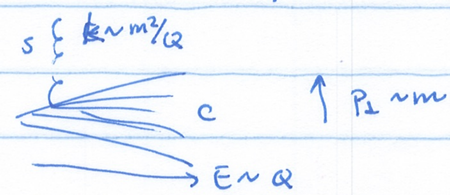
31 MAY 2016 (pm)

## Lecture 3: construction of $SCET_I$ Lagrangian

mimic steps in HQET:

1) factor out large mom. phase
2) project out "large" & "small" spinor components
3) int. out "heavy" field (eq. of mot.)
4) expand $\mathcal{L}$ in powers of $\lambda$
5) new to SCET: coll. Wilson lines, "label operators"

coll & soft d.o.f.'s e.g. in jets




split $\quad p_c = \tilde{p}_c + k$

$k$: ↳ residual $\sim Q\lambda^2$

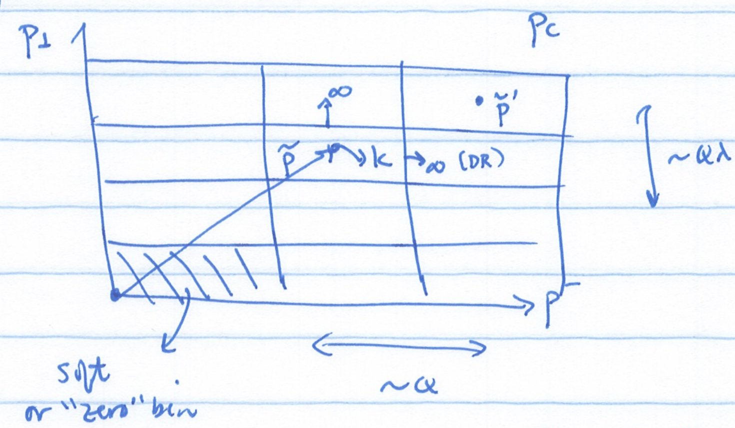
$\tilde{p}_c$ ↓ label $\quad \tilde{p} = \bar{n}\cdot\tilde{p}\,\frac{n^\mu}{2} + \tilde{p}_\perp$

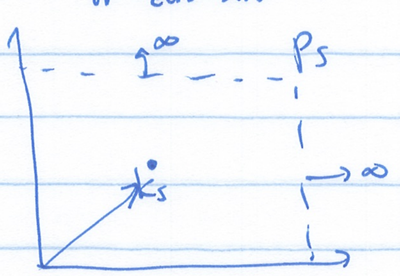
$\bar{n}\cdot\tilde{p}$ ↓ $\mathcal{O}(Q)$ $\qquad \tilde{p}_\perp$ ↓ $\mathcal{O}(Q\lambda)$

like HQET $m v + k$

2 scales in label

momentum space will look like:



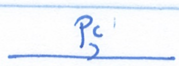




will need "zero bin" subtractions in coll. sector

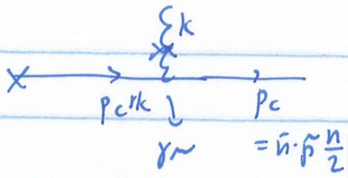
$$\sum_{\tilde{p}\neq 0}\int d^4k \;\longrightarrow\; \int_0 d^4p\,(A_c - A_0)$$

$\sum_{\tilde{p}\neq 0}$: discrete; $\int_0 d^4p$: continuous (DR); $A_c$: coll amp.; $A_0$: $p_c\to 0$ lim of $A_c$

1

coll. prop. 

$= \dfrac{i\,(\tilde{p}\!\!/ + k\!\!/)}{(p+k)^2 + i\epsilon} = \dfrac{i\ \bar{n}\cdot\tilde{p}}{\bar{n}\cdot\tilde{p}\ n\cdot k + \tilde{p}_\perp^2 + i\epsilon}\ \dfrac{n\!\!/}{2}$

soft vertex 

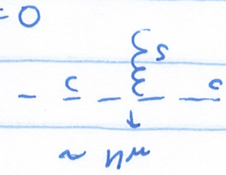
$\dfrac{i\ \bar{n}\cdot\tilde{p}}{\bar{n}\cdot\tilde{p}\ n\cdot k + i\epsilon}\ \dfrac{n\!\!/}{2}\ \gamma_\mu\ \cancel{i\,\bar{n}\cdot\tilde{p}}\ u_n$

$\rightarrow \dfrac{i}{n\cdot k + i\epsilon}\left(-\gamma_\mu \dfrac{n\!\!/}{2} + n_\mu\right) u_n$

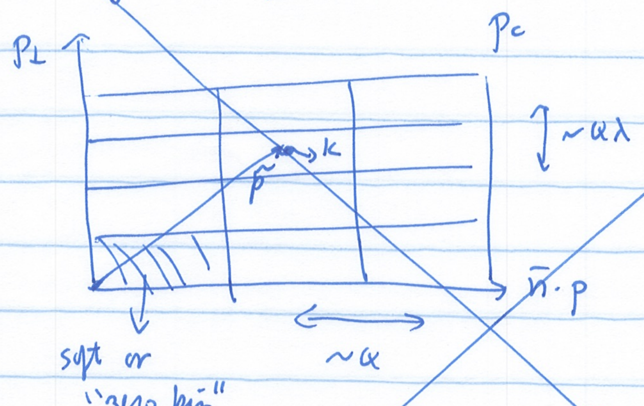
eikonal prop.

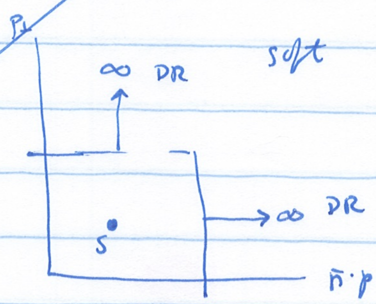
$n\!\!/ u_n = 0$

vertex

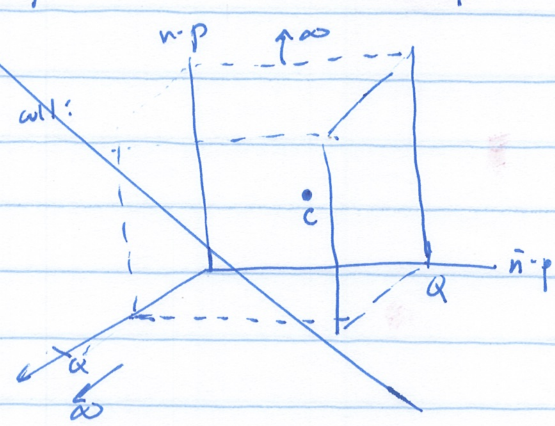



picture of mom. space:




coll:




typically,

$\sum_{\tilde{p}\neq 0} \int d^4k \rightarrow \int d^4 p_c\,(A - A_\phi)$

limit of coll amp as $p_c \to 0$.

"0-bin subtraction"

step 1) $\mathcal{L}_{QCD} = \bar{\psi} i \not{D} \psi$ (quarks)

$\psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \psi_{n,\tilde{p}}(x)$

↳ need sum due to label changing ints. : $\tilde{p} \to \tilde{q}$, $\tilde{p}-\tilde{q}$

gluons $A^{\mu}(x) \to A_c^{\mu}(x) + A_s^{\mu}(x)$

$A_c^{\mu}(x) = \sum_{\tilde{q}} e^{-i\tilde{q}\cdot x} A^{\mu}_{n,q}(x)$

$\Rightarrow \mathcal{L} = \sum_{\tilde{p},\tilde{p}'} \bar{\psi}_{n,p'} e^{i\tilde{p}'\cdot x} i\not{D} e^{-i\tilde{p}\cdot x} \psi_{n,p}$ $\qquad i\not{D} = i\not{\partial} + g(\not{A}_c + \not{A}_s)$

$= \sum_{\tilde{p}\tilde{p}'} e^{i(\tilde{p}'-\tilde{p})\cdot x} \bar{\psi}_{n,p'} (i\not{D} + \tilde{\not{p}}) \psi_{n,p}$

$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$

2) define projectors $P_n = \frac{\not{n}\bar{\not{n}}}{4}$ $\quad P_{\bar{n}} = \frac{\bar{\not{n}}\not{n}}{4}$

$P_n + P_{\bar{n}} = 1$ $\qquad \frac{\not{n}\bar{\not{n}}}{4} + \frac{\bar{\not{n}}\not{n}}{4} = \frac{2n\cdot\bar{n}}{4} = 1$

$P_n P_n = \frac{\not{n}\bar{\not{n}}}{4}\frac{\not{n}\bar{\not{n}}}{4} = \frac{\not{n}\bar{\not{n}}}{16} 2n\cdot\bar{n} = \frac{\not{n}\bar{\not{n}}}{4} = P_n$

$P_{\bar{n}}^2 = P_{\bar{n}}$ $\qquad P_n P_{\bar{n}} = 0$

note

$P_n^{\dagger} = \frac{\bar{\not{n}}^{\dagger}\not{n}^{\dagger}}{4} = \frac{\not{n}\bar{\not{n}}}{4} = P_n$

$P_n^{\dagger}\gamma_0 = \gamma_0 \frac{\bar{\not{n}}\not{n}}{4}$

$\bar{\xi}_n = \bar{\psi}_n \frac{\bar{\not{n}}\not{n}}{4} = \bar{\psi}_n P_{\bar{n}}$

$\psi_{n,p} = \underset{P_n\psi_{n,p}}{\xi_{n,p}} + \underset{P_{\bar{n}}\psi_{n,p}}{\Xi_{n,p}}$

note $\not{n}\xi_{n,p} = 0$ (Dirac eq for $p^{\mu} = \frac{\bar{n}\cdot p}{2} n^{\mu}$)

$$\mathcal{L} = \sum_{\tilde{p}\tilde{p}'} e^{i(\tilde{p}'-\tilde{p})\cdot x} (\bar{\xi}_{n,p'} + \bar{\Xi}_{n,p'})(i\not{D} + \tilde{\not{p}})(\xi_{n,p} + \Xi_{n,p})$$

$\gamma^{\mu} = \bar{\not{n}}\frac{n^{\mu}}{2} + \not{n}\frac{\bar{n}^{\mu}}{2} + \gamma_{\perp}^{\mu}$ $\qquad$ use $\quad P_{\bar{n}}\gamma^{\mu}P_n = \frac{\bar{\not{n}}\not{n}}{4}\gamma^{\mu}\frac{\not{n}\bar{\not{n}}}{4} = \frac{\bar{\not{n}}\not{n}}{4}\bar{\not{n}}\frac{\not{n}\bar{\not{n}}}{4}\frac{n^{\mu}}{2} = P_{\bar{n}}\bar{\not{n}}\frac{n^{\mu}}{2}$

$P_n\gamma^{\mu}P_{\bar{n}} = \frac{\not{n}\bar{\not{n}}}{4}\not{n}\frac{\bar{n}^{\mu}}{2}\frac{\bar{\not{n}}\not{n}}{4} = P_n\frac{\not{n}}{2}\bar{n}^{\mu}$

$P_n\gamma^{\mu}P_n = P_n\gamma_{\perp}^{\mu}P_n$

$P_{\bar{n}}\gamma^{\mu}P_{\bar{n}} = P_{\bar{n}}\gamma_{\perp}^{\mu}P_{\bar{n}}$

$$\Rightarrow \mathcal{L} = \sum_{\tilde{p}\tilde{p}'} e^{i(\tilde{p}'-\tilde{p})\cdot x}\Big[\bar{\xi}_{n,p'}\frac{\bar{\not{n}}}{2} in\cdot D\,\xi_{n,p} + \bar{\Xi}_{n,p'}\frac{\not{n}}{2}(i\bar{n}\cdot D + \bar{n}\cdot\tilde{p})\Xi_{n,p}$$
$$+ \bar{\xi}_{n,p'}(i\not{D}_{\perp} + \tilde{\not{p}}_{\perp})\Xi_{n,p} + \bar{\Xi}_{n,p'}(i\not{D}_{\perp} + \tilde{\not{p}}_{\perp})\xi_{n,p}\Big]$$

3) $\Xi_{n,p}$ is a heavy field w/ "mass" $\bar{n}\cdot\tilde{p} \sim Q$

int. out. : eq of motion

$$(i\bar{n}\cdot D + \bar{n}\cdot\tilde{p})\frac{\not{\bar{n}}}{2}\,\Xi_{n,p} = -(i\not{D}_\perp + \tilde{\not{p}}_\perp)\,\xi_{n,p}$$

$$\frac{\hat{\not{n}}}{2} \qquad\qquad \frac{\hat{\not{n}}}{2}$$

$$\Rightarrow \boxed{\Xi_{n,p} = \frac{i\not{D}_\perp + \tilde{\not{p}}_\perp}{i\bar{n}\cdot D + \bar{n}\cdot\tilde{p}}\,\frac{\not{\bar{n}}}{2}\,\xi_{n,p}}$$

$$\Rightarrow \mathcal{L} = \sum_{p,p'} e^{i(\tilde{p}-\tilde{p}')\cdot x}\,\bar{\xi}_{n,p'}\left[in\cdot D + (i\not{D}_\perp + \tilde{\not{p}}_\perp)\frac{1}{i\bar{n}\cdot D + \bar{n}\cdot\tilde{p}}(i\not{D}_\perp + \tilde{\not{p}}_\perp)\right]\frac{\not{\bar{n}}}{2}\,\xi_{n,p}$$

4) expand in $\lambda$: note $A_c^\mu \sim Q(\overset{-}{1}, \overset{+}{\lambda^2}, \overset{\perp}{\lambda})$ just like their momenta

$A_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$\Rightarrow \quad in\cdot D = in\cdot\partial + gn\cdot A_c + gn\cdot A_s \quad \sim Q\lambda^2$$

$$iD_\perp + \tilde{p}_\perp = \cancel{i\partial_\perp} + \tilde{p}_\perp + g\,n\cdot A_\perp^c \cancel{+ g\,n\cdot A_\perp^s} \quad \sim Q\lambda$$

$$i\bar{n}\cdot D + \bar{n}\cdot\tilde{p} = \cancel{i\bar{n}\cdot\partial} + \bar{n}\cdot\tilde{p} + g\bar{n}\cdot A^c \cancel{+ g\bar{n}\cdot A_s} \quad \sim Q$$

define "label operator" $\mathcal{P}^\mu \phi_{n,p} = \tilde{p}^\mu \phi_{n,p}$

$\mathcal{P}^\mu \phi^\dagger_{n,p} = -\tilde{p}^\mu \phi^\dagger_{n,p}$

and $iD_c^\mu \equiv \mathcal{P}^\mu + gA_c^\mu$

$$\Rightarrow \mathcal{L}_{SCET} = e^{i\mathcal{P}\cdot x}\,\bar{\xi}_{n,p'}\left[in\cdot D + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c\right]\frac{\not{\bar{n}}}{2}\,\xi_{n,p}$$

gives coll. prop

& coll-soft vertex $--\underset{n,p}{\xi}--$

5) can rewrite last term w/ Wilson lines:

$$W_n(x) \equiv P \exp\left( ig \int_{-\infty}^{x} ds\, \bar{n}\cdot A_n(\bar{n}s) \right)$$

obeys $\quad i\bar{n}\cdot D_c W_n = 0$

$$i\bar{n}\cdot D_c (W_n \mathcal{O}) = (\bar{\mathcal{P}} + g\bar{n}\cdot A_n)(W_n \mathcal{O})$$

$$= \underbrace{[i\bar{n}\cdot D_c W_n]}_{=0} \mathcal{O} + W_n \bar{\mathcal{P}} \mathcal{O}$$

$$\Rightarrow \quad i\bar{n}\cdot D_n (W_n \,\bullet) = W_n \bar{\mathcal{P}} (\cdot)$$

$$\Rightarrow \quad W_n^\dagger\, i\bar{n}\cdot D_n W_n = \bar{\mathcal{P}}$$

$$i\bar{n}\cdot D_n = W \bar{\mathcal{P}} W^\dagger$$

$$\Rightarrow \quad \mathcal{L} = e^{-i\mathcal{P}\cdot x} \sum_{pp'} \bar{\xi}_{n,p'} \left( in\cdot D + i\not{D}_\perp^c\, W \frac{1}{\bar{\mathcal{P}}} W^\dagger\, i\not{D}_\perp^c \right) \frac{\not{\bar{n}}}{2} \xi_{n,p}$$

will soft decouple ... next lecture