. C **77**, 014906 (2008)

Corrected Scaling

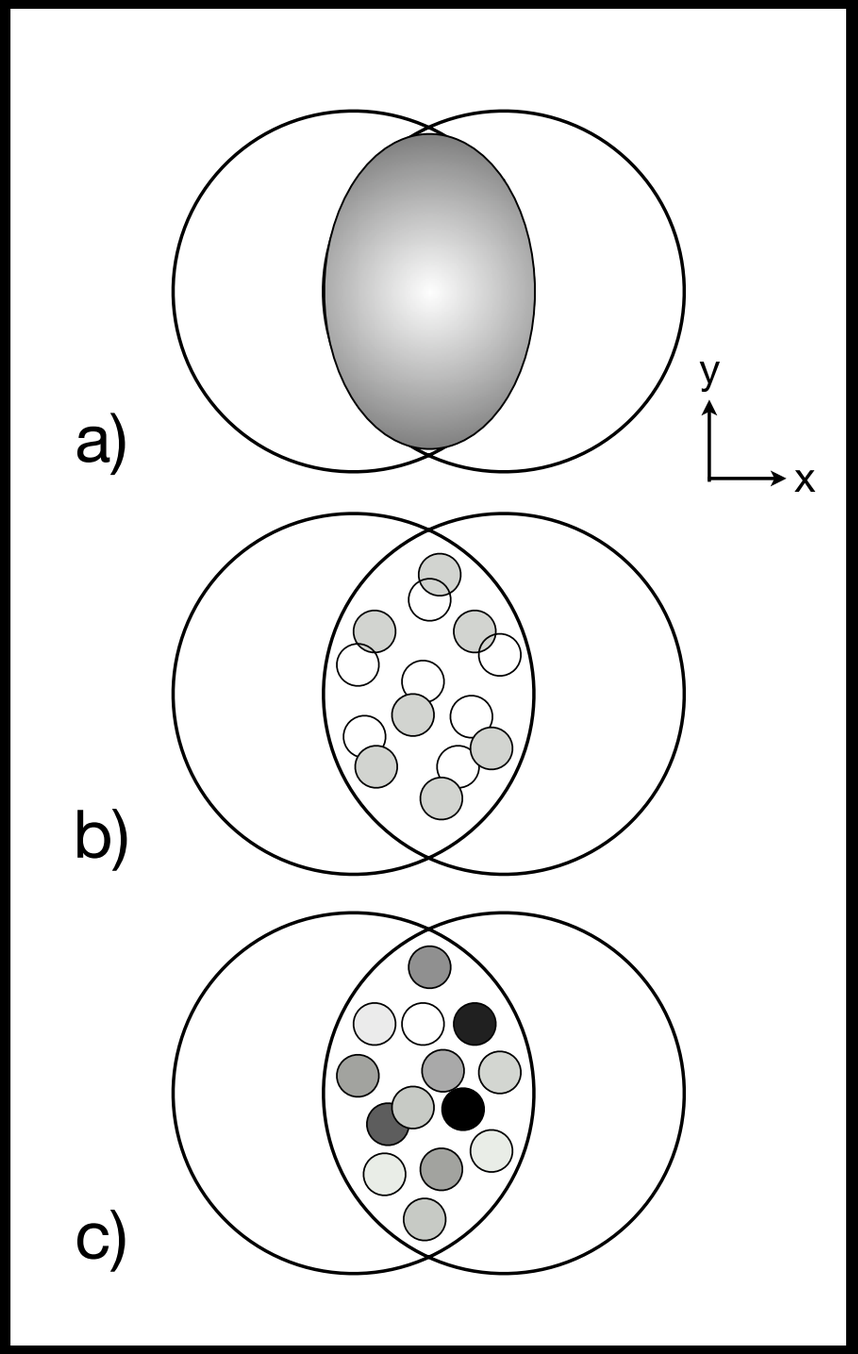




Not a major change, but a slight flatterning at large dN/dy/<S>

Phys. Rev. C **77**, 014906 (2008)

Correlations in A+A



smooth densities
(leading to “standard” eccentricity)

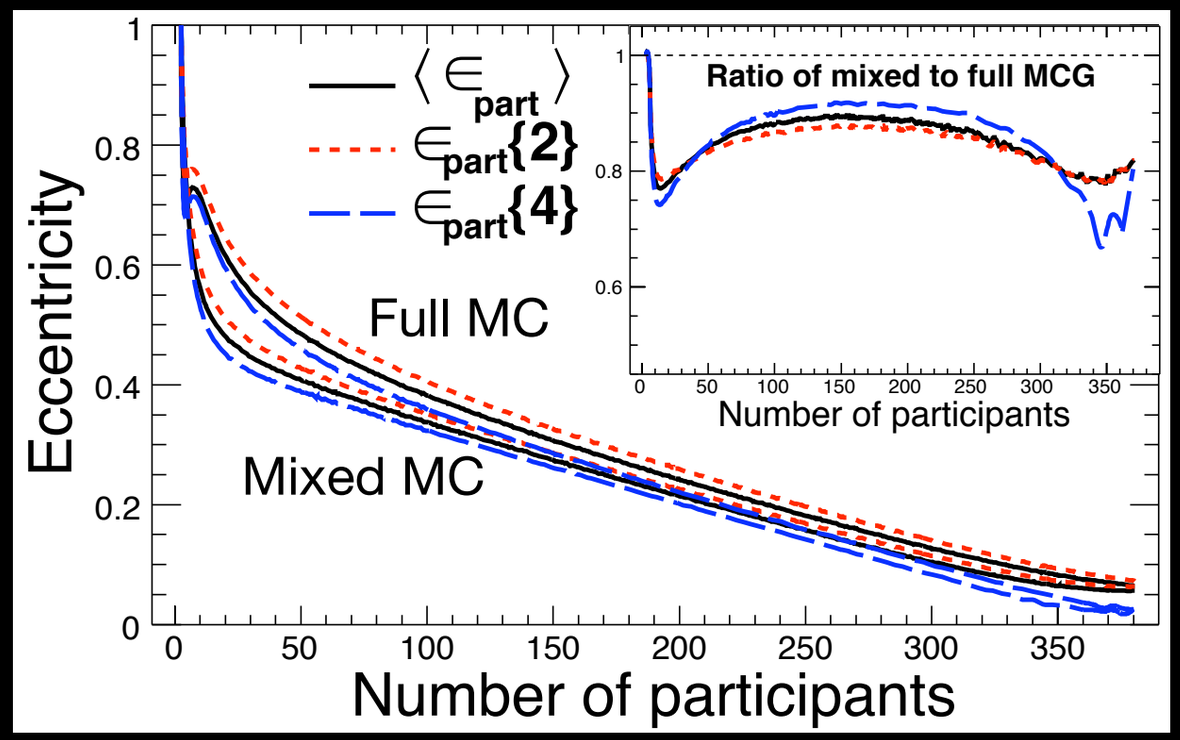
standard Glauber MC
(nucleons collide in <u>pairs</u>,
fluctuations & <u>correlations</u>)

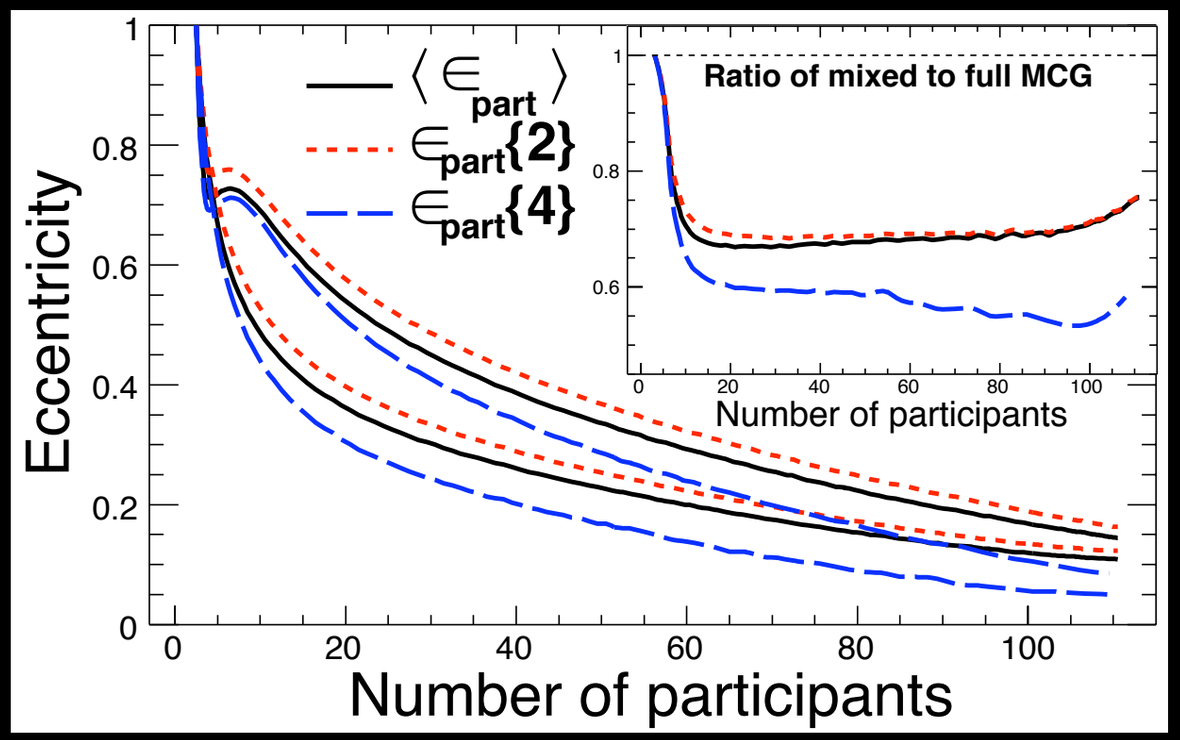
“mixed” Glauber MC
(sample nucleons from
different collisions,
fluctuations & ~~correlations~~)

NB: no correlations <u>between</u> nucleons in a nucleus

Phys. Rev. C **77**, 014906 (2008)

Full MC vs. Mixed MC





2nd Cumulant is higher than mean
4th Cumulant is slightly lower than mean

Mixed events always have lower ε than full MC,
down by 30% in Cu+Cu

Phys. Rev. C **77**, 014906 (2008)

Analytic Calculation

B&O derived a semi-analytic form for the cumulants, esp ε{4}

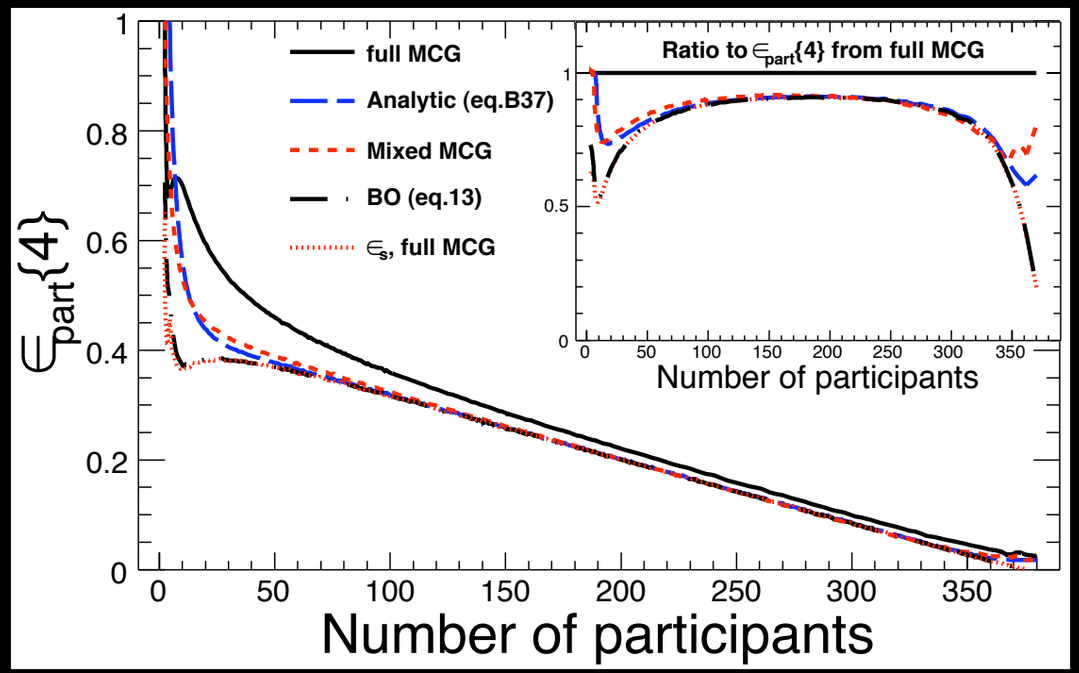
Rederived (by MDB!), keeping all relevant terms

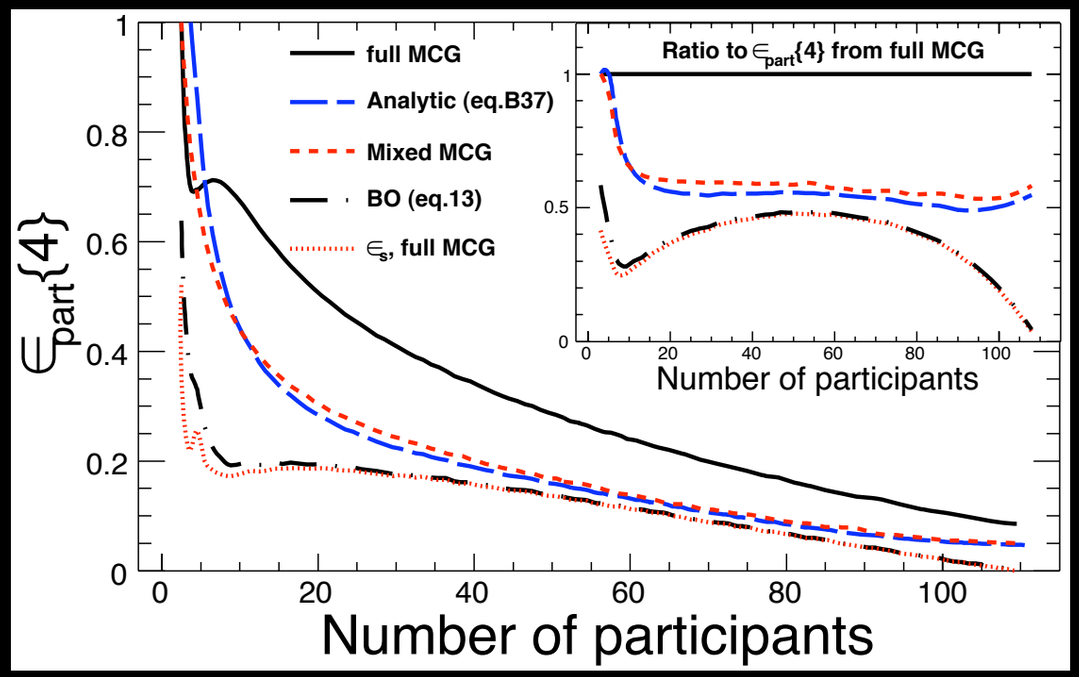
$$
\begin{aligned}
\epsilon_{\rm part}\{4\}^4 \;=\; & \epsilon_{\rm s}^4 + \frac{1}{N\langle r^2\rangle^2}\left[2\epsilon_{\rm s}^4\langle r^4\rangle - 2\epsilon_{\rm s}^2\langle r^4\cos 4\phi\rangle\right] \\
& + \frac{1}{N^2}\left[8\epsilon_{\rm s}^2\frac{\langle r^6\rangle}{\langle r^2\rangle^3} + 4\epsilon_{\rm s}\frac{\langle r^6\cos 2\phi\rangle}{\langle r^2\rangle^3}\right. \\
& \left. - 16\epsilon_{\rm s}^2\frac{\langle r^4\rangle^2}{\langle r^2\rangle^4} - 16\epsilon_{\rm s}\frac{\langle r^4\rangle\langle r^4\cos 2\phi\rangle}{\langle r^2\rangle^4}\right] \\
& + \frac{1}{N^3}\left[\frac{2\langle r^4\rangle^2}{\langle r^2\rangle^4} - \frac{\langle r^8\rangle}{\langle r^2\rangle^4} + \frac{8\langle r^4\rangle\langle r^6\rangle}{\langle r^2\rangle^5}\right. \\
& \left. - \frac{8\langle r^4\rangle^3}{\langle r^2\rangle^6}\right] + \mathcal{O}\left(\frac{\epsilon_{\rm s}^4}{N^2}\right) + \mathcal{O}\left(\frac{\epsilon_{\rm s}^2}{N^3}\right) \\
& + \mathcal{O}\left(\frac{\epsilon_{\rm s}^0}{N^4}\right) + \mathcal{O}\left(\frac{\epsilon_{\rm s}^0}{N^5}\right) + \ldots, \qquad \text{(B37)}
\end{aligned}
$$

B&O (13)

$1/N^2$

$1/N^3$

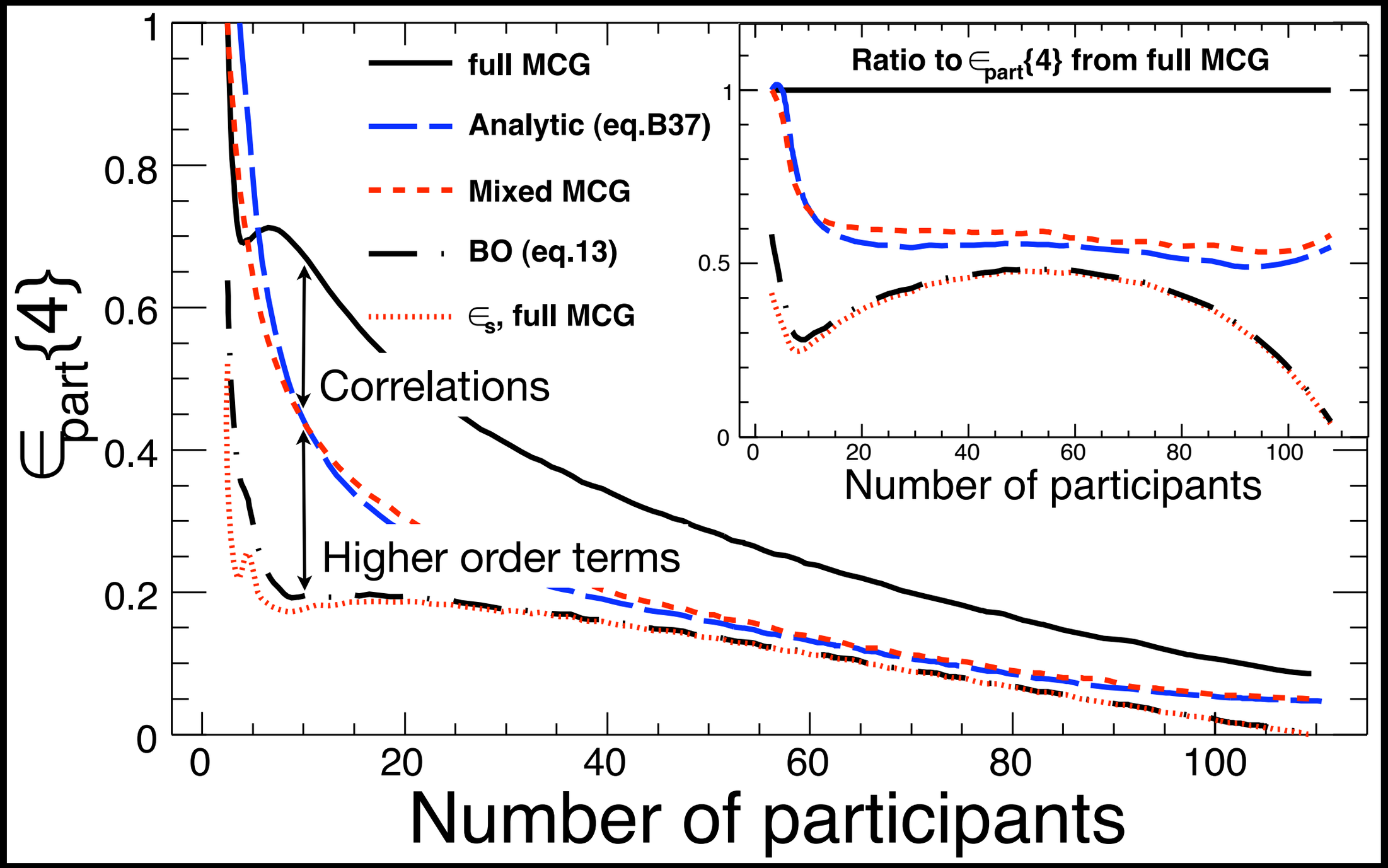




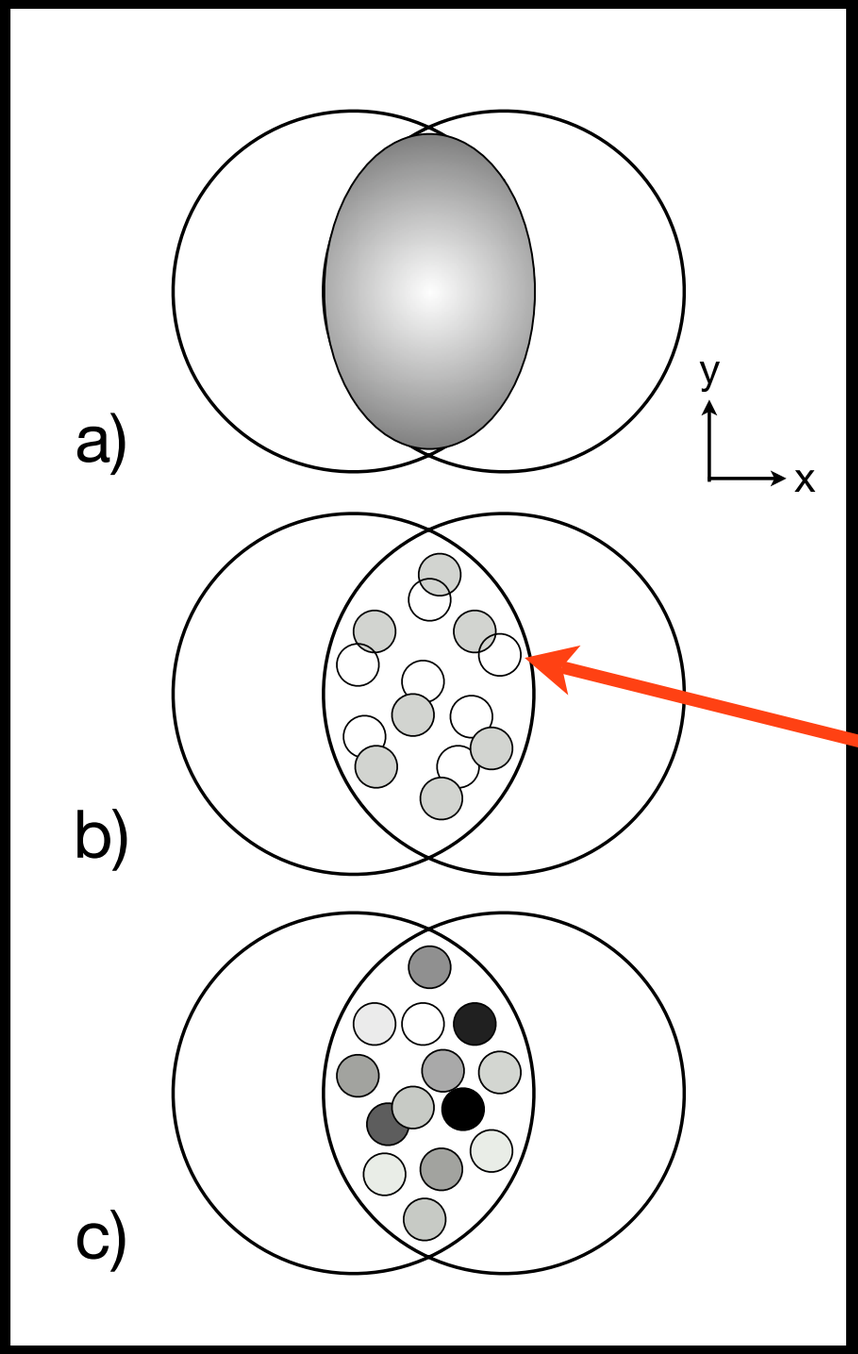
Agrees with Mixed MC

Phys. Rev. C **77**, 014906 (2008)

A Closer Look at Cu+Cu

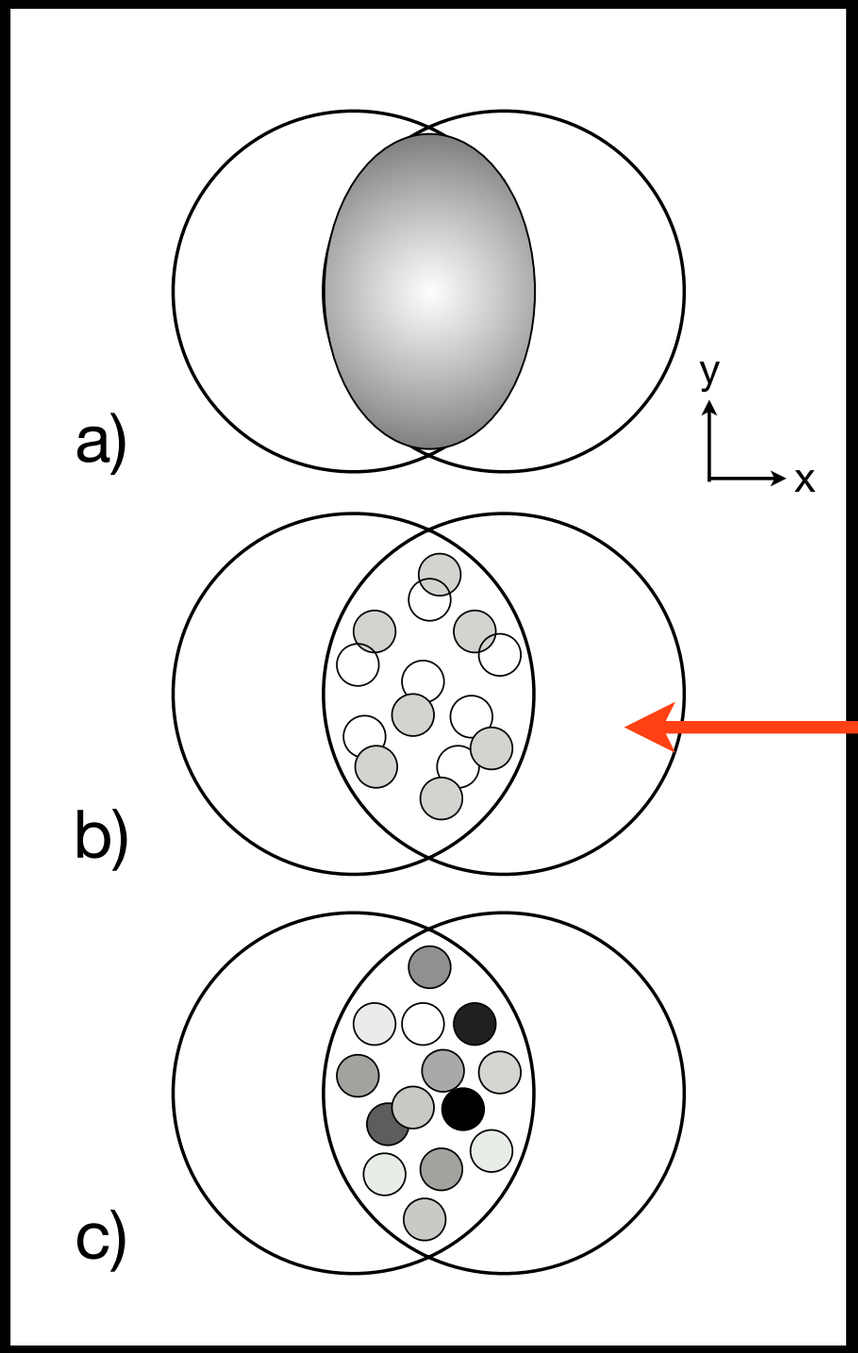


The Difference



The <u>pairwise collisions</u> of nucleons as the nuclei collide induce genuine **<u>spatial correlations</u>** which enhance the fluctuations

Conclusions



These are all different models of how nuclei collide and produce matter

Based on comparisons with data, **Glauber MC** still seems to be the most realistic

There is no simple way (e.g. cumulants) recover the “ideal” geometry

Summary

- **Optical Limit vs. MC approach to Glauber Modeling**

- **Participant eccentricity**
  - Comparisons to experimental data

- **Role of fluctuations & correlations**
  - Irreducible feature of MC approach
  - Not included in most existing models

TGlauberMC

- **PHOBOS Glauber MC has been used to generate most of the results shown here**

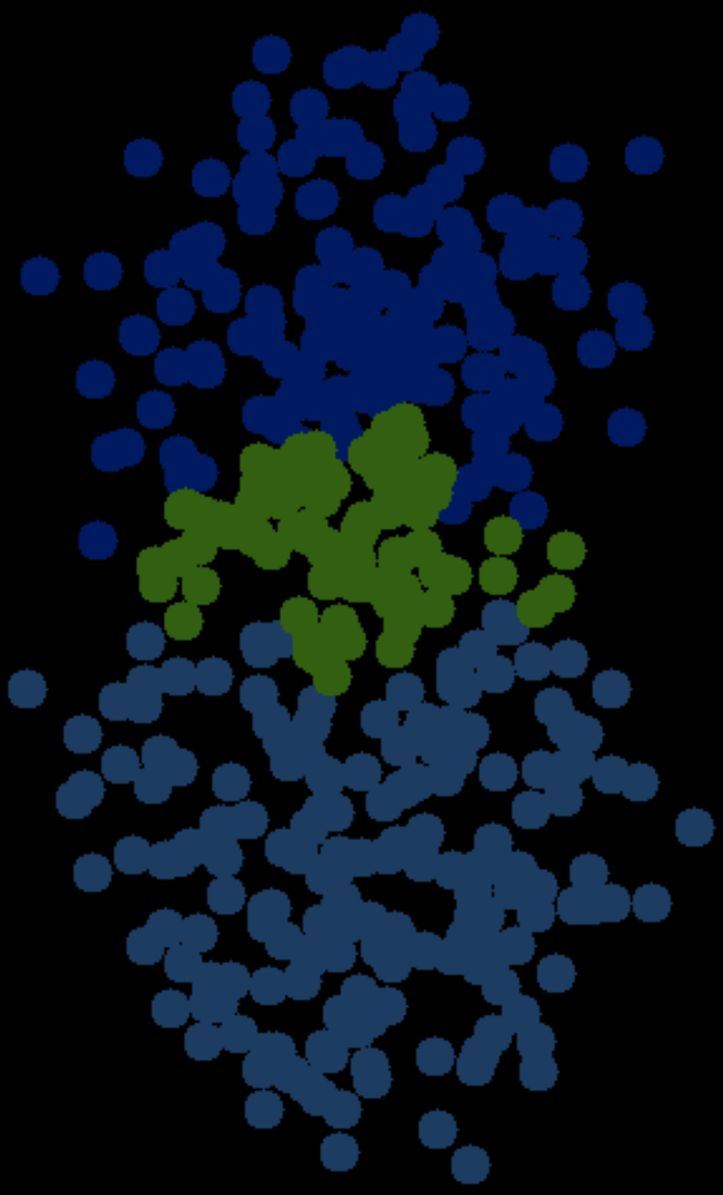
- **Near release (pending PHOBOS approval)**
  - C++ code, interfaced with ROOT system
  - Document explaining use patterns, with a few comparison distributions

A. Running the Code

To generate Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV ($\sqrt{s_{\rm NN}}$ = 42 mb) one would construct a `TGlauberMC` object by issuing the commands:

```
root [0] .L runglauber_X.Y.C+
root [1] TGlauberMC glauber("Au","Au",42);
```

where the first ROOT command compiles, links and loads the compiled macro [2] including the Glauber code as explained in chapter 2 of the ROOT users' guide.

Events can be generated interactively using the two functions

- **Let us know if you are interested in using (or testing) it!**