Entropy at high E and μ_B

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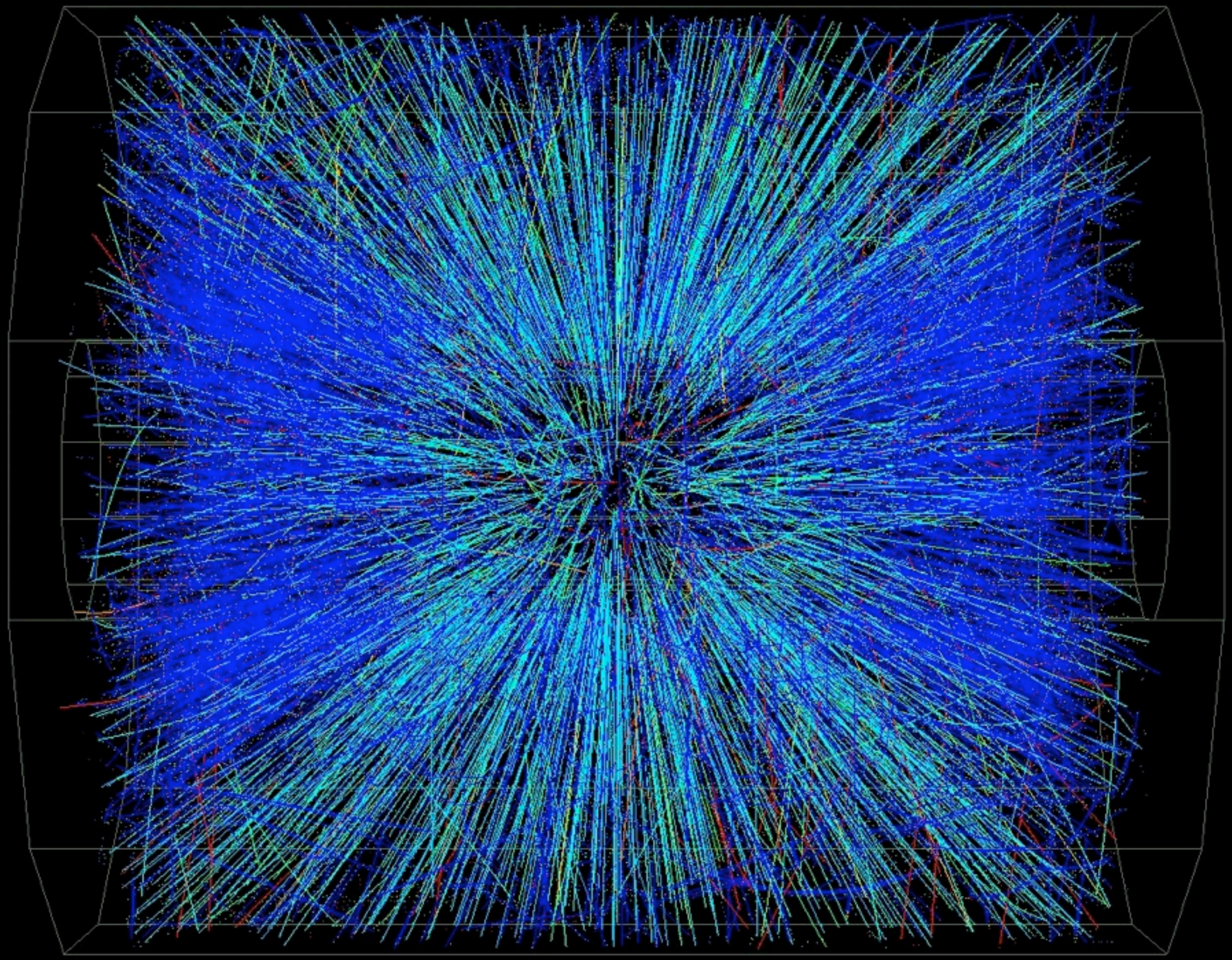
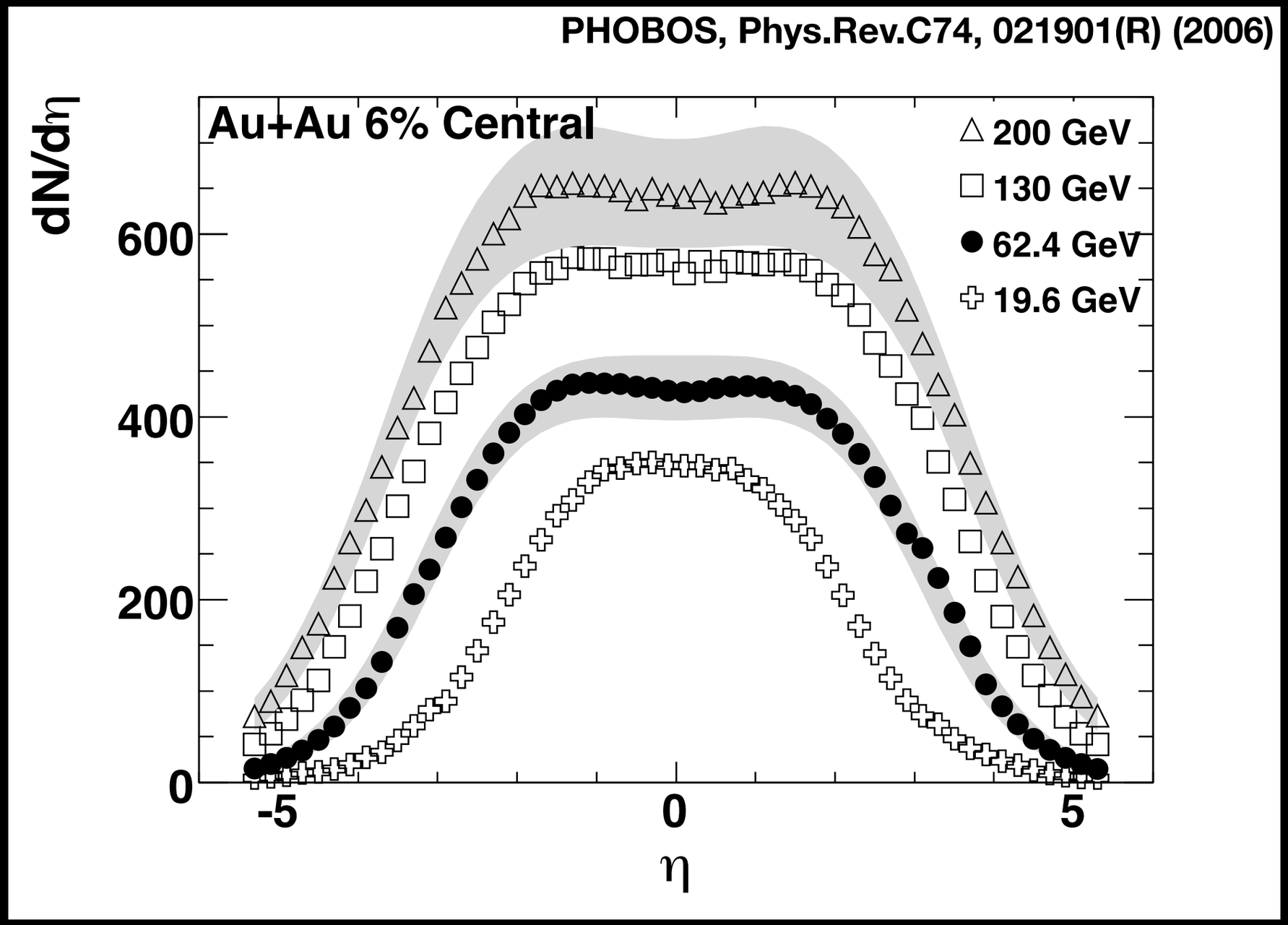
25 January 2009

Brunelleschi's dome:
elegant & simple
on the outside

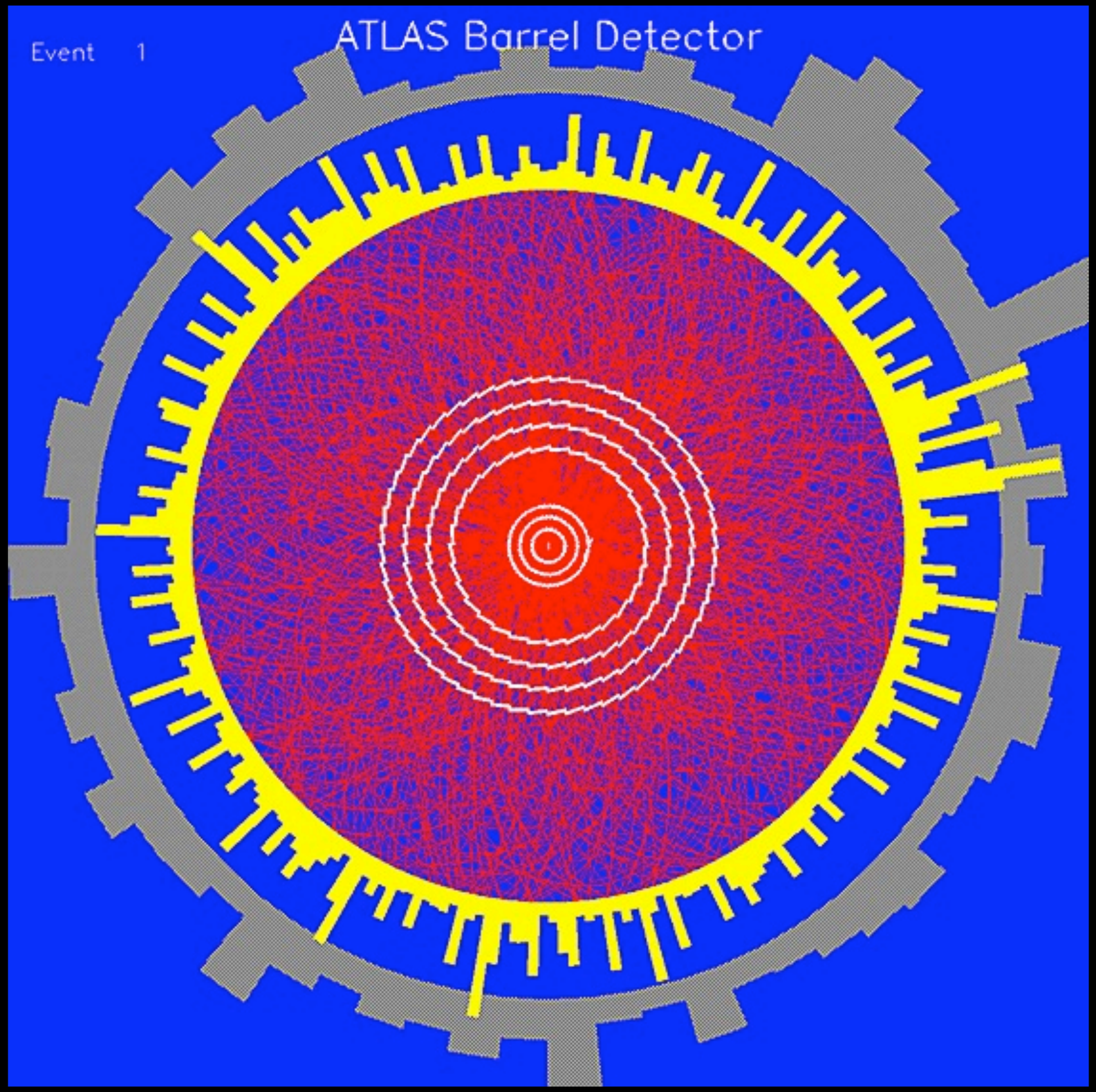
Complicated inside



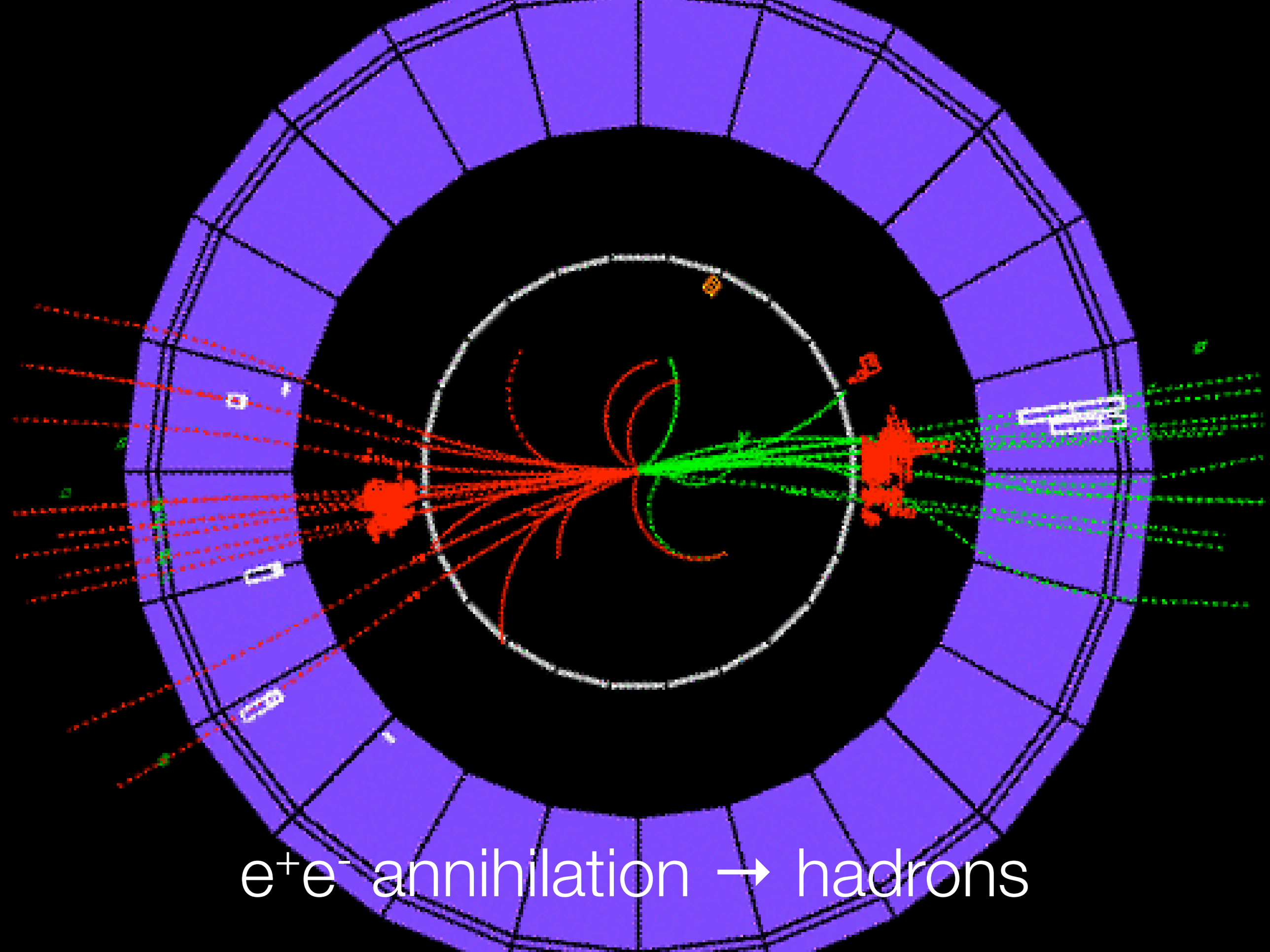
dN/dη vs. Beam Energy



5 years of A+A @ RHIC



Only 2 years until A+A @ LHC...

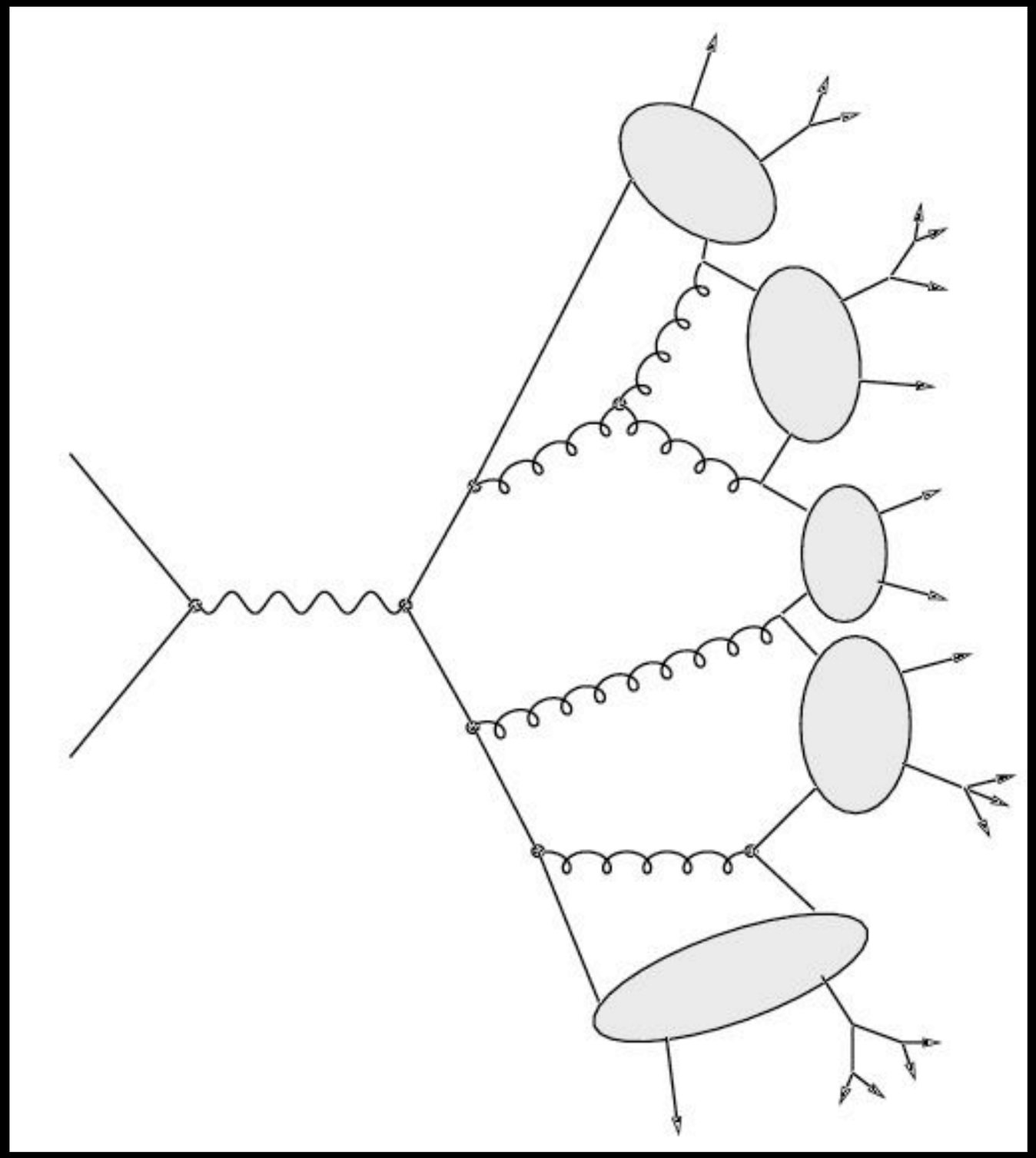


e^+e^- annihilation → hadrons

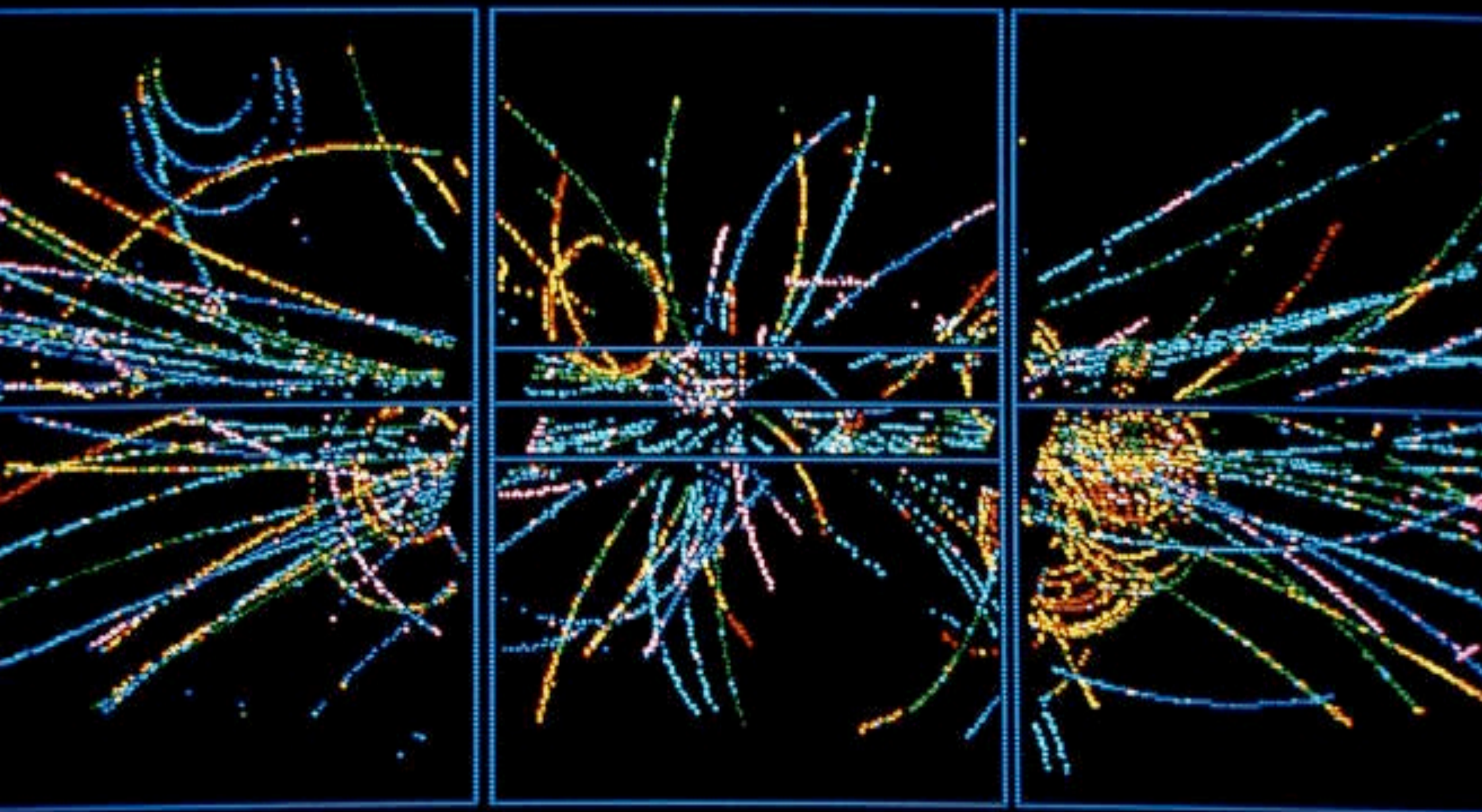
Modelling e+e-

- **String model → Parton showers**
- **Resummed pQCD calculations (MLLA)**
- **pQCD evolution is <u>primary</u> dynamical component of the total multiplicity**

$$n_{ch} \propto \alpha_s^A \exp\left(B\sqrt{\ln(s)}\right)$$



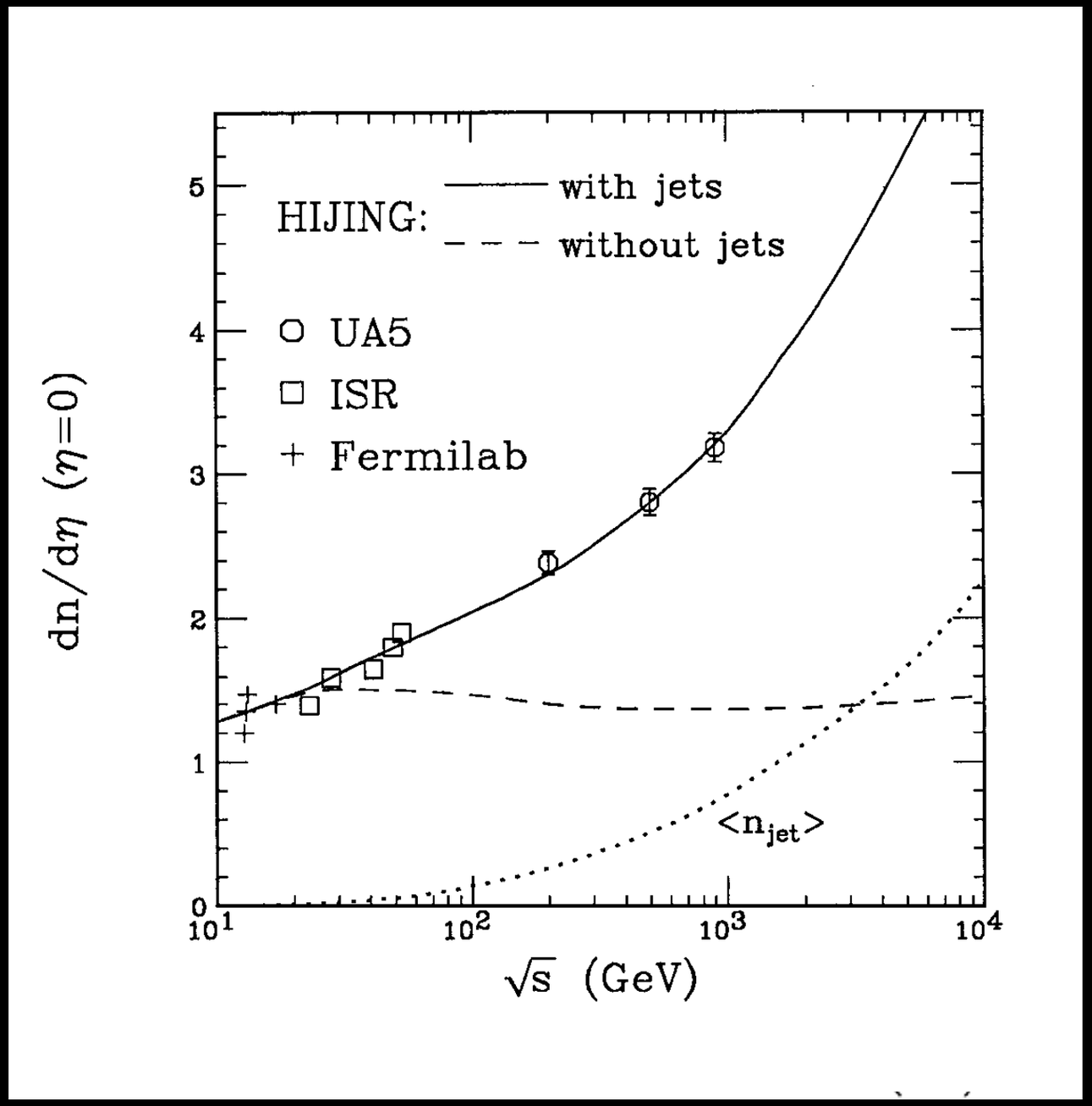
Charged primaries + some secondaries (up to 8% correction)



minimum-bias $\bar{p}$+p → hadrons

Modelling p+p

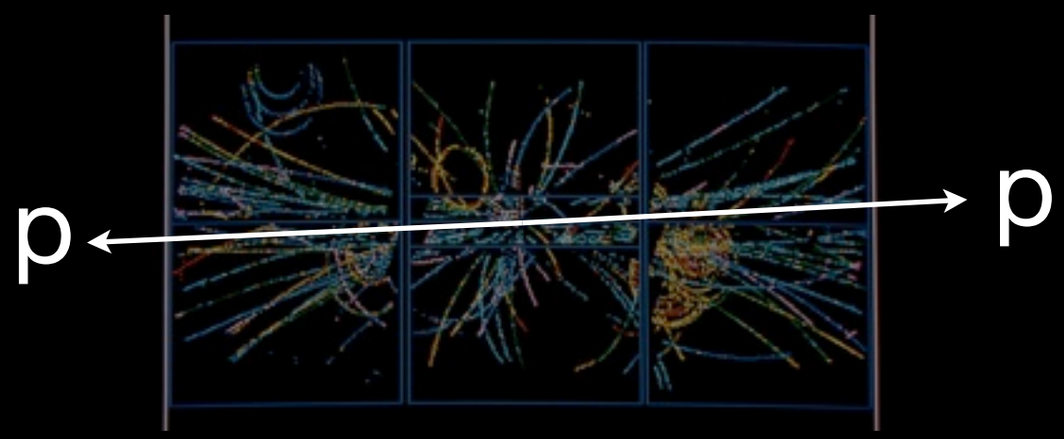
- **Two component picture**
  - Soft component parametrized
  - Hard component from PDFs & pQCD
- **Various implementations**
  - PYTHIA, HERWIG, PHOJET, HIJING
  - Different evolution



I. Dawson, ATLAS

The Leading Effect

“leading” particles keep arbitrary fraction of √s

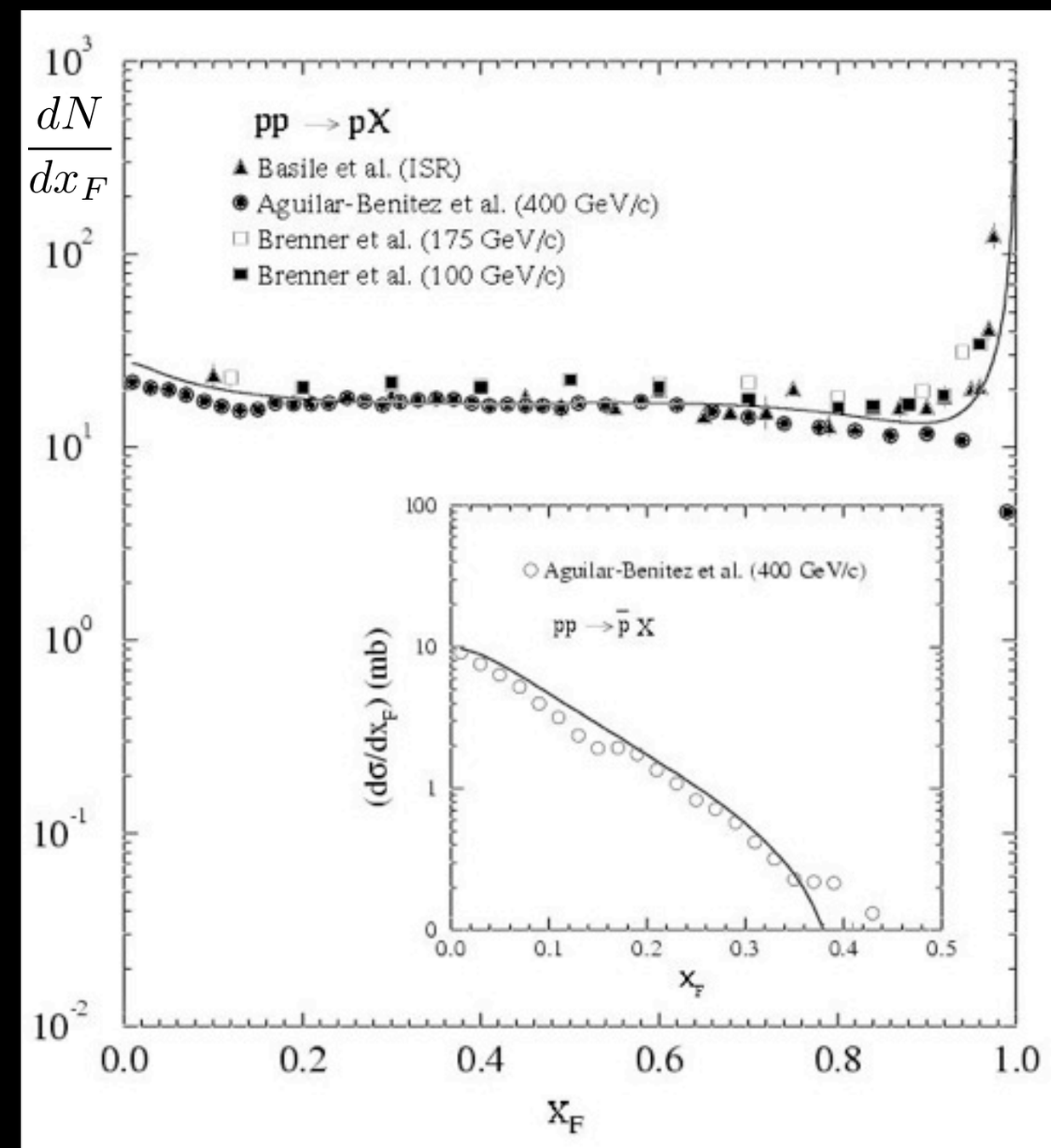


w/ flat probability distribution:

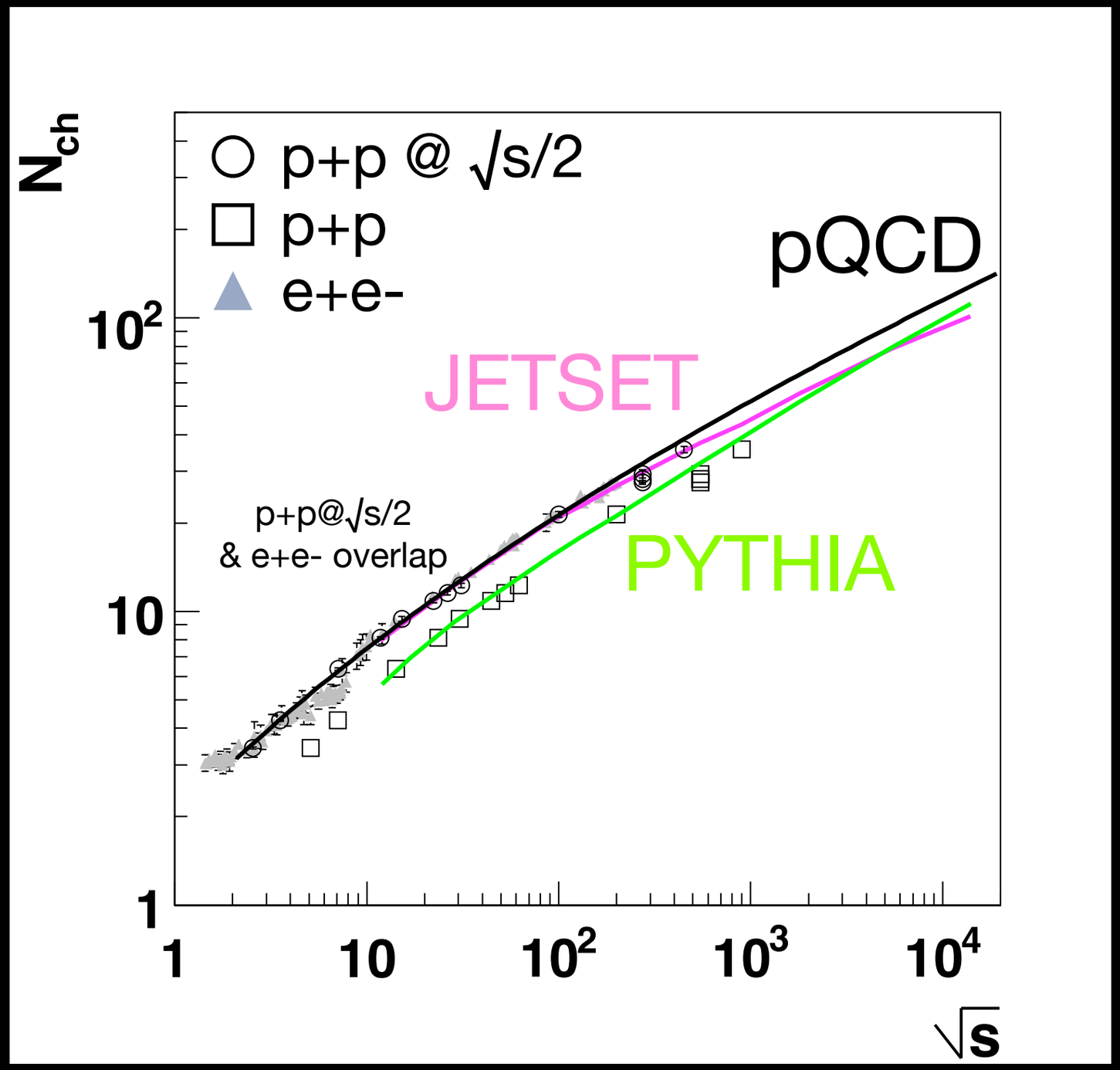
$$x_F = \frac{2p_z}{\sqrt{s}} \qquad \langle x_F \rangle \sim 1/2$$

$$\sqrt{s_{eff}} = \langle x_F \rangle \sqrt{s} = \frac{\sqrt{s}}{2}$$

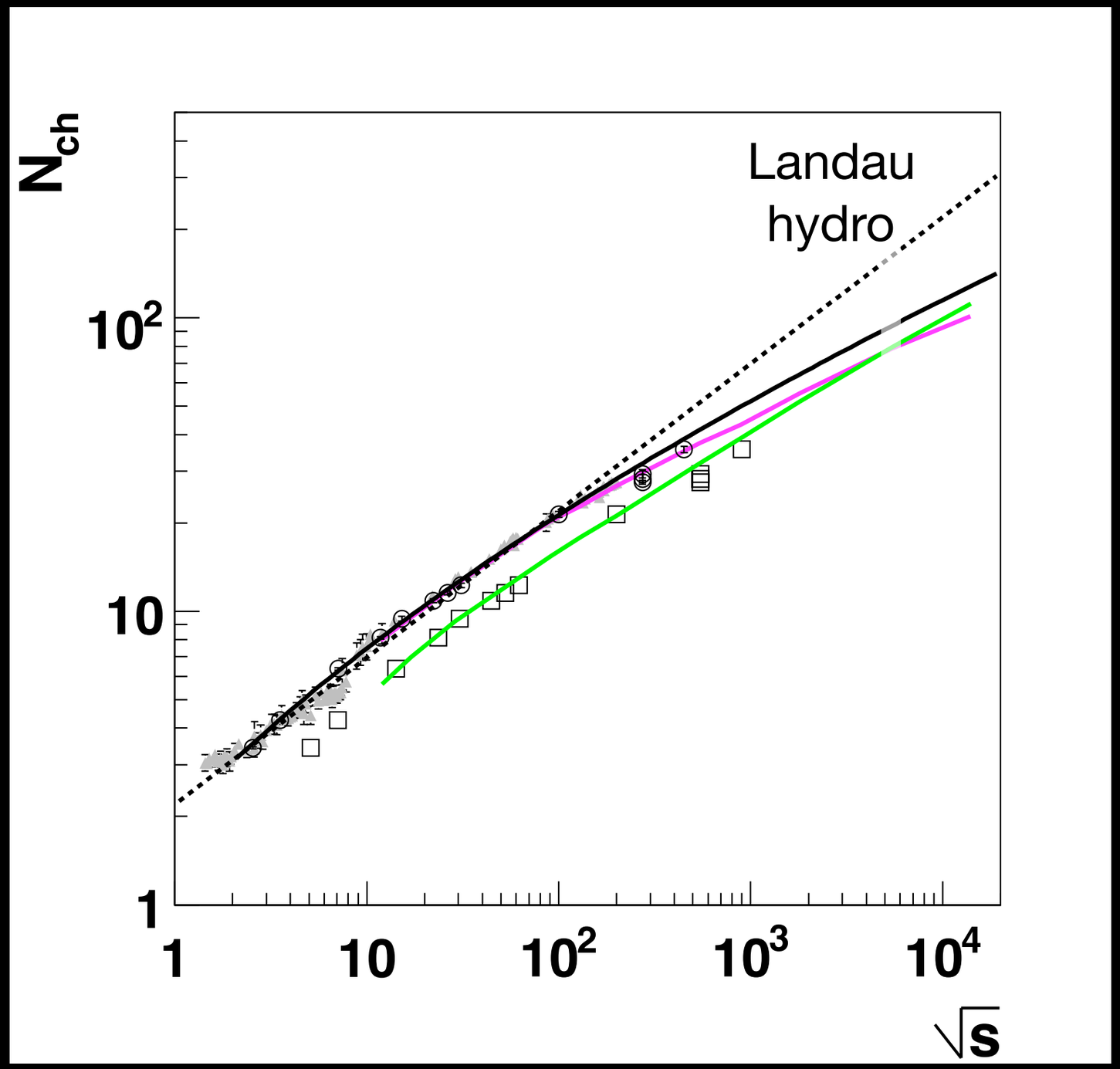
“effective energy” (a la Basile et al)



e^+e^- & p+p vs. Models



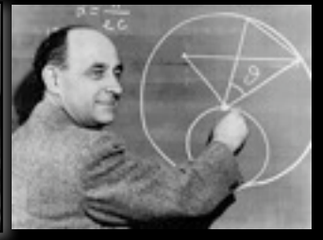
pQCD vs. Landau



Landau evolution

pQCD evolution

are quite close for available data...



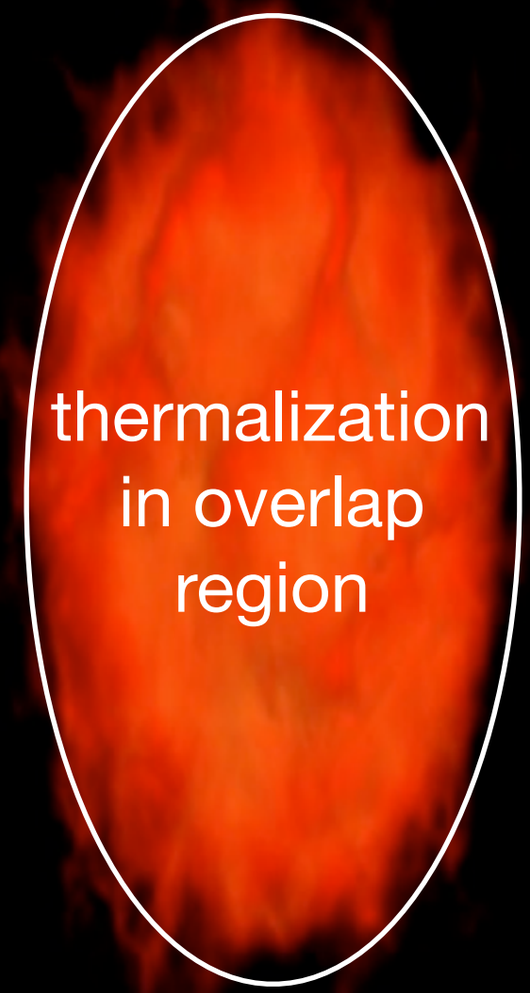
Fermi-Landau Model

$$E = \sqrt{s_{NN}}$$

Total Energy

$$V = \frac{V_0}{\sqrt{s_{NN}}/2m_N}$$

Total Volume



$$\epsilon = \frac{s}{2m_N V_0}$$

Energy Density E/V
(>3 TeV/fm^3 @ RHIC!)

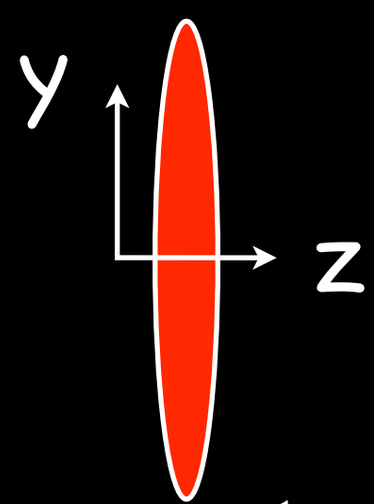
$$p = \frac{1}{3}\epsilon \Rightarrow \sigma \propto \epsilon^{3/4} \propto s^{3/4}$$

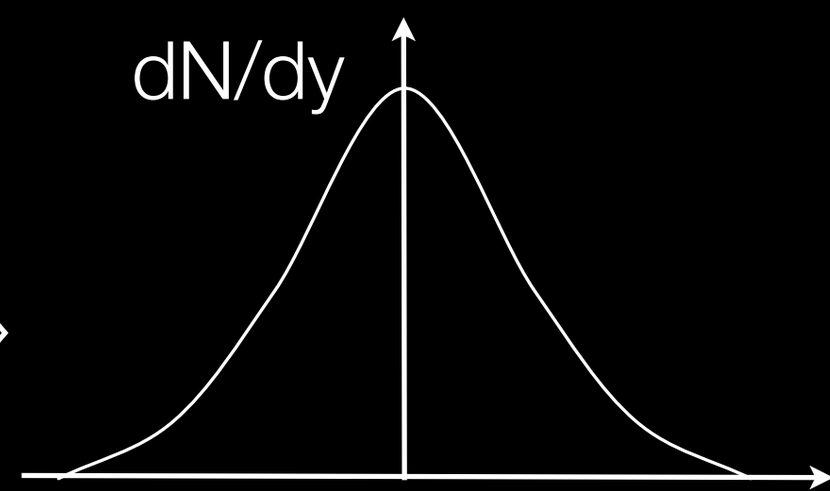
Blackbody thermodynamics

Cooper, Frye, Schonberg (1975)

$$N_\pi \propto E_{\text{c.m.}}{}^a, \quad \frac{1 - c_0{}^2}{1 + c_0{}^2} = a\,.$$

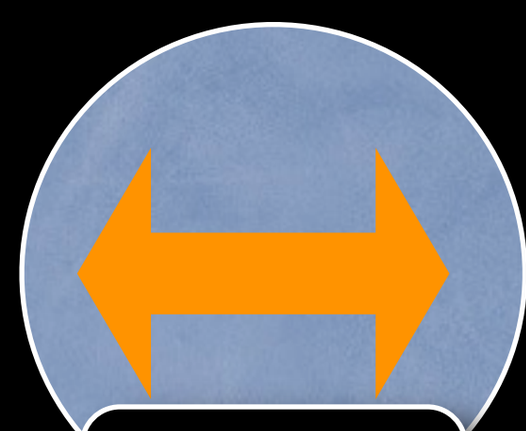
Relativistic Hydrodynamics



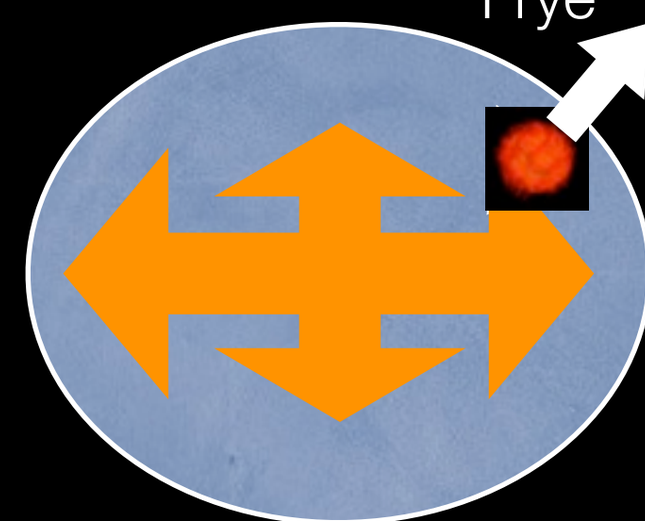


$$\Delta z \sim \frac{1}{\sqrt{s}}$$

$$\sigma_y = \sqrt{\frac{1}{2}\ln\left(\frac{s}{4M_P^2}\right)}$$



$$\partial_\mu T^{\mu\nu} = 0$$
$$P = \frac{\epsilon}{3}$$

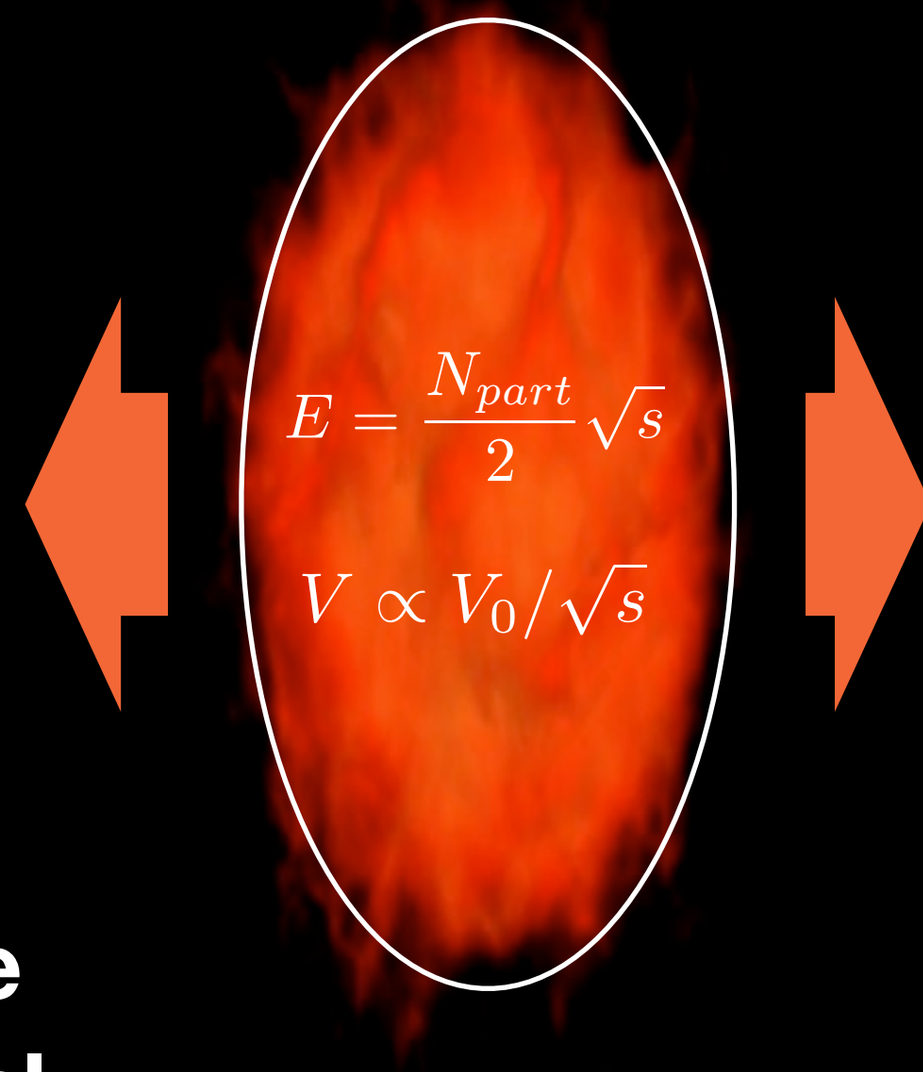


What Landau <u>is</u>

3+1D Dynamics
<u>rapid</u> <u>local</u> <u>equilibration</u>
w/ no free parameters

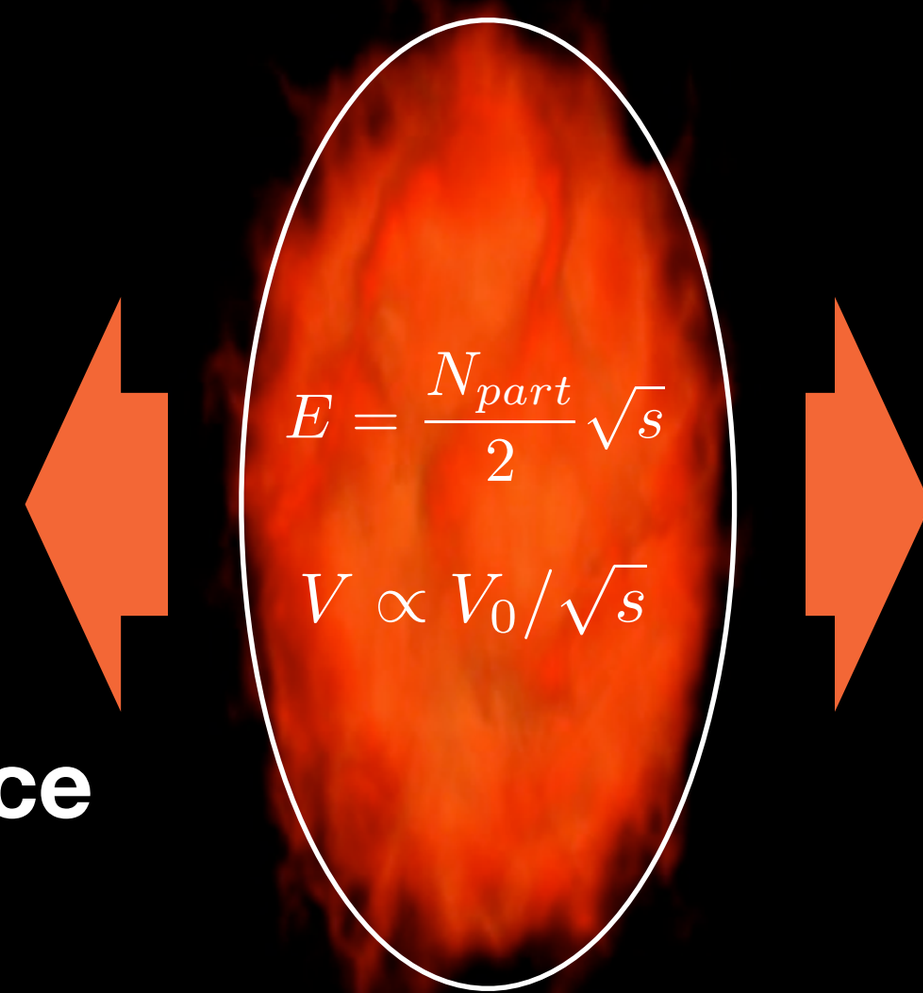
→ Beam energy & T_{ch}
determine initial & final cond
→ Energy dep. of entropy

- **No transparency**
- **No initial boost invariance**
- **Entire system is in thermal contact at t=0**



What Landau is <u>not</u>

- **No net baryons (or any conserved charges)**
- **No phase transitions (just p=ε/3)**
- **No hadronization *per se* (just T=T$_c$)**
- **No thermal freezeout**
- **No trivial mass dependence of dN/dy**
- **No resonance decays**



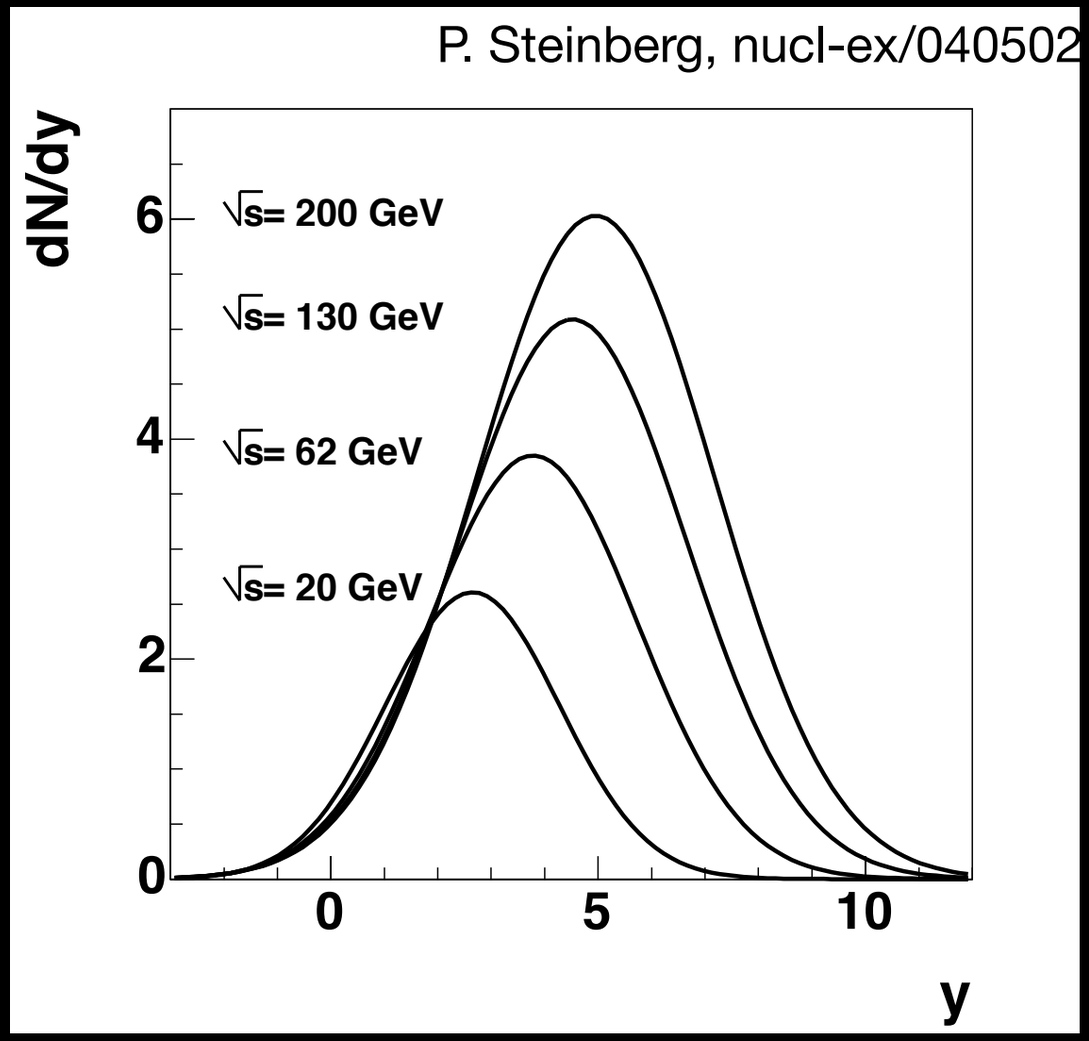
Caveats for all later results

“Extended Longitudinal Scaling”

$$\frac{dN}{dy} = K s^{1/4} \frac{1}{\sqrt{2\pi L}} \exp\left(-\frac{y^2}{2L}\right)$$

$$y' = y + y_{beam} = y + e^L \quad L = \ln\left(\frac{\sqrt{s}}{2m_P}\right)$$

$$\frac{dN}{dy'} \sim \frac{1}{\sqrt{L}} \exp\left(-\frac{y'^2}{2L} - y'\right)$$

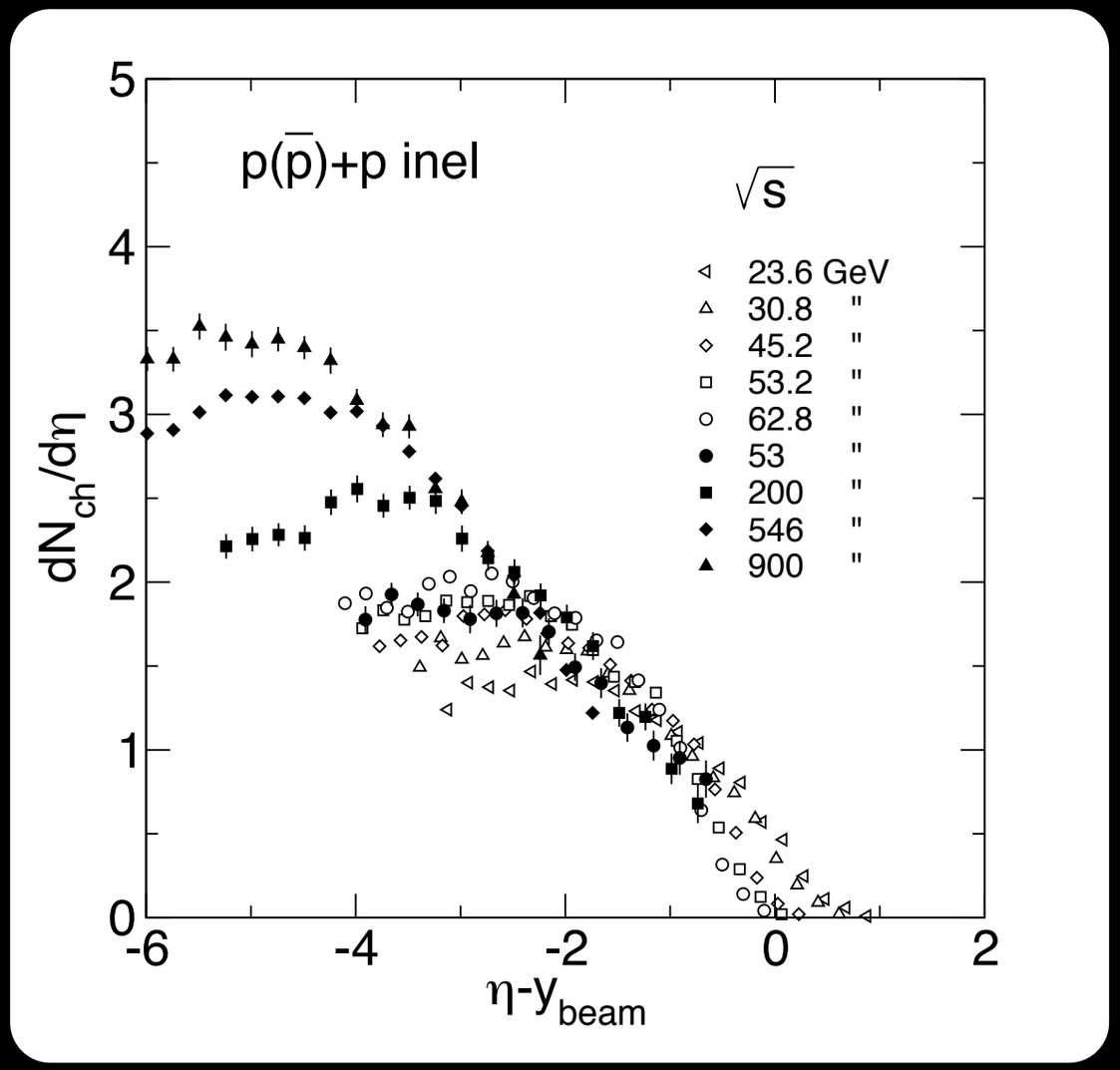


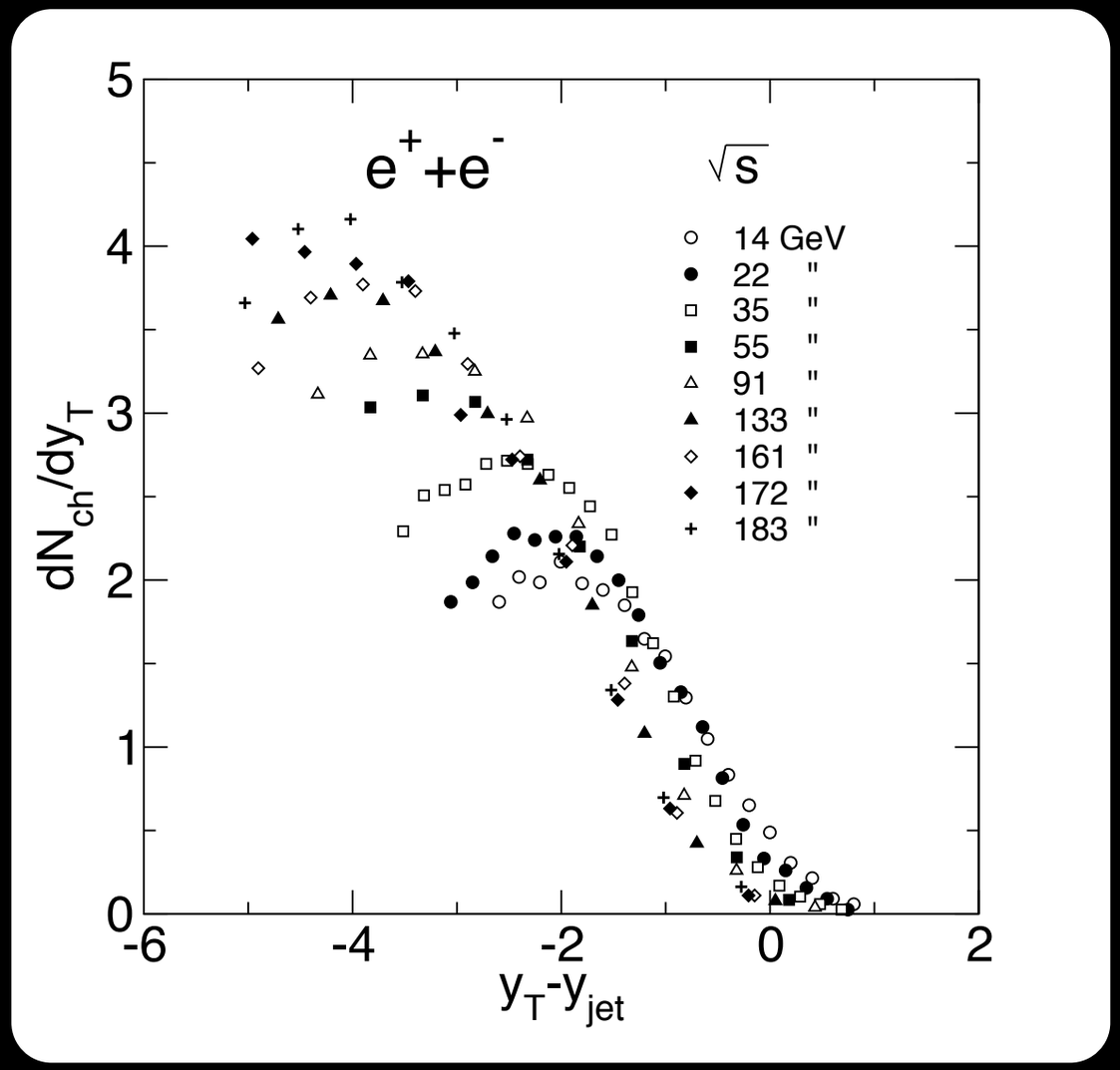
Landau/Fermi multiplicity formula
&
Landau hydro dN/dy

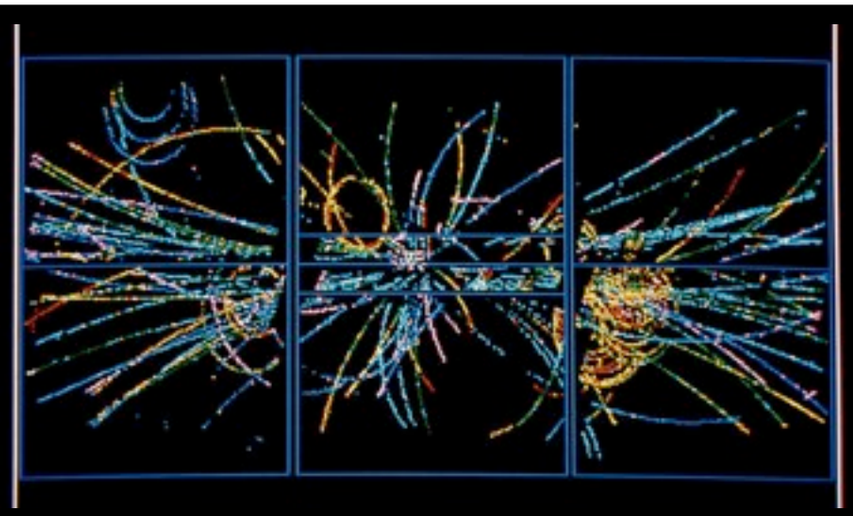
→

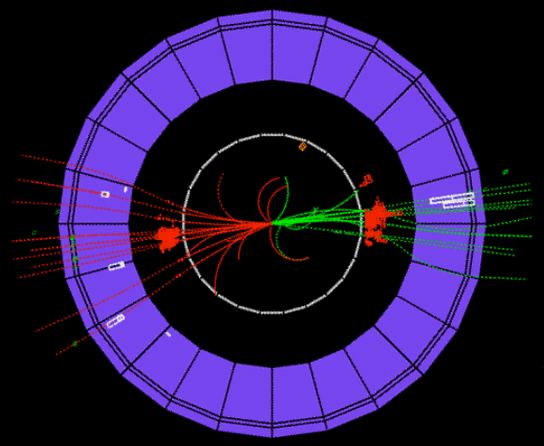
Longitudinal Scaling
(a.k.a. Limiting Fragmentation)

Compiled in PHOBOS White Paper (2004)



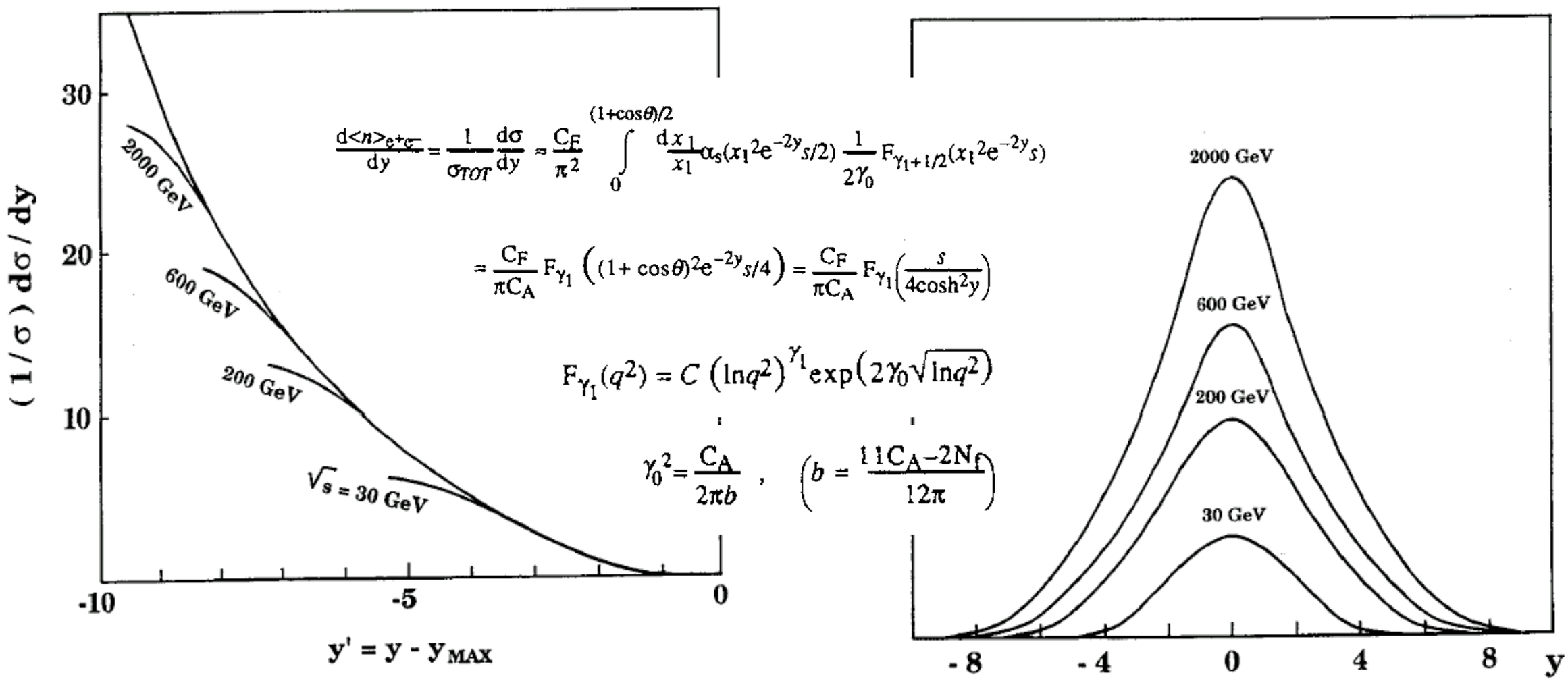






pQCD “vs.” Landau

K. Tesima, Z. Phys. C (1989)



$$\frac{d\langle n\rangle_{e^+e^-}}{dy} = \frac{1}{\sigma_{TOT}}\frac{d\sigma}{dy} \approx \frac{C_F}{\pi^2}\int_0^{(1+\cos\theta)/2} \frac{dx_1}{x_1}\alpha_s(x_1{}^2e^{-2y}s/2)\,\frac{1}{2\gamma_0}F_{\gamma_1+1/2}(x_1{}^2e^{-2y}s)$$

$$= \frac{C_F}{\pi C_A}F_{\gamma_1}\left((1+\cos\theta)^2e^{-2y}s/4\right) = \frac{C_F}{\pi C_A}F_{\gamma_1}\left(\frac{s}{4\cosh^2 y}\right)$$

$$F_{\gamma_1}(q^2) = C\,(\ln q^2)^{\gamma_1}\exp\left(2\gamma_0\sqrt{\ln q^2}\right)$$

$$\gamma_0{}^2 = \frac{C_A}{2\pi b}\,, \quad \left(b = \frac{11C_A - 2N_f}{12\pi}\right)$$

<u>MLLA pQCD</u> shows “limiting fragmentation” & $\sigma_y \propto \sqrt{\ln s}$

parametrically: **pQCD ↔ Landau Hydro** [?!]

p+p & e^+e^- → A+A

Parton distributions,
Nuclear Geometry,
Nuclear shadowing

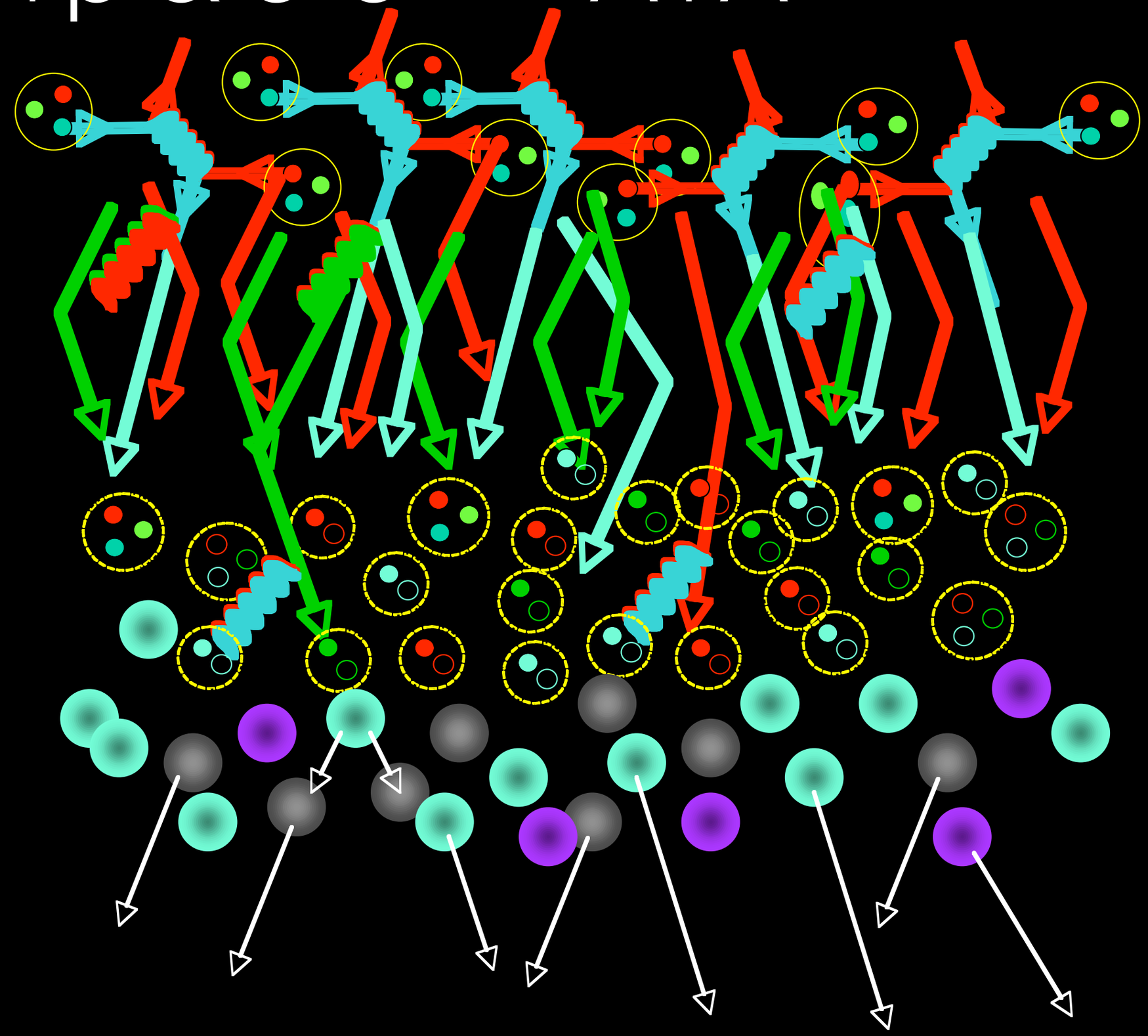
Parton production &
reinteraction

Chemical freezeout
(Quark recombination)

Jet fragmentation functions

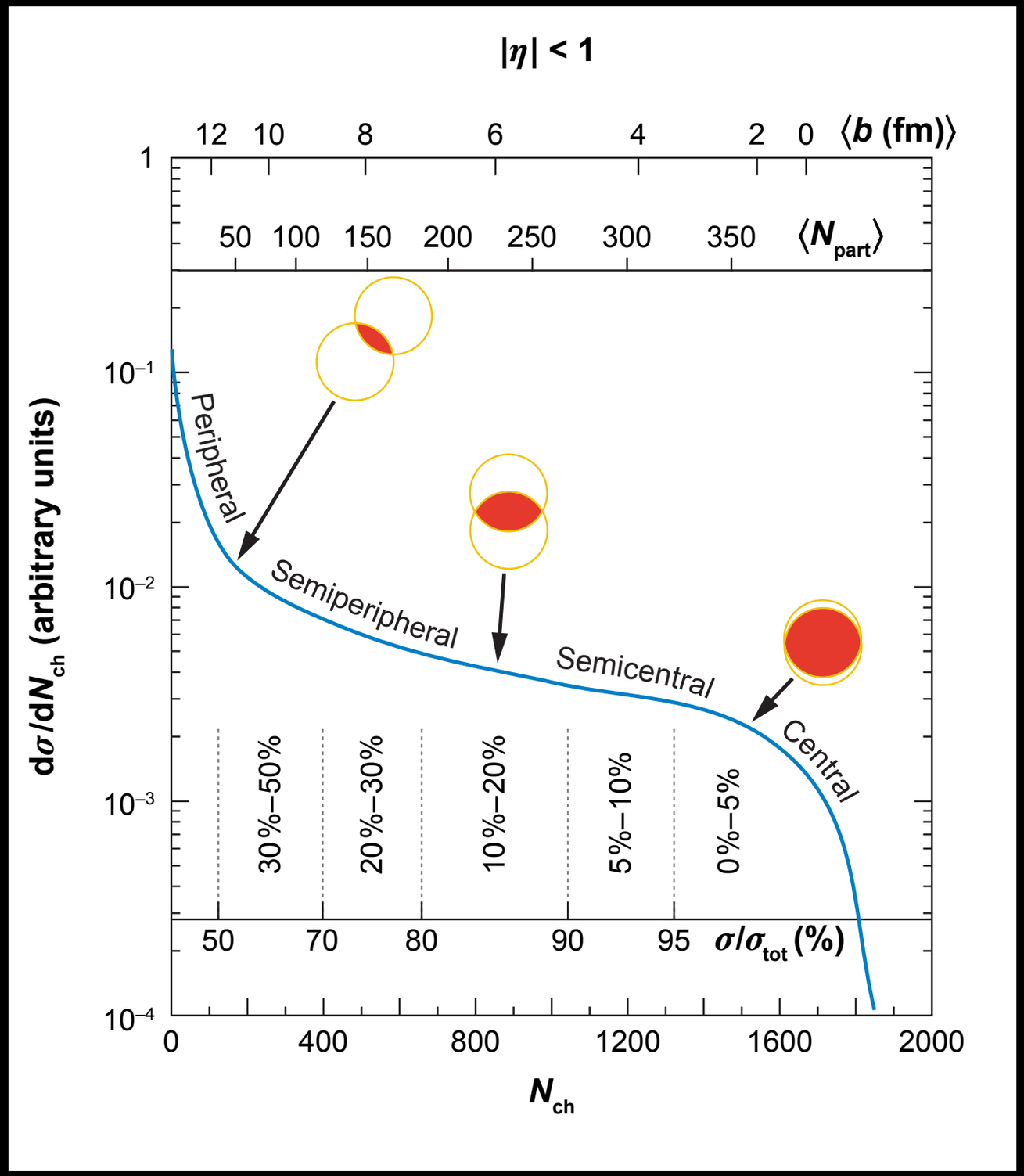
Hadron rescattering

Thermal freezeout &
Hadron decays



Many dynamical stages, all in principle independent...

Nuclear Geometry

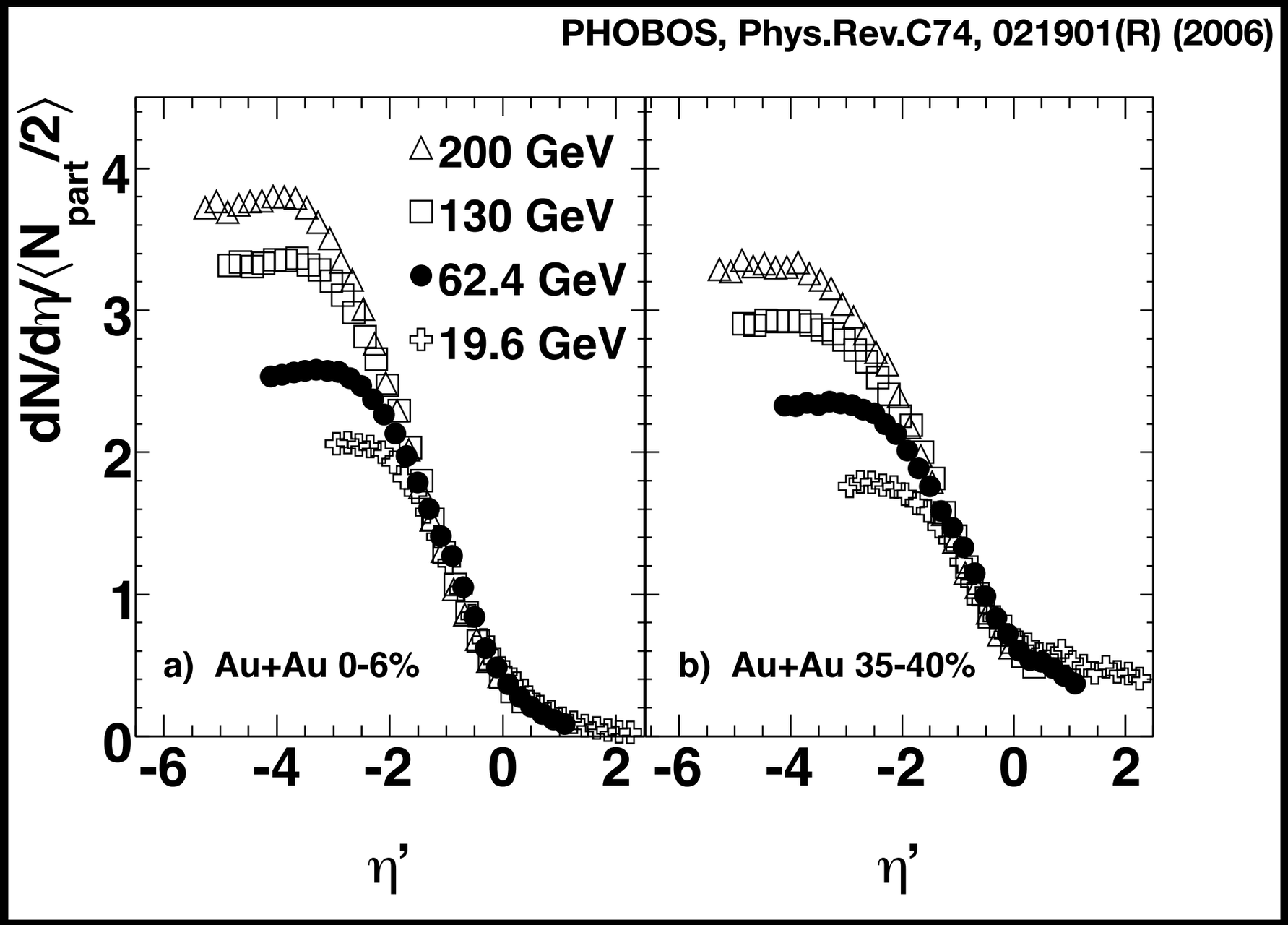


Cannot directly measure N_{part}, N_{spec}, b, ε, etc.

Measure distributions and assume they are monotonic with variable of interest

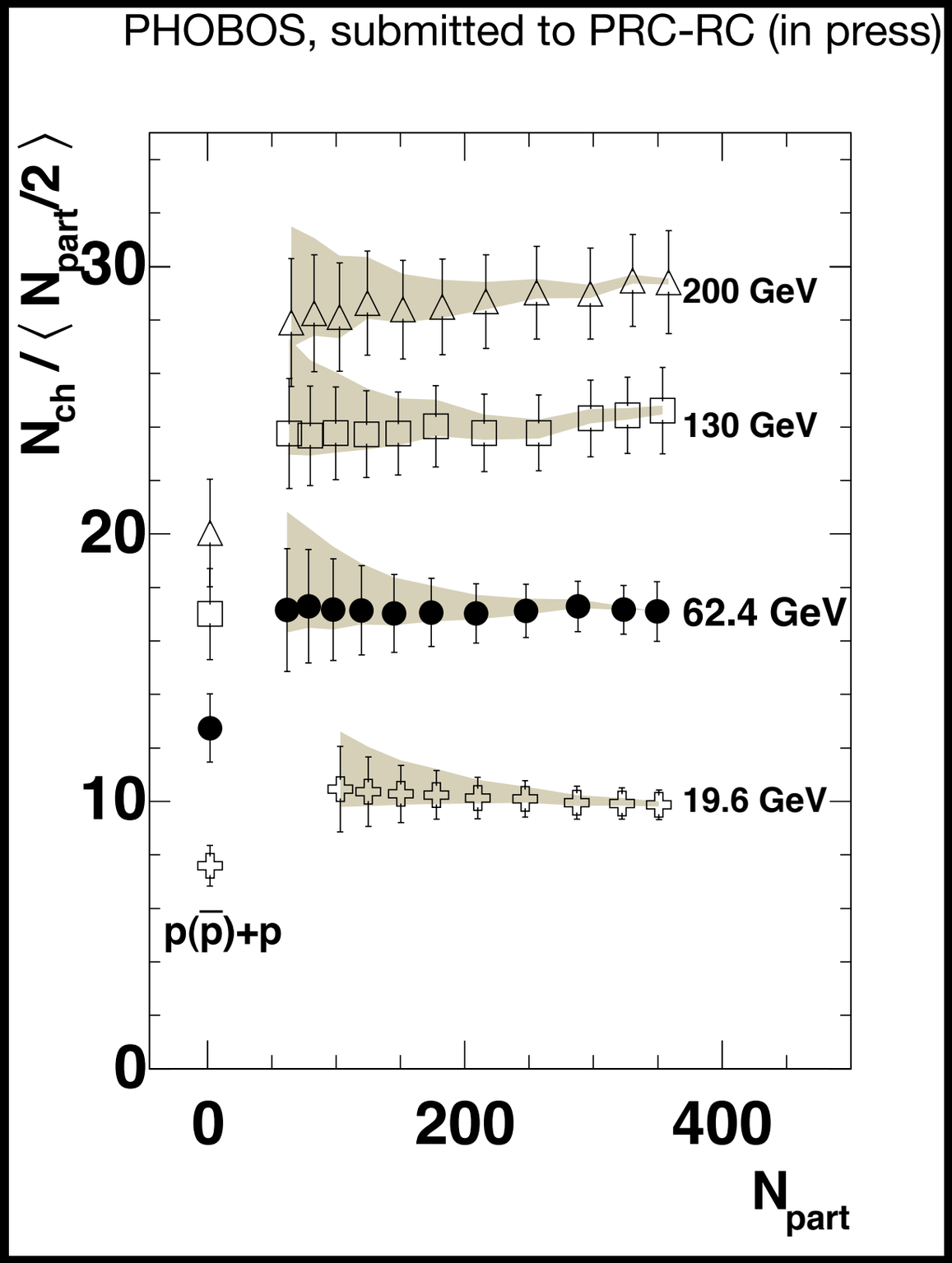
Bin data in "fractions of total inelastic cross section" (requires estimation of σ_{inel}) and relate to same bin in geometric distribution

Extended Longitudinal Scaling in A+A



Just as with p+p, e+e-, extended scaling observed in A+A.
Limiting curve *different* in peripheral events.

N_{part} Scaling



$$\frac{N_{ch}}{N_{part}/2} \sim \frac{N}{V} \sim f(\sqrt{s})$$

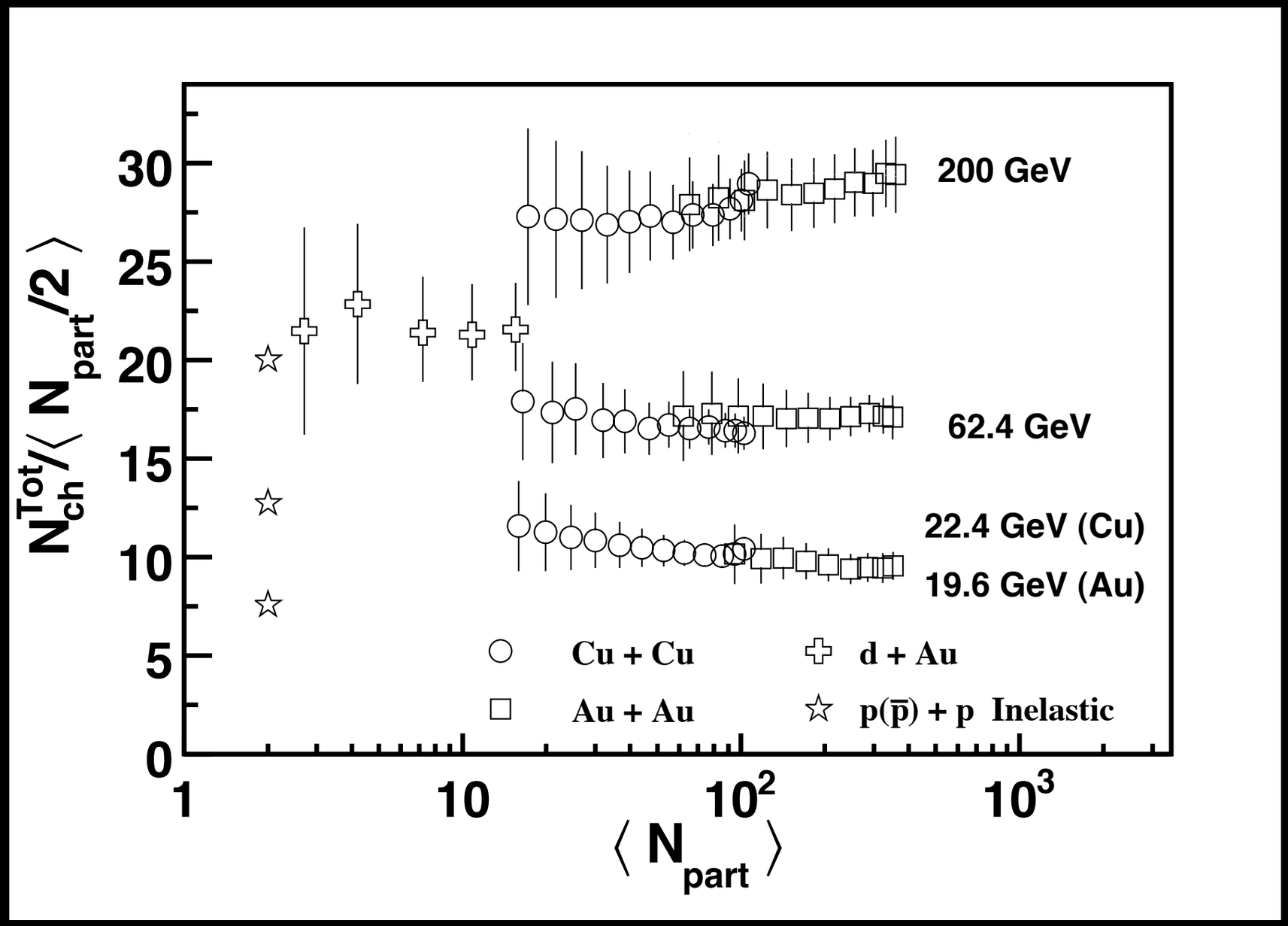
Entropy scales linearly w/ volume (no centrality dependence)

"one-component model"

No simple extrapolation to p+p

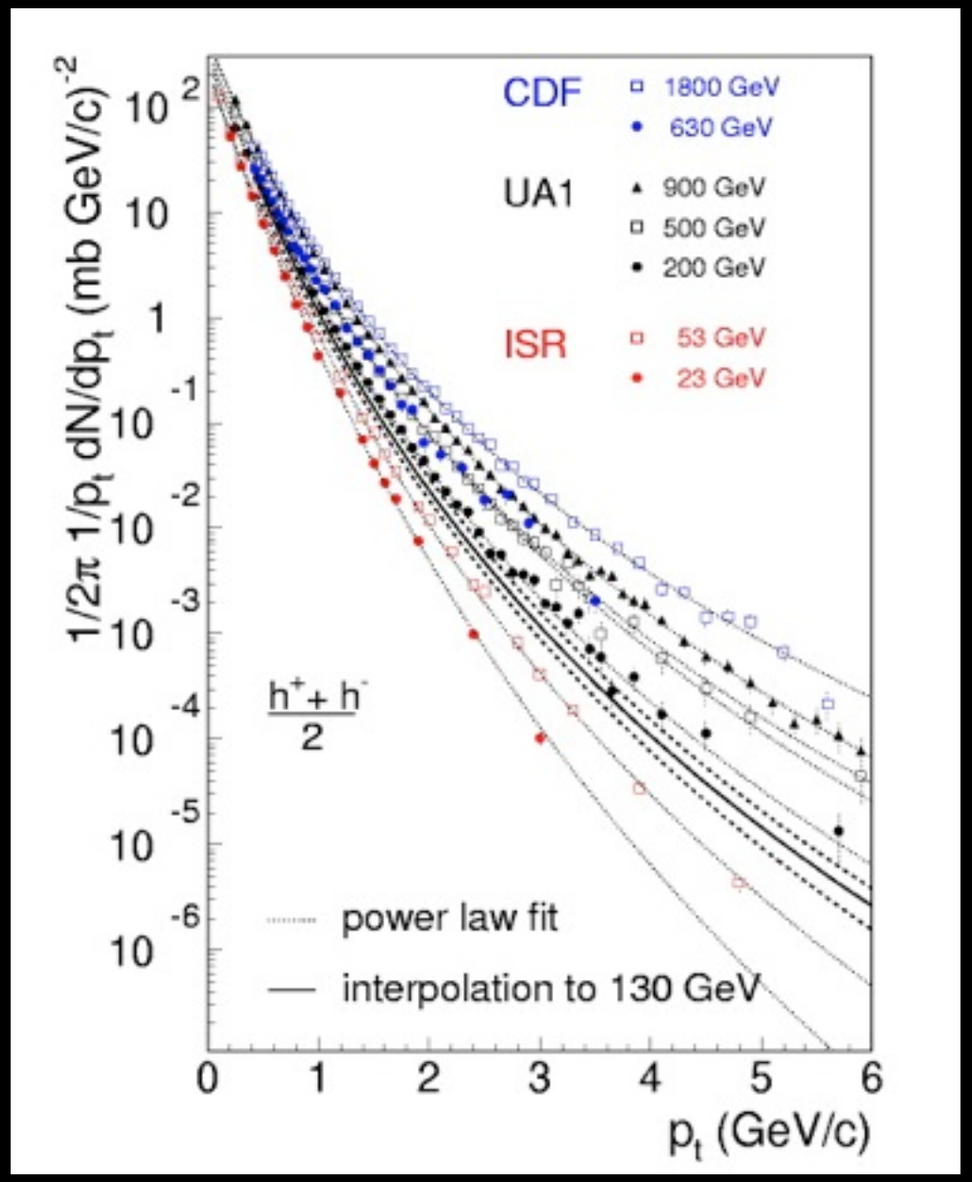
Au+Au & Cu+Cu

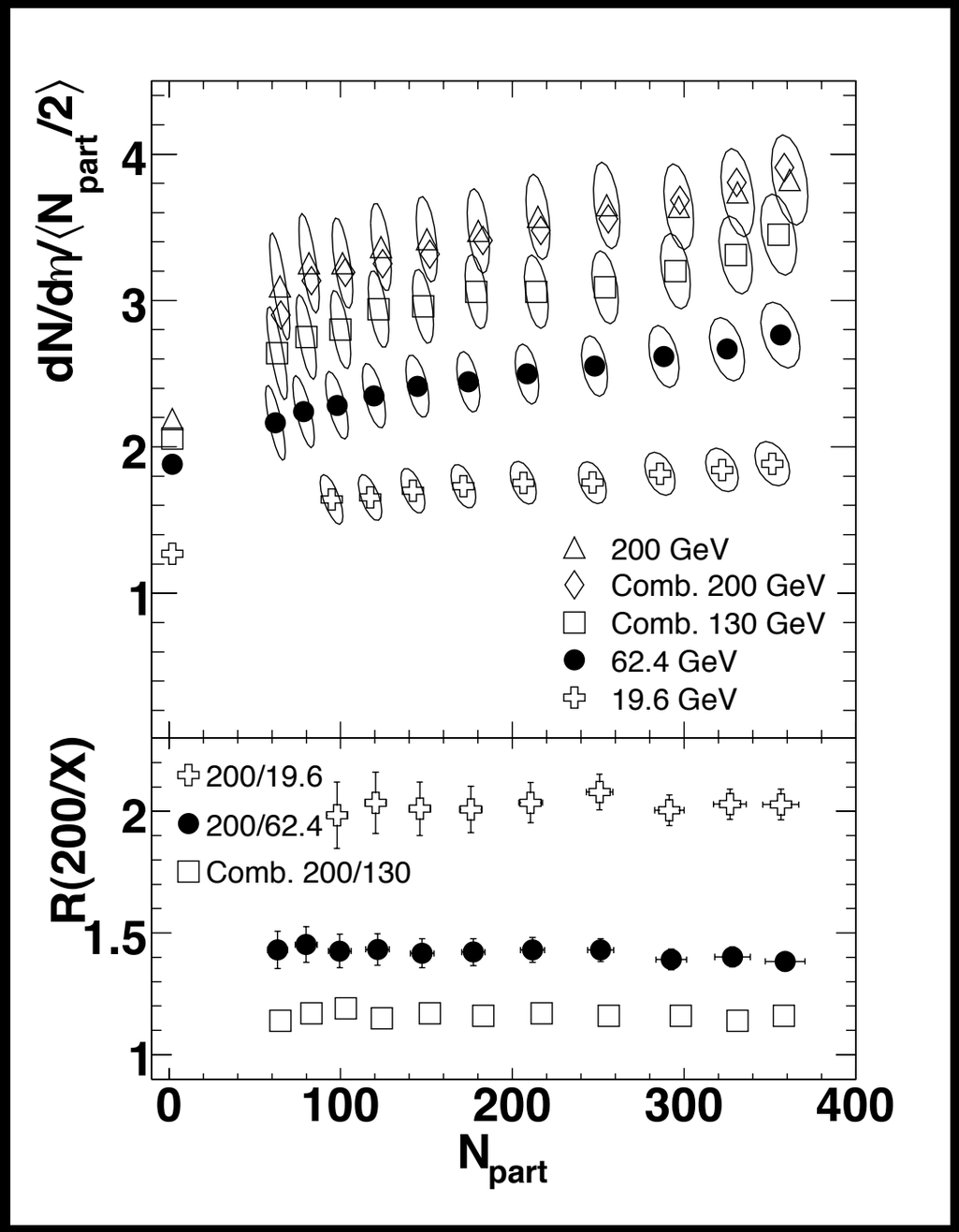
PHOBOS, submitted to PRL (2007)



No significant trend toward p+p observed (N_{part}<20!)
(but systematics preclude definitive statement)

dN/dη @ η=0



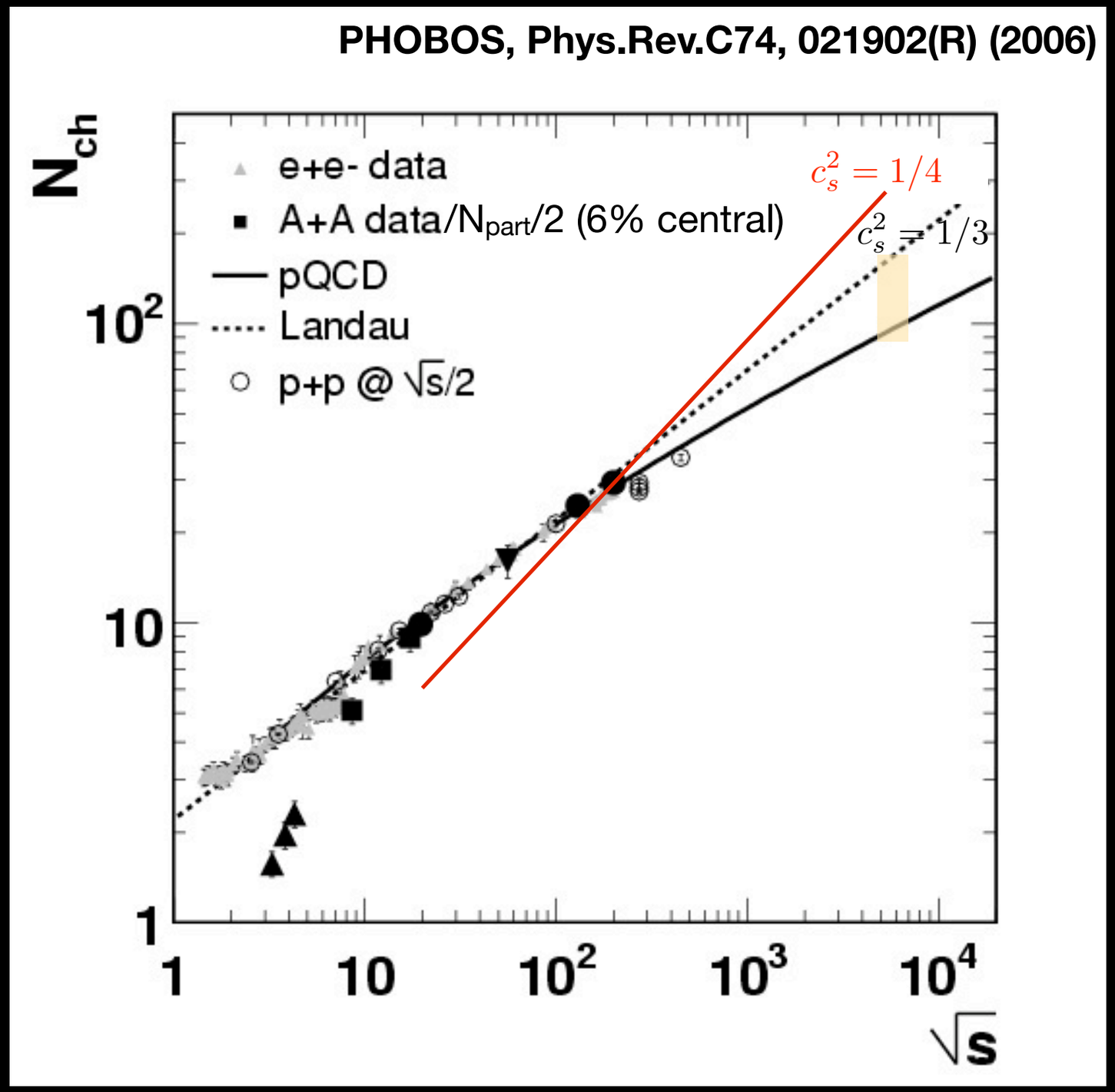


$$\frac{dN}{d\eta} = n_{pp}\left\{x\frac{N_{part}}{2} + (1-x)N_{coll}\right\}$$

"two component model"

where is semi-hard physics?

Comparison with e^+e^-/p+p



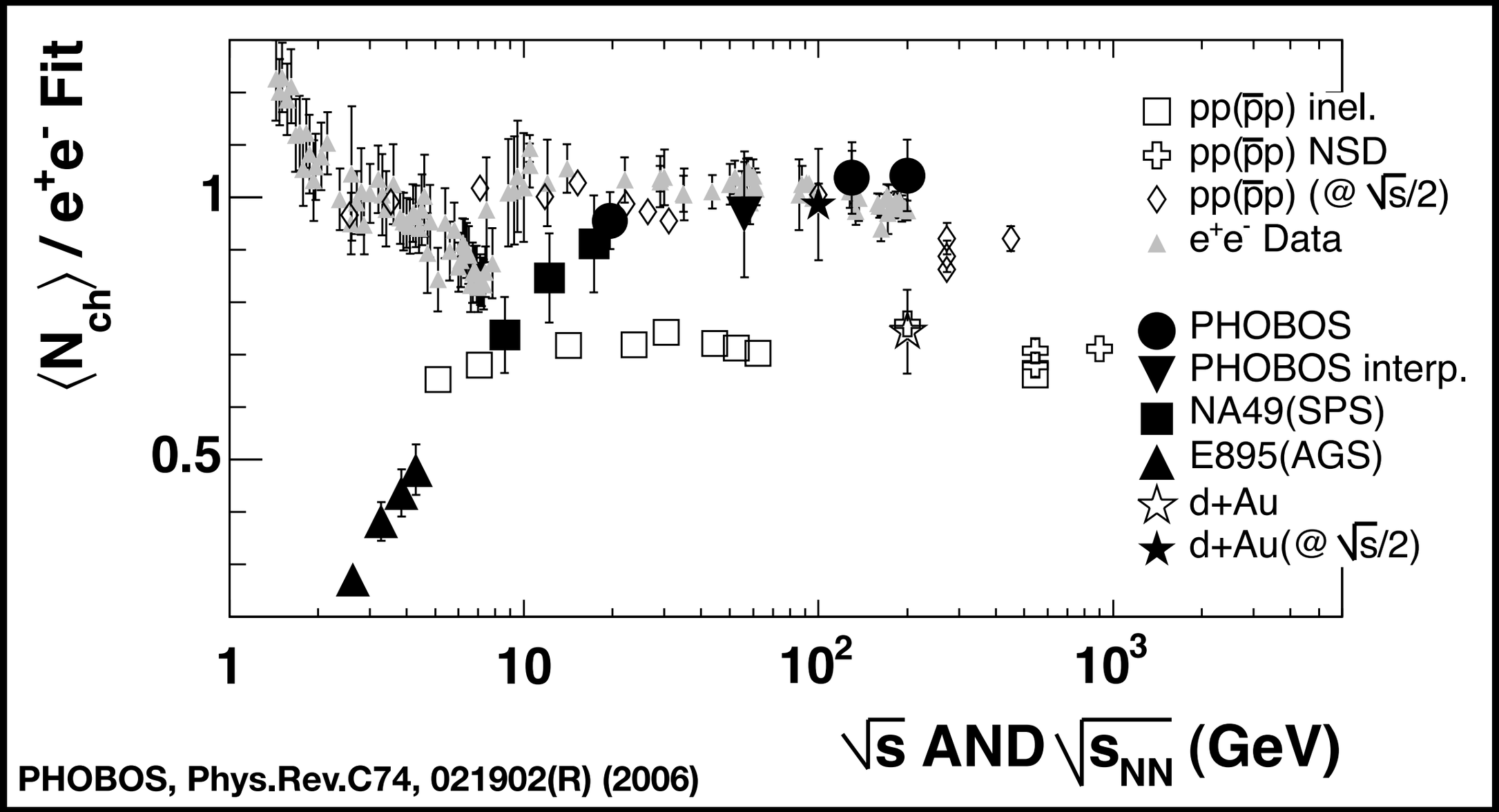
Models diverge at high energy... (LHC?)

A+A same as e^+e^- and p+p above SPS

Sketch of Gubser et al

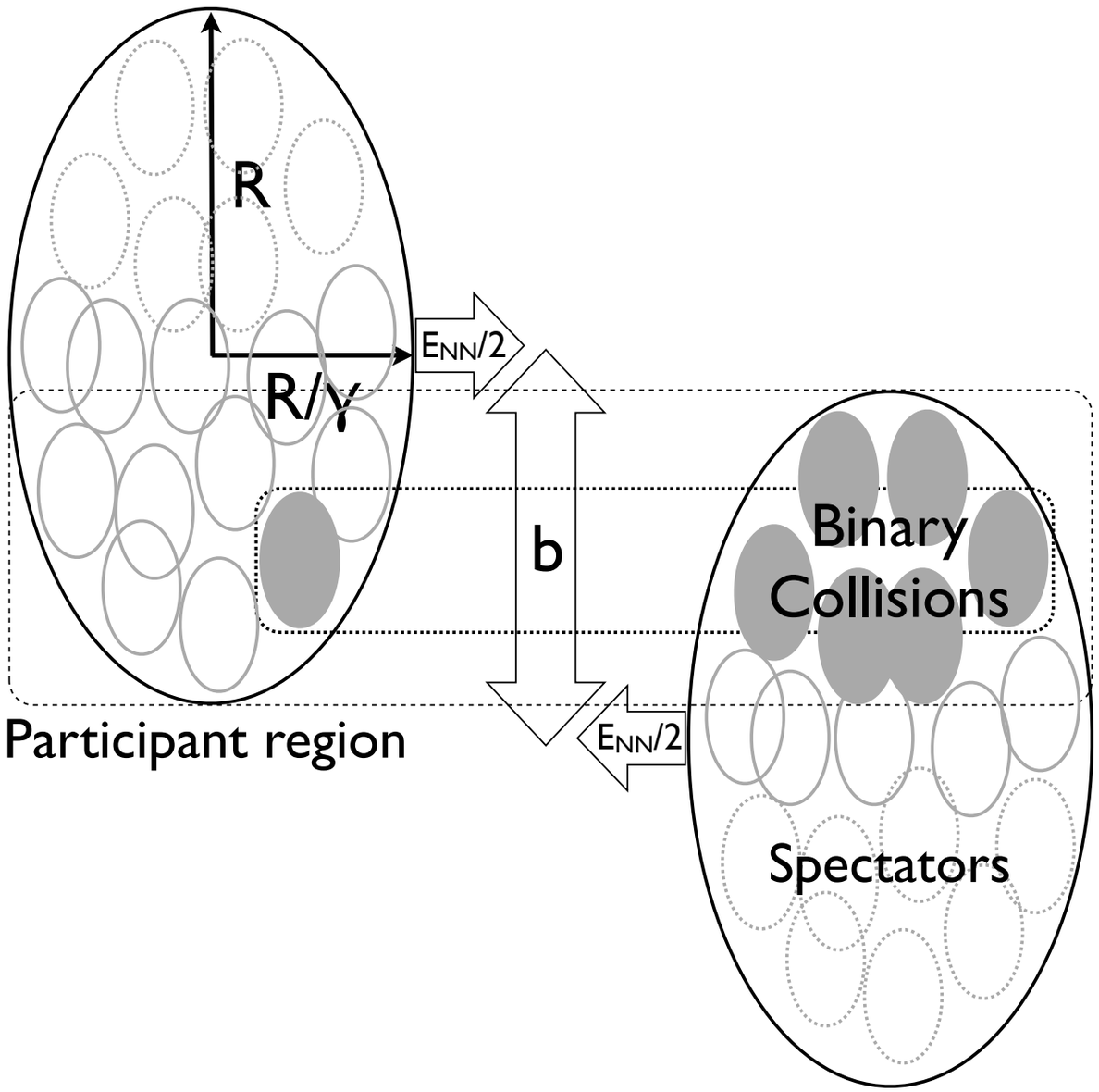
A+A appears "Suppressed" at low energy

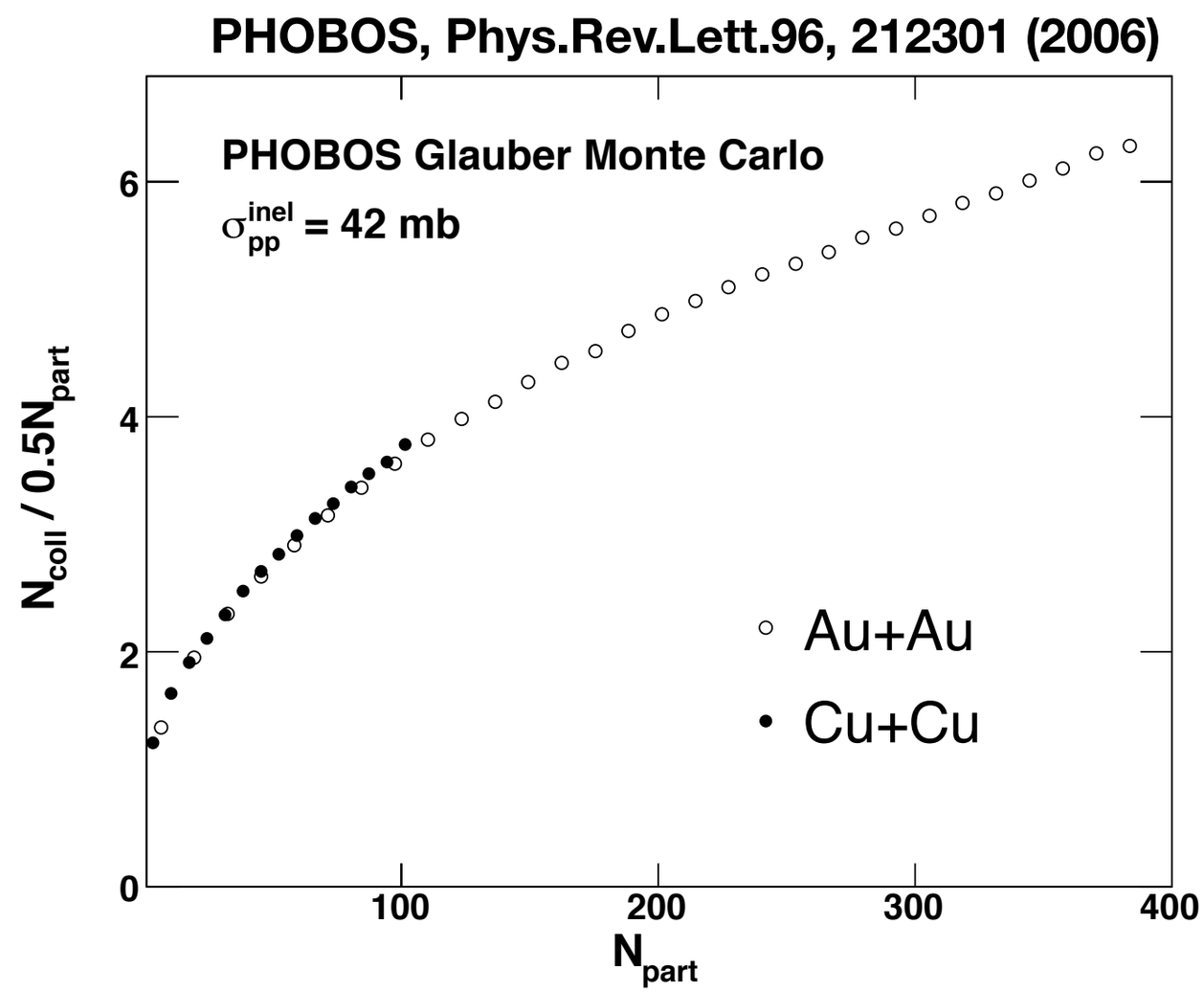
Comparison with e^+e^-/p+p



Strange result: A+A approaches e+e- at 20 GeV, and tracks it for a decade of energy

Geometry of Stopping

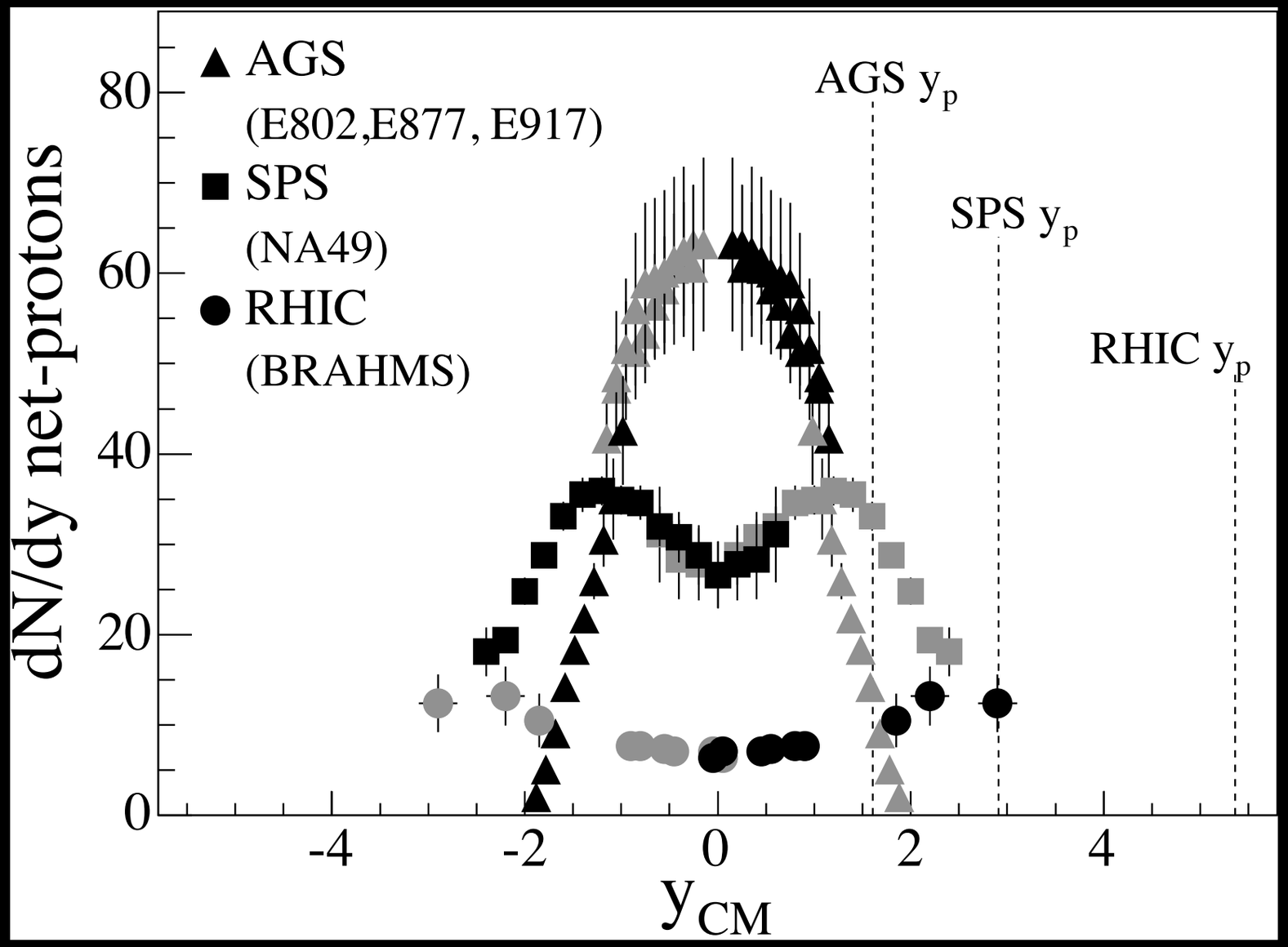




For most heavy ion events, $\nu \equiv \frac{\langle N_{coll} \rangle}{\langle N_{part} \rangle} \geq$ **2-3.** $\nu = 1$ in p+p

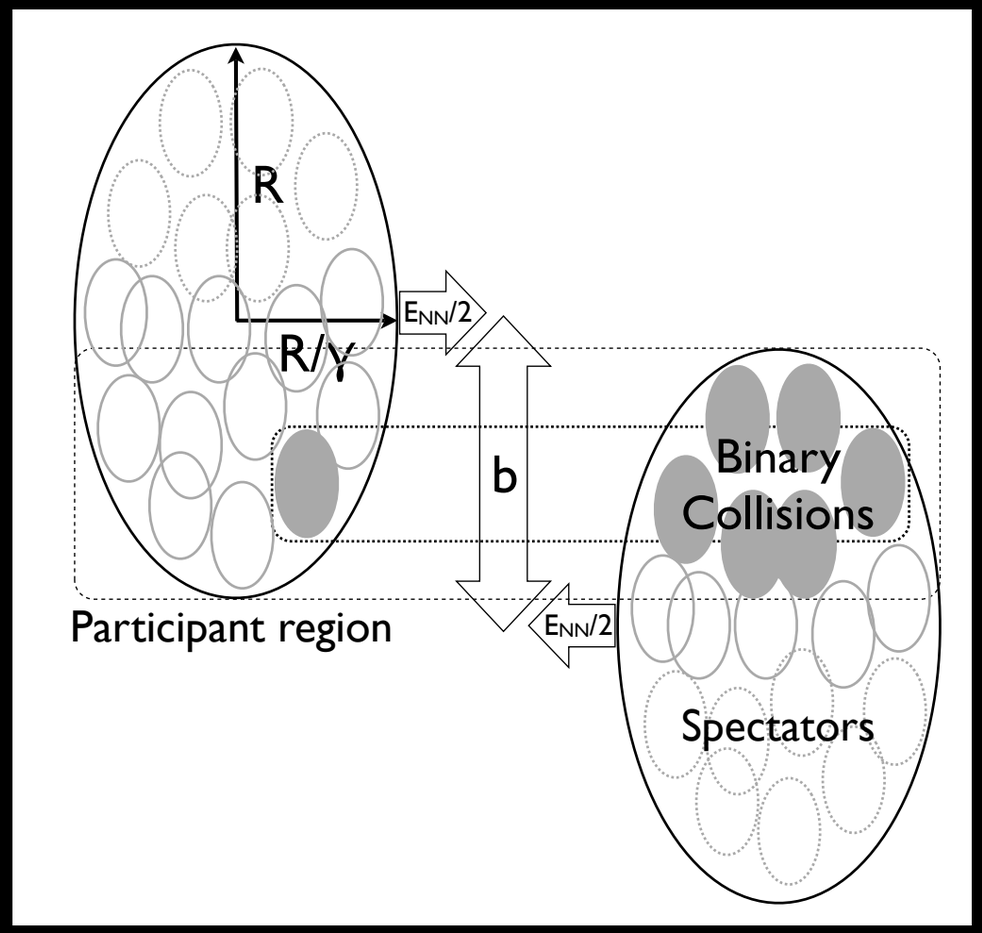
Maybe **>1-2** collisions alleviates leading effect?

Net Baryons

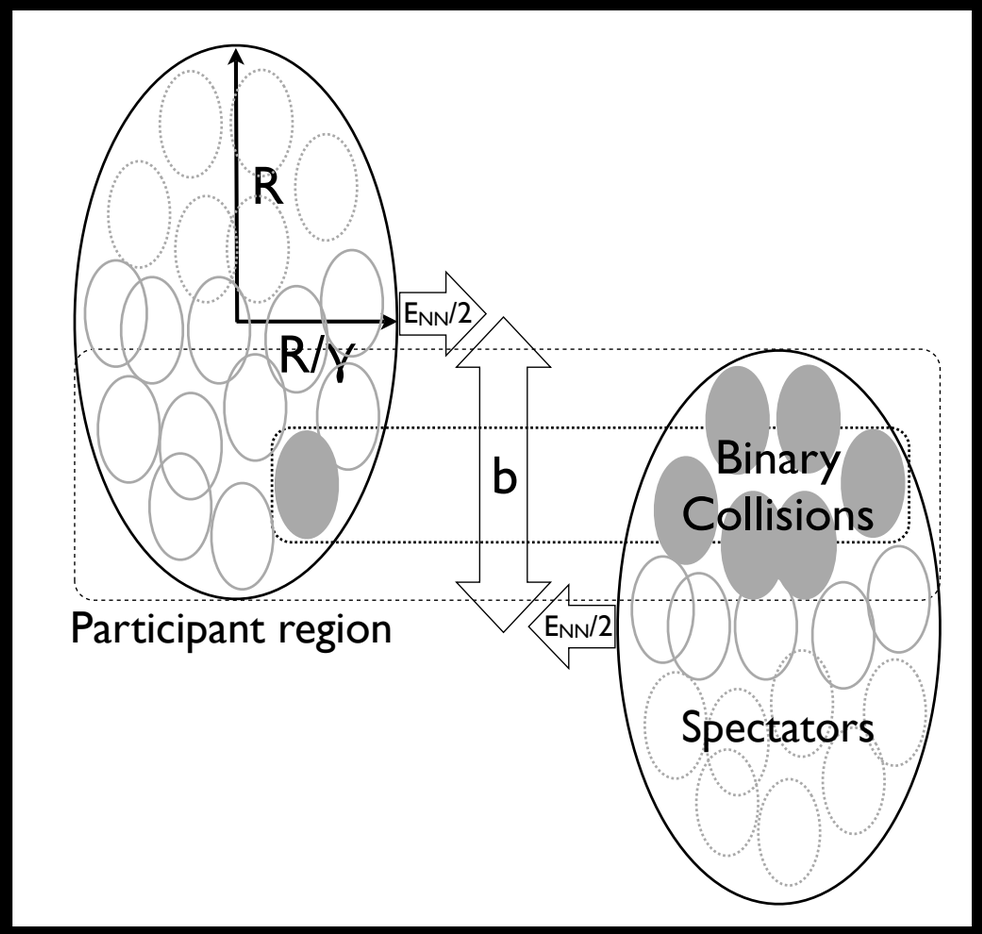


Net momentum (-loss+gain) of net baryon number (not baryons!)

"pile-up" at low energy, "transparency" at high energy?...


R
R/γ
E_NN/2
b
Binary
Collisions
Participant region
E_NN/2
Spectators

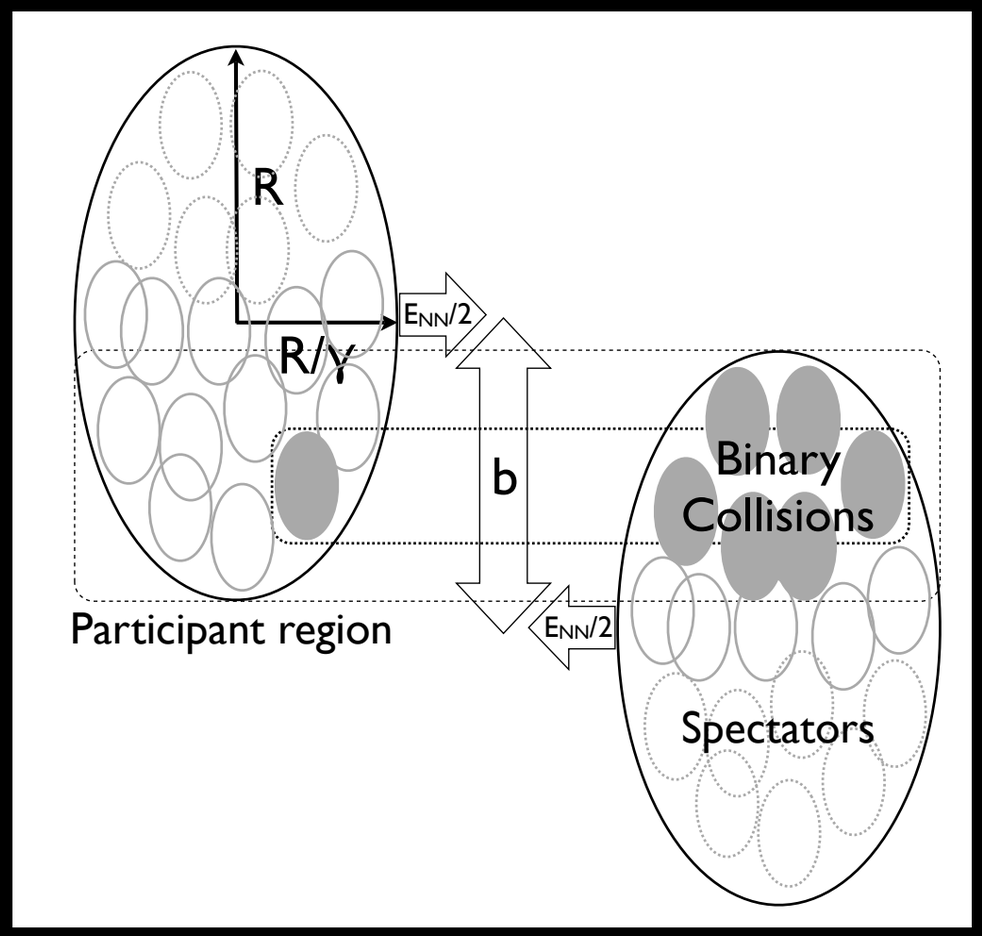
One Possibility...



First collision typically not enough to stop baryons

2nd+ collision is displaced in original direction,

Fireball Sandwich

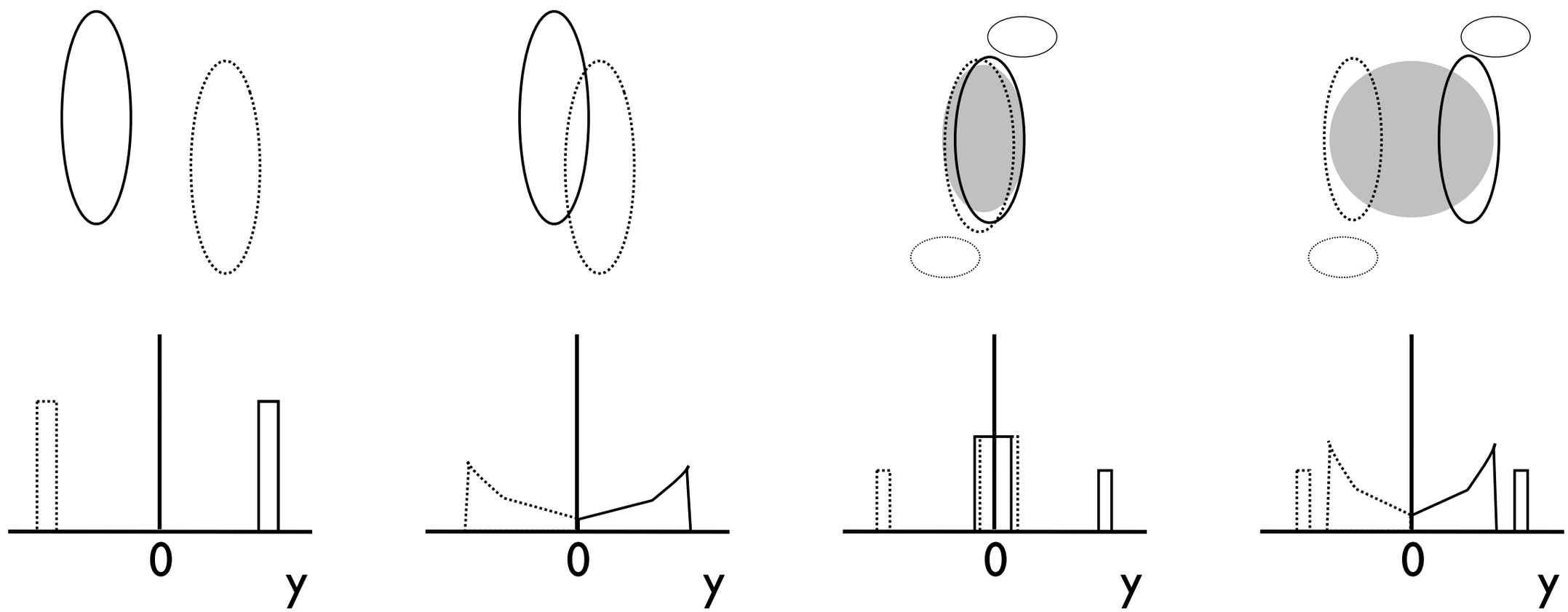


bulk of baryon number near <u>edge</u> of reaction zone at $t=t_0 \sim 1/\sqrt{s}$

→ net baryons are part of entropy generation process

→ hydro evolution <u>pushes</u> them to larger rapidity

Fireball Sandwich



1. Incoming nuclei (beam rapidities)

2. First collision: partial stopping of baryons

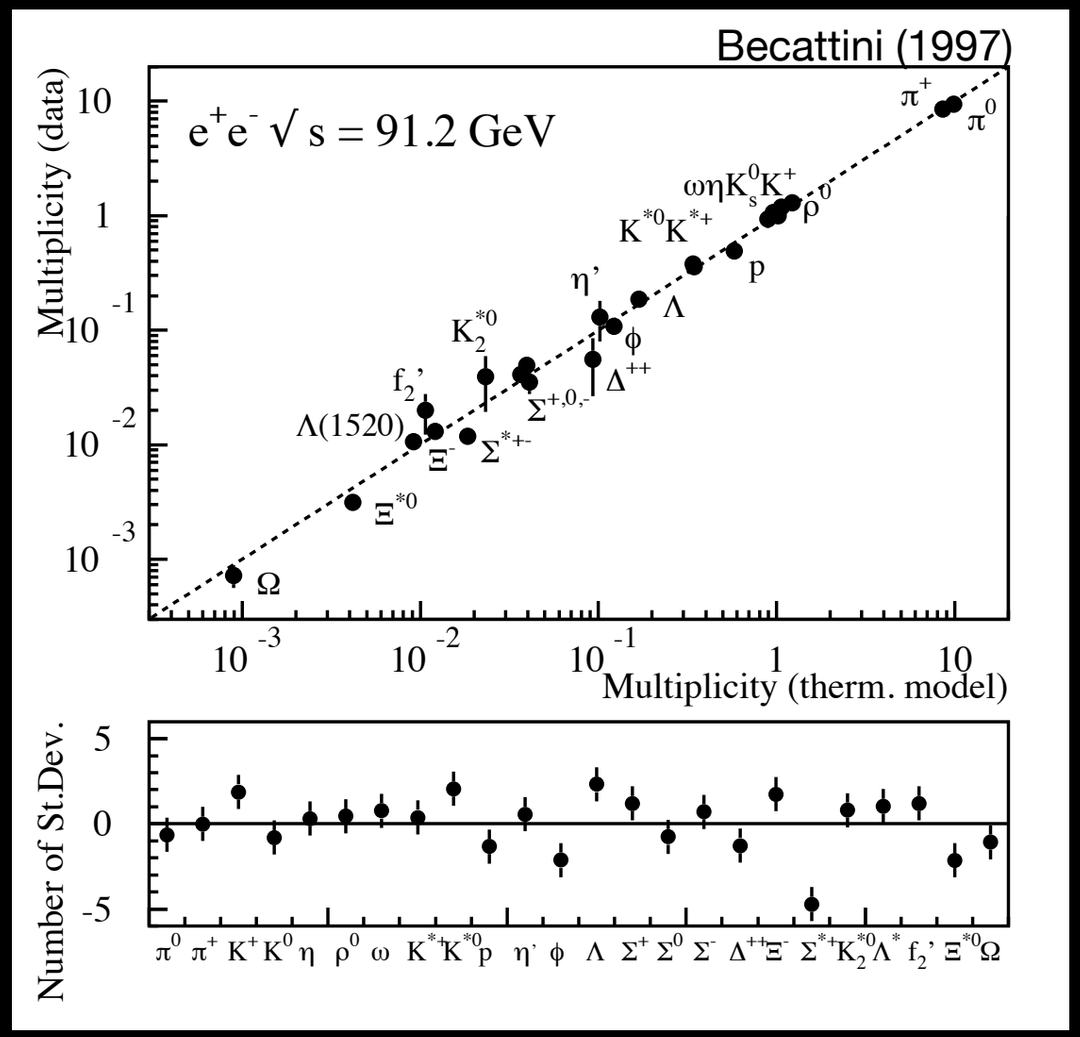
3. Second collisions: full baryon stopping, displaced centroids, thermalization (spectators decouple)

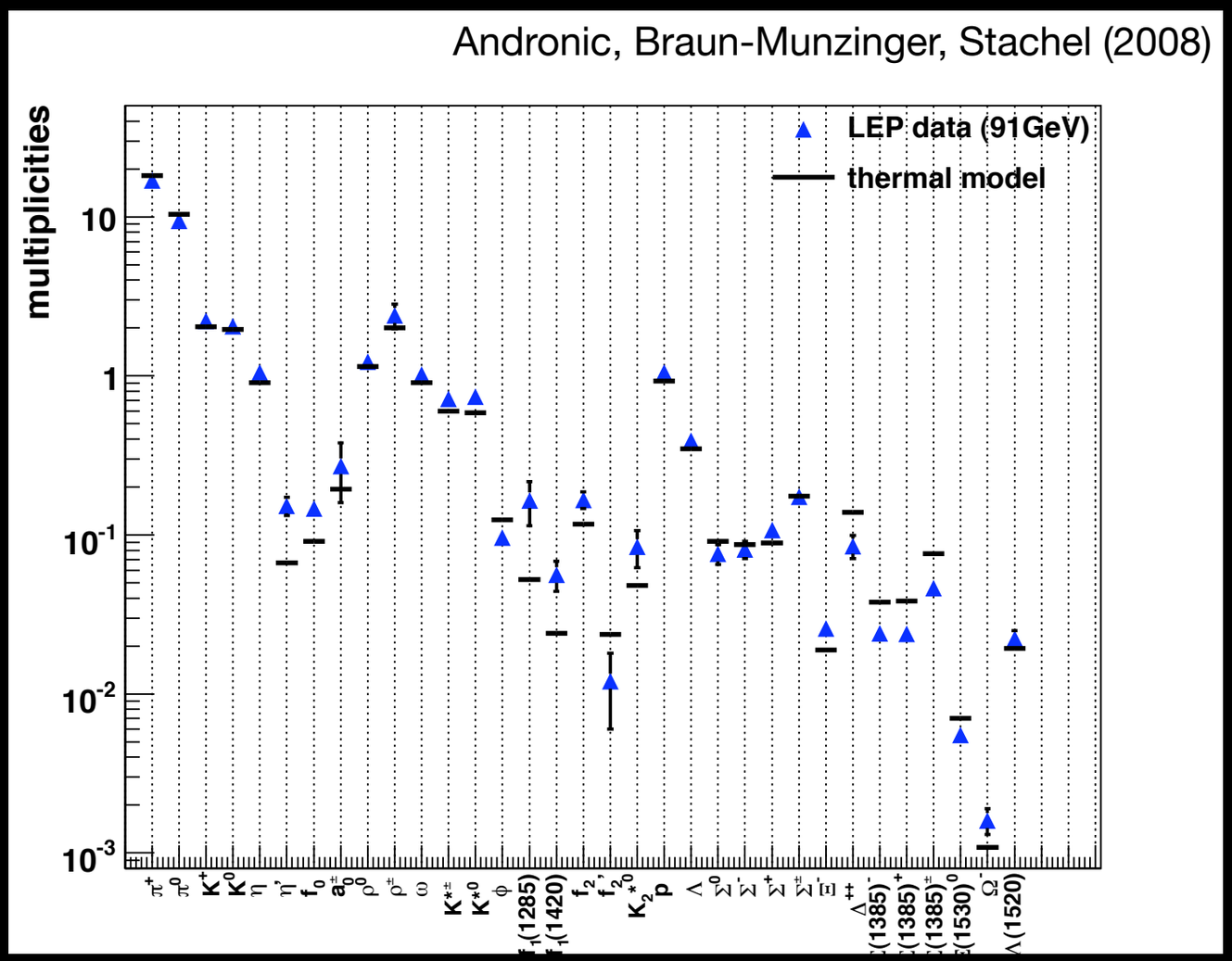
4. Longitudinal expansion of matter, baryons reaccelerated

Conclusions

- **Multiplicity e^+e^-, p+p, A+A described by different mechanisms**
  - Why do the multiplicities come out the same?
- **Problem of early (quasi-)thermalization**
  - Can a pQCD calculation act like Landau hydro?
  - How small is small, anyway?

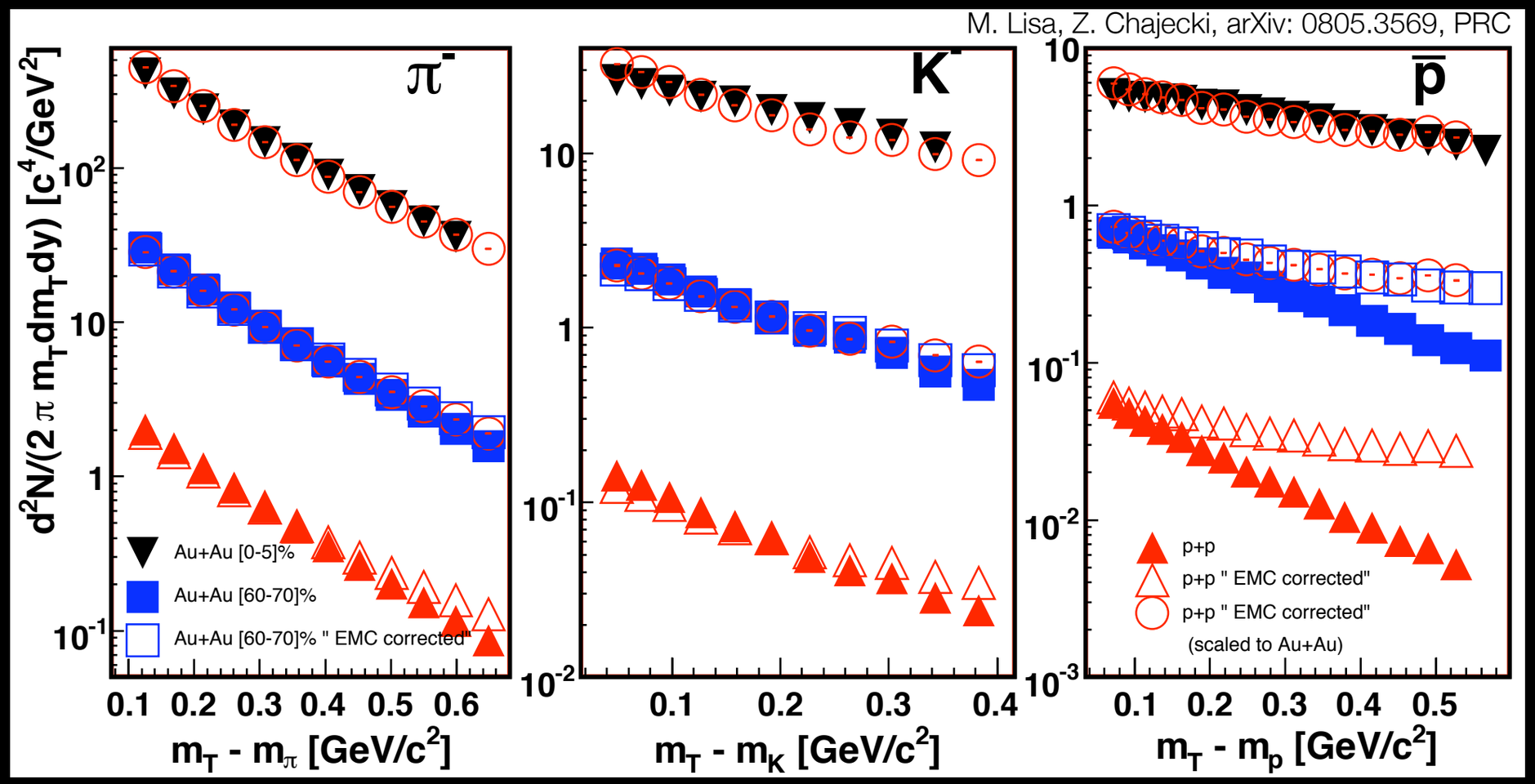
Thermal Fits to e^+e^-





A long-standing issue: thermal fits work well,
but not equally well for all hadrons.
Implementation of local conservation laws matter
(quantum numbers & energy-momentum)

EMC in p+p vs. A+A

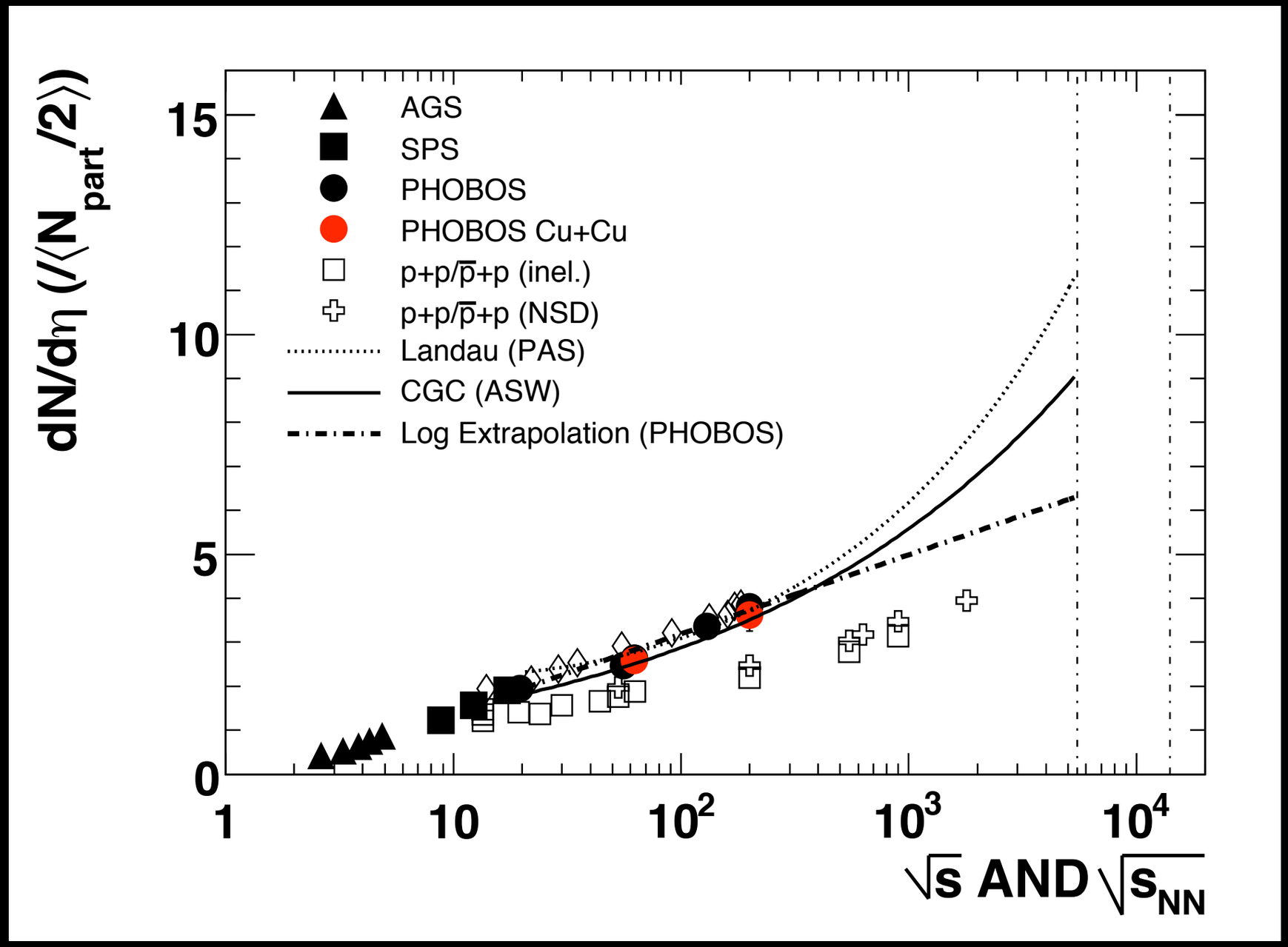


Lisa & Chajecki: energy momentum conservation (EMC) has substantial effect on multiparticle production.
EMC "suppresses" momentum spectrum in p+p collisions.
→ p+p turns out to have same "flow" as A+A

Conclusions

- **Multiplicity e^+e^-, p+p, A+A described by different mechanisms**
  - Why do the multiplicities come out the same?
- **Problem of early (quasi-)thermalization**
  - Can a pQCD calculation act like Landau hydro?
  - How small is small anyway?
- **Brings out deep relation to problem of stopping**
  - What is a baryon? How do they interact “softly”?
- **What can AdS/CFT help us with?**
  - Colliding shock waves: is there a “smallest size”? what is the thermalization time in CFT?
  - Can it really use all 5 dimensions?
- **Experimental prospects bright**
  - LHC extends energy range by x27, results will be rapid

Predictions at mid-rapidity



$\frac{s^{1/4}}{\sqrt{\ln s}}$ Landau-Carruthers

$s^{0.144}$ CGC

$\ln s$ "PHOBOS"

LHC: limited acceptance will require limited checks

What are the microscopic dynamics?

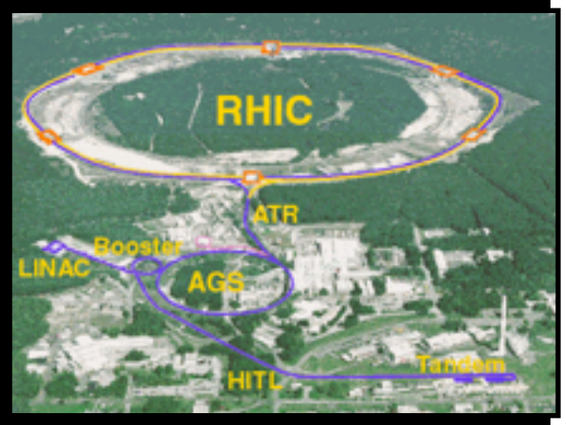
A (relatively)
simple structure...

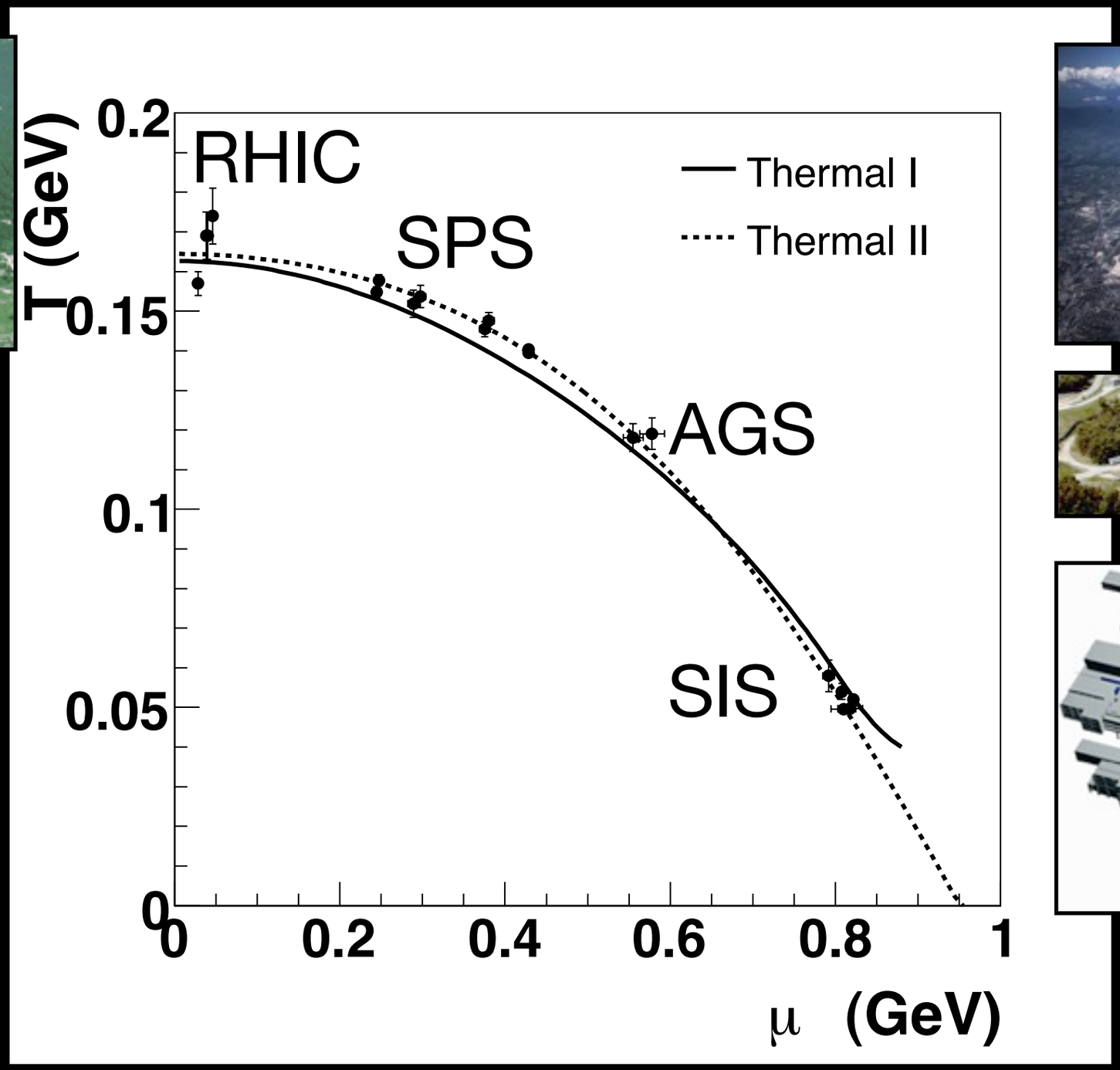
Extra Slides

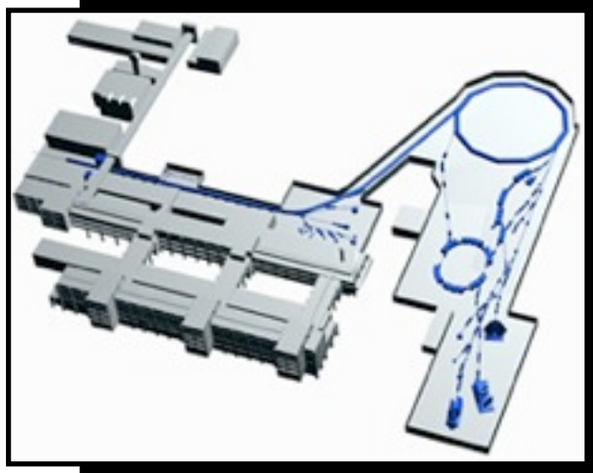
Total Multiplicity at Finite μ_B

(work with Cleymans, Wheaton, Stankiewicz, UCT)

Phase Diagram

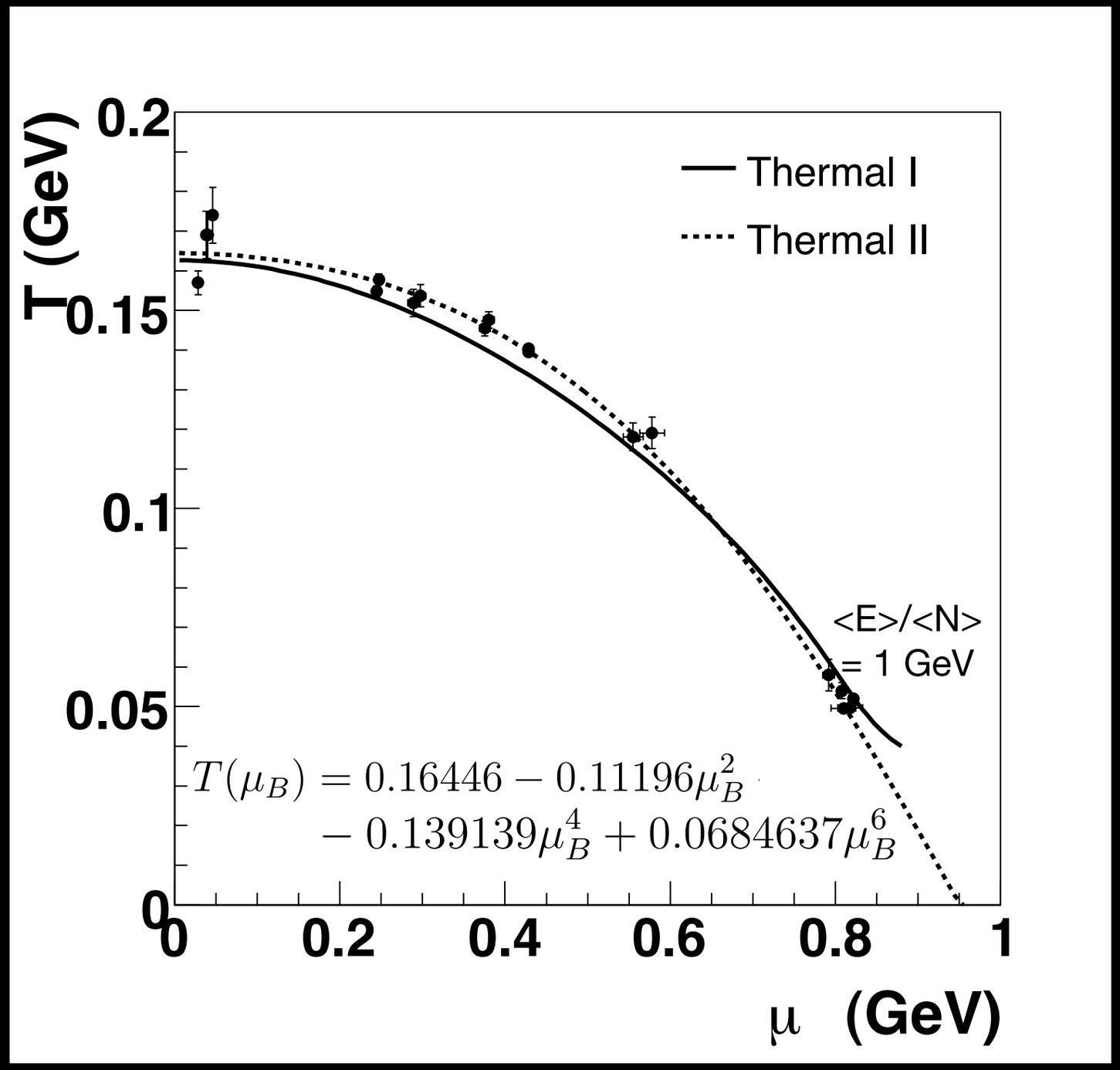






As beam energy decreases, $\mu_B\uparrow$, $T\downarrow$

Two Paths in T-μ_B plane



Thermal I: <E>/<N> =1 GeV
Thermal II: Fit to data

Baryochemistry

In equilibrium:

$$G = E + PV - TS = \mu_B N_B$$

Rearranges to:

$$S = \frac{E + PV}{T} - \frac{\mu_B N_B}{T} \equiv S_0 - S_B$$

So chemical potential reduces entropy, and thus total multiplicity:

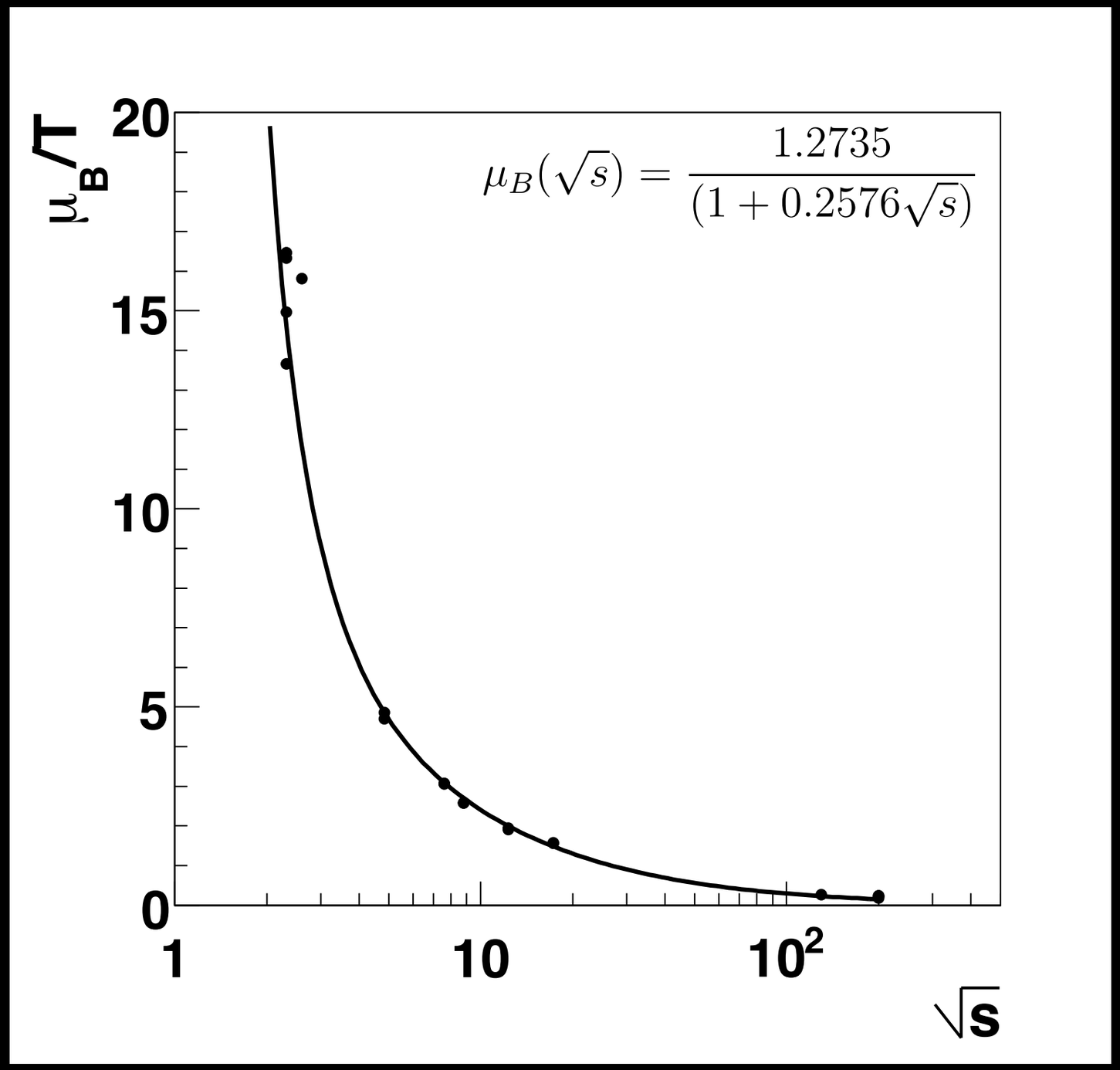
$$\Delta \frac{N_{ch}}{N_{part}/2} \equiv \Delta N_{ch} \propto \frac{S_B}{N_B} \propto \frac{\mu_B}{T}$$

Phenomenology

- **Need a "baryon free" N_{ch} reference**
  - p+p and e+e- show same energy dependence
  - p+p has leading baryons (complicated)
  - *choose* e+e- as reference

- **Two approaches**
  - Correct A+A data for presence of μ_B
  - Thermal model calculation of entropy density

“Correcting” Total Entropy



Not yet applicable to N_{ch}

“Correcting” Total Entropy

Convert change in entropy to change in multiplicity per $N_{part}/2$

$$\Delta N_{ch} = \frac{2}{\alpha\beta N_{part}} \frac{\mu_B N_B}{T}$$
$$= \frac{2}{\alpha\beta} \times \frac{N_B}{N_{part}} \times \frac{\mu_B}{T}$$
$$= \frac{\mu_B}{3T}$$

$N_B \equiv N_{part}$

$\alpha \equiv S/N = 4$

$\beta \equiv N/N_{ch} = 3/2$

$\alpha\beta=6$ is “trivial estimate”.

THERMUS (Cleymans, et al) → $\alpha\beta=7.2$ (15% diff.)

Direct Calculation

$$\frac{2}{N_{part}}\frac{N_{ch}^{A+A}}{N_{ch}^{e^+e^-}} = \frac{C^{A+A}}{C^{e^+e^-}}\frac{V^{A+A}}{V^{e^+e^-}}\frac{s(T,\mu_B)}{s_0}.$$

V_{A+A} = volume
per participant pair

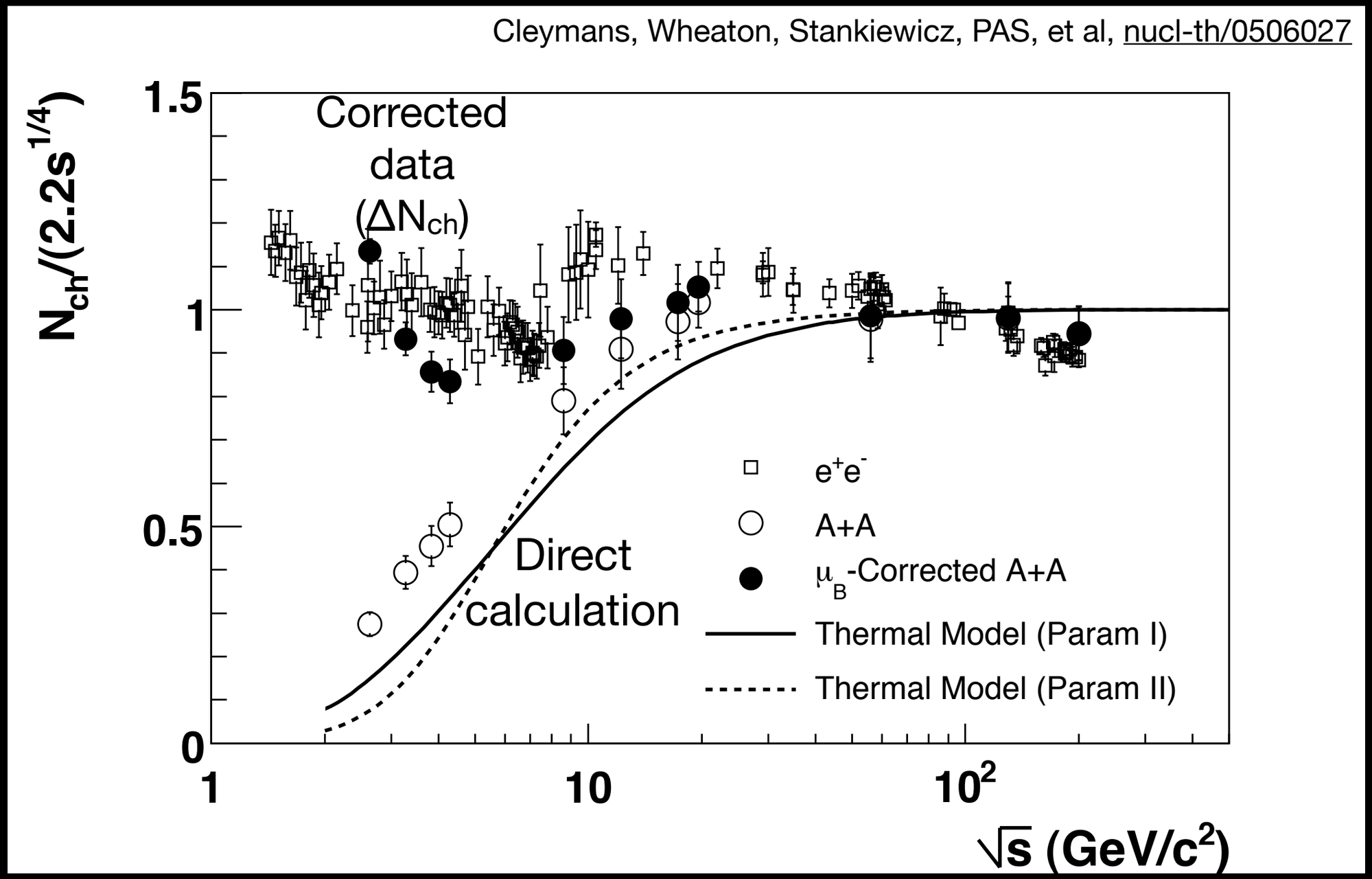
- **Convert √s to** $\mu_B(\sqrt{s})$ & $T(\mu_B(\sqrt{s}))$
- **Use full thermal model to compute**

$$\frac{s(T,\mu_B)}{s(T=T_c,\mu_B=0)} \quad \textbf{vs.} \quad \sqrt{s}$$

- **Predicts what data "should" show if**

$$C^{A+A} = C^{e^+e^-}$$
$$V^{A+A} = V^{e^+e^-}$$

Application to Data



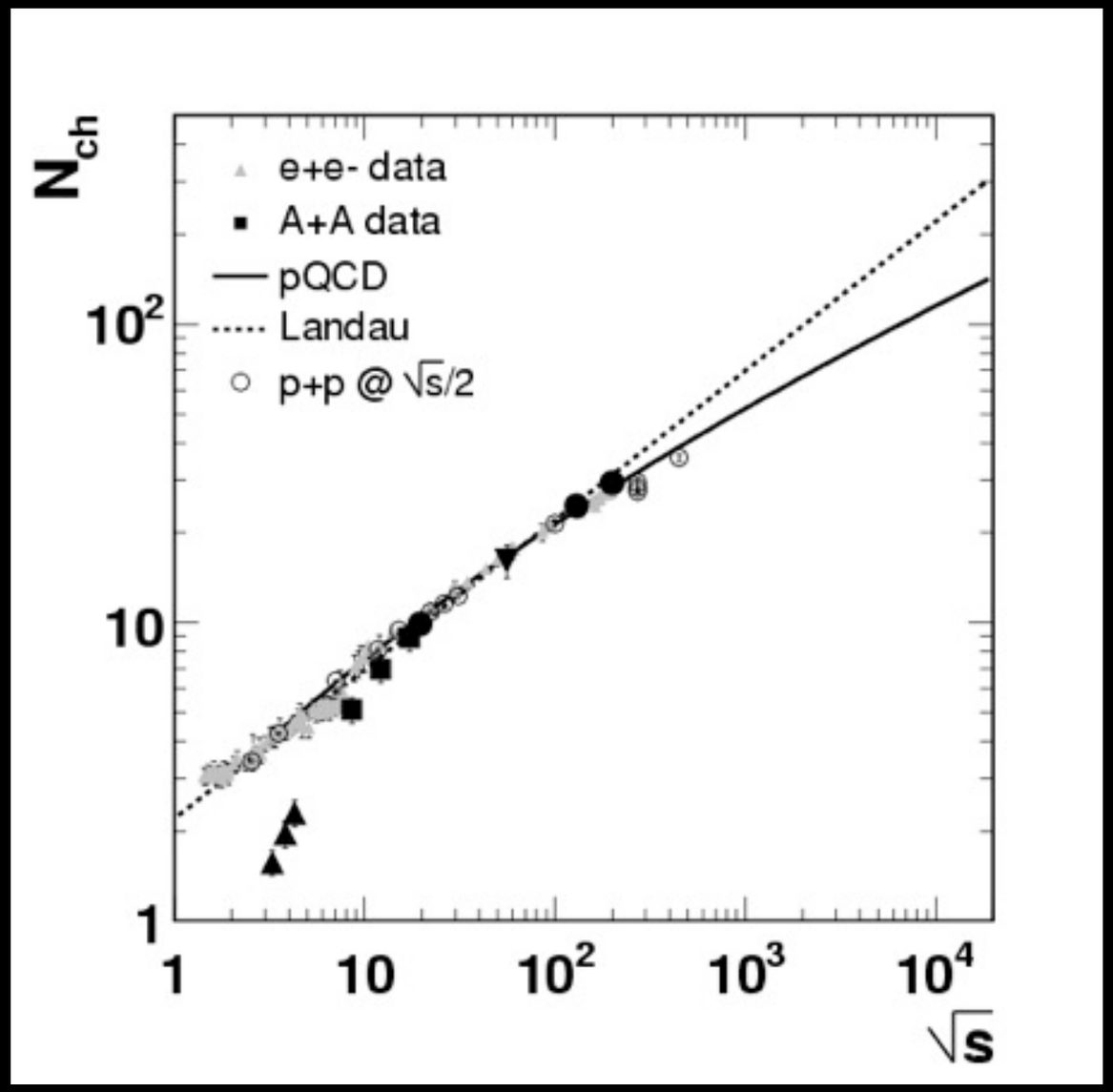
Qualitative model of relationship between baryon density and entropy

Bottom Line

3 rules of thumb:

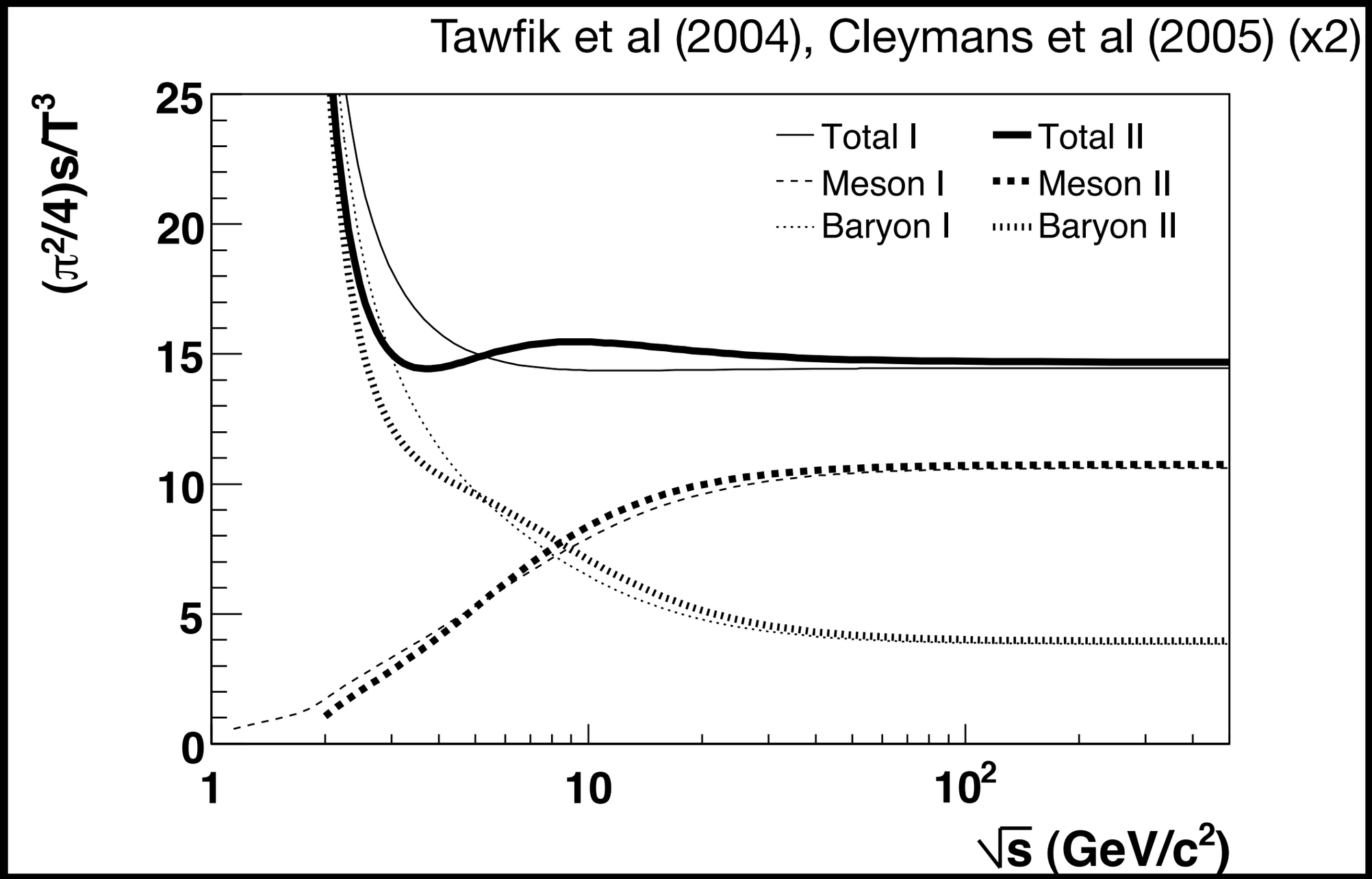
1. Effective energy has a direct relationship to entropy (p+p vs. e+e-)

2. Suppression of entropy by net-baryon density (A+A vs. e+e-)



3. Use N_{ch}: Don't "choose" mesons vs. baryons as true entropy carriers...

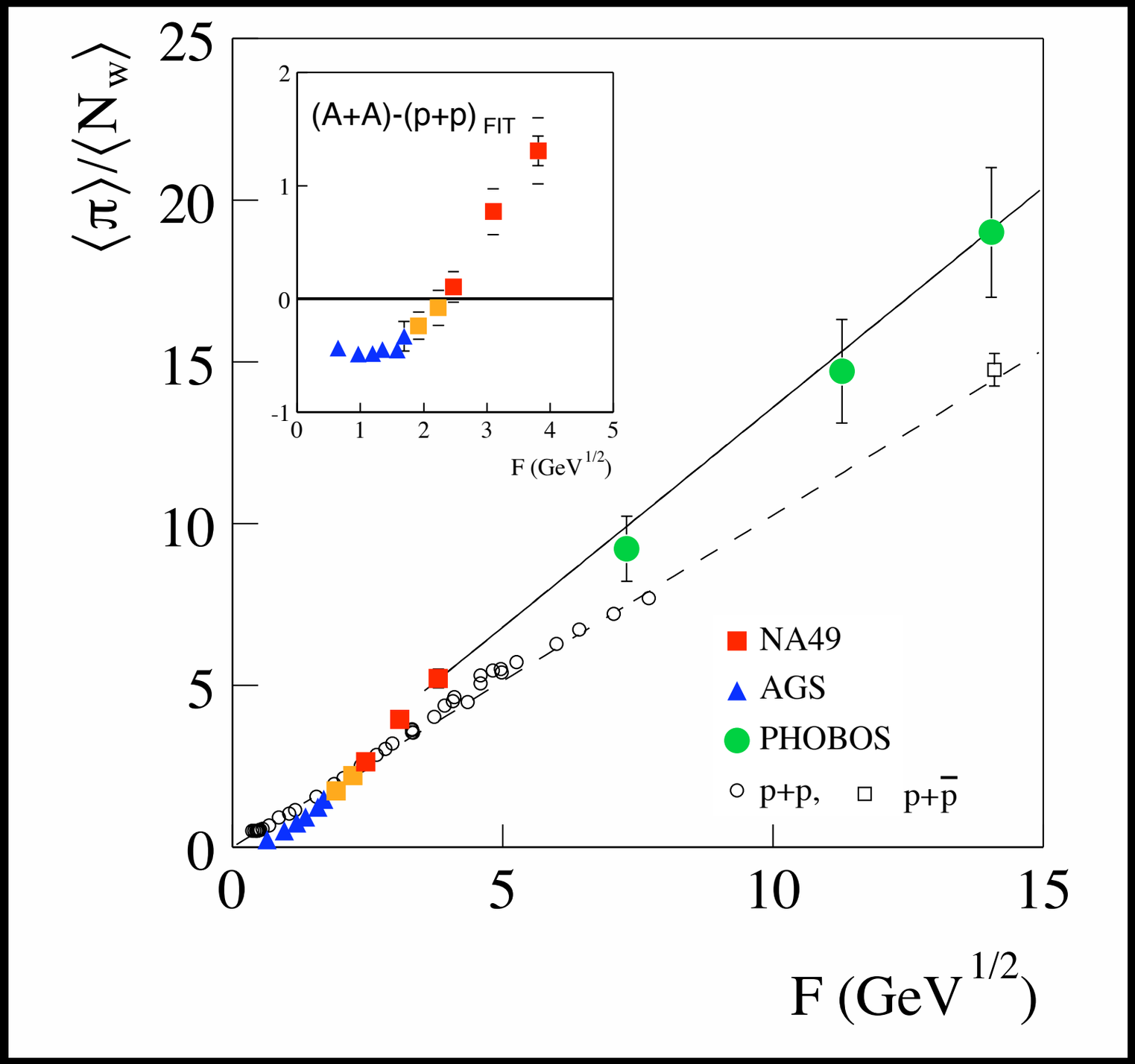
Baryons vs. Mesons



As energy changes, s/T^3 (~degrees of freedom) remain constant if mesons AND baryons are included

NA49 Kink

No baryons



~Landau variable...

no accounting for leading effect