01 Jun 2016 Lecture 4: factorization in SCET$_I$

(see lecture notes online)

remainder of lectures: ⁎ 4) factorization, Wilson lines, soft NP universality

⁎ { 5) 1-loop matching calc.
6) 1-loop soft func. calc., RG & resummation

{ 7) DIS, pp & beam functions, NP universality
8) SCET$_{II}$, σ, + ...

Yesterday ended with

$$\mathcal{L} = e^{iP\cdot x}\, \bar{\xi}_n \left[in\cdot D + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c \right] \frac{\not{\bar{n}}}{2} \xi_n$$

$$\xi_n = \sum_{\tilde{p}} \xi_{n,\tilde{p}} \qquad A_n = \sum_{\tilde{q}} A_{n,\tilde{q}}$$

$$in\cdot D = in\cdot\partial + gn\cdot A_n + gn\cdot A_s$$

$$iD_c^\mu = P^\mu + gA_n^\mu$$

rewrite last term: use coll. Wilson line

$$W_n(x) = \underbrace{P}_{\text{path ordering}} \exp\left[ig \int_{-\infty}^{x} ds\, \bar{n}\cdot A_n(\bar{n}s) \right]$$

$$i\bar{n}\cdot D_c W_n = 0$$

so $$i\bar{n}\cdot D_c (W_n \mathcal{O}_c) = \cancelto{0}{(i\bar{n}\cdot D_c W_n)}\, \mathcal{O}_c + W_n \bar{P} \mathcal{O}_c$$

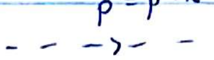
$$\Rightarrow \left[i\bar{n}\cdot D_c W_n \,\cdot\, \right] \overset{\text{operator}}{=} W_n \bar{P}$$

$$\Rightarrow \quad i\bar{n}\cdot D_c = W_n \bar{P} W_n^\dagger$$

$$\Rightarrow \quad \frac{1}{i\bar{n}\cdot D_c} = W_n \frac{1}{\bar{P}} W_n^\dagger$$

$$\Rightarrow \quad \mathcal{L} = e^{iP\cdot x}\, \bar{\xi}_n \left[in\cdot D + i\not{D}_\perp^c W_n \frac{1}{\bar{P}} W_n^\dagger i\not{D}_\perp^c \right] \xi_n$$

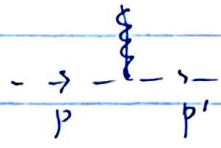
Feynman rules:



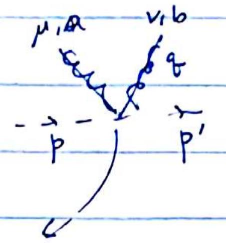
$$= i\frac{\bar{n}\!\!\!/}{2}\,\frac{\bar n\cdot p}{\bar n\cdot p\; n\cdot k + p_\perp^2 + i\epsilon}$$

$$= i g t^a n_\mu \frac{\bar{n}\!\!\!/}{2}$$

($e^{iP\cdot x}$ enforces label mom. conservation at vertices)



$$= i g t^a \left[n_\mu + \frac{\gamma_\mu^\perp p\!\!\!/_\perp}{\bar n\cdot p} + \frac{p\!\!\!/'_\perp \gamma_\mu^\perp}{\bar n\cdot p'} - \frac{p\!\!\!/'_\perp p\!\!\!/_\perp}{\bar n\cdot p\; \bar n\cdot p'}\,\bar n_\mu \right]\frac{\bar{n}\!\!\!/}{2}.$$



$$= \frac{i g^2 t^a t^b}{\bar n\cdot(p-q)}\left[\gamma_\mu^\perp\gamma_\nu^\perp - \frac{\gamma_\mu^\perp p\!\!\!/_\perp}{\bar n\cdot p}\bar n_\nu - \frac{p\!\!\!/'_\perp \gamma_\nu}{\bar n\cdot p'}\bar n_\mu + \frac{p\!\!\!/'_\perp p\!\!\!/_\perp}{\bar n\cdot p\;\bar n\cdot p'}\,\bar n_\mu \bar n_\nu\right]\frac{\bar{n}\!\!\!/}{2}$$

$$+ \frac{1}{\bar n\cdot(q+p')}(a\leftrightarrow b,\ \mu\leftrightarrow\nu)$$

int. out Ξ_n!

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soft-collinear decoupling:

look at $\bar\xi_n (i n\cdot D_s)\frac{\bar{n}\!\!\!/}{2}\xi_n \qquad iD_s^\mu = i\partial^\mu + g A_s^\mu$

~~trans~~ define soft Wilson line $Y_n(x) = P\exp\left[ig\int_{-\infty}^{x} ds\ n\cdot A_s(ns)\right]$

$$i n\cdot D_s Y_n = 0$$

field redefinition : $\xi_n(x) = Y_n(x)\,\xi_n^{(0)}(x) \qquad A_n(x) = Y_n A_n^{(0)} Y_n^\dagger$

$$\Rightarrow W_n = Y_n W_n^{(0)} Y_n^\dagger$$

$$\Rightarrow \bar\xi_n\, i n\cdot D_s \frac{\bar{n}\!\!\!/}{2}\xi_n \to \bar\xi_n^{(0)} Y_n^\dagger\, i n\cdot D_s\, Y_n \frac{\bar{n}\!\!\!/}{2}\xi_n^{(0)}$$

$$= \bar\xi_n^{(0)} Y_n^\dagger Y_n\, i n\cdot\partial \frac{\bar{n}\!\!\!/}{2}\xi_n^{(0)}$$

$$= \bar\xi_n^{(0)}\, i n\cdot\partial\, \frac{\bar{n}\!\!\!/}{2}\xi_n^{(0)}$$

no soft gluons!

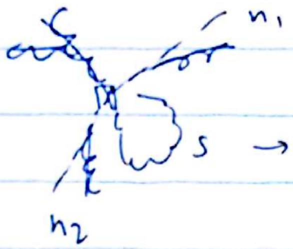
similarly $\bar\xi_n\, g\bar n\cdot A_n \xi_n \to \bar\xi_n^{(0)} Y_n^\dagger Y_n\, \bar n\cdot A_n^{(0)} Y_n^\dagger Y_n\, \xi_n^{(0)} = \bar\xi_n^{(0)}\bar n\cdot A_n^{(0)}\xi_n^{(0)}$

etc.

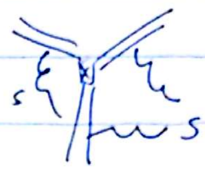
similarly in coll. gluon Lag.

This means $\mathcal{L}_{SCET} = \sum_n \mathcal{L}^{(0)}_{c,n} + \mathcal{L}_s$

all decoupled



$\rightarrow$



$\rightarrow$ cannot resolve collinear splittings
only total n_i & color (3 or 8)

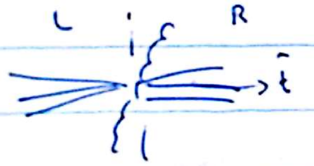
in fact $\mathcal{L}^{(0)}_{c,n}$ is just $\mathcal{L}_{QCD}$ boosted in n direction

so $\mathcal{L}_{SCET}$ is sum of decoupled copies of QCD in each coll. & soft sector, but at different scales . . .

2-jet factorization

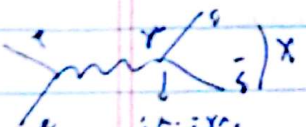
Consider a measurement of "thrust" in e^+e^-:

$e^+e^- \rightarrow X$



$T = \frac{1}{Q} \max_{\hat{t}} \sum_{i \in X} |\hat{t}\cdot\vec{p}_i| = \frac{1}{Q}(\vec{p}_z^{\,L} + \vec{p}_z^{\,R})$

$\rightarrow 1$ for back-to-back $\overset{\longleftrightarrow}{\frac{Q}{2}\ \ \frac{Q}{2}}$



use $\tau = 1 - T \rightarrow 0$

$j^\mu = \bar{q}\gamma^\mu q$

$$\frac{d\Gamma}{d\tau} = \frac{1}{2Q^2} \sum_X |\langle X|\bar{q}\gamma^\mu q|0\rangle L_\mu|^2 (2\pi)^4\delta^4(Q - p_X)\,\delta(\tau - \tau(X))$$

lepton tensor

operator matching $j^\mu = \bar{q}\gamma^\mu q \longrightarrow j^\mu_{SCET} = \sum_{n_1 n_2} C_2(\tilde{p}_1, \tilde{p}_2, \mu)\, \bar{\xi}_{n_2,p_2}\, \Gamma^\mu\, \xi_{n_1,p_1}$

means not just $\bar{\xi}\xi$

$[\bar{\xi}_{n_2} W_{n_2}]_{p_2} \Gamma^\mu [W^\dagger_{n_1} \xi_{n_1}]_{p_1}$

$\chi_n = W^\dagger_n \xi_n$

actually after field redef: $, \bar{\chi}_{n_2,p_2} Y^\dagger_{n_2} \Gamma^\mu Y_{n_1} \chi_{n_1,p_1}$

$j^\mu \rightarrow C_2(\mu) O_2(\mu) = \bar{\chi}_{n_2} Y^\dagger_{n_2} C(\mathcal{P}_{n_1}, \mathcal{P}^\dagger_{n_2}, \mu) Y_{n_1} \chi_{n_1}$

$$\sum_{X_n X_{\bar n} X_s} |X_n X_{\bar n} X_s\rangle\langle X_n X_{\bar n} X_s|$$

$$\Rightarrow \frac{d\Gamma}{d\tau} = \frac{1}{2Q^2} L_\mu L_\nu^* \sum_X \langle 0|\, \bar\chi_{n_1} Y_{n_1}^\dagger\, C(P_1^+, P_2, \mu)\, \Gamma^\nu\, Y_{n_2}\chi_{n_2}|X\rangle$$
$$\times \langle X|\, \bar\chi_{n_2} Y_{n_2}^\dagger\, C(P_1, P_2, \mu)\, \Gamma^\mu\, Y_{n_1}\chi_{n_1}|0\rangle$$
$$\times (2\pi)^4 \delta^4(Q - p_X)\ \delta\!\left(\tau - 1 + \frac{|p_z^L| + |p_z^R|}{Q}\right)$$

n_1, n_2 back-to-back

$$(Q, \vec 0) = p_{n_1} + p_{n_2} + p_s$$

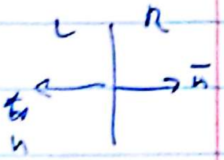
$$(Q, +Q, \vec 0_\perp) = \underset{Q\ \ Q\lambda^2\ \ Q\lambda}{(p_1^-, p_1^+, p_1^\perp)} \Rightarrow \tilde P_1^- = Q$$
$$+ \underset{Q\lambda^2\ \ Q\ \ Q\lambda}{(p_2^-, p_2^+, p_2^\perp)} \qquad \tilde P_2^+ = Q$$
$$\tilde P_1^\perp = -\tilde P_2^\perp \to 0$$
$$+ \underset{Q\lambda^2}{(p_s^+, p_s^-, p_s^\perp)}$$

residual & soft momenta unconstrained. $(k \to \infty)$
(by $\delta^4(Q-p_X)$, anyway)

~~$p_{n_1} = \tilde P_1 + k_1$~~

measurement: $\tau = 1 - \frac{p_z^L + p_z^R}{Q}$

$$Q\tau = Q - p_z^L - p_z^R \qquad p_z = \frac{p^- - p^+}{2}$$
$$= Q - (p_{c_1}^L + p_{c_2}^L + p_s^L)$$
$$- (p_{c_1}^R + p_{c_2}^R + p_s^R)$$
$$= Q - \left(\frac{p_L^- - p_L^+}{2}\right) - \left(\frac{p_R^+ - p_R^-}{2}\right)$$
$$= Q - \Big(\tilde P_1^- -$$



split measurement into coll & soft parts:

$$\tau = \tau_n + \tau_{\bar n} + \tau_s$$
$$\int d\tau_n d\tau_{\bar n} d\tau_s\, \delta(\tau - \tau_n - \tau_{\bar n} - \tau_s)$$

$$\frac{d\Gamma}{d\tau} = \frac{1}{2Q^2} L_\mu L_\nu\, \gamma_\mu^{\perp\alpha\beta} \gamma_\nu^{\perp\delta\epsilon} \sum_{X_{\bar n}} \langle 0|\bar\chi_{\bar n}^\delta |X_{\bar n}\rangle\langle X_{\bar n}|\chi_{\bar n}^\beta|0\rangle\, \delta(\tau_{\bar n} - \tau(X_{\bar n}))$$
$$\times \sum_{X_n} \langle 0|\chi_n^\alpha |X_n\rangle\langle X_n|\bar\chi_n^\epsilon|0\rangle\, \delta(\tau_n - \tau(X_n))$$
$$\times \sum_{X_s} \langle 0|\, Y_n^\dagger Y_{\bar n}\, |X_s\rangle\langle X_s|\, Y_{\bar n}^\dagger Y_n\,|0\rangle\, \delta(\tau_s - \tau(X_s))$$
$$\times C^*(Q^2, \mu)\, C(Q^2, \mu)$$

4

$$\tau(X_{n\bar{n}}) \approx Q\tau = Q - \frac{(\overset{Q}{\overset{\shortparallel}{p_L^+}} - p_L^-)}{2} - \frac{(\overset{Q}{\overset{\shortparallel}{p_R^-}} - p_R^+)}{2}$$

$$= p_L^- + p_R^+$$ (small compn. rel. + s/A)

$$\Rightarrow \frac{d\sigma}{d\tau} = \frac{1}{4Q^2} L^2 \gamma_\mu^{\perp\alpha\beta}\gamma_\perp^{\mu\,\delta\epsilon} \int d\tau_n d\tau_{\bar{n}} d\tau_s\, \delta(\tau-\tau_n-\tau_{\bar{n}}-\tau_s)$$

$$\times \sum_{X_{\bar{n}}} \langle 0|\bar{\chi}_{\bar{n}}^{\delta}|X_{\bar{n}}\rangle\langle X_{\bar{n}}|\chi_{\bar{n},\alpha}^{\beta}|0\rangle\, \delta\!\left(\tau_{\bar{n}} - \frac{p_{X_{\bar{n}}}^-}{Q}\right)$$

$$\times \sum_{X_n} \langle 0|\chi_n^{\alpha}|X_n\rangle\langle X_n|\bar{\chi}_n^{\epsilon}|0\rangle\, \delta\!\left(\tau_n - \frac{p_{X_n}^+}{Q}\right)$$

$$\times \sum_{X_s} \langle 0|Y_n^\dagger Y_{\bar{n}}|X_s\rangle\langle X_s|Y_{\bar{n}}^\dagger Y_n|0\rangle\, \delta\!\left(\tau_s - \frac{k_{X_s}^{R+}}{Q} - \frac{k_{X_s}^{L-}}{Q}\right)$$

$$\times |C|^2$$

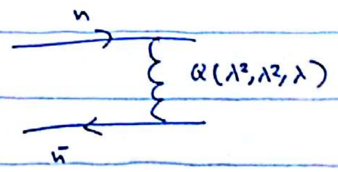
$$\rightarrow \sigma_0 \cdot H(Q^2,\mu) \int d\tau_n d\tau_{\bar{n}} d\tau_s\, \delta(\tau-\tau_n-\tau_{\bar{n}}-\tau_s)$$

$$\times J_n(\tau_n,\mu)\, J_{\bar{n}}(\tau_{\bar{n}},\mu)\, S(\tau_s,\mu)$$

$\downarrow$

1-loop calc. & RGE tomorrow.

subtlety : Glaubers



off-shell
can violate fact.
can include in SCET.

$$\frac{1}{N_c}\mathrm{Tr}\sum_{X_n}\langle 0|\chi_{n,\alpha}^{\beta}|X_n\rangle\langle X_n|\bar{\chi}_n^{\epsilon}|0\rangle\,\delta\!\left(\tau_n - \frac{p_{X_n}^+}{Q}\right) = J(Q^2\tau_n,\mu)\left(\frac{\not{n}}{2}\right)^{\beta\epsilon}$$

$\downarrow$ $p^- p^+$

sim. for $\bar{n}$

$$\frac{1}{N_c}\mathrm{Tr}\sum_{X_s}\langle 0|Y_n^\dagger Y_{\bar{n}}|X_s\rangle\langle X_s|Y_{\bar{n}}^\dagger Y_n|0\rangle\,\delta\!\left(\tau_s - \frac{k_s^{R+}}{Q} - \frac{k_s^{L-}}{Q}\right) = S(k_s^{L-}+k_s^{R+},\mu)$$

$$= S(Q\tau,\mu)$$

$\downarrow$ $k_L^- + k_R^+$

	p^2
H	Q^2
J	$Q^2\tau$
S	$Q^2\tau^2$