The Lagrangian of QCD theory

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{fer} + \mathcal{L}_{gf} + \mathcal{L}_{FP}$$

$$\mathcal{L}_{kin} = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$= \frac{-1}{2}\left(\partial_\mu A_\nu^a \partial_\mu A_\nu^a - \partial_\mu A_\nu^a \partial_\nu A_\mu^a\right) - \frac{1}{2} g f_{abc}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right) A_\mu^b A_\nu^c$$

$$- \frac{1}{4} g^2 f_{abc} f_{ade} A_\mu^b A_\nu^c A_\mu^d A_\nu^e$$

$$\mathcal{L}_{fer} = \bar{\psi}(\not{D} - m)\psi$$

$$= \bar{\psi}(\not{\partial} - m)\psi - ig\frac{\lambda^a}{2} A_\mu^a \bar{\psi}\gamma^\mu\psi$$

where

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A_\mu^a$$

$$F_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu],$$

$$\left(\begin{array}{c} Tr(\lambda^a \lambda^b) = 2\delta_{ab} \\ \left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i f_{abc}\frac{\lambda^c}{2} \end{array} \right)$$

(1) In R_ξ gauge,

$$\mathcal{L}_{gf} = \frac{-1}{2\alpha}\left(-\partial_\mu A^{a\mu}\right)^2$$

$$\mathcal{L}_{FP} = -(\partial_\mu \bar{\chi}^a)(\partial^\mu \chi^a) + g f_{abc}(\partial_\mu \bar{\chi}^a)\chi^b A^{c\mu}$$

(2) In axial gauge,

$$\mathcal{L}_{gf} = \frac{-1}{2\lambda}\left(n_\mu A^{a\mu}\right)^2$$

$\Rightarrow$ There is no ghost propagator.

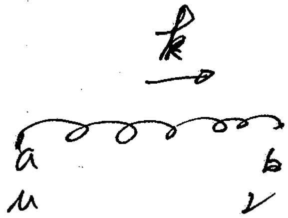
Feynman rules

(in Bjorken-Drell notation $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$)

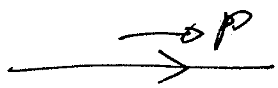
(1) In R_ξ gauge:

$$\mathrm{Tr}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right) = \frac{1}{2}\delta_{ab}$$

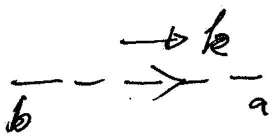
$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right]_- = i f_{abc}\left(\frac{\lambda^c}{2}\right)$$



$$(i)\frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{(1-\alpha)k_\mu k_\nu}{k^2}\right]$$

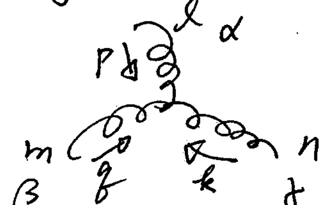


$$(i)\frac{1}{\not p - m} = (i)\frac{\not p + m}{p^2 - m^2}$$

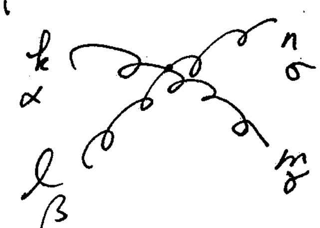


$$(i)\frac{1}{k^2}\delta_{ab}$$

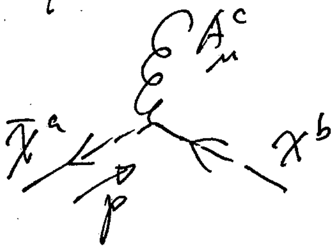
$$(i)\, g\, \gamma_\mu \left(\frac{\lambda^a}{2}\right)^i{}_j$$



$$g f^{lmn}\left\{(p-q)_\gamma g_{\alpha\beta} + (q-k)_\alpha g_{\beta\gamma} + (k-p)_\beta g_{\alpha\gamma}\right\}$$



$$-(i) g^2 \left\{ \begin{array}{l} f^{akl} f^{amn}(g_{\alpha\gamma} g_{\beta\sigma} - g_{\alpha\sigma} g_{\beta\gamma}) \\ + f^{akm} f^{aln}(g_{\alpha\beta} g_{\sigma\gamma} - g_{\beta\gamma} g_{\alpha\sigma}) \\ + f^{akn} f^{alm}(g_{\alpha\beta} g_{\sigma\gamma} - g_{\alpha\sigma} g_{\beta\gamma}) \end{array} \right\}$$



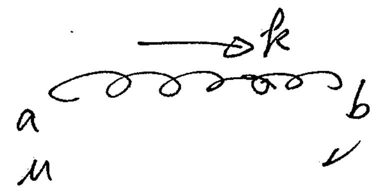
$$-g f^{abc} p_\mu$$

Note: $\alpha = 0 \Rightarrow$ Landau gauge
$\alpha = 1 \Rightarrow$ 't Hooft-Feynman gauge

(2) In axial gauge.

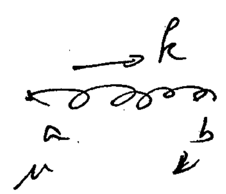
There is no ghost propagator.

The propagator of the gluon field (A_μ^a) is



$$(i)\frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu}+\frac{n_\mu k_\nu+k_\mu n_\nu}{(n\cdot k)}-\frac{(\lambda k^2+n^2)k_\mu k_\nu}{(n\cdot k)^2}\right]$$

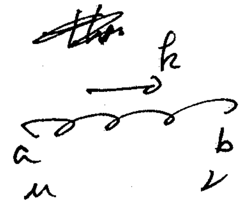
① In the case that $\lambda=0$, then



$$(i)\frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu}+\frac{n_\mu k_\nu+k_\mu n_\nu}{(n\cdot k)}-\frac{n^2 k_\mu k_\nu}{(n\cdot k)^2}\right]$$

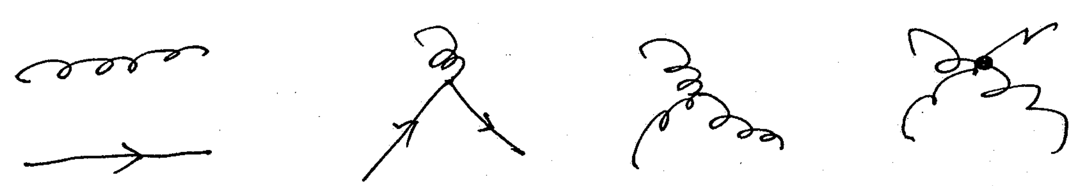
② When $\lambda=0$ <u>and</u> $n^2=0$ (light-cone gauge),

~~the~~

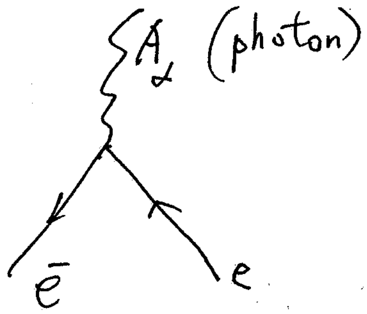


$$(i)\frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu}+\frac{n_\mu k_\nu+k_\mu n_\nu}{(n\cdot k)}\right].$$

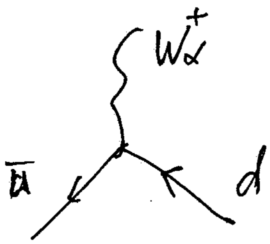
<u>Note</u>: the complete set of Feynman rules in the axial gauge consists of



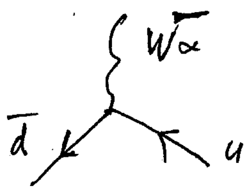
Some relevant Feynman rules
in the Electroweak interactions.



$-(i)|e|\,\gamma_\alpha$



$(i)\,\frac{g}{\sqrt{2}}\,\gamma_\alpha\,\frac{1}{2}(1-\gamma_5)$



$(i)\,\frac{g}{\sqrt{2}}\,\frac{1}{2}(1-\gamma_5)$

Some useful formula

$$\Gamma(z) = (z-1)\,\Gamma(z-1) \qquad \Gamma(n) = (n-1)!$$

$$\Gamma(1) = 1$$

$$\Gamma(2) = 1$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$\gamma \equiv \gamma_E = 0.5772\ldots$ = Euler's constant

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + \frac{1}{2}\varepsilon\left[\frac{\pi^2}{6} + \gamma^2\right] + O(\varepsilon^2)$$

$$\Gamma(1-\varepsilon) = (1-\varepsilon-1)\,\Gamma(1-\varepsilon-1) = -\varepsilon\,\Gamma(-\varepsilon)$$

$$= 1 + \varepsilon\gamma + \frac{1}{2}\varepsilon^2\left(\frac{\pi^2}{6} + \gamma^2\right) + O(\varepsilon^3)$$

$$\frac{1}{a^\alpha b^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_0^1 dx\, \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[ax + b(1-x)]^{\alpha+\beta}}$$

$$\int \frac{d^n l\, l_\mu}{(l^2+m^2)^\alpha} = 0$$

$$\int \frac{d^n l\, l_\mu l_\nu}{(l^2+m^2)^\alpha} = \int \frac{d^n l \left(\frac{l^2 g_{\mu\nu}}{n}\right)}{(l^2+m^2)^\alpha}$$