QCD and Its Success

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Slides prepared by
Wu-Ki Tung @ MSU
Passed away in 2009

useful references

- The CTEQ Pedagogical Page

http://www.physics.smu.edu/~olness/cteqpp/

- CTEQ Summer School

http://www.phys.psu.edu/~cteq/

Perturbative QCD:

from basic principles to current applications

I : Basic Ideas – Qualitative Descriptions

- What is Quantum Chromodynamics (QCD)?
- Why do we believe in Quarks and Gluons?
  Long-distance phys. – the Quark Model
  Short-distance phys. – the Parton Picture
- How can a Strong-interaction theory, QCD, give rise to the simple Parton Picture?
  The importance of *scaling* and *factorization*:
  Renormalization and Assymptotic Freedom;
  Infra-red Safety and *Factorization Thms.*
- Precision Tests of PQCD in e^+e^- Interactions

II : Quantitative case study – Deep Inelastic Scattering at NLO

- LO and NLO calculations
- The origin of *collinear singularities*
- The separation of long- and short- distances
- How does *Factorization* form the foundation of the QCD-Parton-Model?

III : General Formalism of PQCD

- From NLO to Higher Orders
- Universal Parton Dist. & QCD evolution
- How are Hard X-sect's actually calculated?
- Scale- & Scheme-dependencies
- Renormalization vs. Factorization – a head-to-head comparison and correspondence

IV : Survey of Hard Scattering Processes

- Q-Qbar annihilation and Drell-Yan processes: lepton-pair, W- and Z- production
- g-Q scattering and Dir. Photon Prod.
- g-g scattering and Jet Production
- Two-scale hard-scattering processes
- A 2-scale process: Heavy Quark Production

V : Global QCD Analysis

- Overview of Physical Processes and Expts.
- Overview of NLO QCD Theory Input
- Global QCD Analysis
- Parton Distribution Functions

Open Questions and Challenges

Basic Elements of Quantum Chromodynamics

–Non-abelian Gauge Field Theory with SU(3) color Gauge Symmetry

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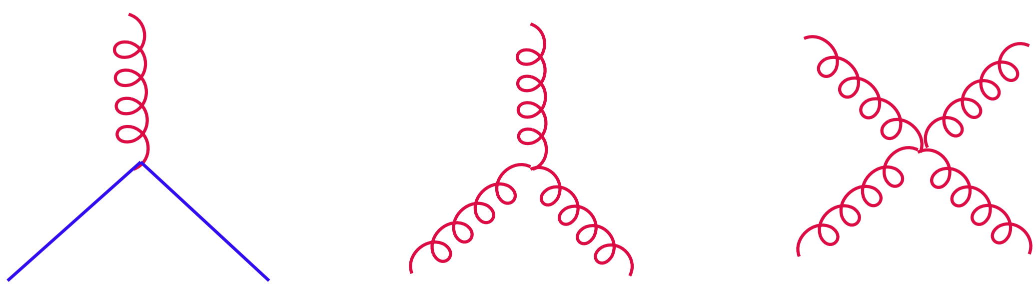
Fields: Quarks ψ^{color}_{flavor} and Gluon $G^{color}(A{\cdot}T, g)$.

Basic Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\not\partial - g\not A \cdot t - m)\psi - \frac{1}{4}G(A \cdot T, g) \cdot G(A \cdot T, g)$$

- g: gauge Coupling Strength
- m_i: quark masses
- t & T: color SU(3) matrices in the fundamental and adjoint representations.
  Group factors: $C_F = \frac{4}{3}$; $T_F = \frac{1}{2}$; $C_A = 3$

Interaction Vertices:



Why do we believe Hadrons are made of Quarks?

– Strong Interaction at *long-distance* scale –

Hadron Spectroscope $\Rightarrow$ the *Quark Model*

Quantum #'s of Mesons given by: L=0,1,2

SU(3)$_{flv}$ Octets (nonets) of q-qbar bound states.

Addition of Charm Q.N. $\Rightarrow$ *SU(4)*

– see plots (PDG)

Combining of *SU(3)*$_{flavor}$ & *SU(2)*$_{spin}$ $\Rightarrow$ *SU(6)*

Quantum #'s of Baryons given by: L=0,1,2

SU(3)$_{flv}$ Octets & decuplets of q-q-q bound states.

– see plots (PDG)

These are the *Constituent Quarks*

However:

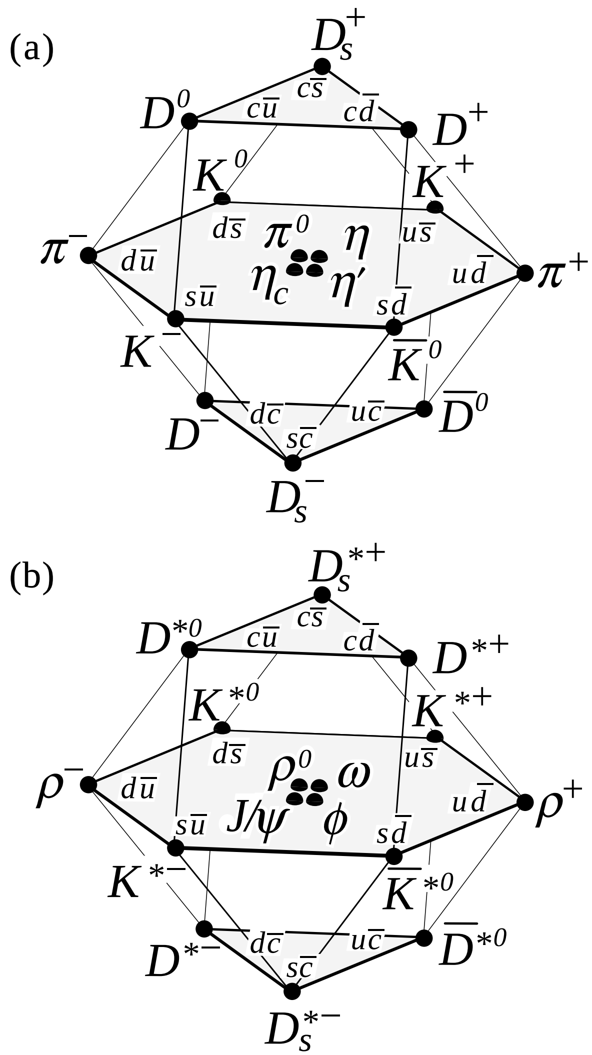
Quarks have not been found in nature.

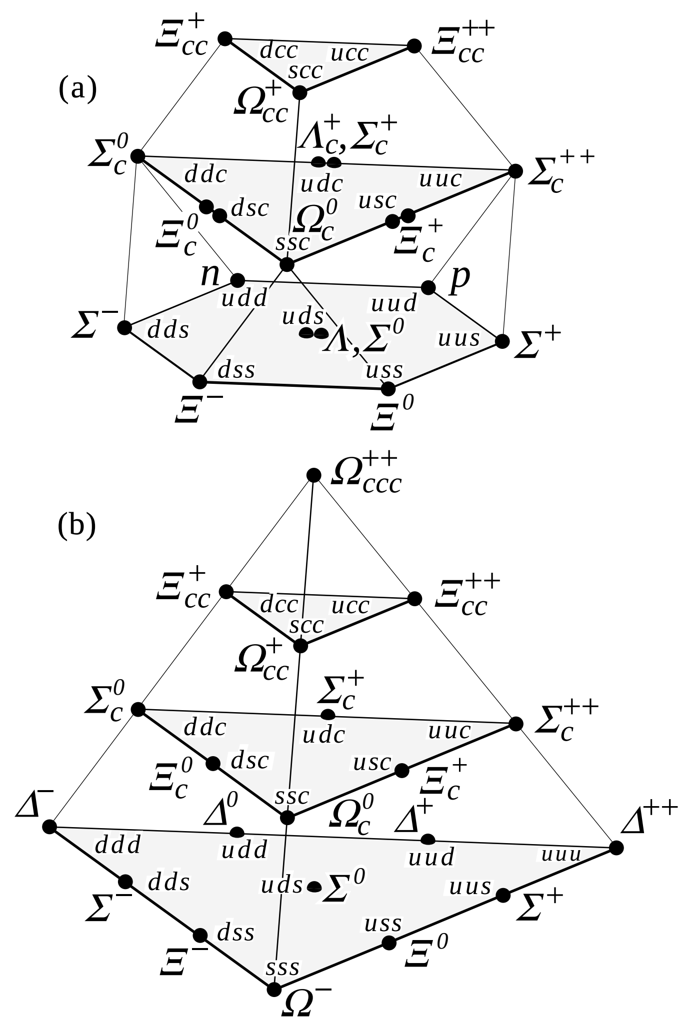
Interaction is very (infinitely?) strong at long-distances.

QCD at Low Energy (long-distance) Scales
-- Confinement, Bound-states
=> the Quark Model

Mesons

Baryons





Why is Hadron Interactions at High Energies described by the QCD Parton Picture?

– Strong Interaction at *short -distance* scale –
Hard Scattering Probes $\Rightarrow$ the *Parton Model*

Evidences for the existence of Partons:

"direct": Most Hard Sc. events contain visible "jets" $\Rightarrow$ fragments of underlying partons?

Are they point-like? "Rutherford expts"

- (Bjorken) Scaling in DIS;
- annihilation into hadrons;
- Hadron-hadron scattering, ...

Properties of Partons:

2-Jet angular distributions in e^+e^-, DIS, DY proc. are the same as for scattering into leptons $\Rightarrow$ underlying partons are fermionic

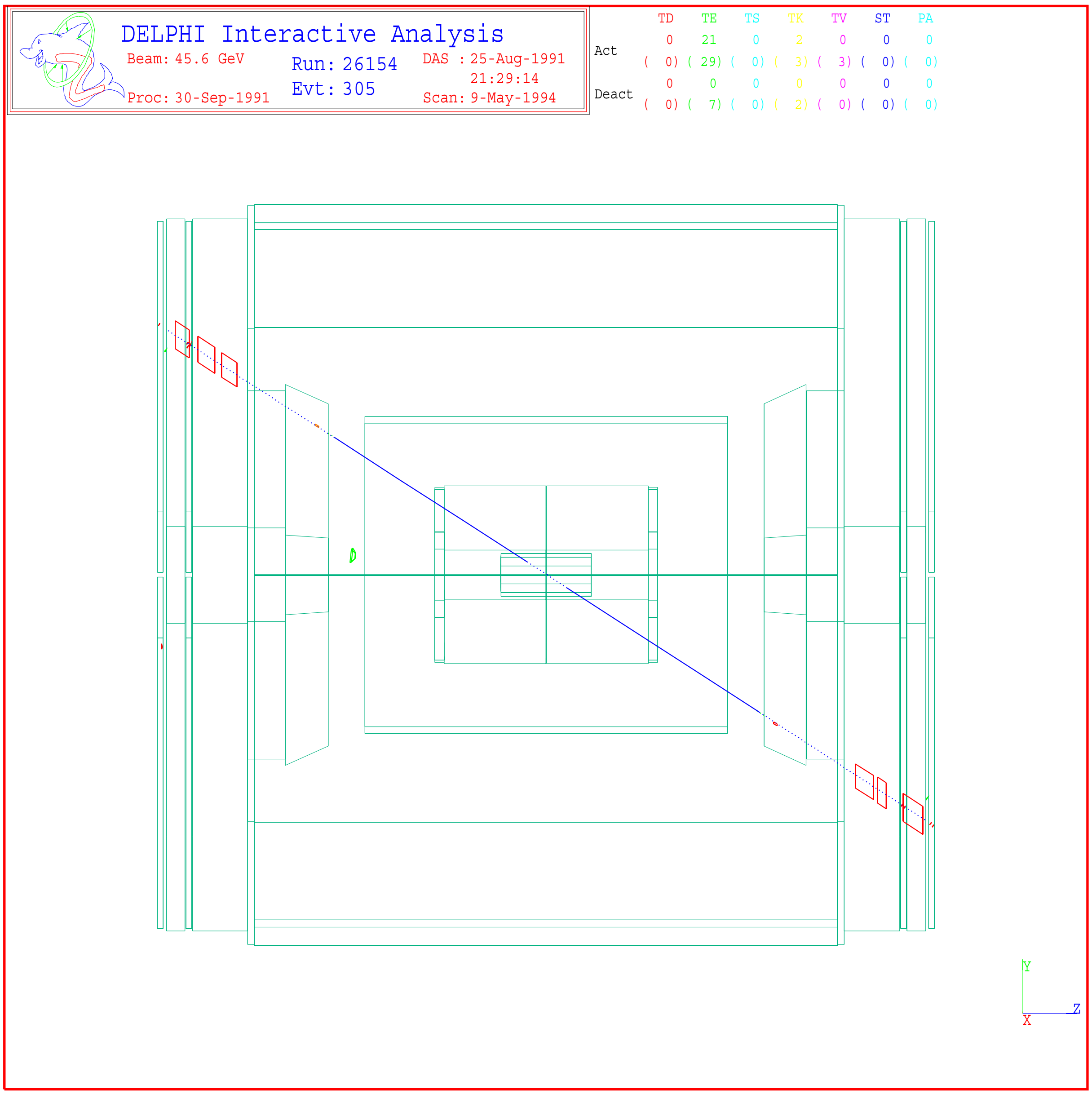
Expts. : EM & Weak Isospin couplings of partons = those of leptons $\Rightarrow$ "Current Quarks"

3-jet events and their detailed properties prove the existence, and spin of gluons

$\Rightarrow$ QCD-parton Model complete.

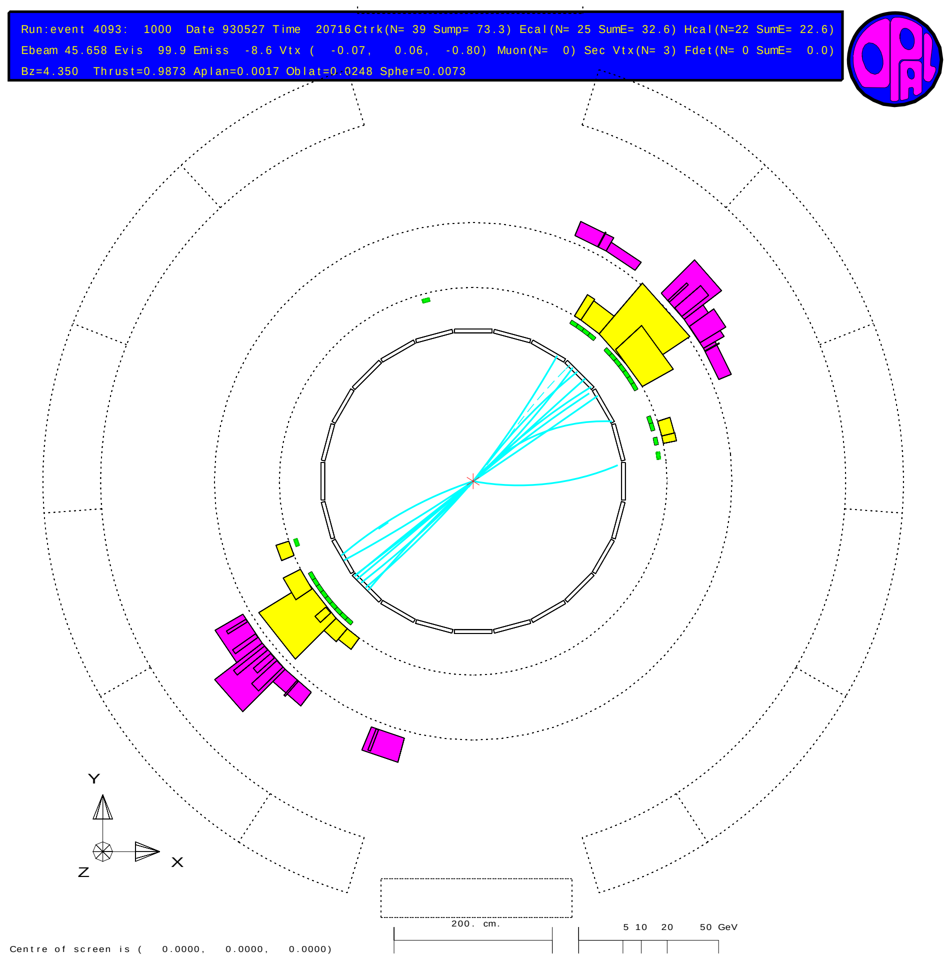
Big Question .....

An elementary particle event
$- e^+ e^- \longrightarrow \mu^+ \mu^-$

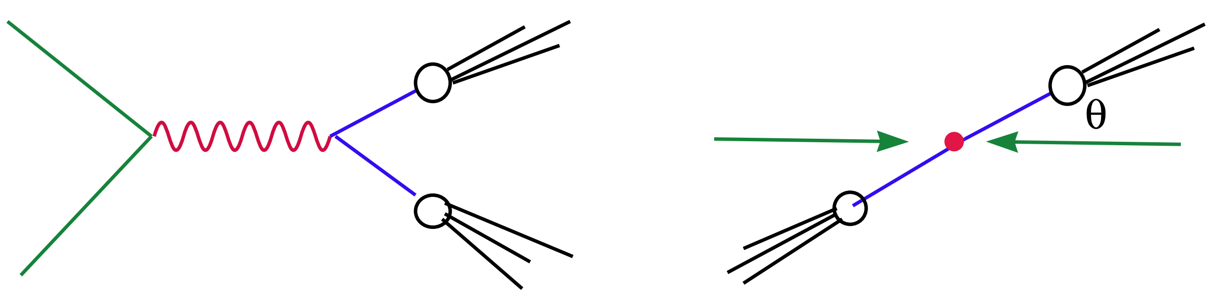


Two-jet Events:
Quark – anti-quark Pair Production

Typical $e^+ e^-$ event with hadron final states:

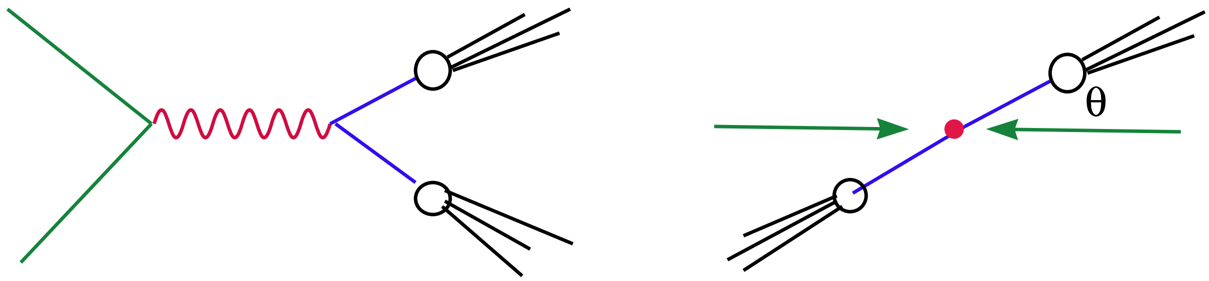


Parton process underlying 2-jet events

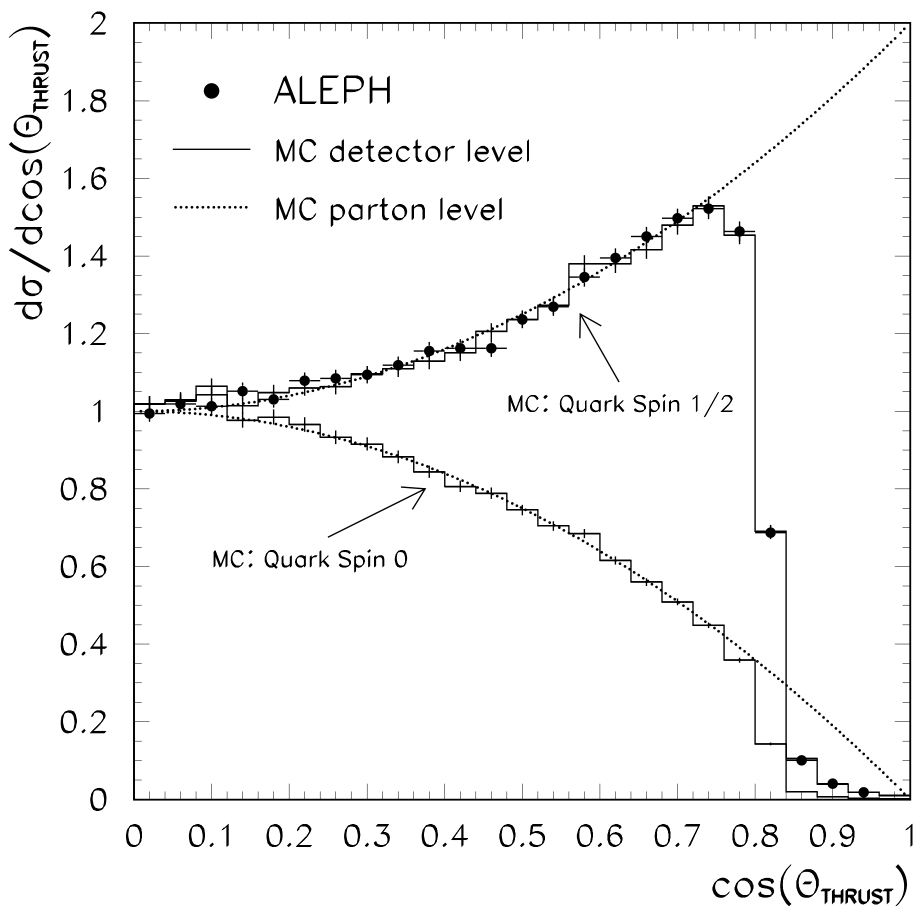


Measuring the Spin of the Quark Parton

Use the angular distribution of 2-jet events

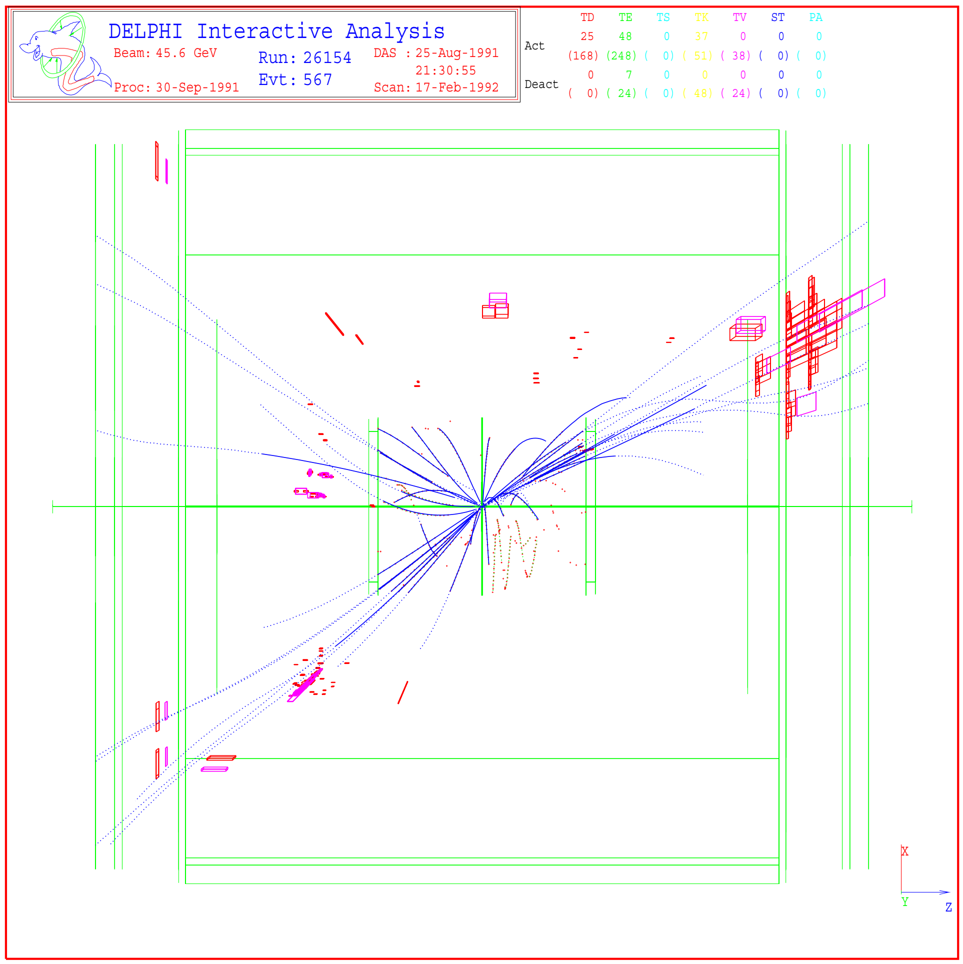


Experimental result (ALEPH):

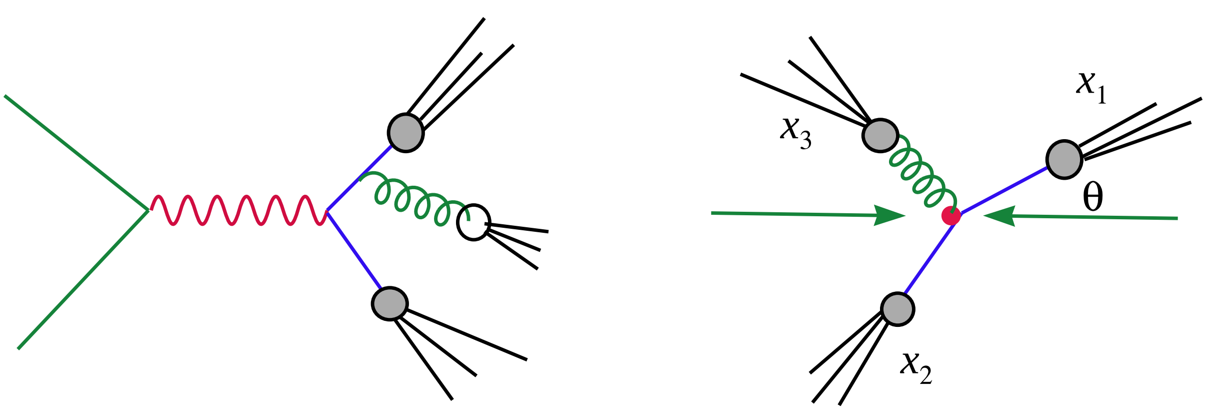


3 Jet Events and the Gluon Parton

A typical 3-jet event ($\sim$ 10% prob.):

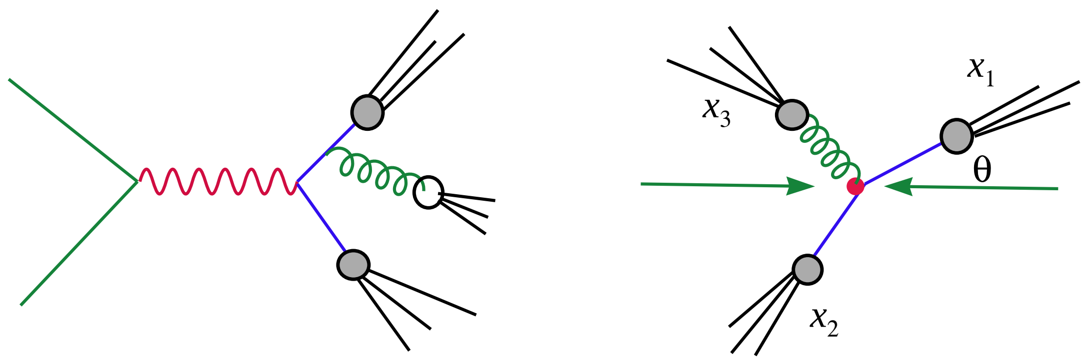


Parton process underlying 3-jet events

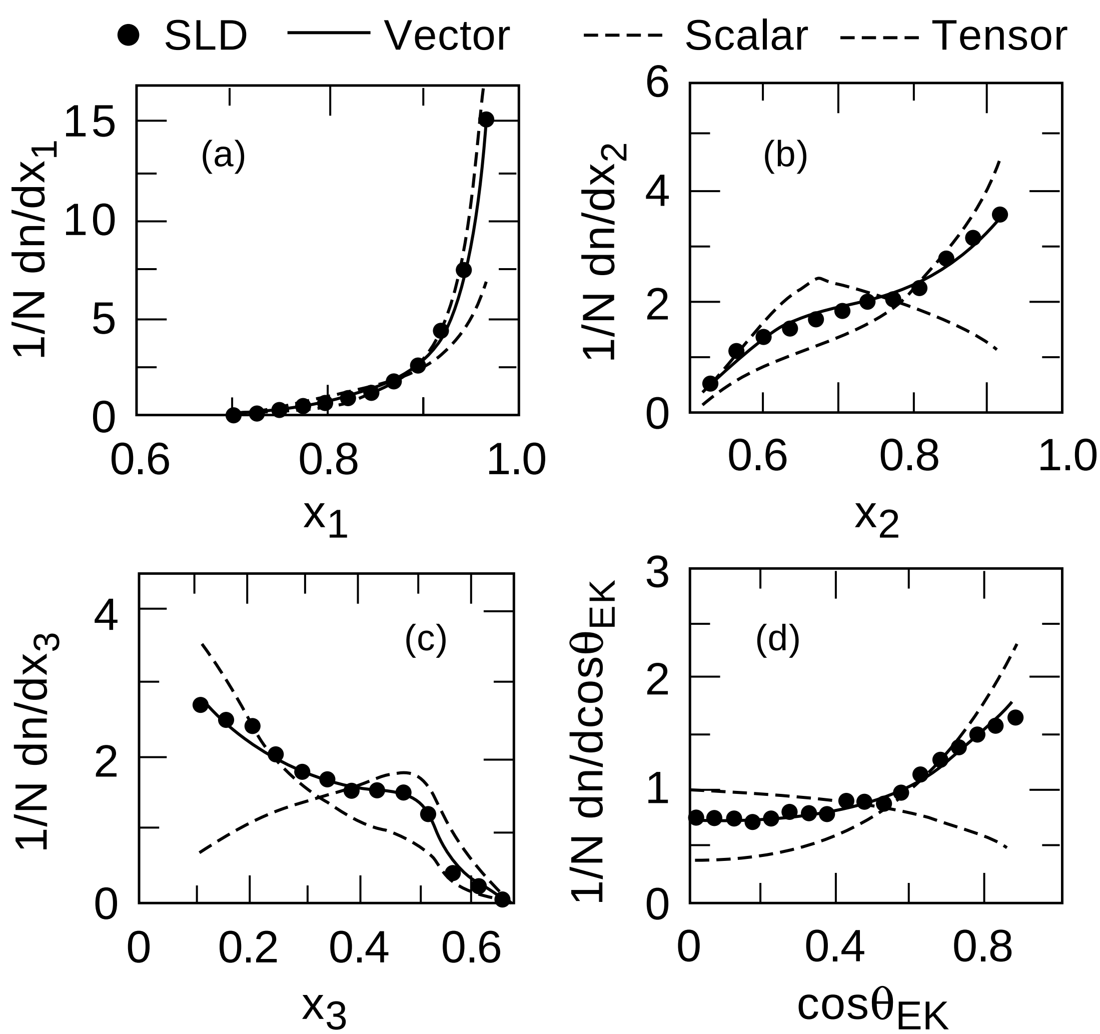


Measuring the Spin of the Gluon

Use the angular distributions of 3-jet events



Experimental result from SLD:



Question:

How could the *simple parton picture* possibly hold in a strongly interacting gauge field theory such as Quantum Chromodynamics?

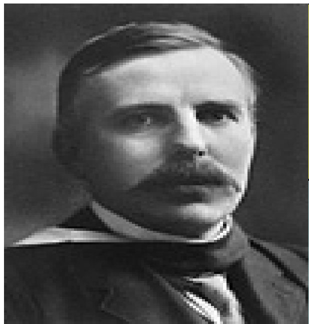
Answers:

1. Asymptotic Freedom:
   A strongly interacting theory at long-distance can become weakly interacting at short-distance.
2. Infra-red Safety:
   There are classes of "infra-red safe" quantities which are independent of long-distance physics, hence are calculable in PQCD.
3. Factorization:
   There are an even wider class of physical quantities which can be *factorized* into long-distance pieces (not calculable, but *universal*) and short-distance pieces (process-dependent, but infra-red safe, hence *calculable*).

"Bottom Line"

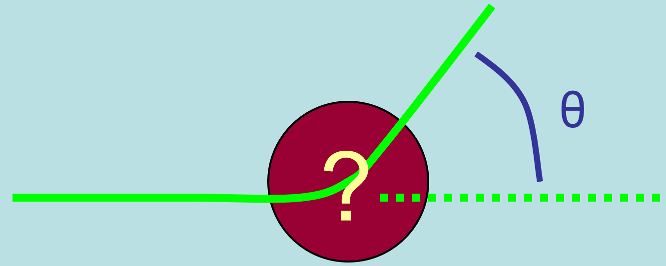
PQCD does not give all the answers; but it does cover quite a lot of ground!

QCD and Its Success



Rutherford Scattering

Rutherford taught us the most important lesson:
use a scattering process to learn about the structure of matter



This story is well known:

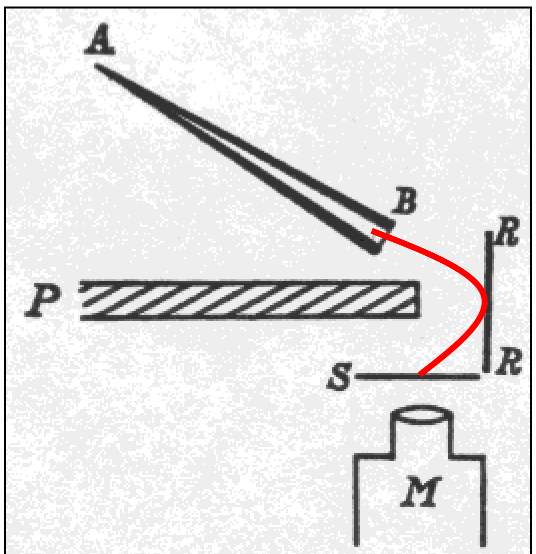
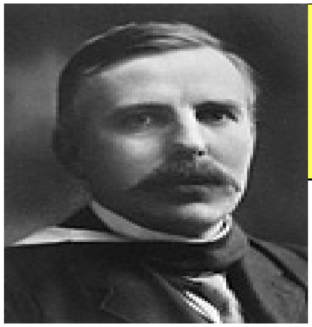


Fig. 1.

H. Geiger and E. Marsden observed that α-particles were sometimes scattered through very large angles.

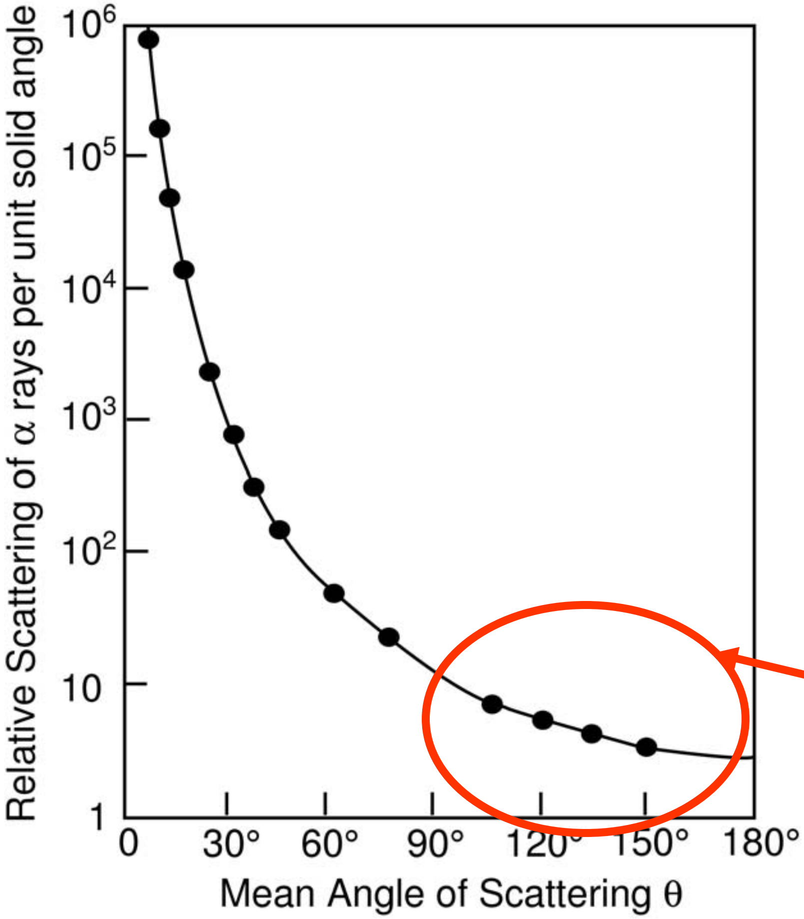
Rutherford interpreted these results as due to the coulomb scattering of the α-particles with the atomic nucleus:

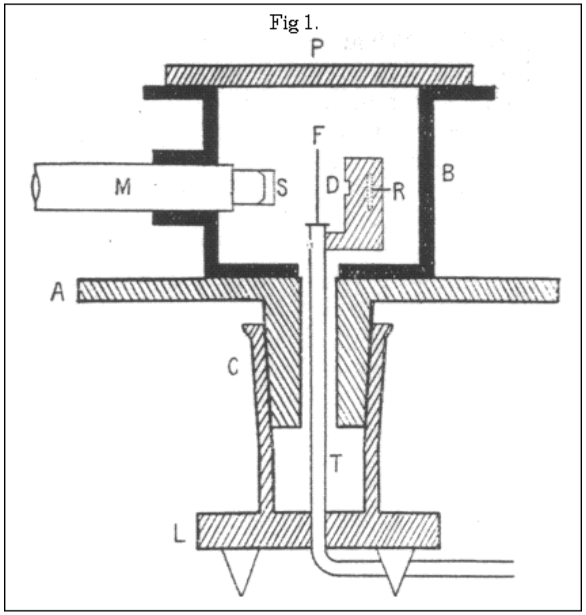
$$\sigma(\theta) = \frac{z^2 Z^2 e^4}{16E^2} \frac{1}{\sin^4 \frac{1}{2}\theta}$$



Rutherford Scattering

In a subsequent paper Geiger/Marsden precisely verified Rutherford theory





Discovery of atomic nucleus

→ N. Bohr Old Quantum theory…

Developments…

-Quantum mechanics rapidly developed in the years 1924-27

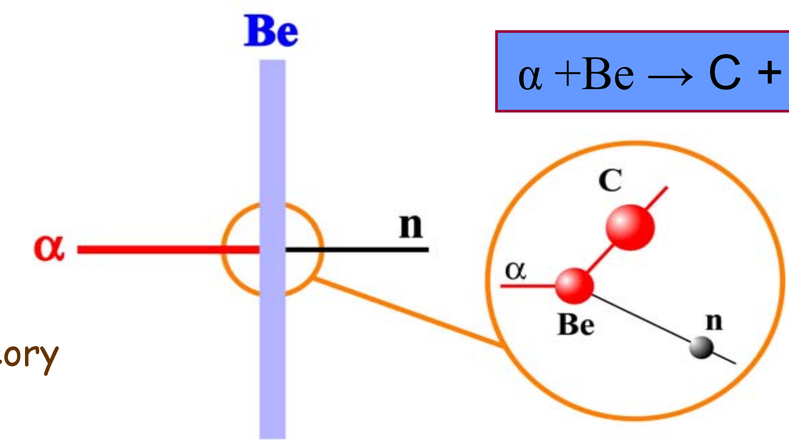
-The nucleus composition remained a mistery (e.g. N_7^{14}) till…

Discovery of neutron

(Chadwick 1932)

Instrumental to the Fermi's beta decay ($n \rightarrow pe\nu$) theory

$\alpha + Be \rightarrow C + n$



Main information concerning geometric detailes of nuclear structure (mirror nuclei, fast neutron capture, binding energies etc) could be summed up in:

$$R = r_0 \times A^{1/3} \text{ fm} \quad \text{with } r_0 = 1.45 \text{ fm}$$

$$\rho_m = 0.08 \text{ nucl/fm}^3 \quad \text{and} \quad \rho_c = (Z/A) \times 0.08 \text{ (prot. charges)/fm}^3$$

The nucleus form factor

Stimulated by accelerators technology advances and fully muture QED various theoreticians (Rose (48), Elton(50)) started to calculate cross sections for elastic electron-Nucleus scattering

$$\sigma_M(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}$$

Mott

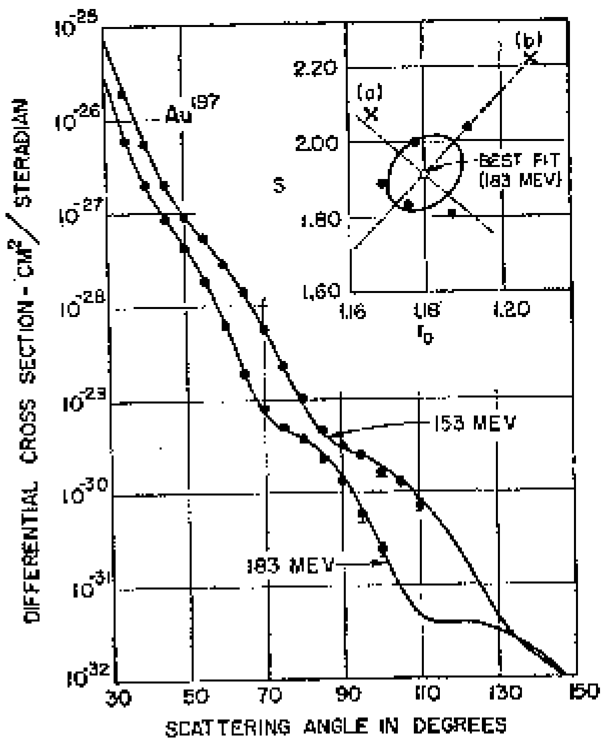
$$\sigma_s(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta} \left| \int_{\substack{\text{nuclear}\\ \text{volume}}} \rho(r) e^{i\mathbf{q}\cdot\mathbf{r}} d\tau \right|^2$$

$$\sigma_s(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta} \left[\int_0^\infty \rho(r) \frac{\sin qr}{qr} 4\pi r^2 dr \right]^2$$

$$F = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin(qr) r dr$$

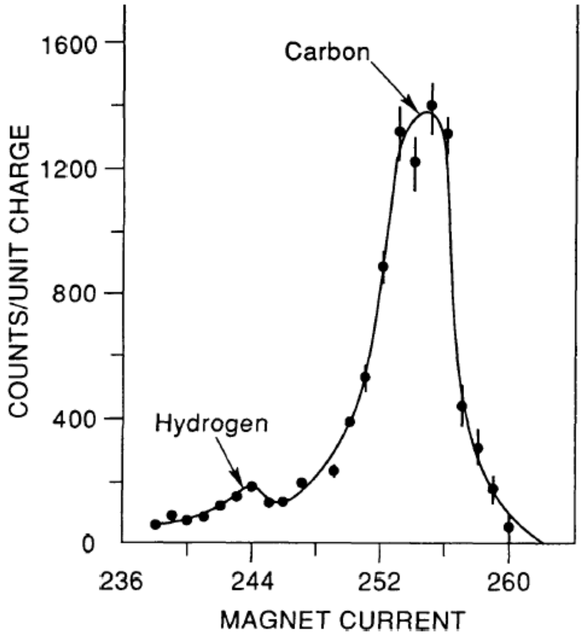
Nucleus form factor

Interference between the scattered wavelets arising from the different parts of the same,finite,nucleus

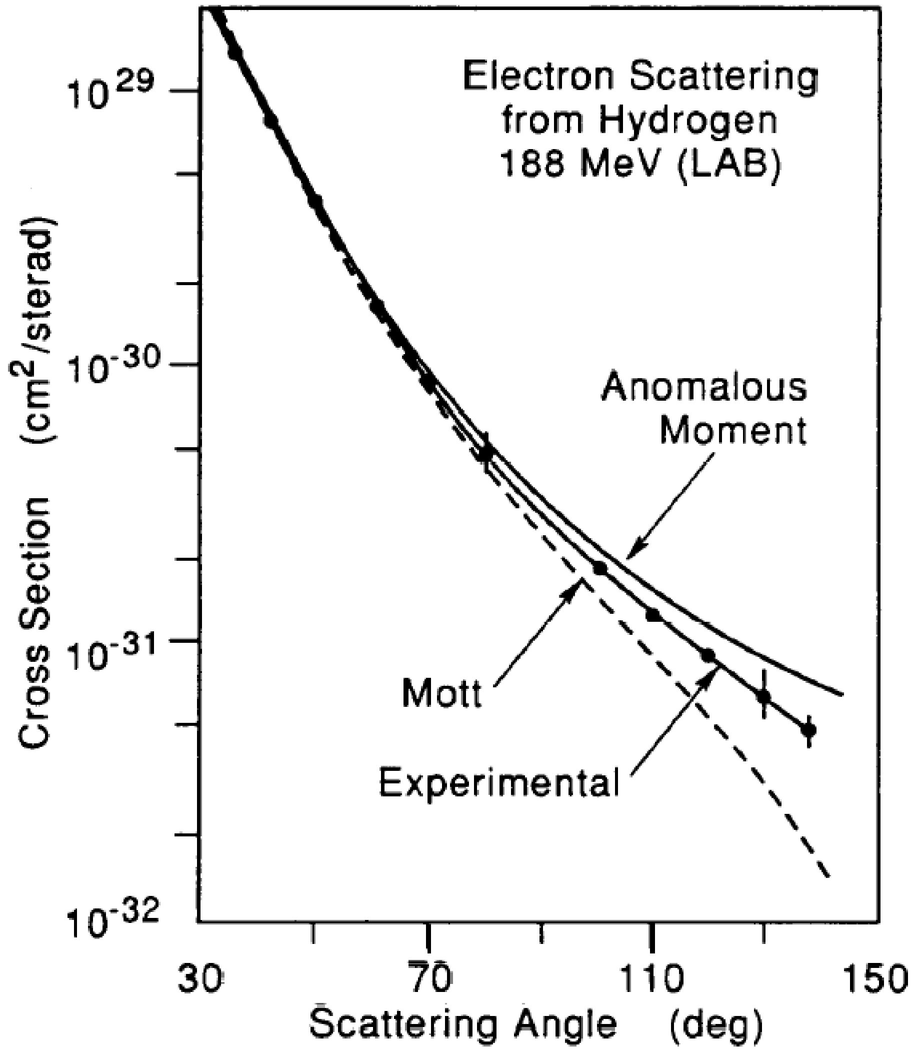


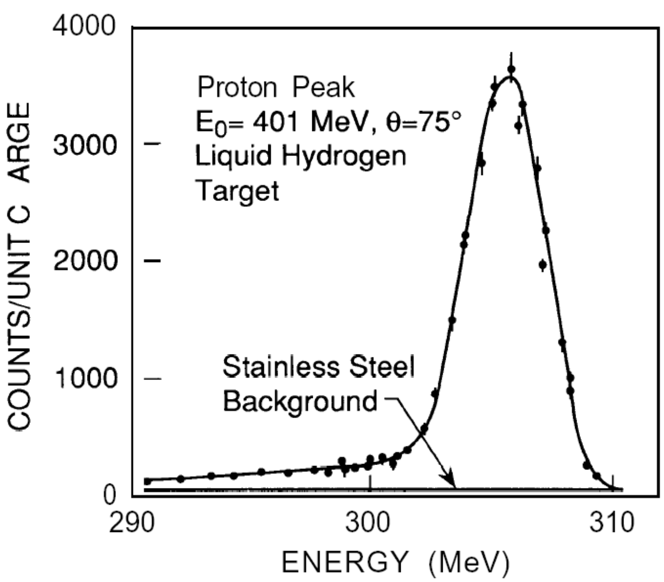
Phase-shift analysis

R. Hofstadter: e-p elastic scattering



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_0^2 \sin^4 \theta/2} \cdot \cos^2 \theta/2 \cdot \frac{E'}{E_0} \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2 \theta/2 \right]$$



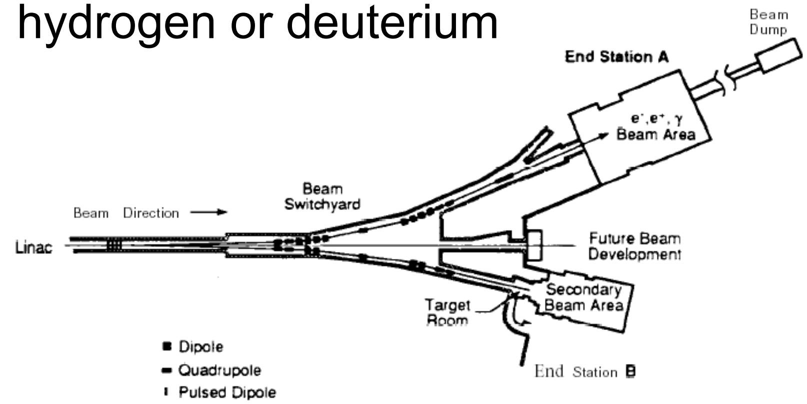


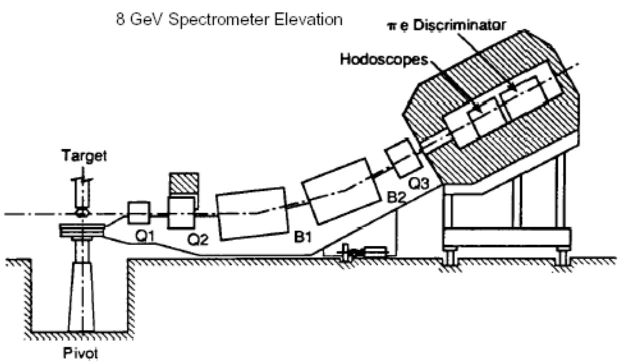
1990 Nobel Prize

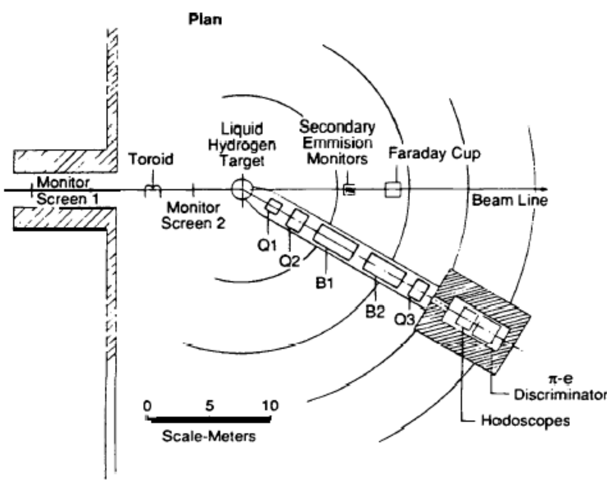
The SLAC-MIT Experiment

Under the leadership of Taylor, Friedman, Kendall
~ 1969

Electron scattered by
liquid hydrogen or deuterium



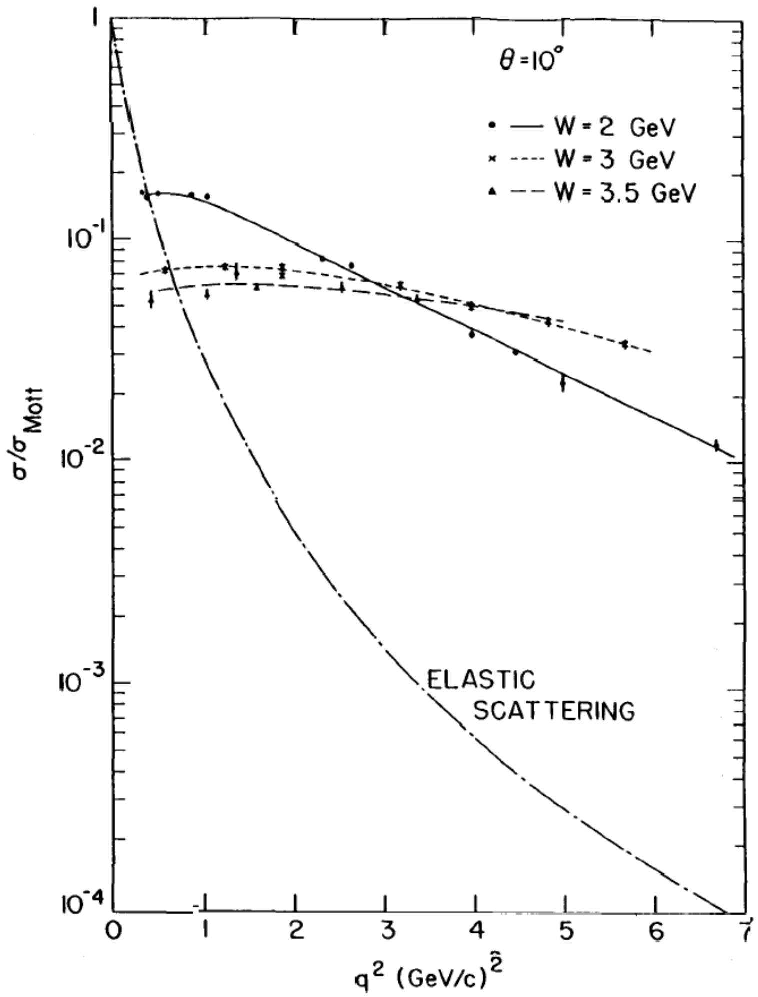


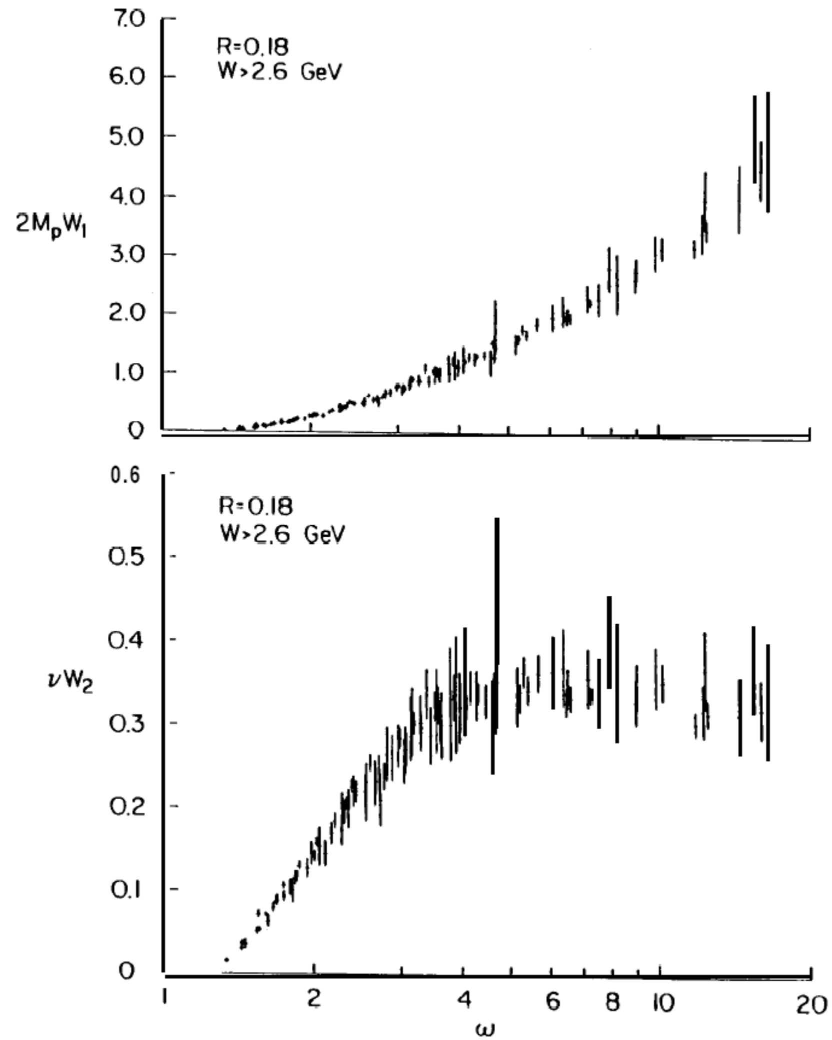


First SLAC-MIT results

Two unexpected results…

Deep-inelastic scattering (DIS)

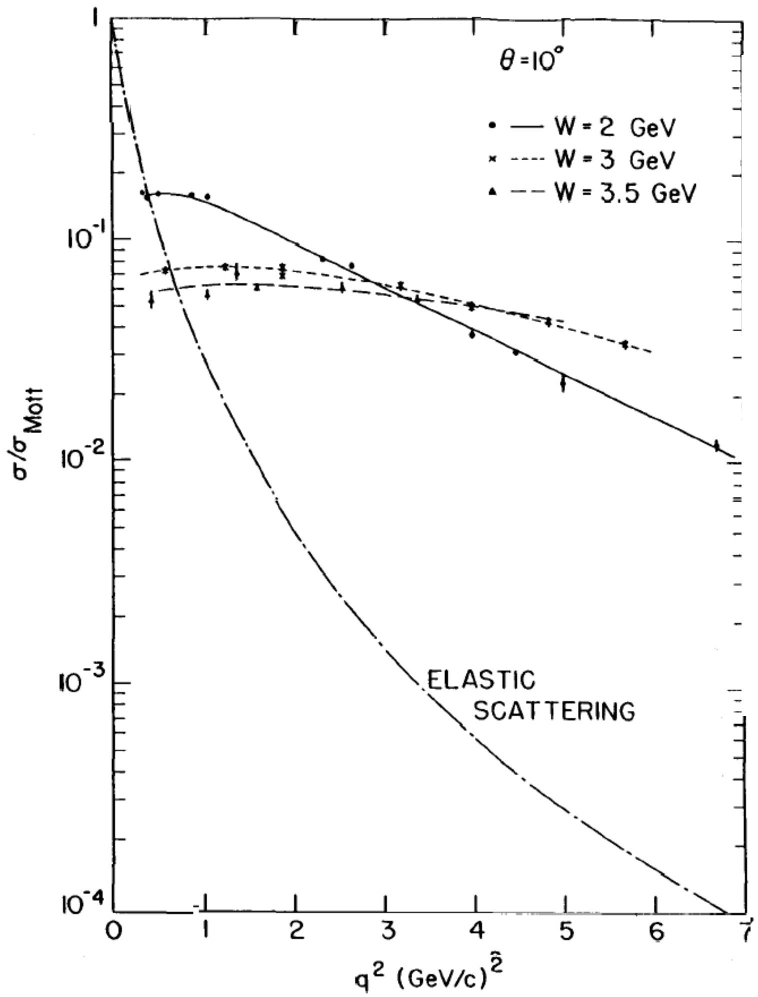




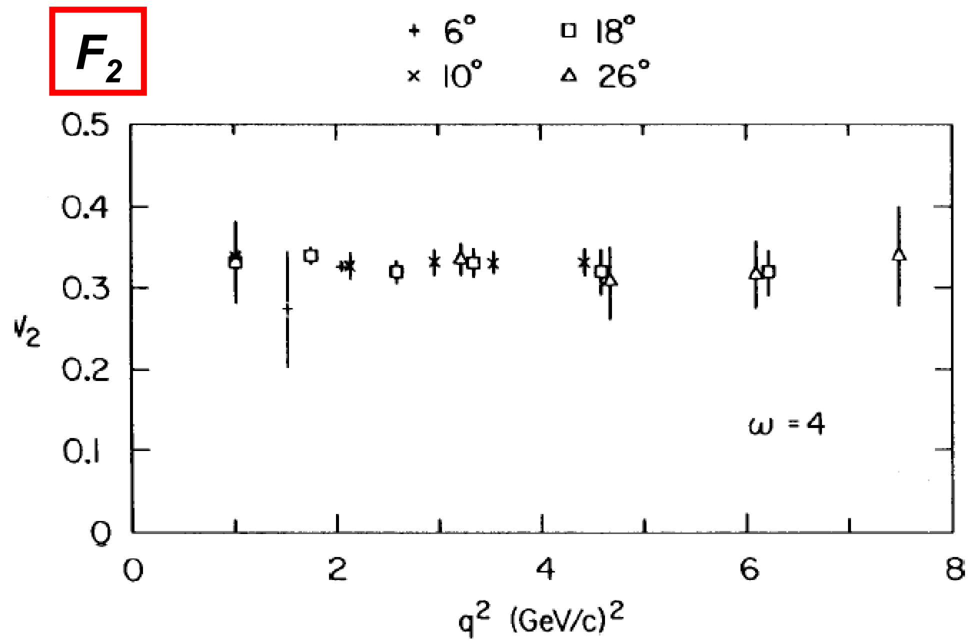
First SLAC-MIT results

Two unexpected results…

Deep-inelastic scattering (DIS)

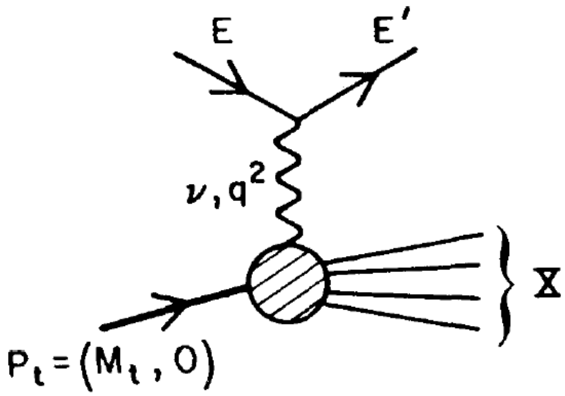


Scaling behavior



$\omega = 1/x$

Deep inelastic scattering (DIS) and structure Fs



Kinematic variables

$$\nu = \frac{P \cdot q}{\sqrt{P \cdot P}} = E_1 - E_2$$

$$x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}$$

$$Q^2 = 2E_1E_2(1-\cos\theta)$$

$$\frac{d^2\sigma}{d\Omega\, dE'}(E, E', \theta) = \sigma_M \left[W_2(\nu, q^2) + 2W_1(\nu, q^2)\tan^2(\theta/2)\right]$$

$$2MW_1(\nu, q^2) = F_1(\omega)$$
$$\nu W_2(\nu, q^2) = F_2(\omega)$$

Bjorken scaling (1969)
(Predicted prior to data)

$\omega = 1/x$

Quantum Chromodynamics

Fields: Quarks ψ^{color}_{flavor} and Gluon $G^{color}(A{\cdot}T, g)$.

Basic Lagrangian:

$$\mathcal{L} = \bar{\psi}(i \not\partial - g \not A \cdot t - m)\psi - \frac{1}{4} G(A \cdot T, g) \cdot G(A \cdot T, g)$$

- g: gauge Coupling Strength
- m_i: quark masses
- t & T: color SU(3) matrices in the fundamental and adjoint representations.

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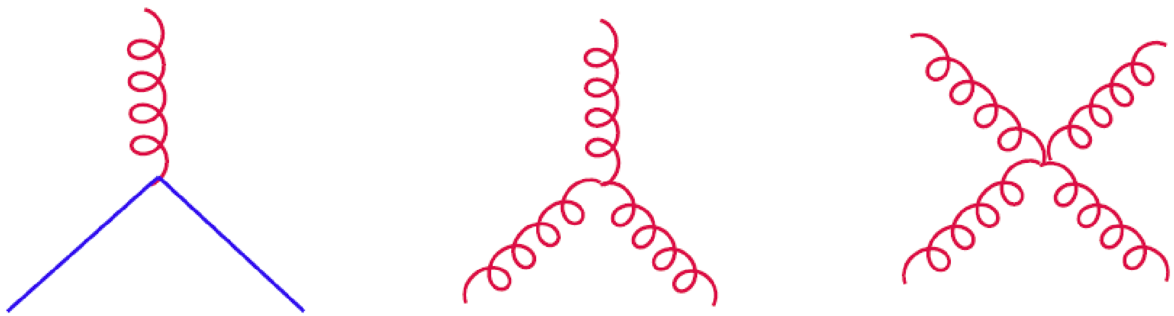
Interaction Vertices:



Why does QCD play such a crucial role in High Energy Phenomenology?

- The parton picture language provides the foundation on which all modern particle theories are formulated, and all experimental results are interpreted.
- The validity of the parton picture is based empirically on an overwhelming amount of experimental evidence collected in the last 30-40 years, and theoretically on the Factorization Theorems of PQCD.

How could the *simple* (almost non-interacting) *parton picture* possibly hold in QCD — a strongly interacting quantum gauge field theory?

Answer: 3 unique features of QCD:

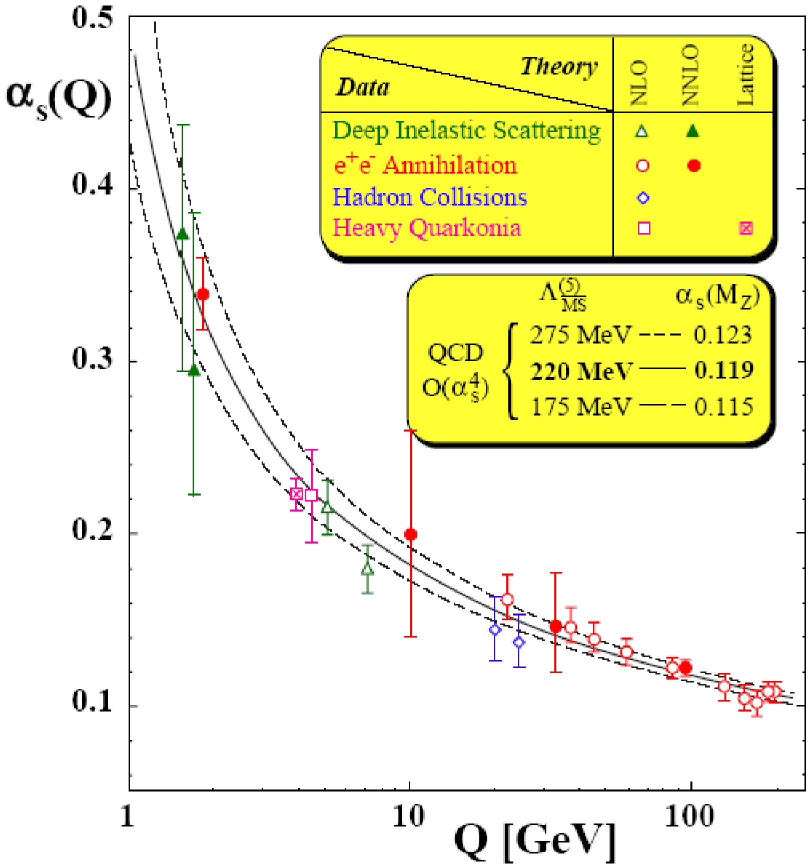
1. Asymptotic Freedom:
   A strongly interacting theory at long-distance can become weakly interacting at short-distance.
2. Infra-red Safety:
   There are classes of *infra-red safe* quantities which are independent of long-distance physics, hence are calculable in PQCD.
3. Factorization:
   There are an even wider class of physical quantities which can be *factorized* into a long-distance piece (not calculable, but *universal*) and short-distance piece (process-dependent, but infra-red safe, hence *calculable*).

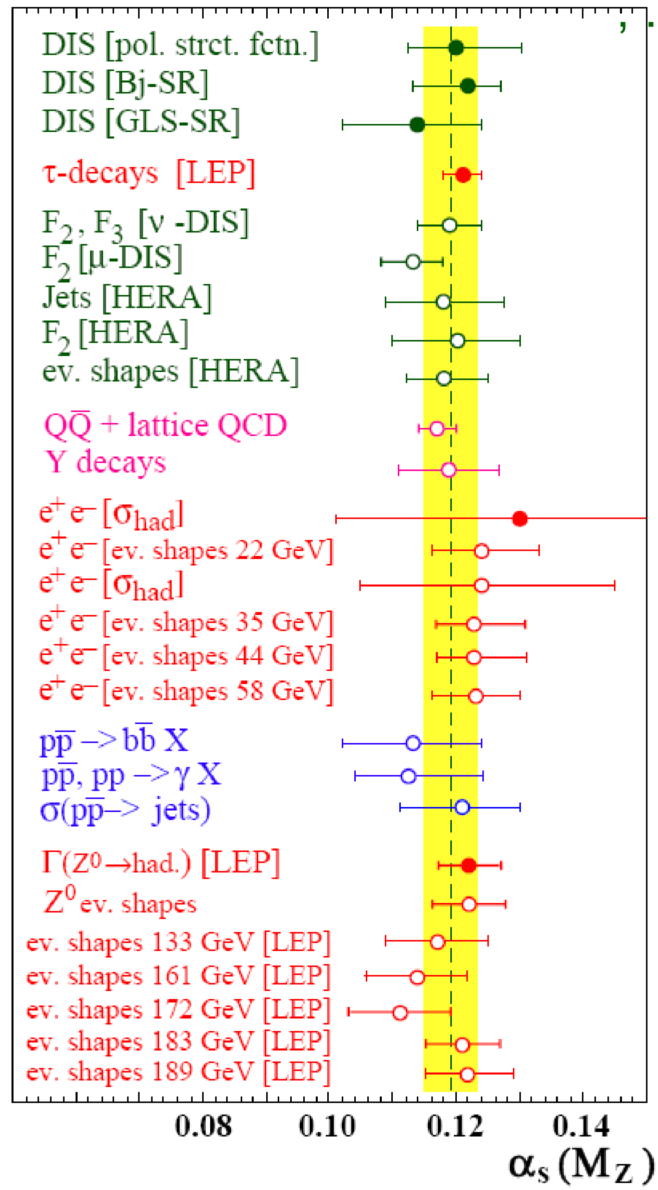
Key concepts: Ultra-violet divergences, bare Green fns, renormalization, RGE, anomalous dimensions, renormalized G.Fs ; … etc.

Asymptotic Freedom

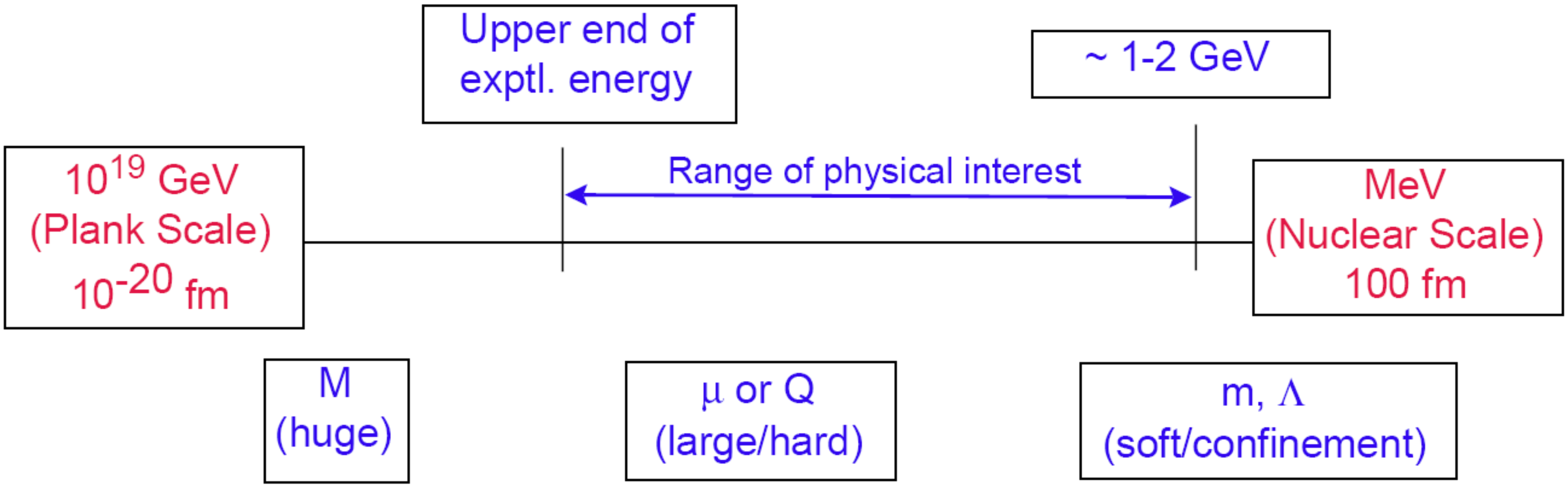
Universal (running) coupling:

$$\alpha_s(Q) = \frac{g^2}{4\pi} = \frac{b}{\ln(Q/\Lambda)}(1 + \ldots)$$





The importance of ***Scales*** -- Renormalization and Factorization



What to do with the long-distance physics associated with colinear/soft singularities in PQCD?

1st strategy:

Identify physical observables which are insensitive to the singularities! (Infra-red-safe (IRS) quantities)

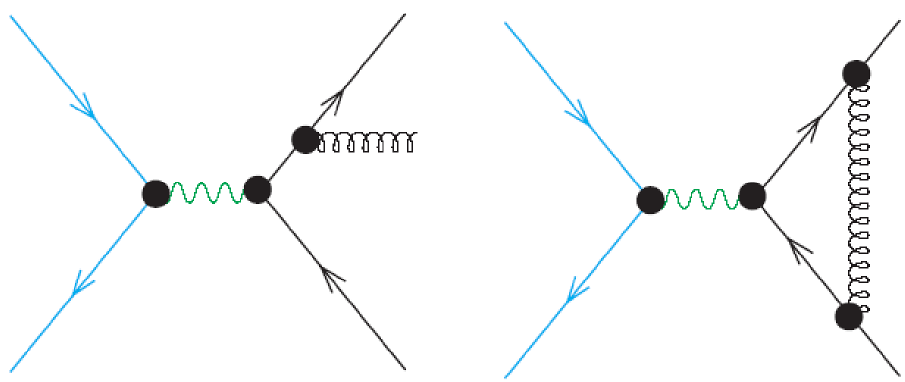
Total Hadronic Cross-section (*inclusive*):

$$\sigma_{tot}(s) = \sigma_0(s)[1 + \alpha_s(s)\, c_1 + \ldots]$$

Block – Nordsieck Thm $\rightarrow$ $c_{1,2,\ldots}$ *are finite, i.e. IRS*
(unitarity)

Order α_s:

Cancellation of the colinear/soft singularities between real and virtual diagrams



Infra-Red-Safe observables:

Total hadronic Cross-section $\sigma_{tot}/\sigma_{\mu+\mu-}$

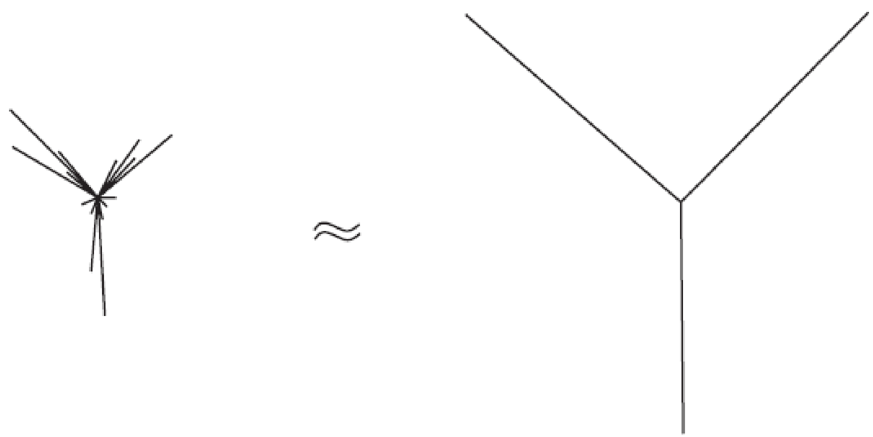
Sterman-Weinberg jet cross-sections and their modern variations (*Jade-, Durham-, ... algorithms*);

Jet shape observables: Thrust, ... ;

energy-energy correlation ; ....

Essential feature of a general IRS physical quantity:

the observable must be such that it is insensitive to whether n or n+1 particles contributed -- if the n+1 particles has n-particle kinematics



e.g. a IRS "jet algorithm"

σ and R in e^+e^- Collisions

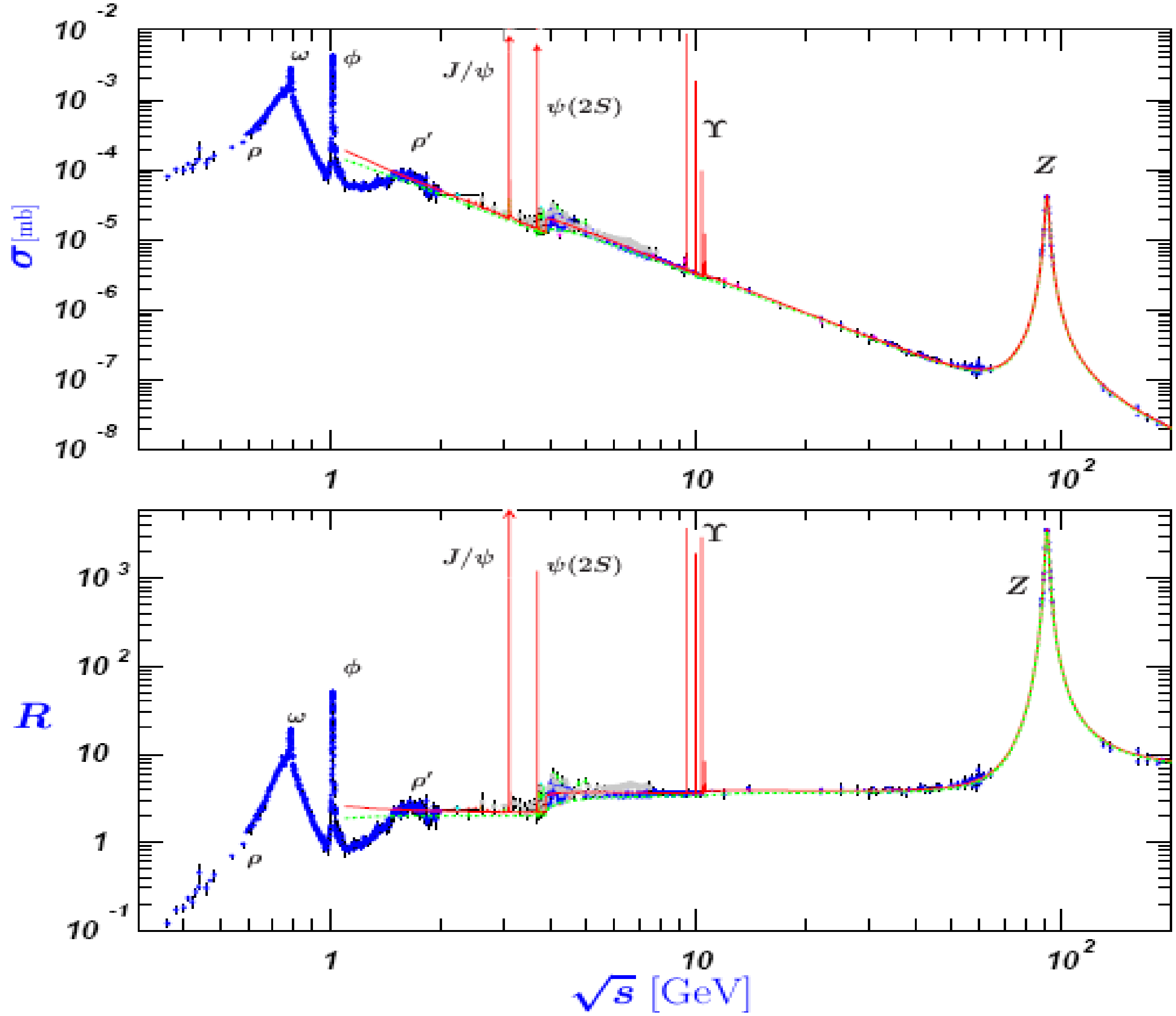
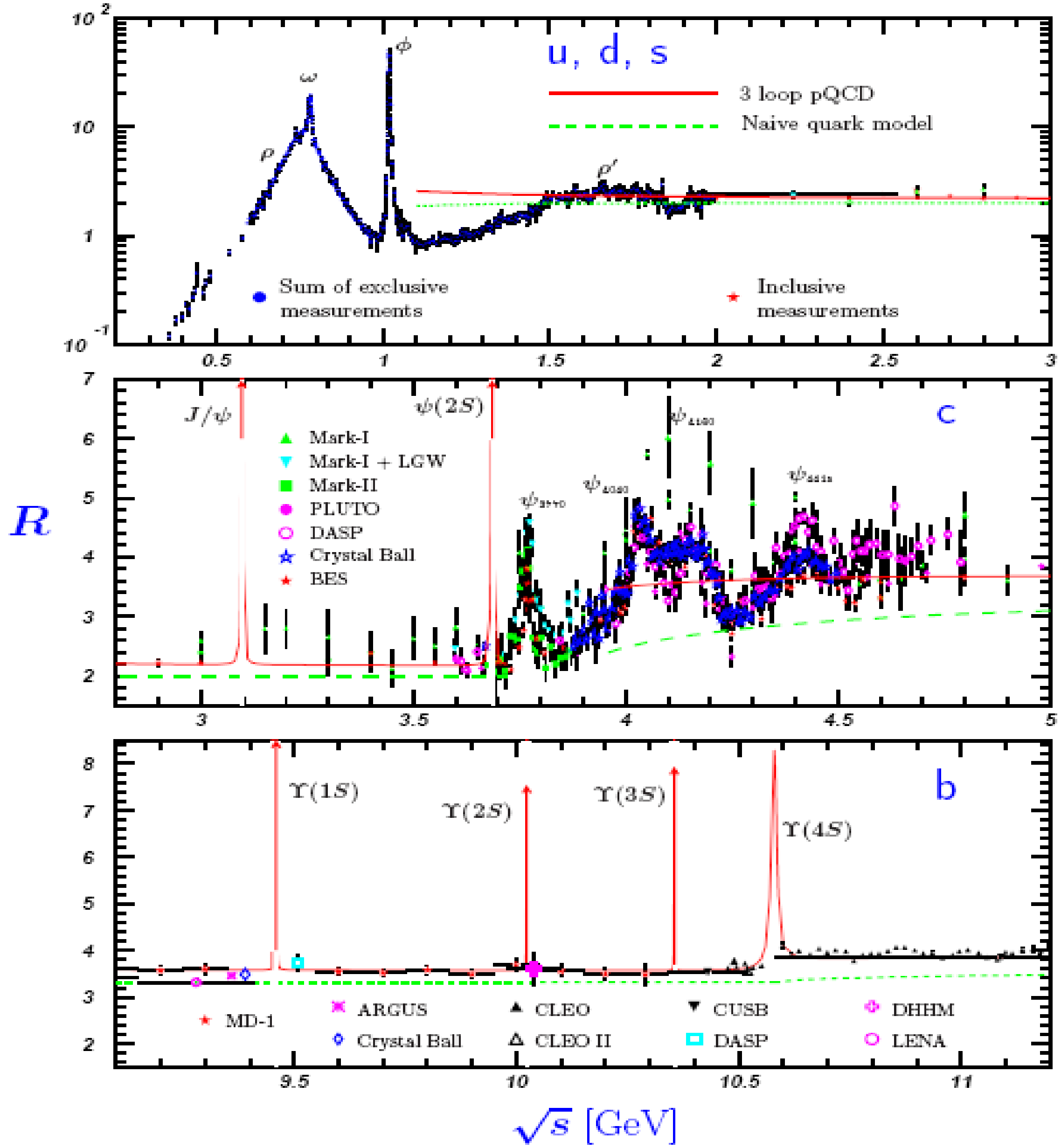


Figure 40.6: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.12) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. B586, 56 (2000) (Erratum *ibid.* B634, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in `[arXiv:hep-ph/0312114]`. Corresponding computer-readable data files are available at `http://pdg.ihep.su/xsect/contents.html`. (Courtesy of the COMPAS(Protvino) and HEPDATA(Durham) Groups, August 2005. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.)) See full-color version on color pages at end of book.

R in Light-Flavor, Charm, and Beauty Threshold Regions

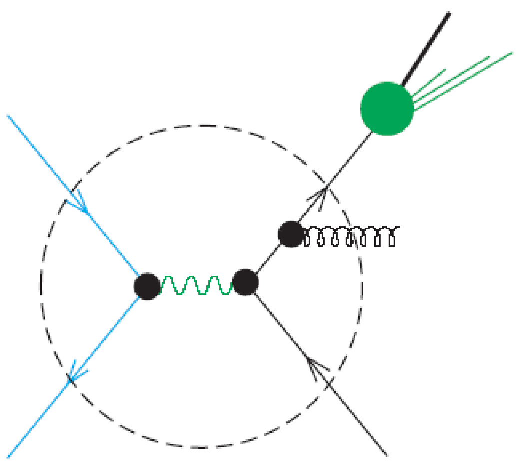


The 2nd strategy:

Factorization ⟶ QCD Parton model

Factorize the physical observable into a calculable ***IRS*** part and a non-calculable but ***universal*** piece.

Example: One particle inclusive cross-section



Fragmentation function:
Long-distance physics;
Universal.

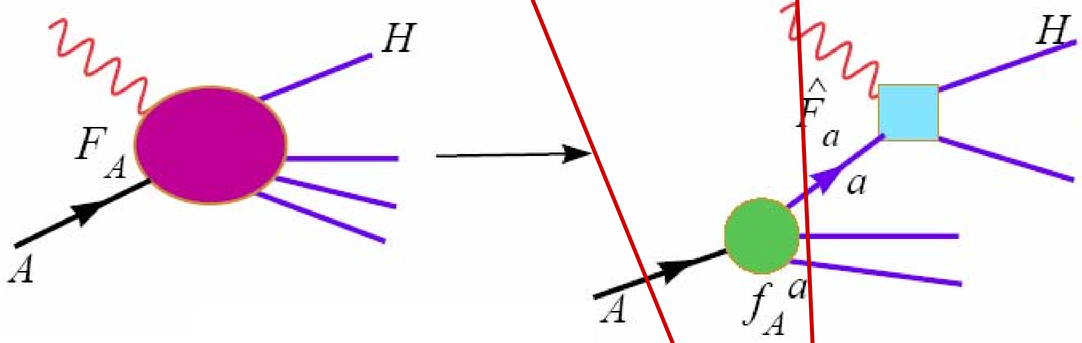
Hard scattering:
Short distance physics;
IRS, perturbatively cal.

$$\sigma(s,z) = \int_z^1 \frac{d\varsigma}{\varsigma}\, \hat{\sigma}^a\left(\frac{s}{\mu}, \frac{z}{\varsigma}, \alpha_s(\mu)\right) \cdot D_a(\varsigma, \mu)$$

QCD and DIS

Master Equation for QCD Parton Model
– the Factorization Theorem

$$F_A^\lambda(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{\mu}) \otimes \hat{F}_a^\lambda(x, \frac{Q}{\mu}, \frac{M}{Q}) + \mathcal{O}((\frac{\Lambda}{Q})^2)$$



A physical observable is independent of μ,
i.e., renormalization group invariant.

μ is the factorization scale.
Usually choose $\mu = Q$: that is how *f(x,Q)* acquires *Q*-dep.

“Renormalization” and “Factorization”

UV renormalization			Collinear/soft factorization	
A:	Bare Green Func.	$G_0(\alpha_0, m_0, ..)$	Partonic X-sect	F_a
B:	Ren. constants	$Z_i(\mu)$	Pert. parton dist.	$f_a^b(\mu)$
C:	Ren. Green Fun.	$G_R = G_0/Z$	Hard X-sect	$\hat{F} = F\,/\,f$
D:	Anomalous dim.	$\gamma = \frac{\mu}{Z}\frac{d}{d\mu}Z$	Splitting fun.	$P = \frac{\mu}{f}\frac{d}{d\mu}f$
E:	Phys. para. α, m	$\alpha_0 Z_i \ldots$	Had. parton dist. f_A	resummed
F:	Phys sc. amp.	$\alpha(\mu)\, G_R(m, \mu)$	Hadronic S.F.'s F_A	$f_A(\mu) \times \hat{F}(\mu)$

Some common features:

A : divergent; but, independent of “scheme” and scale μ;

B : divergent; scale and scheme dependent;
universal; absorbs all ultra-violet/soft/collinear divergences;

C & D : finite; scheme-dependent;
D controls the μ dependence of E & F;

E : physical parameters to be obtained from experiment;

F : Theoretical “prediction”; μ-indep. to all orders,
but μ-dep. at finite order n; $\mu\frac{d}{d\mu} \sim \mathcal{O}(\alpha^{n+1})$

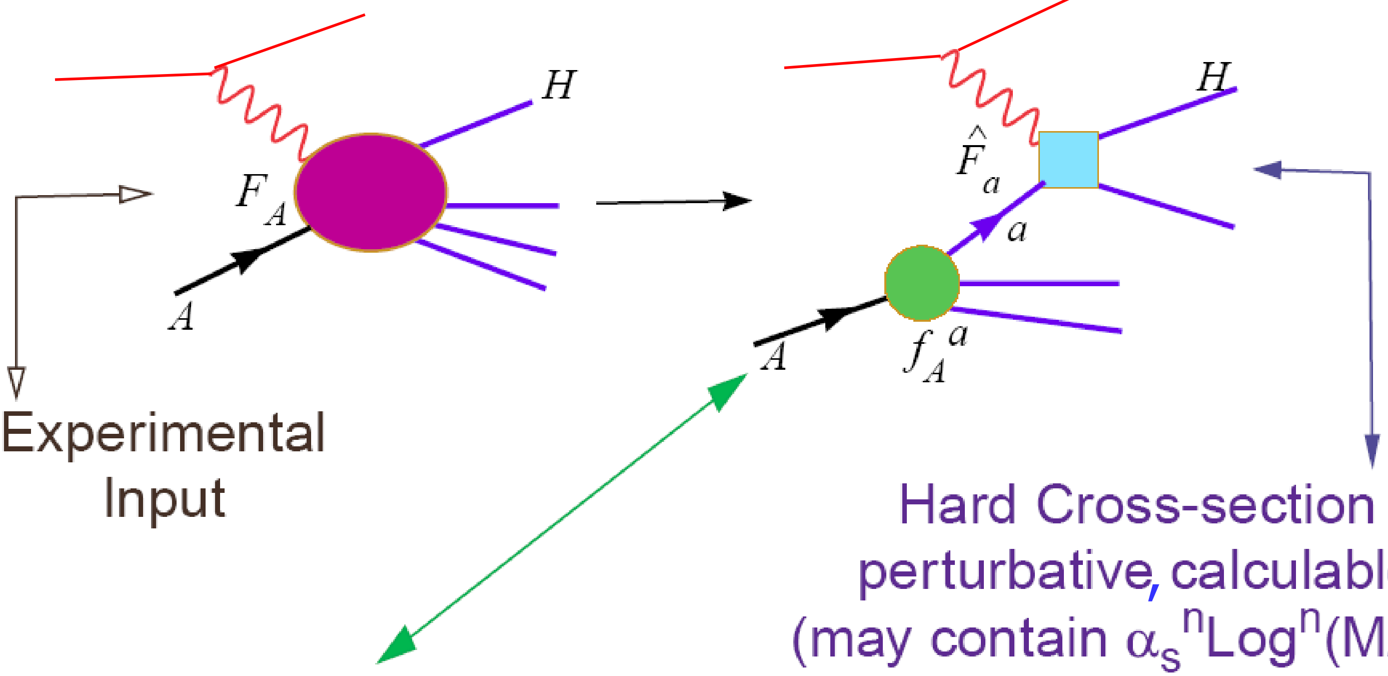
Note: “Renormalization” is factorization (of UV divergences);
“factorization” is renormalization (of soft/collinear div.)

Lepton-hadron Sc.

Master Equation for QCD Parton Model

– the Factorization Theorem

$$F_A^{\lambda}(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{\mu}) \otimes \widehat{F}_a^{\lambda}(x, \frac{Q}{\mu}, \frac{M}{Q}) + \mathcal{O}((\frac{\Lambda}{Q})^2)$$



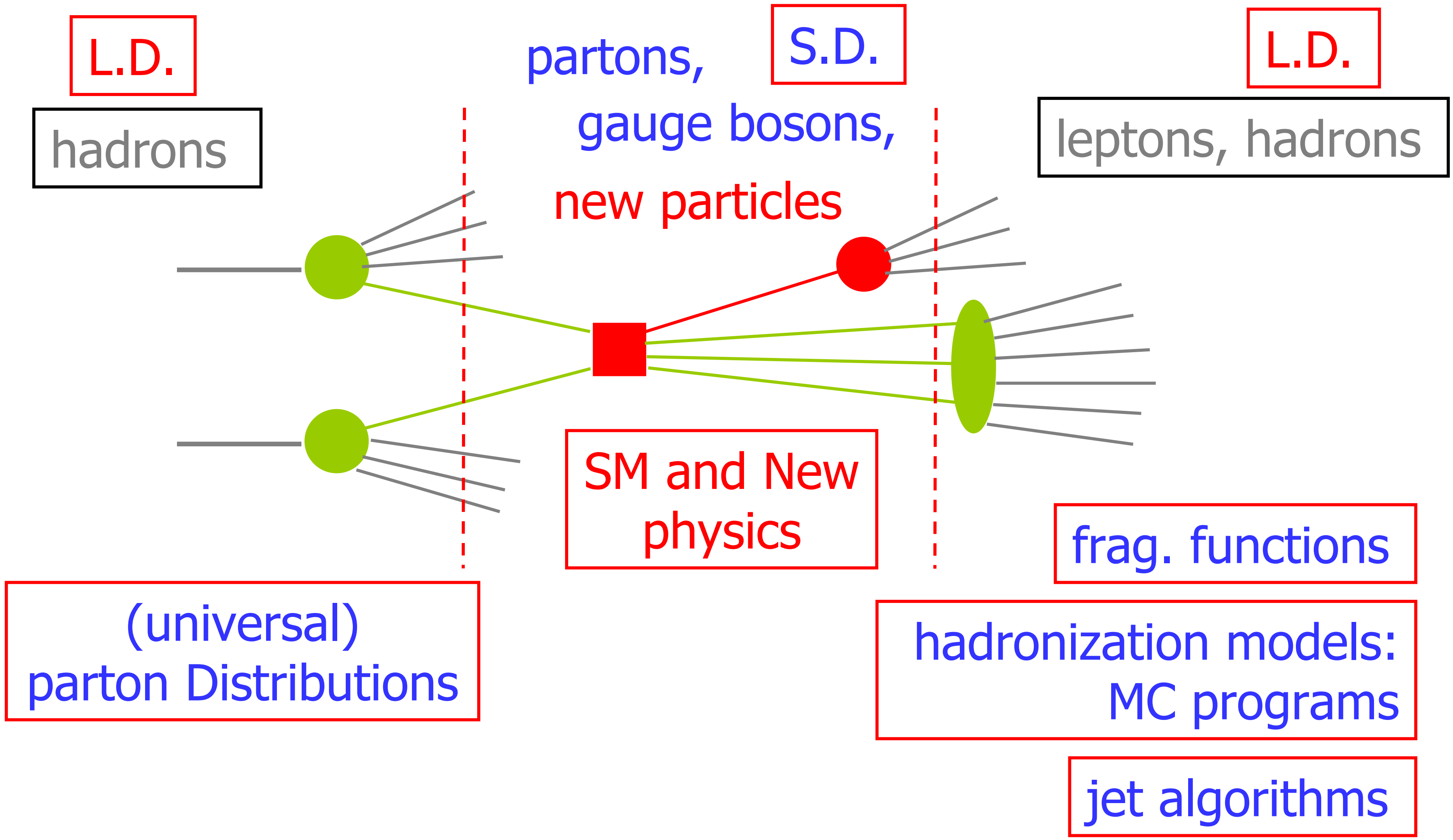
Experimental Input

Theory Input

Hard Cross-section
perturbative, calculable
(may contain $\alpha_s^n \mathrm{Log}^n(M/Q)$)

universal Parton Dist. Fn.
Non-Perturbative Parametrization at Q_0
GLAP Evolution to Q
extracted by global analysis

Hadron Collider Physics



Deep Inelastic Scattering (DIS) in Lepton-Hadron Collisions

Probing the Parton Structure of the Nucleon with Leptons

Deep Inelastic Scattering in Lepton-Hadron Collisions
—Probing the Parton Structure of the Nucleon with Leptons

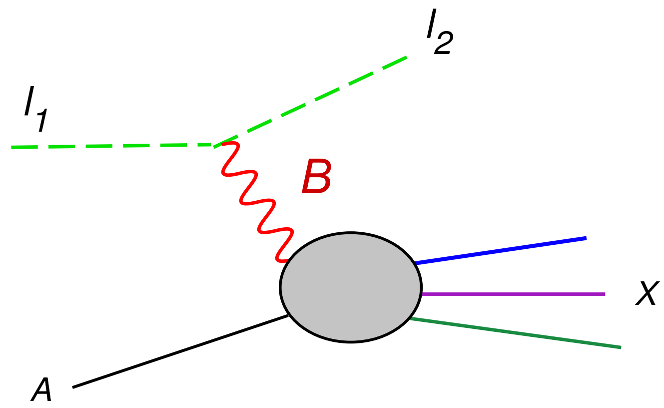
- Basic Formalism
  (indep. of strong dynamics and parton picture)
- Experimental Development
  - Fixed target experiments
  - HERA experiments
- Parton Model and QCD
  - Parton Picture of Feynman-Bjorken
  - Asymptotic freedom, factorization and QCD
- Phenomenology
  - QCD parameters
  - Parton distribution functions
  - Other interesting topics

Basic Formalism

(leading order in EW coupling)

Lepton-hadron scattering process

$$\ell_1(\ell_1) + N(P) \longrightarrow \ell_2(\ell_2) + X(P_X)$$



(B = g, W, Z)

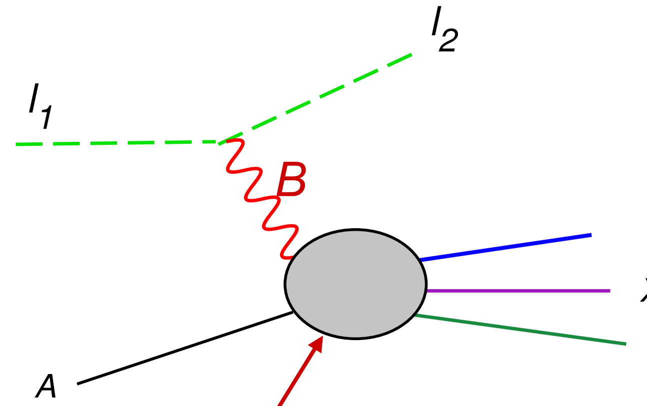
Effective fermion-boson electro-weak interaction Lagrangian:

$$\mathcal{L}_{\rm int}^{\rm EW} = -g_B \left[j_\mu^{(\ell)}(x) + J_\mu^{(h)}(x) \right] \; V_B^\mu(x)$$

EW SU(2)xU(1) gauge coupling constants

B	γ	$W^\pm$	Z
g_B	$-e$	$\frac{g}{2\sqrt{2}}$	$\frac{g}{2\cos\theta_W}$

Scattering Amplitudes



$$\mathcal{M} = J^*_\mu(P,q)\frac{g_B^2 G^\mu{}_\nu}{Q^2+M_B^2} j^\nu(q,\ell)$$

Spin 1 projection tensor $\quad G^\mu{}_\nu = g^\mu{}_\nu - q^\mu q_\nu / M_B^2.$

Lepton current amplitude (known):

$$j^\mu(q,\ell) = \langle \ell_2 | j^\mu | \ell_1 \rangle = \overline{u}(\ell_2)\gamma^\mu[g_R(1+\gamma^5)+g_L(1-\gamma^5)]u(\ell_1)$$

Hadron current amplitude (unknown):

$$J^*_\mu(P,q) = \langle P_X | J^\dagger_\mu | P \rangle$$

Object of study:

* Parton structure of the nucleon; (short distance)
* QCD dynamics at the confinement scale (long dis.)

Basic Formalism: Cross section

Cross section

(amplitude)^2 phase space / flux

$$d\sigma = \frac{G_1\, G_2}{2\Delta(s, m_{\ell_1}^2, M^2)}\; 4\pi Q^2\, L_\nu^\mu\, W_\mu^\nu\, d\Gamma$$

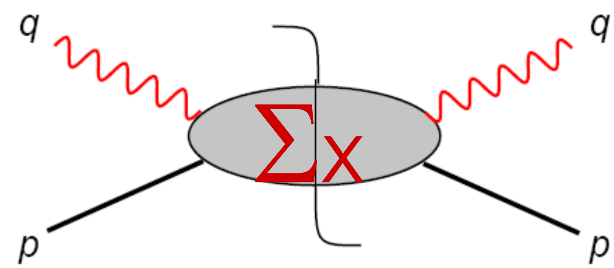
$$G_i = g_{B_i}^2 / (Q^2 + M_{B_i}^2)$$

Lepton tensor (known):

$$L^\mu{}_\nu = \frac{1}{Q^2} \overline{\sum_{\text{spin}}} \langle \ell_1 | j_\nu^\dagger | \ell_2 \rangle \langle \ell_2 | j^\mu | \ell_1 \rangle$$

Hadron tensor (unknown):

$$W^\mu{}_\nu = \frac{1}{4\pi} \overline{\sum_{\text{spin}}} (2\pi)^4 \delta^4(P + q - P_X) \langle P | J^\mu | P_X \rangle \langle P_X | J_\nu^\dagger | P \rangle$$



Object of study:

- * Parton structure of the nucleon;
- * QCD dynamics at the confinement scale

Expansion of $W^{\mu}{}_{\nu}$ in terms of *independent* components

$$W^{\mu}{}_{\nu} = -g^{\mu}{}_{\nu} W_1 + \frac{P^{\mu} P_{\nu}}{M^2} W_2 - i \frac{\epsilon^{Pq\mu}{}_{\nu}}{2M^2} W_3 + \\ + \frac{q^{\mu} q_{\nu}}{M^2} W_4 + \frac{P^{\mu} q_{\nu} + q^{\mu} P_{\nu}}{2M^2} W_5 + \frac{P^{\mu} q_{\nu} - q^{\mu} P_{\nu}}{2M^2} W_6$$

Cross section in terms of the structure functions

$$\frac{d\sigma}{dE_2\, d\cos\theta} = \frac{2E_2^2}{\pi M} \frac{G_1 G_2}{n_\ell} \left\{ g_{+\ell}^2 \left[2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2} \right] \pm g_{-\ell}^2 \left[\frac{E_1 + E_2}{M} W_3 \sin^2\frac{\theta}{2} \right] \right\}$$

Charged Current (CC) processes (neutrino beams):
W-exchange (diagonal); left-handed coupling only; ….

Neutral Current (NC) processes (e,μ scat.)---low energy:
(fixed tgt): γ-exchange (diagonal); vector coupling only; …

Neutral Current (NC) processes (e,μ scat.)---high energy
(hera): γ & Z exchanges: G_1^2, $G_1 G_2$, G_2^2 terms; ….

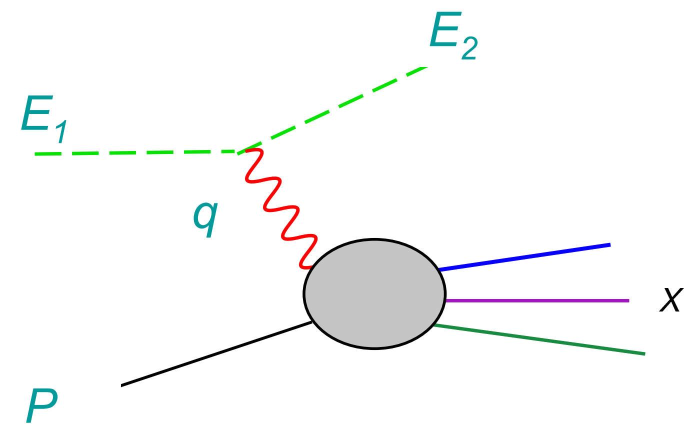
Basic Formalism: Scaling structure functions

Kinematic variables

$$\nu = \frac{P\cdot q}{\sqrt{P\cdot P}} = E_1 - E_2$$

$$x = \frac{-q^2}{2P\cdot q} = \frac{Q^2}{2M\nu}$$

$$y = \frac{P\cdot q}{P\cdot \ell_1} = \frac{\nu}{E_1}$$



Scaling (dimensionless) structure functions

$$F_1(x,Q) = W_1$$
$$F_2(x,Q) = \frac{\nu}{M} W_2$$
$$F_3(x,Q) = \frac{\nu}{M} W_3$$

Scaling form of cross section formula:

$$(\; g^2_{\pm\,\ell} = g^2_{L\,\ell} \pm g^2_{R\,\ell} \;)$$

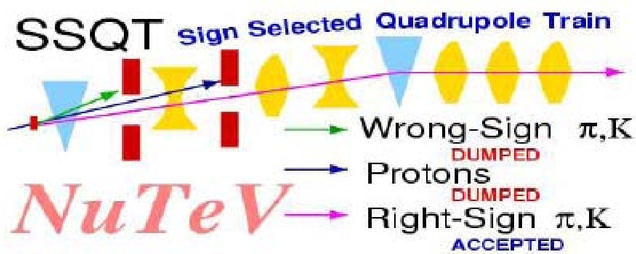
$$\frac{d\sigma}{dx\,dy} = \frac{2ME_1}{\pi}\,\frac{G_1 G_2}{n_\ell}\left\{ g^2_{+\,\ell}\left[xF_1\,y^2 + F_2\left[(1-y) - \left(\frac{Mxy}{2E_1}\right)\right]\right] \pm g^2_{-\,\ell}\left[xF_3\,y(1-\frac{y}{2})\right]\right\}$$

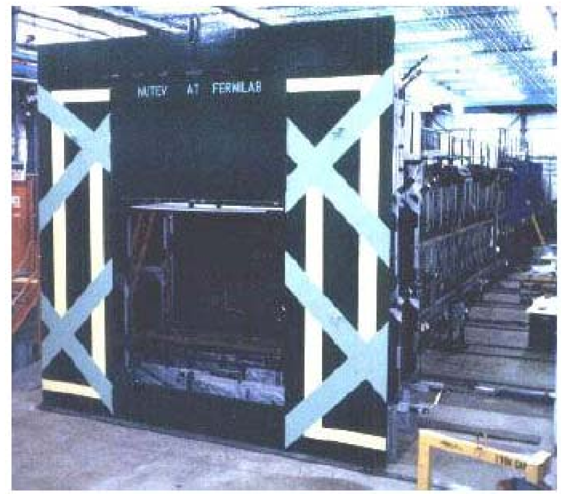
n_ℓ is the number of polarization states of the incoming lepton.

The highest energy (anti-) neutrino DIS experiment

The NuTeV experiment at FNAL

ν-N DIS, sign-selected beam $\langle E_\nu \rangle \sim 120$ GeV
and continuous test beam calibration

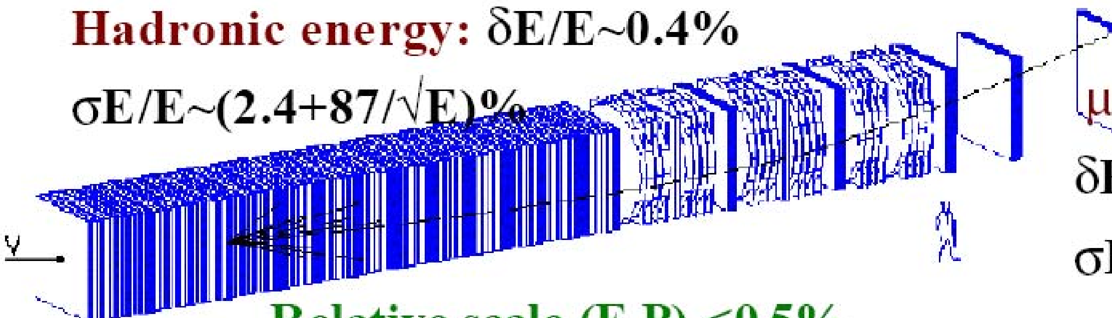




Data taken during 1996-97

Hadronic energy: $\delta E/E \sim 0.4\%$

$\sigma E/E \sim (2.4+87/\sqrt{E})\%$



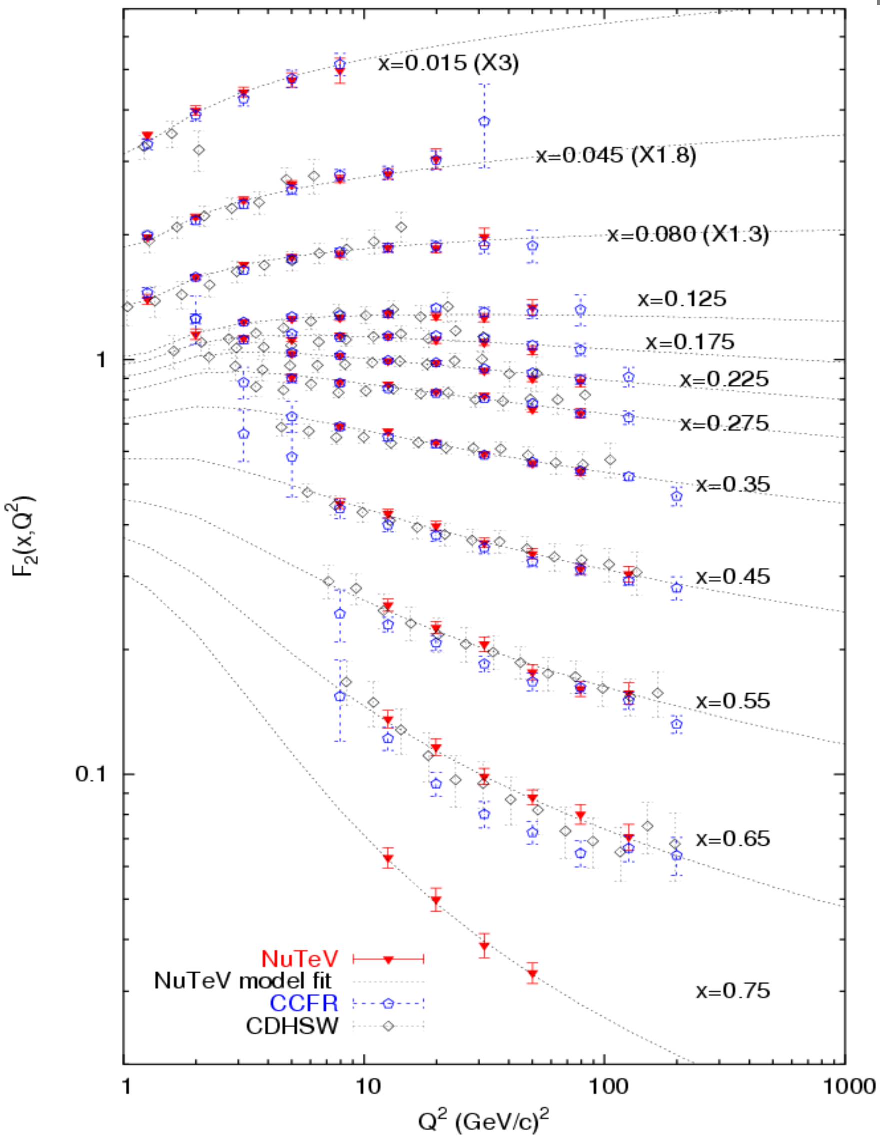
μ momentum

$\delta P/P \sim 1\%$

$\sigma P/P \sim 11\%$

Relative scale (E,P) <0.5%

F_2 Measurement

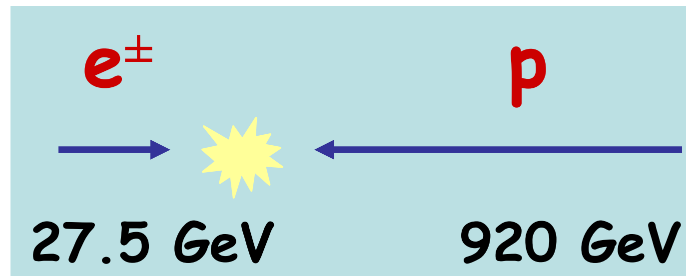


- **Isoscalar ν-Fe F_2**
- **NuTeV F_2 is compared with CCFR and CDHSW results**
  - **the line is a fit to NuTeV data**
- **All systematic uncertainties are included**
- **All data sets agree for *x<0.4*.**
- **At *x>0.4* NuTeV agrees with CDHSW**
- **At *x>0.4* NuTeV is systematically above CCFR**

The HERA Collider

The first and only ep collider in the world

Located in Hamburg



$\sqrt{s}$ = 318 GeV

Equivalent to fixed target experiment with 50 TeV $e^{\pm}$

The Collider Experiments

H1 Detector

Complete 4π detector with

Tracking
- **Si-μVTX**
- **Central drift chamber**

Liquid Ar calorimeter

→ $\delta E/E = 12\%/\sqrt{E[\mathrm{GeV}]}$ (e.m.)

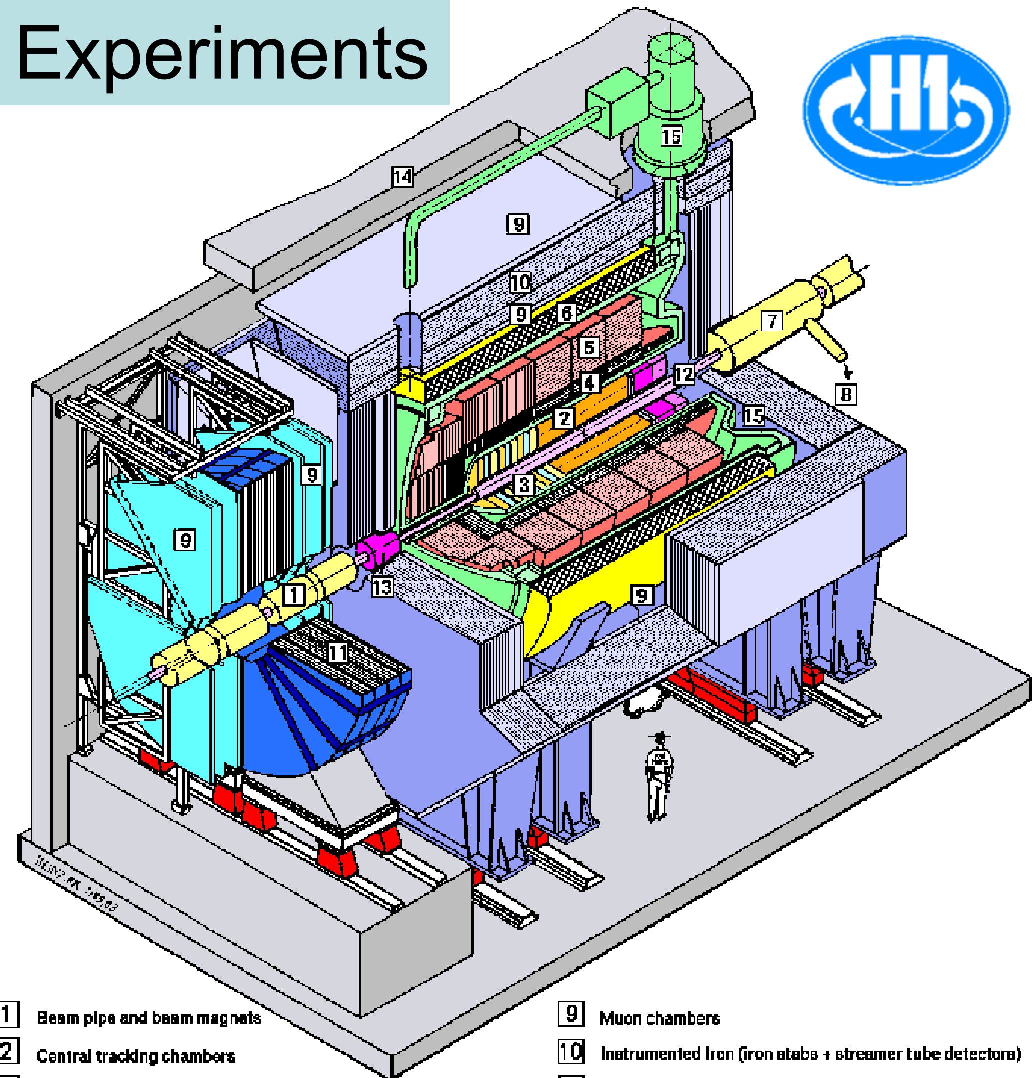
$\delta E/E = 50\%/\sqrt{E[\mathrm{GeV}]}$ (had)

Rear Pb-scintillator calorimeter

→ $\delta E/E = 7.5\%/\sqrt{E[\mathrm{GeV}]}$ (e.m.)

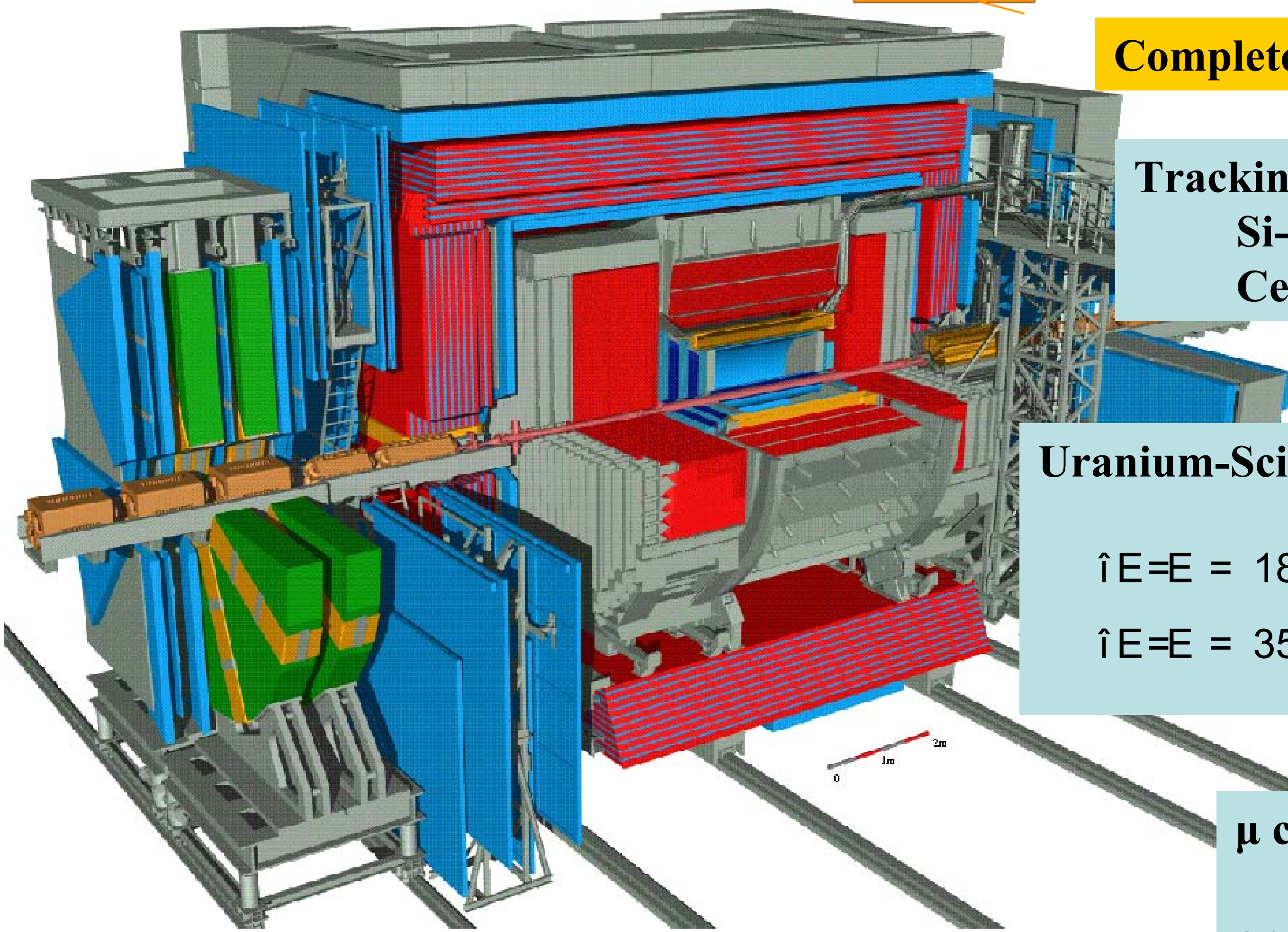
μ chambers

and much more…



1. Beam pipe and beam magnets
2. Central tracking chambers
3. Forward tracking and Transition radiators
4. Electromagnetic Calorimeter (lead) } Liquid Argon
5. Hadronic Calorimeter (stainless steel) } Liquid Argon
6. Superconducting coil (1.2T)
7. Compensating magnet
8. Helium cryogenics
9. Muon chambers
10. Instrumented Iron (iron slabs + streamer tube detectors)
11. Muon toroid magnet
12. Warm electromagnetic calorimeter
13. Plug calorimeter (Cu, Si)
14. Concrete shielding
15. Liquid Argon cryostat

ZEUS Detector



Complete 4π detector with

Tracking
- **Si-μVTX**
- **Central drift chamber**

Uranium-Scintillator calorimeter

$$\Delta E/E = 18\%/\sqrt{E[GeV]}\ (e.m.)$$

$$\Delta E/E = 35\%/\sqrt{E[GeV]}\ (had)$$

μ chambers

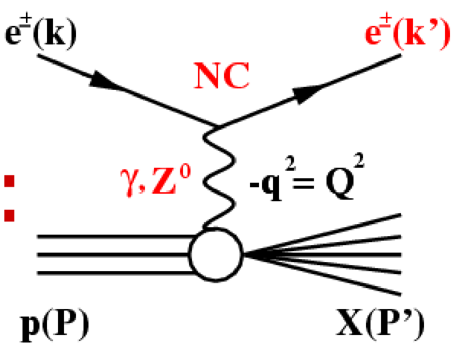
and much more…

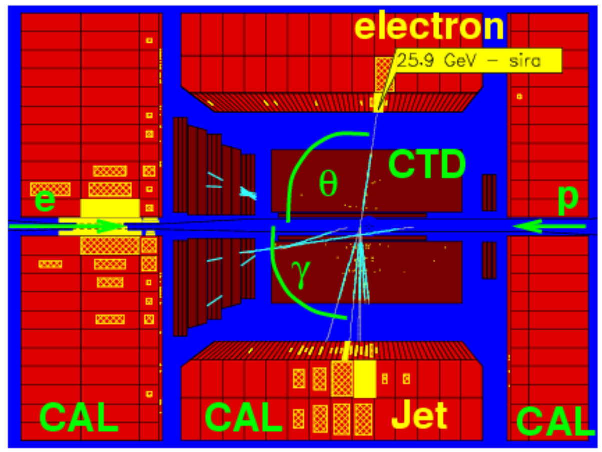
Both detectors asymmetric

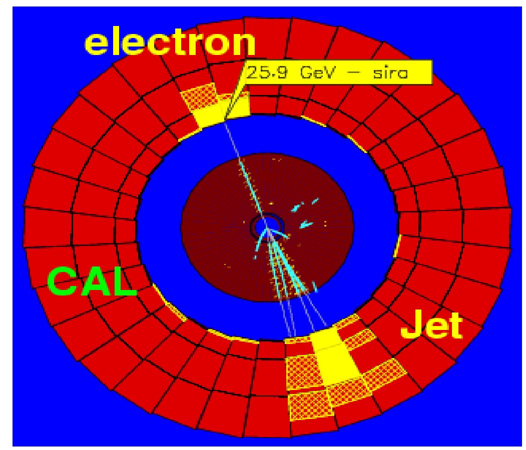
NC and CC incl. Processes measured at HERA

$$\mathrm{NC}: \quad e^{\pm} + p \rightarrow e^{\pm} + X, \qquad \mathrm{CC}: \; e^{\pm} + p \rightarrow \bar{\nu}_e(\nu_e) + X$$

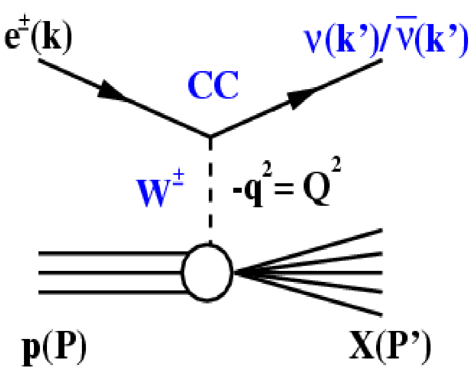
NC:

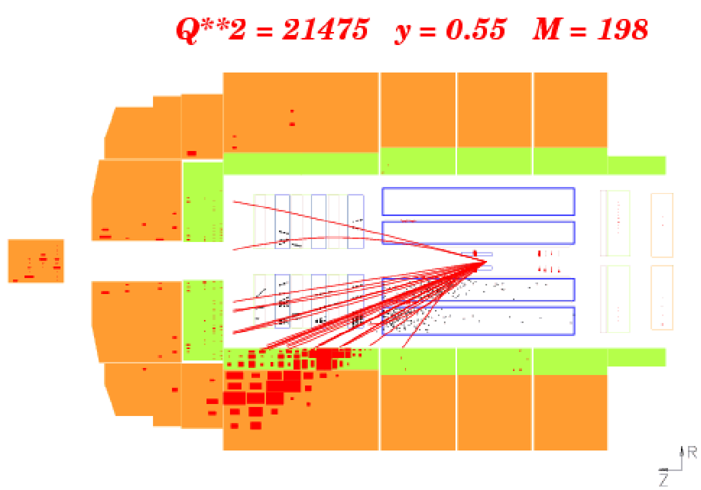


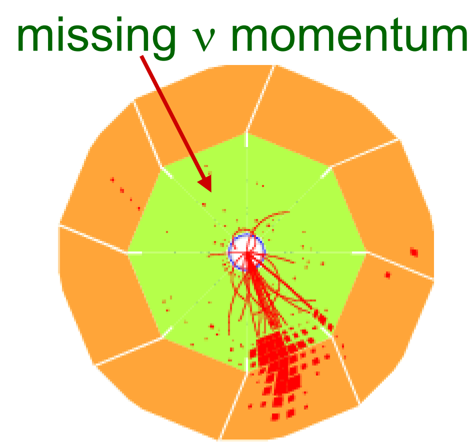




CC:

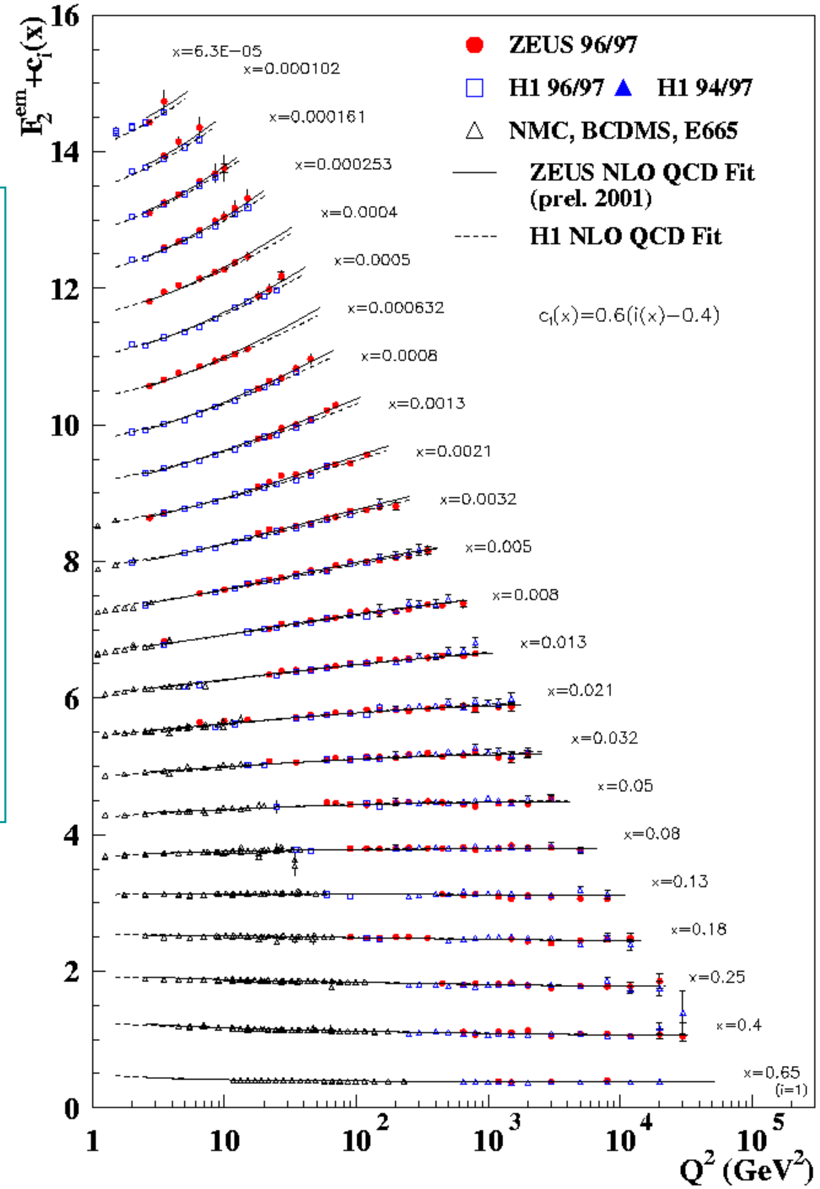






Measurement of $F^{\gamma}_2(x,Q^2)$

- For $Q^2 \ll M_Z^2 \rightarrow xF_3$ negligible;
- F_L only important at high y;
- Both F_L and xF_3 ~ calculable in QCD
- Correct for higher order QED radiation
- Extract $F_2(x,Q^2)$ from measurement of $\frac{d^2\hat{\sigma}^{ep}}{dx\,dQ^2}$

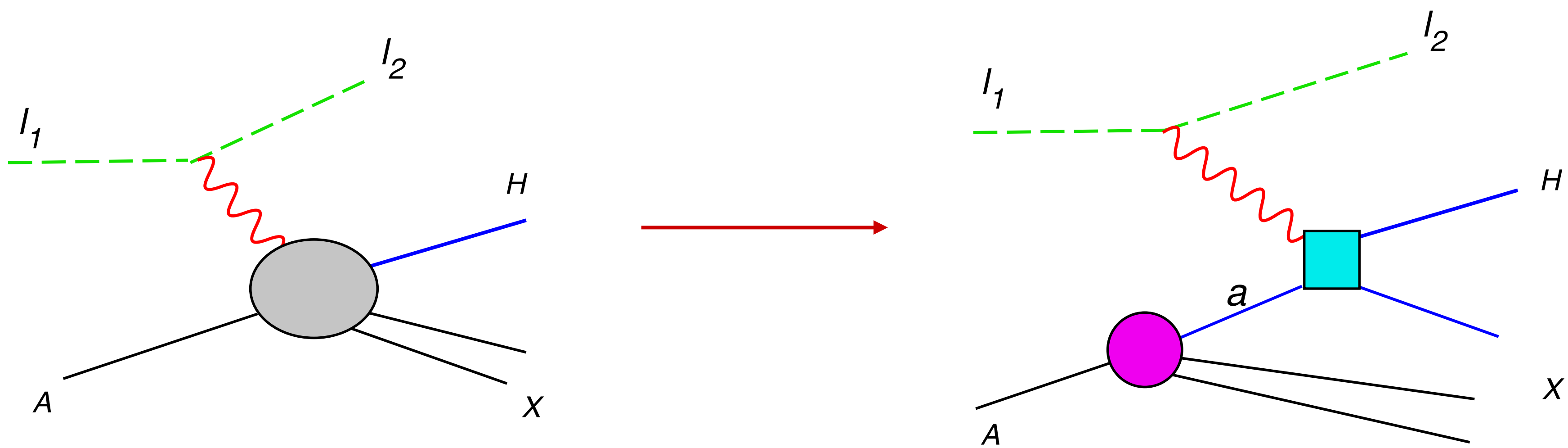


These are difficult measurements: nevertheless precision level has reached: errors of 2-3%

Physical Interpretations of DIS Structure Function measurements

- The Parton Model (Feynman-Bjorken)
- Theoretical basis of the parton picture and the QCD improved parton model

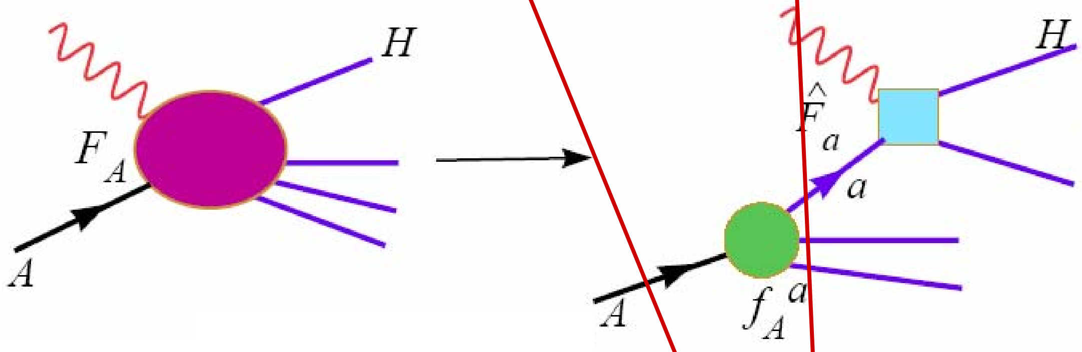
High energy (Bjorken) limit:
(large Q^2 and ν, for a fixed x value)



QCD and DIS

Master Equation for QCD Parton Model
– the Factorization Theorem

$$F_A^{\lambda}(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{\mu}) \otimes \hat{F}_a^{\lambda}(x, \frac{Q}{\mu}, \frac{M}{Q}) + \mathcal{O}((\frac{\Lambda}{Q})^2)$$



A physical observable is independent of μ,
i.e., renormalization group invariant.

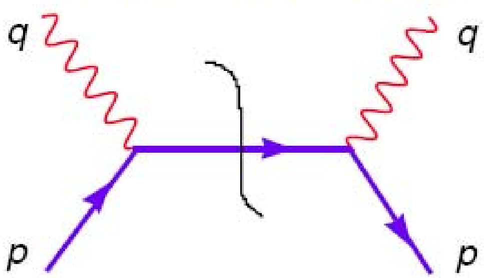
μ is the factorization scale.
Usually choose $\mu = Q$: that is how $f(x,Q)$ acquires Q-dep.

Parton Model results on Structure Functions

$$F_\lambda(x,Q^2) \sim \int_0^1 \frac{d\xi}{\xi} \sum_a \, f_A^a(\xi) \, \hat{F}_\lambda^a(x/\xi, Q^2) + \mathcal{O}(\frac{m}{Q}).$$

where $\hat{F}_\lambda^a(z,Q^2)$ is the "partonic structure function" for DIS on the parton target a.

The Feynman diagram contributing to this elementary quantity and the result of a straightforward calculation are (for electro-magnetic coupling case):



$$\begin{array}{lcc} \hat{F}_T^a(x/\xi,Q^2) & = & Q_a^2 \, \delta(x/\xi \Leftrightarrow 1) \\ \hat{F}_L^a(x/\xi,Q^2) & = & 0 \\ \hat{F}_{PV}^a(x/\xi,Q^2) & = & 0 \end{array}$$

$\Longrightarrow$ the simple scaling parton model results:

$$\begin{array}{lcll} F_T(x,Q^2) & = & \sum_a \, Q_a^2 \, f_A^a(x) & \text{(Bj.} \Leftrightarrow \text{Feynman)} \\ F_L(x,Q^2) & = & 0 & \text{(Callan} \Leftrightarrow \text{Gross)} \end{array}$$

Structure functions: Quark Parton Model

Quark parton model (QPM) NC SFs for proton target:

$$[F_2^{\gamma}, F_2^{\gamma Z}, F_2^{Z}] = x \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2]\{q + \bar{q}\}$$

$$[xF_3^{\gamma Z}, xF_3^{Z}] = 2x \sum_q [e_q a_q, v_q a_q]\{q - \bar{q}\} = 2x \sum_{q=u,d} [e_q a_q, v_q a_q] q_v$$

QPM CC SFs for proton targets:

$$xF_{2,W+}^{CC} = x\{\bar{u} + \bar{c} + d + s\}, \qquad xF_{3,W+}^{CC} = x\{d + s - (\bar{u} + \bar{c})\}$$

$$xF_{2,W-}^{CC} = x\{u + c + (\bar{d} + \bar{s})\}, \qquad xF_{3,W-}^{CC} = x\{u + c - (\bar{d} + \bar{s})\}$$

For neutron targets, invoke (flavor) isospin symmetry:

$u \Leftrightarrow d$ and $\bar{u} \Leftrightarrow \bar{d}$

continued

Consequences on CC Cross sections (parton model level):

$$\frac{d\sigma^{\nu/\bar{\nu}}}{dxdy} \propto W \cdot L \propto F_{\nu/\bar{\nu}} \left(\frac{1 \pm \cosh\psi}{2} \right)^2$$

$$\cosh\psi = \frac{2-y}{y} \quad \longrightarrow \quad \frac{1 \pm \cosh\psi}{2} \propto \begin{cases} 1 \\ 1-y \end{cases}$$

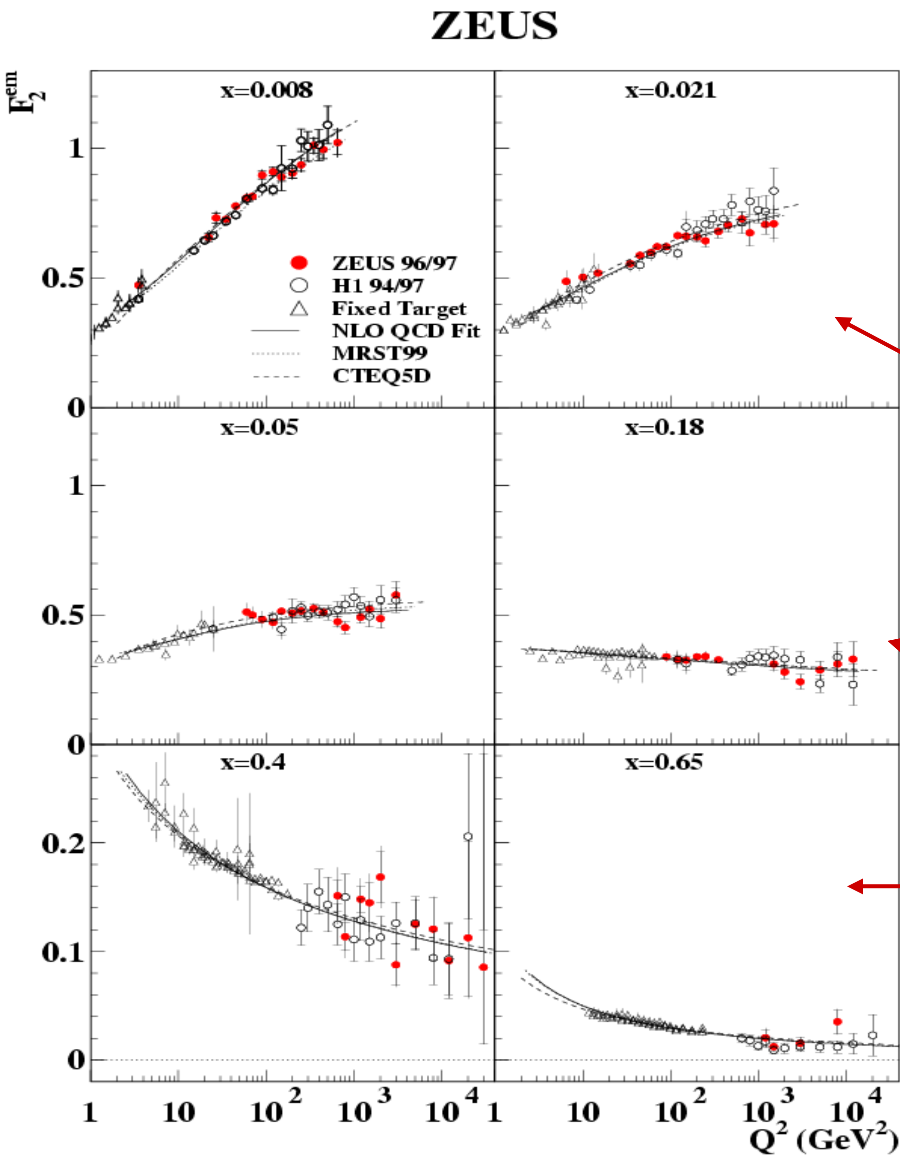
$$\longrightarrow \quad \frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} = \int dy \frac{d\sigma^{\bar{\nu}}}{dy} \Big/ \int dy \frac{d\sigma^{\nu}}{dy} \approx \frac{1}{3}$$

These qualitative features were verified in early (bubble chamber) high energy neutrino scattering experiments.

Gargamelle (CERN)

Refined measurements reveal QCD corrections to the approximate naïve parton model results. These are embodies in all modern “QCD fits” and “global analyses”.

F_2 : "Scaling violation" — Q-dependence inherent in QCD



Renormalization group equation governs the scale dependence of parton distributions and hard cross sections. (DGLAP)

Rise with increasing Q at small-x

Flat behavior at medium x

decrease with increasing Q at high x

QCD evolution

Evolution performed in terms of (1/2/3) non-singlet, singlet and gluon densities:

$$\frac{\partial}{\partial \ln \mu_F^2} q_{NS}^{\pm} = P_{NS}^{\pm} \otimes q_{NS}^{\pm}$$

$$\frac{\partial}{\partial \ln \mu_F^2} \begin{Bmatrix} \Sigma \\ g \end{Bmatrix} = \begin{Bmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{Bmatrix} \otimes \begin{Bmatrix} \Sigma \\ g \end{Bmatrix} = \mathrm{P} \otimes \mathrm{q}$$

Where

$$P(x) = a_S P^{(0)}(x) + a_S^2 \left[P^{(1)}(x) - \beta_0 \ln \frac{\mu_F^2}{\mu_R^2} P^{(0)}(x) \right] \qquad \text{with} \qquad a_S = \frac{\alpha_S(\mu_R^2)}{4\pi}$$

$$\frac{da_S}{d \ln \mu_R^2} = \beta(a_S) = \sum_{l=0}^{\infty} a_S^{l+2} \beta_l \cong a_S^2 \beta_0 + a_S^3 \beta_1 \qquad \text{where} \qquad \beta_0 = 11 - \frac{2}{3} N_F$$

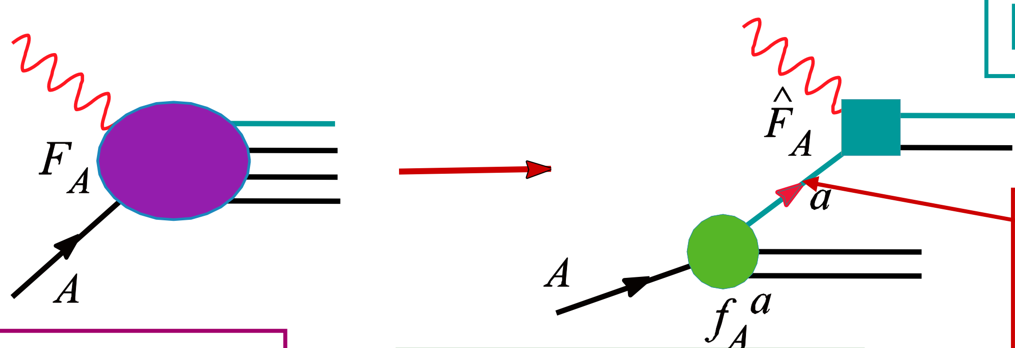
$$\text{and} \qquad \beta_1 = 102 - \frac{38}{3} N_F$$

Parton Distribution Functions (PDF): most significant physical results derived from DIS (with help from other hard scattering processes)

A common misconception:

Parton distribution functions $\not\approx$ “Structure functions”

These are the hard Xsecs.



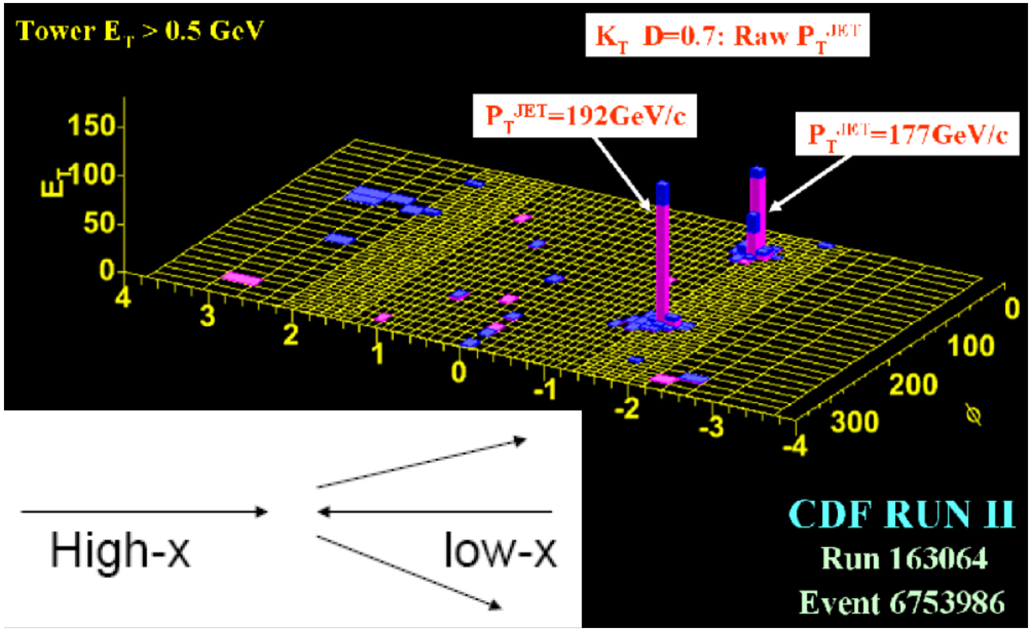
There is a convo-lution integral and a summation over partons here!

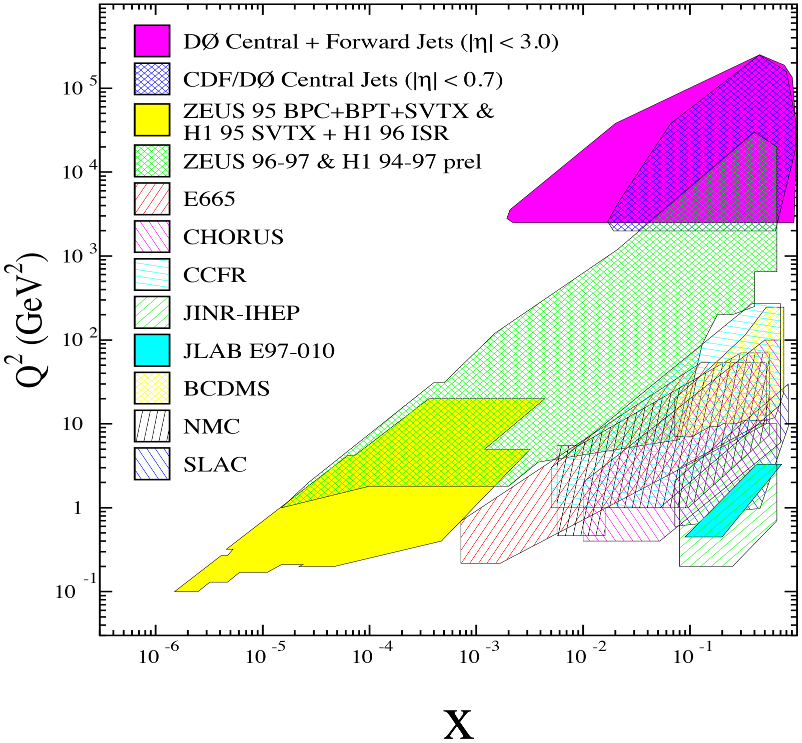
These are the (process-dep) S.F.s

These are the (universal) PDFs

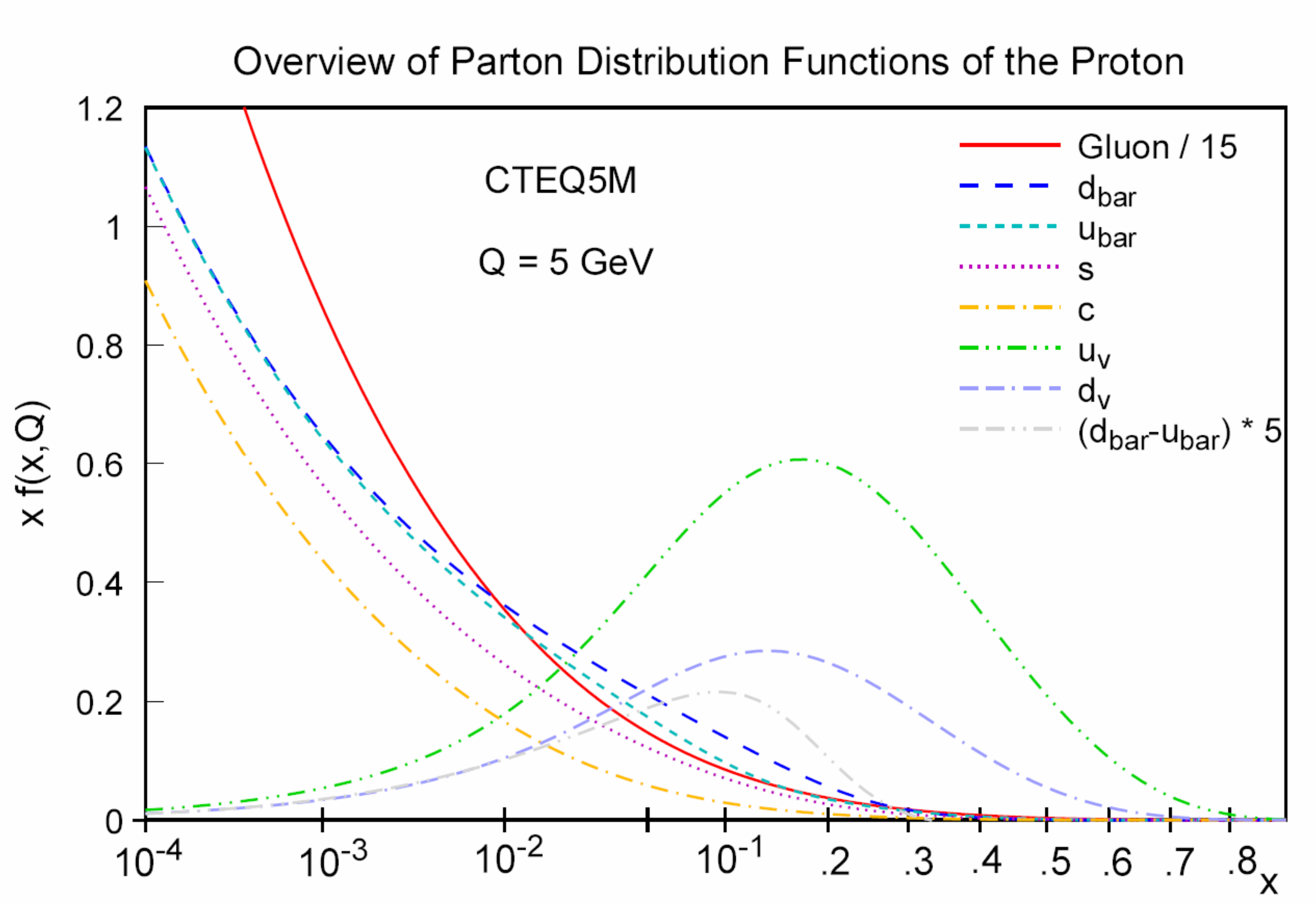
Forward Jet Measurement

- Forward jets probe high-x at lower Q^2 (= $-q^2$) than central jets
  - Q^2 evolution given by DGLAP
  - Essential to distinguish PDF and possible new physics at higher Q^2
- Also, extend the sensitivity to lower x



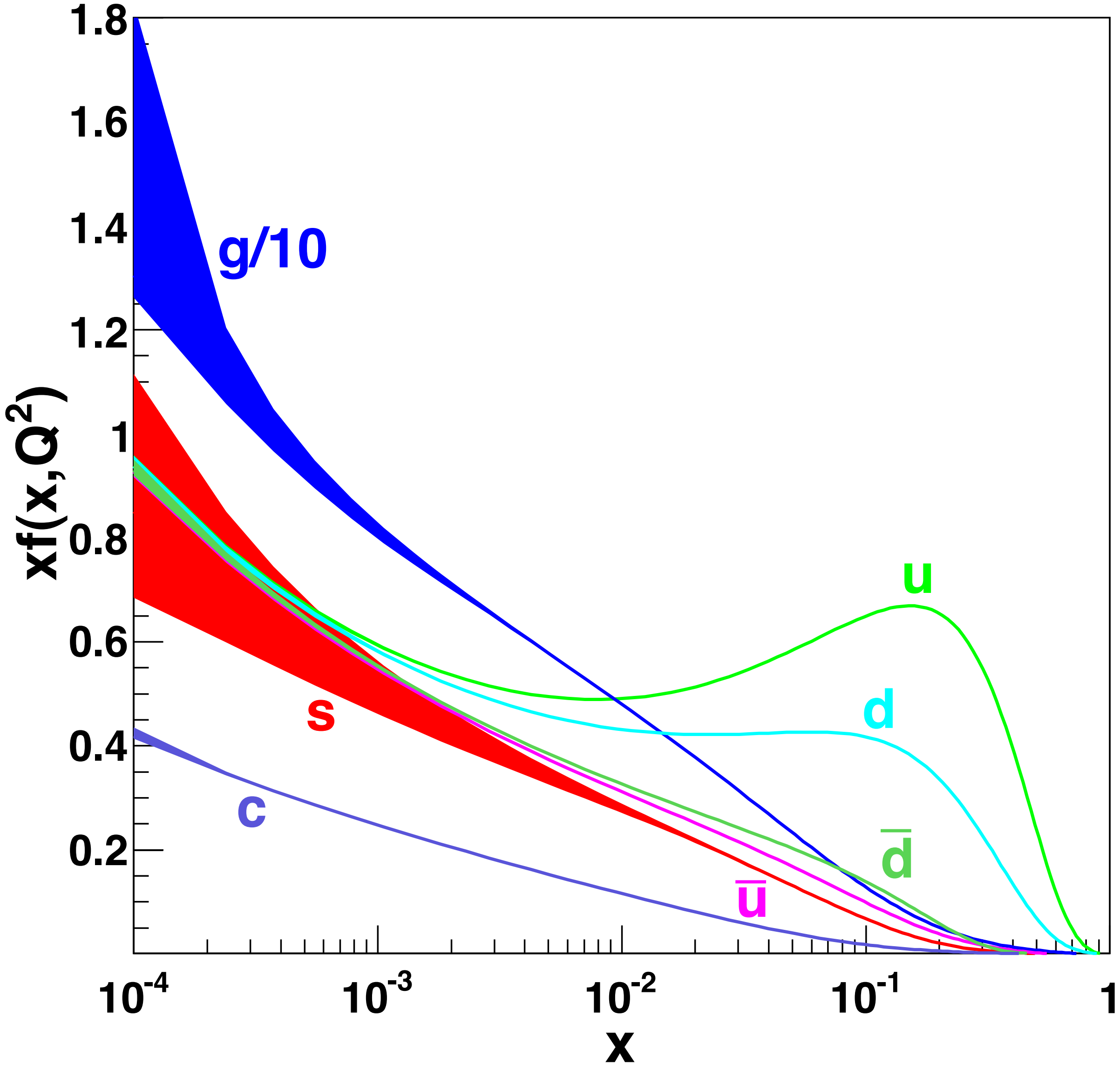


Parton Distributions: one example

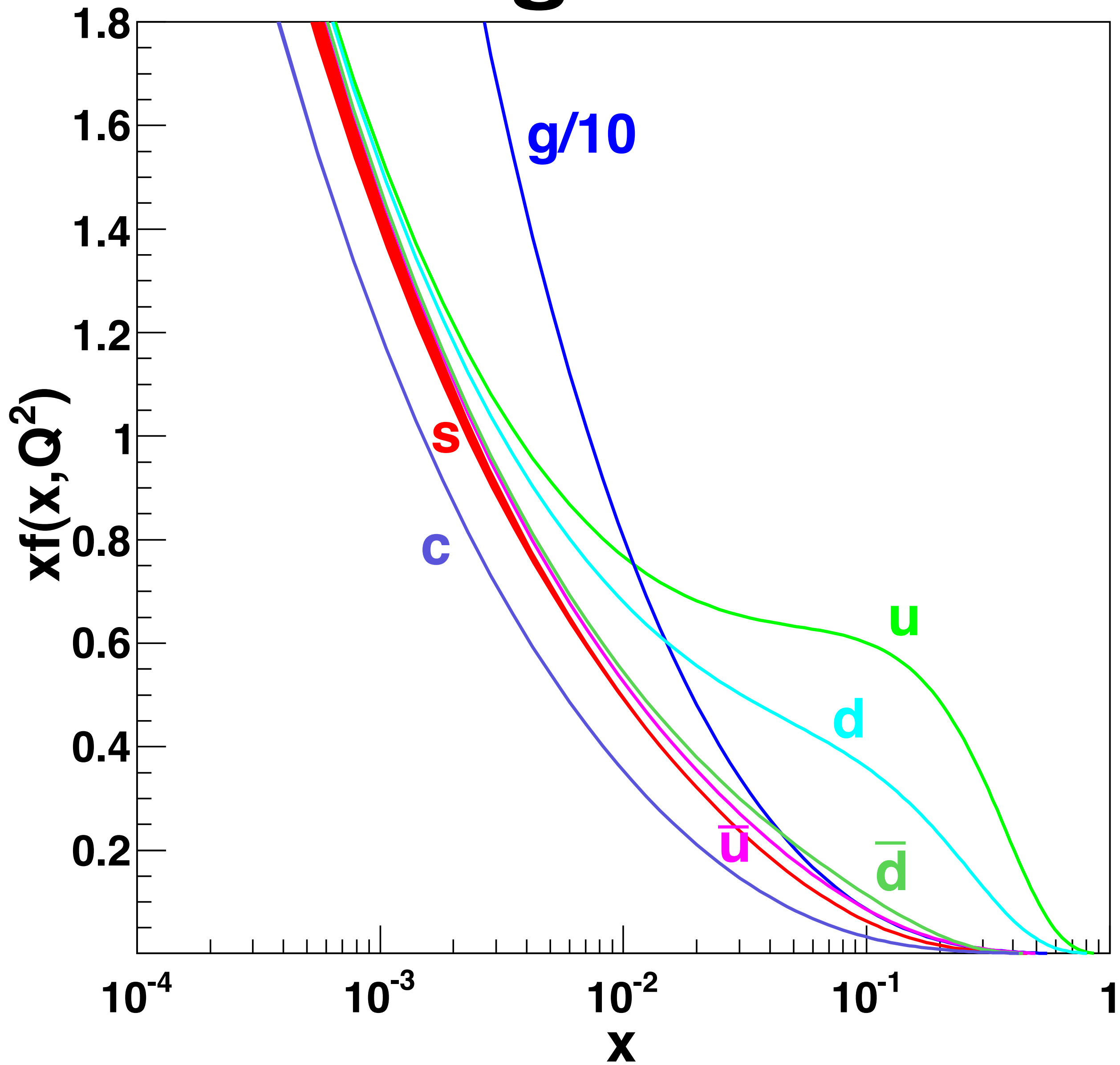


CT10 PDF plots

Low Scale



High Scale

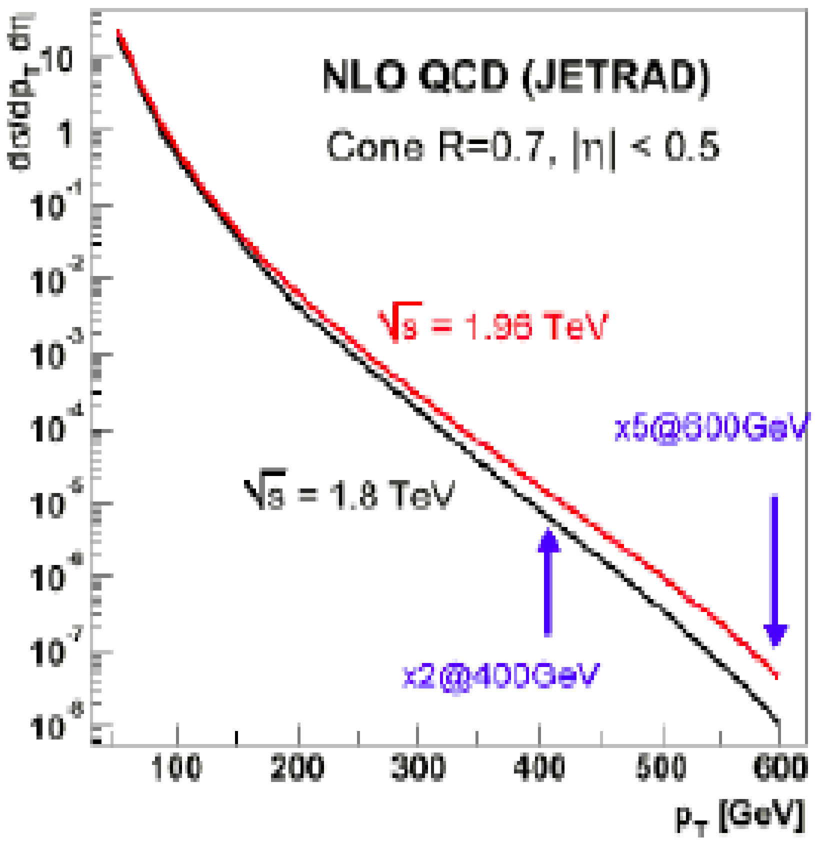


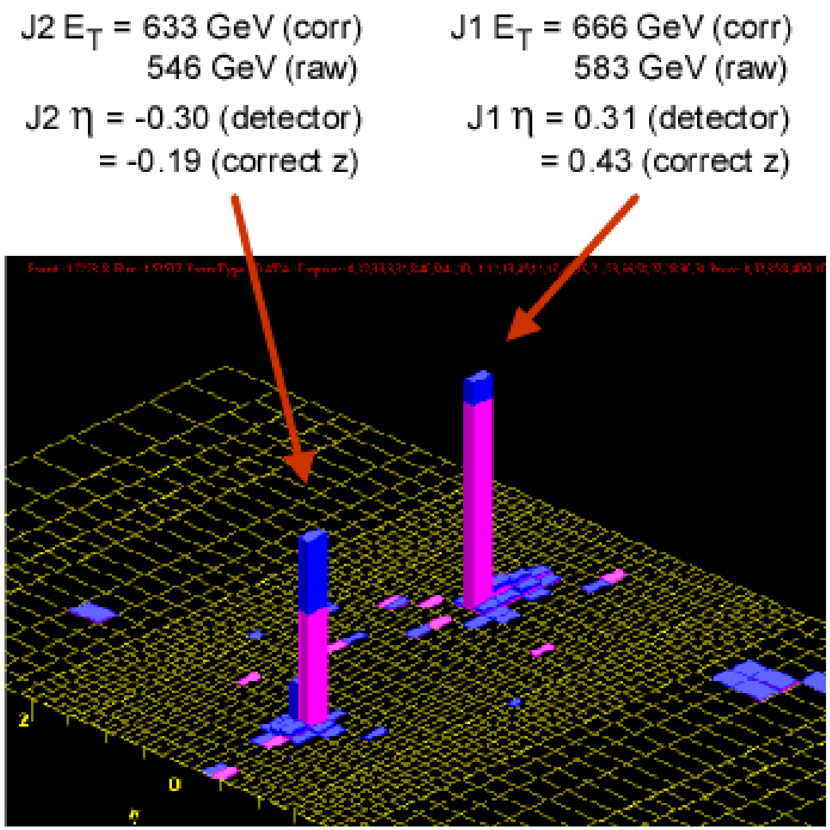
QCD in Hadron Collisions

Jets

Inclusive Jet Production

- Nowhere is the increase in center-of-mass energy more appreciated





CDF Run 2 Preliminary

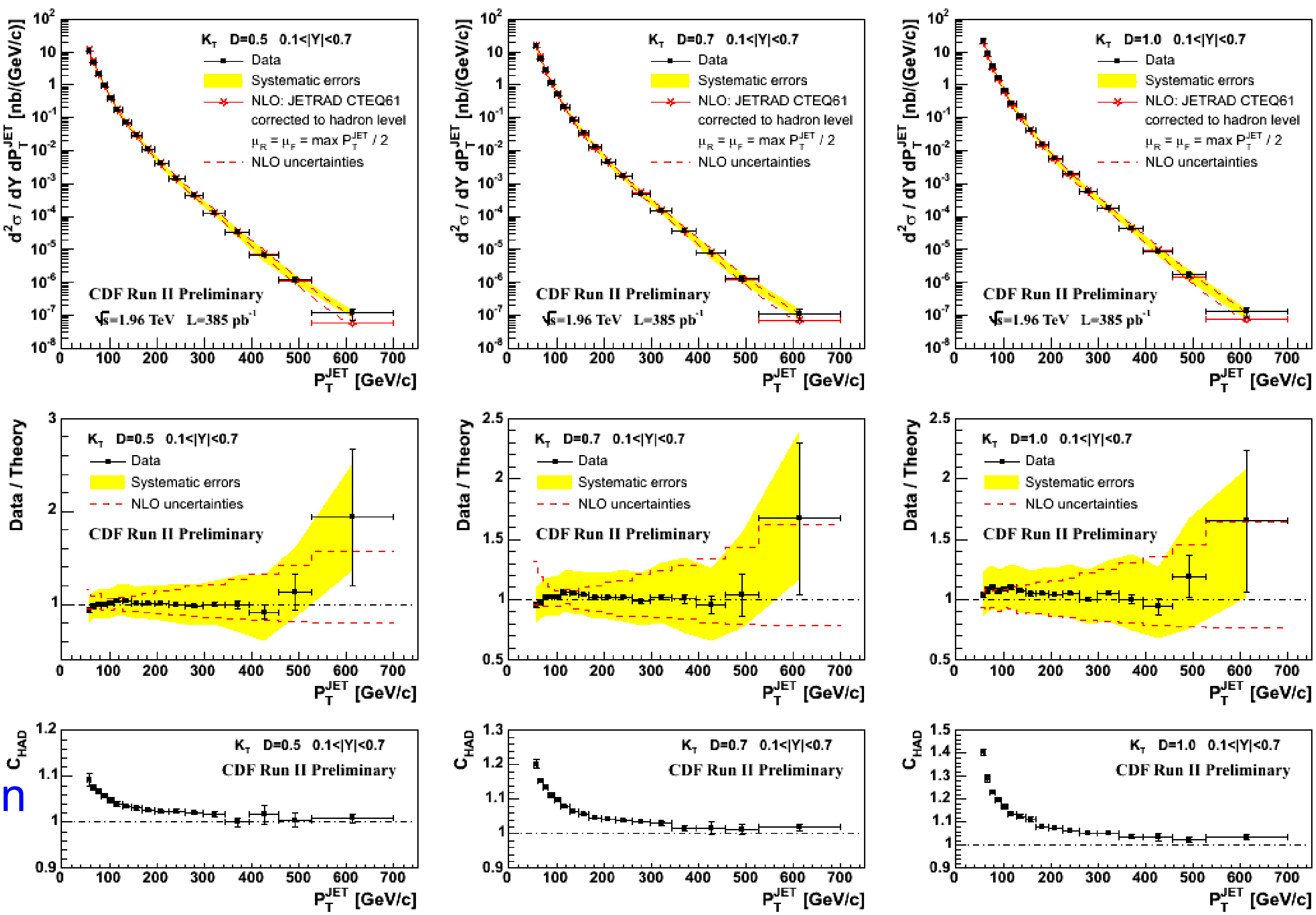
CDF: k_T jet cross section results

$$d_{ij} = \min(P^2_{T,i}, P^2_{T,j}) \frac{\Delta R^2}{D^2}$$

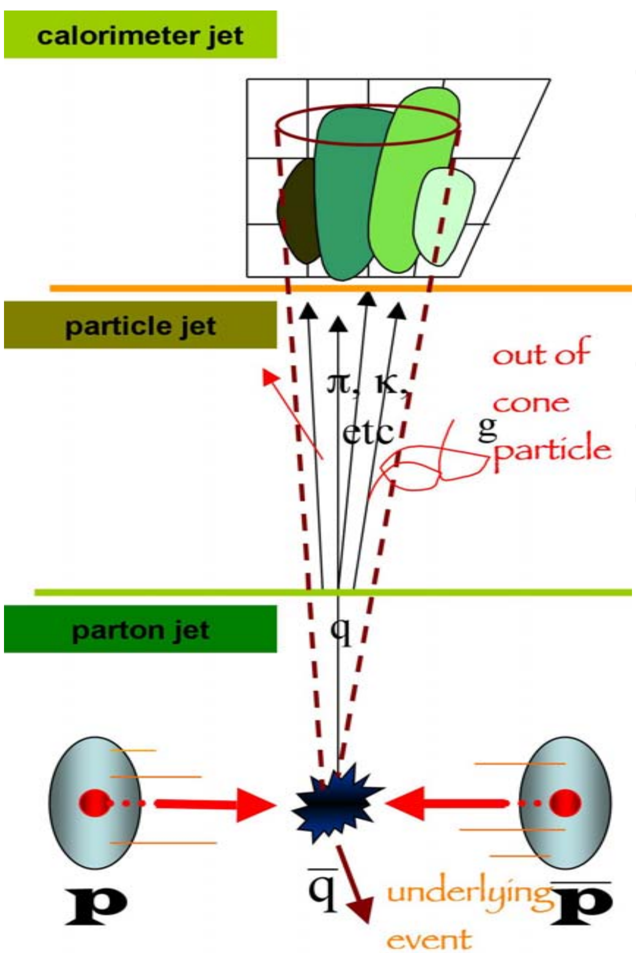
$$d_i = (P_{T,i})^2$$

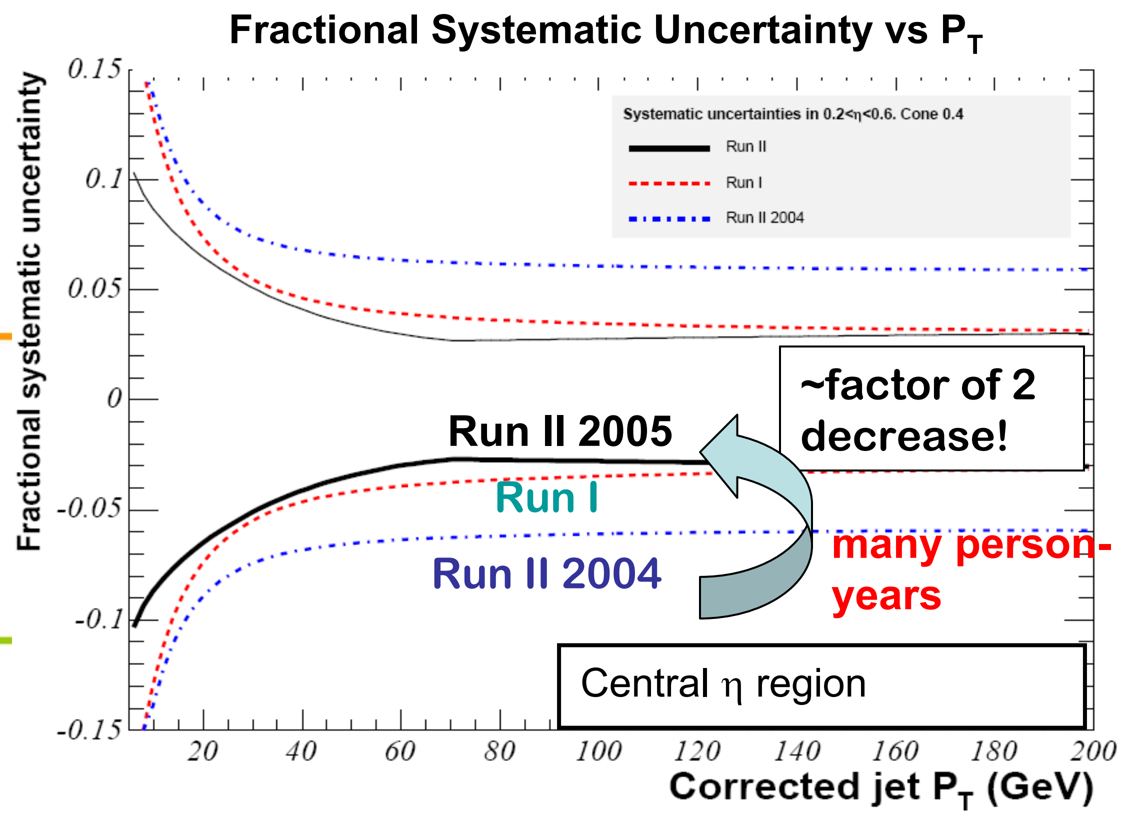
k_T algorithm seems to work well at a hadron collider

underlying + hadronization correction



CDF Jet Energy Scale: from Run-1 to Run-2

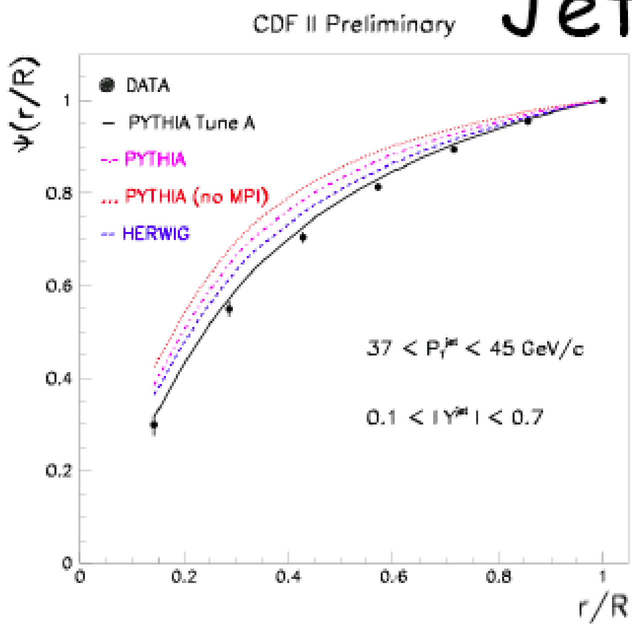


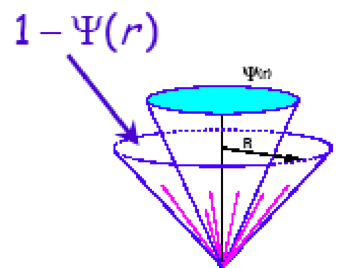


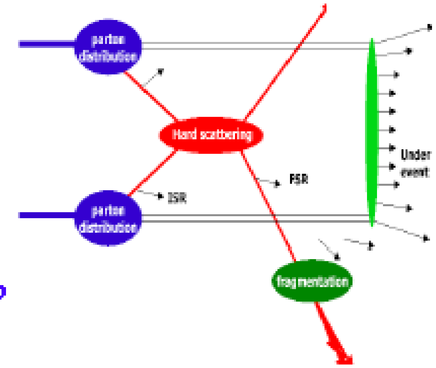
have to correct calorimeter energy depositions for detector, algorithm and physics effects to obtain "true" jet energy

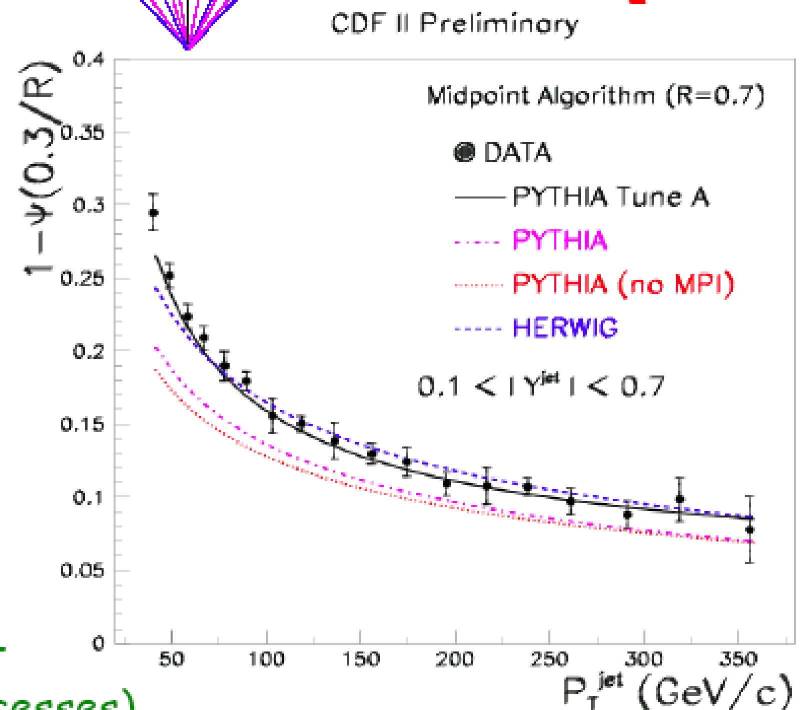
Jet Fragmentation

Jet shapes









- PYTHIA Tune A describes the data (enhanced ISR + MPI tuning)
- PYTHIA default too narrow
- MPI are important at low Pt
- HERWIG too narrow at low Pt

We know how to model the UE at 2 TeV (at least for QCD jet processes)