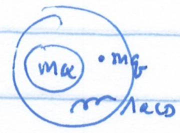
31 MAY 2016 (am) SCET in QCD

Lecture 2: Jets & building blocks of SCET

... finish HQET:



$p_Q = m_Q v + k$ $\quad k \sim \Lambda_{QCD} \ll m_Q$

$$\Longrightarrow \quad i\,\frac{1+\slashed{v}}{2}\,\frac{1}{v\cdot k + i\epsilon} \qquad\qquad \text{soft } k: \quad i g v_\mu T^A$$

construct Lag:

step 1: $\psi(x) = e^{-i m_Q v\cdot x}\,\psi_v(x)$

step 2: $\psi_v = h_v + H_v$

$$h_v = \frac{1+\slashed{v}}{2}\psi_v \qquad H_v = \frac{1-\slashed{v}}{2}\psi_v$$

step 3:

$$\mathcal{L}_{\substack{\text{Dirac}\\ QCD}} = \bar\psi(i\slashed{D} - m_Q)\psi$$
$$= \bar\psi_v\, e^{i m_Q v\cdot x}\,(i\slashed{D} - m_Q)\, e^{-i m_Q v\cdot x}\,\psi_v$$
$$= \bar\psi_v\,(i\slashed{D} + m_Q\slashed{v} - m_Q)\,\psi_v$$

$$(1-\slashed{v})\,h_v = 0$$
$$(1-\slashed{v})\,H_v = 2H_v$$

$$= (\bar h_v + \bar H_v)\left[\, i\slashed{D}\,h_v + (i\slashed{D} - 2m_Q)\,H_v\,\right]$$

$$\frac{1+\slashed{v}}{2}\,i\slashed{D}\,\frac{1+\slashed{v}}{2} = \frac{1+\slashed{v}}{2}\,i v\cdot D \qquad \frac{1-\slashed{v}}{2}\,i\slashed{D}\,\frac{1-\slashed{v}}{2} = \frac{1-\slashed{v}}{2}\,(-i v\cdot D)$$

$$= \bar h_v\, i v\cdot D\, h_v - \bar H_v\,(i v\cdot D + 2m_Q)\,H_v$$
$$+ \bar h_v\, i\slashed{D}\,H_v + \bar H_v\, i\slashed{D}\,h_v \qquad (\bar h_v\, m_Q\, H_v = 0)$$

actually only $\slashed{D}_\perp$ where $x_\perp^\mu = x^\mu - x\cdot v\, v^\mu$

$$\frac{1+\slashed{v}}{2}\slashed{D}\frac{1-\slashed{v}}{2}$$
$$= \frac{1+\slashed{v}}{2}\left(\frac{1-\slashed{v}}{2}\slashed{D} - v\cdot D\right)$$
$$= \frac{1+\slashed{v}}{2}(\slashed{D} - v\cdot D)$$
$$= \frac{1+\slashed{v}}{2}\,\gamma^\mu D_\mu (g_{\mu\nu} - v_\mu v_\nu)$$

$$\frac{1+\slashed{v}}{2}(\slashed{D} - v\cdot D\,\slashed{v} + v\cdot D\,\slashed{v})\frac{1-\slashed{v}}{2}$$
$$= \frac{1+\slashed{v}}{2}\left(\slashed{D}_\perp\frac{1-\slashed{v}}{2} + \frac{\slashed{v}-1}{2}\,v\cdot D\right)$$
$$\downarrow 0$$

step 4: H_v is "heavy", mass $2m_Q$

$\Rightarrow$ integrate out, solve classical eq. of motion

$$\Rightarrow \quad (i v\cdot D + 2m_Q)\,H_v = i\slashed{D}_\perp h_v$$

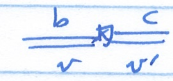
$$\Rightarrow \mathcal{L} = \bar h_v\left(i v\cdot D + i\slashed{D}_\perp\,\frac{1}{i v\cdot D + 2m_Q}\,i\slashed{D}_\perp\right)h_v$$
$$= \bar h_v\left(i v\cdot D - \frac{1}{2m_Q}\slashed{D}_\perp\slashed{D}_\perp + \cdots\right)h_v$$
$$= \bar h_v\left(i v\cdot D - \frac{D_\perp^2}{2m_Q} - g\,\frac{\sigma^{\mu\nu}G_{\mu\nu}}{4m_Q}\right)h_v + \cdots$$
$$\qquad\quad \mathcal{O}(1) \qquad\quad \mathcal{O}(1/m_Q) \qquad\qquad \mathcal{O}\!\left(\tfrac{1}{m_Q^2}\right)$$

$\Rightarrow$ reproduces Feyn rules

(some higher order terms constrained by RPI

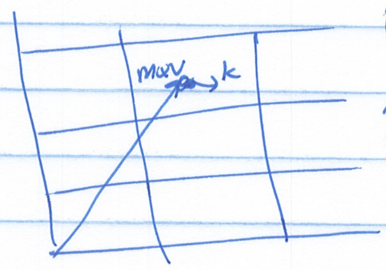
$p_Q = m_Q v + k$ inv. to $v \to v + \frac{\varepsilon}{m_Q}$

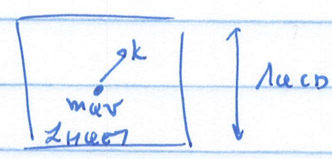
$k \to k - \varepsilon$)

if more than one heavy quark e.g. ($B \to D e \bar\nu$) 

$\mathcal{L}_{HQET} = \sum_v \mathcal{L}_v$

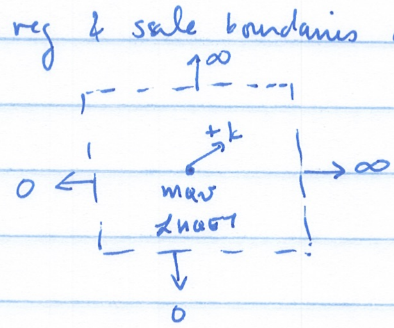
picture in momentum space :



HQET lives in the box 

but cutoffs violate Lorentz & gauge invariance

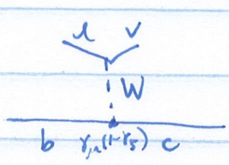
we use dim. reg & scale boundaries away.



$\mathcal{L}_{HQET}$ reproduces QCD in the box for $k \sim \Lambda_{QCD}$ (IR)

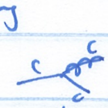
but is bad for large k ~~$k \to \infty$~~ and different UV divergence structure as $k \to \infty$

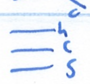
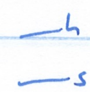
↳ take care of by matching & renormalization

 $\longrightarrow$ 

$\mathcal{L}_{QCD,SM} \longrightarrow C(v \cdot v', \mu) \, \langle O_{vv'} \rangle (\mu)$

UV matching renorm. low-energy ME

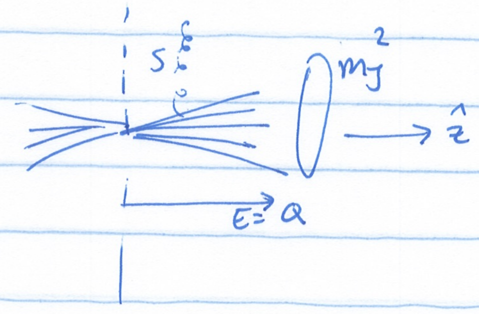
in SCET will find • labels can change 

• multiple scales  HQET 

II Jets & SCET bldg. blocks

Degrees of freedom:

measure jet mass



light-cone coords: $n=(1,\hat{z})$ $\quad n^2=\bar{n}^2=0$

$\bar{n}=(1,-\hat{z})$ $\quad n\cdot\bar{n}=2$

$$p^\mu = \frac{\bar{n}\cdot p}{2}\,n^\mu + \frac{n\cdot p}{2}\,\bar{n}^\mu + p_\perp^\mu$$

$$p^2 = \bar{n}\cdot p\; n\cdot p + p_\perp^2 \qquad p^- = \bar{n}\cdot p = E+p_z$$

$$= p^+p^- - \vec{p}_\perp^{\,2} \qquad p^+ = n\cdot p = E-p_z$$

$$p = (p^-, p^+, p_\perp)$$

measure inv. mass of particles in a hemisphere:

$$p_c^2 = p^+ \underbrace{p^-}_{\sim Q} + p_\perp^2 \sim m^2$$

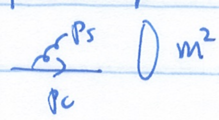
$$\Rightarrow p^+ \sim \frac{m^2}{Q} \qquad \Downarrow\; p_\perp \sim m \qquad p_c \sim \left(Q, \frac{m^2}{Q}, m\right)$$

$$= Q(1,\lambda^2,\lambda) \qquad \lambda = m/Q$$

but I can add: $p_c + p_s$

$$p_s \sim \left(\frac{m^2}{Q}, \frac{m^2}{Q}, \frac{m^2}{Q}\right) = Q(\lambda^2,\lambda^2,\lambda^2)$$

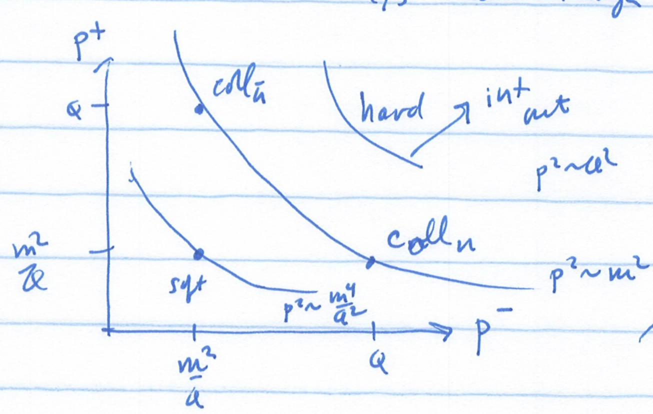
and preserve jet mass $\quad p_J \overset{\text{still}}{\sim} \left(Q, \frac{m^2}{Q}, \frac{m}{Q}\right)$



c, s "talk" through p^+ comp. only

modes in "SCET$_I$":

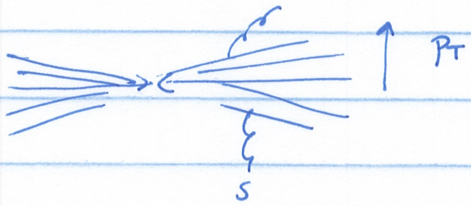
e.g. back-to-back jets



separated in virtuality

dim reg & scale μ alone regulates.

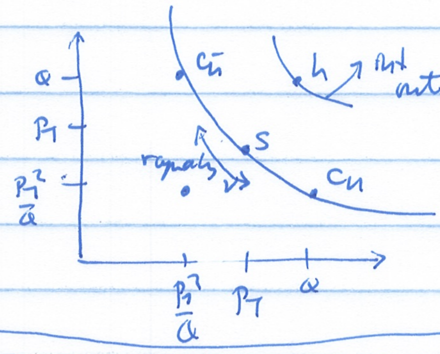
consider measuring jet / beam p_T:



$$p_c = \left(Q, \frac{p_T^2}{Q}, p_T\right)$$
$$= Q(1, \eta^2, \eta) \qquad \eta \sim \frac{p_T}{Q}$$

$$p_s = (p_T, p_T, p_T) = Q(\eta, \eta, \eta)$$
fixed by meas.

modes in "$SCET_{II}$":

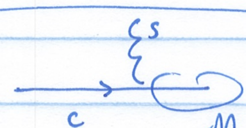


c, s "talk" through $p_\perp$

virtuality μ not enough!

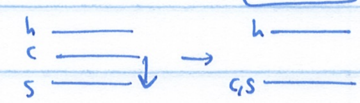
---

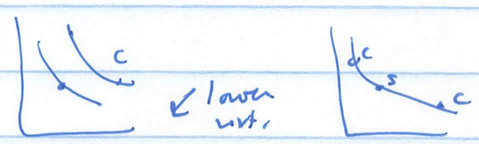
end Lecture 2



off shell in $SCET_{II}$

can match $QCD \to SCET_I \to SCET_{II}$





label collinear momenta:
$$p_c = \tilde{p}_c + k$$
label; residual $\sim Q\lambda^2$

like $m_Q v$ in HQET

but now 2 pieces: $\tilde{p}_c = \bar n \cdot \tilde{p}_c \frac{n}{2} + \tilde{p}_\perp$

$\sim Q$ $\qquad \sim Q\lambda$

collinear fields: $\psi \to \psi_c(x) = \sum_{\tilde p \neq 0} e^{-i\tilde{p}\cdot x}\psi_{n,p}(x)$

↳ sum (not in HQET) needed because of coll. gluons

$$A^\mu \to A_c^\mu = \sum_{\tilde q \neq 0} e^{-i\tilde{q}\cdot x} A^\mu_{n,q}(x)$$

label $\tilde{p} \neq 0$ avoid overlap with soft

$$+ A_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$$