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Soft Collinear Effective Theory in QCD

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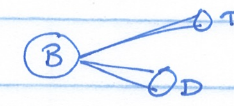
Lecture 1: Intro to EFT

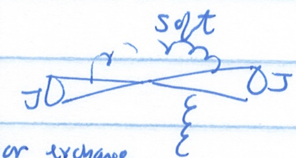
What is SCET?

- Effective field theory of QCD applicable to processes with energetic, lightlike d.o.f. $E \gg m$ (collinear)

  e.g. jets  $m_J^2 \ll E^2$

  light hadrons  $m_\pi^2 \ll E_\pi^2$

  $m_B - m_D \gg m_\pi$

  interacting with soft (wide-angle) radiation 

  $\Rightarrow$ color exchange, hadronization

- invented ~~in~~ early 2000s

  (early LEET had no coll. gluons)

- many new applications to jets | $SCET_I$

  in 2010s. jet substructure | $SCET_+$

  TMD physics | $SCET_{II}$

  small-x | SCET + glaubers

  heavy ions | $SCET_G$

SCET is QCD, organized in power expansion in $\lambda \sim \frac{m}{E}$ formulated as an EFT, aiding in

- factorization of physics at separated scales
- systematic power corrections
- RG evolution to resum logs in pert. th.
- finding universal non-pert. effects.

You can do all this in full QCD, but may be more challenging to organize esp. at higher orders (p.c.'s, log resum.)

SCET has helped achieve:
- ~~[illegible]~~ N^3LL resummations of certain jet observables (event shapes)
- rigorous proof of universal NP corrections in event shapes
- new approaches to resum non-global logs, ~~jet~~ calculate jet substructure, small-x, TMDs, heavy ions...

Plan of Lectures

月 Intro to EFT

火 Jets and SCET building blocks

水 i) $SCET_I$ Lagrangian

ii) factorization in $SCET_I$ (e^+e^- thrust)

木 i) RG evolution & resummation

ii) DIS, pp collisions, beam functions

金 i) $SCET_{II}$ & TMD physics

ii) $SCET_{+,++}$ & jet substructure

I Effective Field Theory

To construct EFT, need
- field content for known (or accessible) degrees of freedom
- any known symmetries
- regularization / renormalization scheme (DR)

If you know the "full" theory, can derive $\mathcal{L}_{EFT}$ by power expansion and matching. If not, unknown coeffs in $\mathcal{L}_{EFT}$ need to be measured.

Every theory is an EFT.

Even Standard Model:

$$\mathcal{L}_{SM} = \underbrace{\sum_i \bar\psi i \not{D} \psi}_{\text{quarks \& leptons}} + \underbrace{\sum \tfrac{1}{4} \mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu}}_{\substack{\text{gauge bosons} \\ SU(3)\times SU(2)\times U(1)}} + \underbrace{\partial_\mu H^\dagger \partial^\mu H - V(H)}_{\text{Higgs}} +$$

$$+ \underbrace{\sum y_{ij} \bar\psi_L^i \psi_R^j H}_{\text{Yukawa}} \qquad (+\text{ gauge fixing})$$

- field content:

	dim	
spin-0	$[H]=1$	$[\mathcal{L}]=4$ (renormalizable)
spin-$\frac{1}{2}$	$[\psi]=\frac{3}{2}$	$\downarrow$ could be complete theory
spin-1	$[A^\mu]=1$	

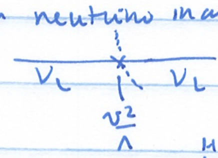
- Lorentz sym.
- gauge sym. $\underbrace{SU(3)\times SU(2)_L \times U(1)_Y}_{\hookrightarrow U(1)_{EM}}$

but we know there is more. SM is an EFT of the higher scale "full" theory:

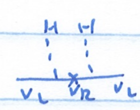
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_5}{\Lambda}(L_i^T C \epsilon H)(H^T \epsilon L) + \sum \frac{c_6}{\Lambda^2} O_i^{(6)}$$

dim 5

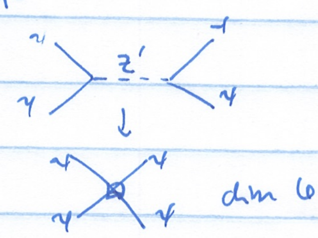
Majorana neutrino mass



generated by heavy ν_R



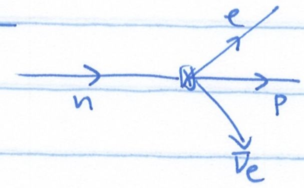
e.g.



w/o knowing full theory you can still compute/constrain its effects at low energy $E \ll \Lambda$.

Fermi theory of weak ints.

β decay :



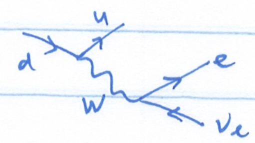
$$\mathcal{L}_{EFT} = \sum_{ij} \frac{c_{ij}}{\Lambda_W^2} \; \bar{p}\Gamma_i n \; \bar{e}\Gamma_j \nu_e \quad + h.c.$$

Γ_i, Γ_j can be $1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \to 16$
S P V A T $\times 16$

$\mathcal{L}$ real, Lorentz inv $\Rightarrow$ $S \cdot S^\beta_\phi$ reduces to 10
$V \cdot / \{ \gamma_A$

C, P, or T inv can reduce further

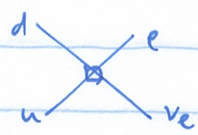
Full theory (SM) : β decay mediated by



$$\mathcal{A} = \frac{g^2}{8} V_{ud} \frac{i}{k^2 - M_W^2} \; \bar{u}\gamma_\mu(1-\gamma_5) d \; \bar{e}\gamma^\mu(1-\gamma_5)\nu_e$$

at low engies $k \ll M_W$: $\frac{i}{k^2-M_W^2} \to -\frac{i}{M_W^2}\left(1 + \frac{k^2}{M_W^2} - \frac{1}{2}\left(\frac{k^2}{M_W^2}\right)^2 + \cdots\right)$

$\underbrace{1}_{\text{lowest order reproduced by}}$



$$\hat{\mathcal{O}} = \bar{u}\gamma_\mu(1-\gamma_5) d \; \bar{e}\gamma^\mu(1-\gamma_5)e$$

$\hat{\mathcal{O}}$: "V-A"

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} \hat{\mathcal{O}}$$

$\Rightarrow$ SM predicts $\frac{G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2}$ & operator content.

(still need non-pert matrix elts.
$\langle p | \bar{u}\gamma_\mu d | n \rangle = g_V \, \bar{u}_p \gamma_\mu u_n$, $g_V \to 1$ EM current
$\langle p | \bar{u}\gamma_\mu\gamma_5 d | n \rangle = g_A \, \bar{u}_p \gamma_\mu \gamma_5 u_n$, $g_A \to \sim 1.27$

we can now do same for BSM eft.

In this kind of EFT, just "integrate out" a heavy particle completely from the theory.

In some EFTs we integrate out only the large momentum ~~modes~~ but keep the particle (HQET, SCET, NRQCD)

simplest

HQET (Heavy Quark Effective Theory)

consider B meson (D)

m_b m_q $k \sim \Lambda_{QCD}$

$P_Q = m_Q v + k$

$k \ll m_Q$

$v = (1, \vec{0})$ in Q rest frame

goal: integrate out m_Q while leaving $k \sim \mathcal{O}(\Lambda_{QCD})$ fluctuations in mom.

what do we expect EFT to look like?

heavy quark propagator $\xrightarrow{P_Q}$

$$i \frac{\not{p}_Q + m_Q}{p^2 - m_Q^2 + i\epsilon} = i \frac{m_Q \not{v} + \cancel{\not{k}} + m_Q}{\cancel{m_Q^2} + 2 m_Q v\cdot k + \cancel{k^2} - \cancel{m_Q^2} + i\epsilon}$$

$$\rightarrow i \frac{1+\not{v}}{2 v\cdot k + i\epsilon}$$

note: indep of m_Q $\boxed{\text{flavor sym.}}$ $c \leftrightarrow b$

proj. tr. $\frac{1+\not{v}}{2}$

gluon vertex: k, γ_μ $\rightarrow \frac{1+\not{v}}{2}\gamma_\mu \frac{1+\not{v}}{2} = \frac{1+\not{v}}{2}\cancel{\frac{1-\not{v}}{2}}\gamma^\mu + \frac{1+\not{v}}{2} v_\mu$

0

$\Rightarrow \gamma_\mu \rightarrow v_\mu$ v_μ $\rightarrow$ simpler! $\boxed{\text{spin sym.}}$)

HQ sym: relates e.g. $B \rightarrow D^{(*)} e\nu$ decay

m_Q dep effects go like $\sim \frac{1}{m_Q}$.

Construct the HQET Lagrangian:

Step 1: take out large mom: $\psi(x) = e^{-im_Q v\cdot x}\, \psi_v(x)$

$\downarrow$ ∂_μ $\to$ $m_Q v$; $\psi_v \to \Lambda_{QCD}$

Step 2: project "large" & "small" comps:

$$h_v = \frac{1+\not{v}}{2}\psi_v \qquad H_v = \frac{1-\not{v}}{2}\psi_v$$

h_v obeys $\not{v}\, h_v = h_v$ (Dirac eq. for fermion w/ mom. $m_Q v$)

end Lect. I

Step 3: expand QCD Lag: $\psi = h_v + H_v$

$$\mathcal{L}_{Dir.} = \bar{\psi}(i\not{D} - m_Q)\psi \qquad \bar{h}_v = \psi_v^\dagger \frac{1+\not{v}^\dagger}{2}\gamma_0 = \bar{\psi}_v \frac{1+\not{v}}{2}$$

$$= (\bar{h}_v + \bar{H}_v)(i\not{D} - m_Q)(h_v + H_v) \qquad = \psi_v^\dagger \gamma_0\left(\frac{1-\not{v}}{2} + 2v_0\right)\psi_v$$

$\neq$

$$= (\bar{h}_v + \bar{H}_v)\, e^{+im_Q v\cdot x}\, (i\not{D} - m_Q)\, e^{-im_Q v\cdot x}\, (h_v + H_v)$$

$$= (\bar{h}_v + \bar{H}_v)\, e^{im_Q v\cdot x} e^{-im_Q v\cdot x}\, (i\not{D} + m_Q\not{v} - m_Q)\,(h_v + H_v)$$

$$(1-\not{v})h_v = 0$$

$$(1-\not{v})H_v = 2H_v$$

$$= (\bar{h}_v + \bar{H}_v)\left[i\not{D}\, h_v + (i\not{D} - 2m_Q)H_v\right]$$

$$\frac{1+\not{v}}{2}\not{D}\frac{1+\not{v}}{2} = v\cdot D \qquad = -\frac{1-\not{v}}{2}\not{D}\frac{1-\not{v}}{2} = -\frac{1-\not{v}}{2}\left(\cancel{\frac{1+\not{v}}{2}}\not{D} - v\cdot D\right)$$

$$\cancel{\frac{1+\not{v}}{2}\gamma^\mu\frac{1-\not{v}}{2} = \frac{1+\not{v}}{2}\left(\frac{1+\not{v}}{2}\gamma^\mu - v^\mu\right) = \frac{1+\not{v}}{2}(\gamma^\mu - v^\mu)}$$

$$= \bar{h}_v\, iv\cdot D\, h_v - \bar{H}_v\,(iv\cdot D + 2m_Q)H_v$$

$\hookrightarrow$ effective heavy mass $2m_Q$ for H_v

$$+ \bar{h}_v\, i\not{D}\, H_v + \bar{H}_v\, i\not{D}\, h_v$$

actually only $\not{D}_\perp$ where $X_\perp^\mu = X^\mu - X\cdot v\, v^\mu$

$$\frac{1+\not{v}}{2}\not{v}\frac{1-\not{v}}{2} = 0$$

Step 4: "integrate out H_v". at tree level just classical eq. of mot:

$$(iv\cdot D + 2m_Q)H_v = i\not{D}_\perp h_v$$

$$\Rightarrow \mathcal{L} = \bar{h}_v\left(iv\cdot D + i\not{D}_\perp \frac{i\not{D}_\perp}{iv\cdot D + 2m_Q}\right)h_v$$

$$= \bar{h}_v\left(iv\cdot D - \frac{1}{2m_Q}\not{D}_\perp\not{D}_\perp + \cdots\right)h_v$$

$$\mathcal{L} = \bar{h}_v\left(iv\cdot D - \frac{D_\perp^2}{2m_Q} - g\frac{\sigma^{\mu\nu}G_{\mu\nu}}{4m_Q}\right)h_v + \cdots$$

reproduces LO Feyn. rules $\longrightarrow$ $\frac{1}{m_Q}$ corrs.