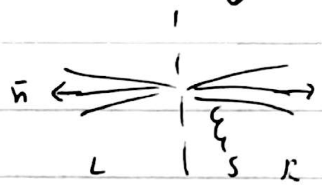
02 JUN 2016 Lecture 5–6: Matching QCD → SCET

e^+e^- thrust



$\tau = 1 - T$

$T = \frac{1}{Q} \max_{\hat{t}} \sum_{i \in X} |\hat{t} \cdot \vec{p}_i|$

$= \frac{1}{Q}\left(|\vec{p}_L^{\,z}| + |\vec{p}_R^{\,z}|\right)$

$$\frac{d\sigma}{d\tau} = \frac{1}{2Q^2} \sum_X |\langle X | j^\mu | 0 \rangle L_\mu|^2 (2\pi)^4 \delta^4(Q - p_X)\, \delta(\tau - \tau(X))$$

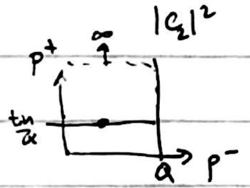
$$j^\mu_{QCD} = \bar{q}\gamma^\mu q \longrightarrow \sum_{\substack{n_1 n_2 \\ \tilde{p}_1 \tilde{p}_2}} C_2(\tilde{p}_1, \tilde{p}_2, \mu)\, O^\mu_{2\, n_1 n_2}(\tilde{p}_1, \tilde{p}_2)$$

$$O^\mu_{2} = \bar{\chi}_{n_1 \tilde{p}_1} \gamma^\mu_\perp \chi_{n_2 p_2} \xrightarrow{BPS} \bar{\chi}_{n_1} Y^\dagger_{n_1} \gamma^\mu_\perp Y_{n_2} \chi_{n_2}$$

C_2: compute today

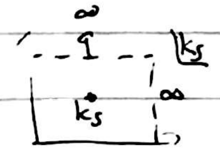
$$\Rightarrow \frac{d\sigma}{d\tau} = \sigma_0\, H(Q^2, \mu) \int dt_n\, dt_{\bar{n}}\, dk_s\, \delta\left(\tau - \frac{t_n + t_{\bar{n}}}{Q^2} - \frac{k_s}{Q}\right) J_n(t_n) J_{\bar{n}}(t_{\bar{n}}, \mu) \times S(k_s, \mu)$$

$H = |C_2|^2$



$$J_n(t_n) = \sum_{X_n} \frac{1}{N_C} \mathrm{Tr} \langle 0 | \chi_n | X_n \rangle \langle X_n | \bar{\chi}_n | 0 \rangle\, \delta(t_n - Q p^+_{X_n}) \times \delta(Q - p^-_{X_n})\, \delta^2(p^\perp_{X_n})$$

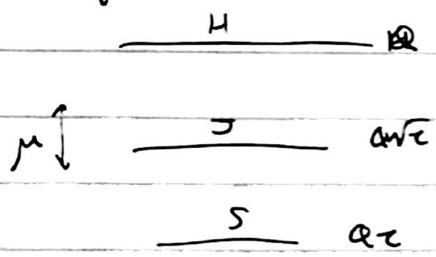
sim. $J_{\bar{n}}$



$$S(k_s, \mu) = \sum_{X_s} \frac{1}{N_C} \mathrm{Tr} \langle 0 | \bar{T}[Y^\dagger_{\bar{n}} Y^\dagger_n] | X_s \rangle \langle X_s | T[Y_n Y_{\bar{n}}] | 0 \rangle \times \delta(k_s - p^{L\,-}_{X_s} - p^{R\,+}_{X_s})$$

↳ 2nd lecture today.

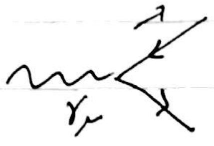
First, compute hard matching.



MES $\langle \bar{q}\gamma^\mu q\rangle_{QCD}$ should agree with $C_2 \cdot \langle O_2\rangle_{SCET}$

choose any ext. states w/ overlap w/ operators.

$\langle q_n \bar{q}_{\bar{n}} | j^\mu | 0\rangle$



$p_{\bar{n}} = \frac{Q}{2}\bar{n}$

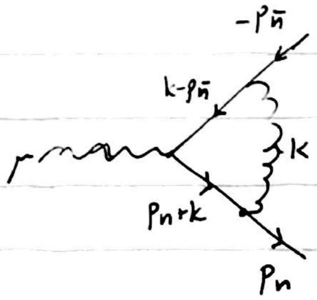
$p_n = \frac{Q}{2} n$

$p_n\cdot p_{\bar{n}} = \frac{Q^2}{4} n\cdot\bar{n} = \frac{Q^2}{2}$

tree : $\bar{u}_n \gamma_\mu^\perp v_{\bar{n}}$ $\qquad \not{n}\, u_n = 0 \qquad \not{\bar{n}}\, v_{\bar{n}} = 0$

SCET : $\langle q_n \bar{q}_{\bar{n}} | \bar{\chi}_n \Gamma^\mu \chi_{\bar{n}} | 0\rangle$ = same $\Rightarrow C_2^{tree} = 1.$

1-loop:



$$= \bar{u}_n\, ig\mu^\epsilon \gamma^\alpha t^a \int\frac{d^Dk}{(2\pi)^D} \frac{i(\not{p}_n+\not{k})}{(p_n+k)^2+i\epsilon}\gamma^\mu \frac{i(\not{k}-\not{p}_{\bar{n}})}{(k-p_{\bar{n}})^2+i\epsilon}\, ig\mu^\epsilon t^a \gamma_\alpha\, v_{\bar{n}} \times \frac{-i}{k^2+i\epsilon}$$

$$= -ig^2\mu^{2\epsilon} C_F \int d^Dk \frac{\bar{u}_n\gamma^\alpha(\not{p}_n+\not{k})\gamma^\mu(\not{k}-\not{p}_{\bar{n}})\gamma_\alpha v_{\bar{n}}}{(k^2+i\epsilon)(k^2+2p_n\cdot k+i\epsilon)(k^2-2p_{\bar{n}}\cdot k+i\epsilon)}$$

numerator: $\bar{u}_n\left[-2(\not{k}-\not{p}_{\bar{n}})\gamma^\mu(\not{k}+\not{p}_n) + 2\epsilon(\not{k}+\not{p}_n)\gamma^\mu(\not{k}-\not{p}_{\bar{n}})\right]v_{\bar{n}}$

$$= \bar{u}_n\Big[2(\not{k}-\not{p}_{\bar{n}})(\not{k}+\not{p}_n)\gamma^\mu - 4\underbrace{(\not{k}-\not{p}_{\bar{n}})}_{0}(k^\mu+p_n^\mu) + 2\epsilon\not{k}\gamma^\mu\not{k}\Big]v_{\bar{n}}$$

$$= \bar{u}_n\Big[\Big(2k^2 + \cancel{2\not{k}\not{p}_{\bar{n}}} - 4p_{\bar{n}}\cdot k - 2\cancel{\not{p}_n\not{k}} + 4p_n\cdot k + 2\cancel{\not{p}_n\not{p}_{\bar{n}}} - 4p_n\cdot p_{\bar{n}}\Big)\gamma^\mu - 4(k^\mu+p_n^\mu)\not{k} + 2\epsilon(-k^2\gamma^\mu + 2\not{k}k^\mu)\Big]v_{\bar{n}}$$

$$= \bar{u}_n\Big[(2k^2 - 2Q\bar{n}\cdot k + 2Qn\cdot k - 2Q^2)\gamma^\mu - (4-4\epsilon)k^\mu\not{k} - 4p_n^\mu\not{k} - 2\epsilon k^2\gamma^\mu\Big]v_{\bar{n}}$$

$$V^{(1)} = -2ig^2\mu^{2\epsilon} C_F\, \bar{u}_n \int d^Dk \Big[\big((1-\epsilon)k^2 - Qk^- + Qk^+ + Q^2\big)\gamma_\perp^\mu - \big((2-2\epsilon)k^\mu + 2p_n^\mu\big)\not{k} \Big]$$

$$\times \frac{1}{k^2+i\epsilon}\,\frac{1}{k^2+Qn\cdot k+i\epsilon}\,\frac{1}{k^2-Q\bar{n}\cdot k+i\epsilon}$$

$\downarrow$ result

$$\underset{\overline{MS}}{=} \frac{\alpha_s C_F}{4\pi}\bar{u}_n\gamma_\perp^\mu v_n \left[\cancel{\frac{1}{\epsilon_{UV}}} - \frac{2}{\epsilon_{IR}^2} - \frac{2\ln\frac{\mu^2}{-Q^2} + \cancel{4}^{\,3}}{\epsilon_{IR}} - \ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

$\hookrightarrow$ could use another IR regulator e.g. $\frac{1}{\epsilon_{IR}} \to \ln\frac{p^2}{Q^2}$

$$\left(V^{(0)}+V^{(1)}\right)\times\frac{1}{(\sqrt{Z_2})^2} \qquad \frac{1}{Z_2} = 1 - \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)$$

j^μ vertex not renormalized due to EM current conservation.

graphs in EFT: n-coll

(W.f. remain same as in QCD)

in dim reg actually all scaleless $\Rightarrow 0$

$$A_n + A_{\bar{n}} + A_s \propto \left(\frac{2}{\epsilon_{UV}^2} - \frac{2}{\epsilon_{IR}^2} + \left(3 - \ln\frac{\mu^2}{-Q^2}\right)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)\right)$$

$$\Rightarrow \quad Z_{O_2} = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{2}{\epsilon_{UV}^2} - \frac{3}{\epsilon_{UV}} - \frac{2}{\epsilon_{UV}}\ln\frac{\mu^2}{-Q^2}\right)$$

$$C_2 = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right)$$

more instructive to put in explicit IR regulator.

$p_n = Q\frac{n}{2} + \frac{p^2}{Q}\bar{n}$
$p_{\bar{n}} = Q\frac{\bar{n}}{2} + \frac{p^2}{Q}n$ $\leftarrow$ — e.g. keep quark off-shellness $p^2 \neq 0$ in denominators:

$$V_{\text{tree}} + \underset{p^2\neq 0}{\overset{p^2\neq 0}{\text{loop}}} = \bar{u}\gamma_\perp^\mu v\left\{1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \ln\frac{Q^2}{\mu^2} - 2\ln^2\frac{p^2}{Q^2} - 4\ln\frac{p^2}{Q^2} - \frac{2\pi^2}{3}\right]\right\}$$

$$\times\frac{1}{(\sqrt{Z_2})^2} \to \left[1 - \frac{\alpha_s}{4\pi}C_F\left[\frac{1}{\epsilon_{UV}} + 1 - \ln\frac{-p^2}{\mu^2}\right]\right]$$

$$V_{QCD} = \bar{u}\gamma_\perp^\mu v\left\{1 + \frac{\alpha_s C_F}{4\pi}\left[-\ln\frac{Q^2}{\mu^2} - 2\ln^2\frac{p^2}{Q^2} - 4\ln\frac{p^2}{Q^2} + \ln\frac{-p^2}{\mu^2} - 1 - \frac{2\pi^2}{3}\right]\right\}$$

$\ln\frac{+p^2}{Q^2}$

$-3\ln\frac{p^2}{Q^2}$

Scanned by CamScanner

n-coll:

$$p_n = Q\frac{n}{2} + \frac{p^2}{\alpha}\frac{\bar n}{2} \qquad p_{\bar n} = Q\frac{\bar n}{2} + \frac{p^2}{\alpha}\frac{n}{2}$$

$$\Rightarrow \quad (k+p_n)^2 = k^2 + Q\, n\cdot k + \frac{p^2}{\alpha}\bar n\cdot k + p^2 \xrightarrow{\text{soft}} Q n\cdot k + p^2$$

$$(k+p_{\bar n})^2 = k^2 + Q\bar n\cdot k + \frac{p^2}{\alpha} n\cdot k + p^2 \qquad Q\bar n\cdot k + p^2$$

$\boxed{\bar n \text{ soft}}$

coll $\quad k^2 + Qn\cdot k + \frac{p^2}{\alpha}\bar n\cdot k + p^2 \xrightarrow{\text{o bin}} Qn\cdot k + p^2$

$\cancel{k}$

$$\Rightarrow \tilde{I}_n = 2ig^2\mu^{2\epsilon} C_F\, \bar u \gamma_\perp^\perp v \int d^D k \; \frac{Q(Q+k^-)}{k^2+i\epsilon}\; \frac{1}{k^2 + Qn\cdot k + \frac{p^2}{\alpha}\bar n\cdot k + p^2}\; \frac{1}{-\bar n\cdot k + i\epsilon}$$

o-bin

$$\to I_{n0} = 2ig^2\mu^{2\epsilon} C_F\, \bar u \gamma_\perp^\perp v \int d^D k \; \frac{Q^2}{k^2+i\epsilon}\; \frac{1}{Qn\cdot k + p^2}\; \frac{1}{-\bar n\cdot k + i\epsilon}$$

$$\tilde I_n - I_{n0} = 2ig^2\mu^{2\epsilon} C_F\, \bar u\gamma_\perp^\perp v \int d^D k\; \frac{1}{k^2+i\epsilon}\;\frac{1}{-k^-+i\epsilon}\;\frac{1}{k^2+Qk^+ + \frac{p^2}{\alpha}k^- + p^2 + i\epsilon}\;\frac{1}{k^+ + p^2/\alpha + i\epsilon}$$

$$\times \left\{ (Q+k^-)\left(k^+ + \frac{p^2}{\alpha}\right) - k^2 - Qk^+ - \frac{p^2}{\alpha}k^- - p^2 \right\}$$

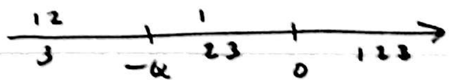
$$\cancel{Qk^+} + k^+k^- + \cancel{p^2} + \cancel{\frac{p^2}{\alpha}k^-} - k^2 - \cancel{Qk^+} - \cancel{\frac{p^2}{\alpha}k^-} - \cancel{p^2}$$

$$= +\vec k_\perp^{\,2}$$

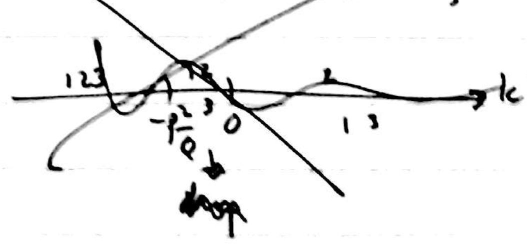
$$= 2ig^2\mu^{2\epsilon} C_F\, \bar u \gamma_\perp^\perp v \int d^D k\; \frac{\vec k_\perp^{\,2}}{k^+k^- - \vec k_\perp^{\,2} + i\epsilon}\;\frac{1}{-k^- + i\epsilon}\;\frac{1}{\underbrace{k^+k^-}_{-\vec k_\perp^2} + Qk^+ + \frac{p^2}{\alpha}k^- + p^2 + i\epsilon}\;\frac{1}{k^+ + p^2/\alpha + i\epsilon}$$

do k^+ int: $\cancel{\text{poles } k^- = \frac{\vec k_\perp^2 - i\epsilon}{k^+},\ i\epsilon,\ \frac{\vec k_\perp^2 - Qk^+ - p^2 - i\epsilon}{k^+}}$

poles $\quad k^+ = \underbrace{\frac{\vec k_\perp^2 - i\epsilon}{k^-}}_{1},\ \underbrace{\frac{\vec k_\perp^2 - \frac{p^2}{\alpha}k^- - p^2 - i\epsilon}{Q + k^-}}_{2},\ \underbrace{-\frac{p^2}{\alpha} - i\epsilon}_{3}$



$= \cancel{2ig^2\mu^{2\epsilon}}$ $\quad \cancel{k^- = \underbrace{\frac{\vec k_\perp^2 - i\epsilon}{k^+}}_{1},\ \underbrace{i\epsilon}_{2},\ \underbrace{\frac{\vec k_\perp^2 - Qk^+ - p^2 - i\epsilon}{k^+ + p^2/\alpha}}_{3}}$



$$A_n = 2 i g^2 \mu^{2\epsilon} C_F\, \bar u \gamma^\mu_\perp v \Big\{ -\frac{2\pi i}{2(2\pi)^2} \int_{-\infty}^{-Q} dk^- \int \frac{d^d k_\perp}{(2\pi)^d} \frac{\vec k_\perp^{\,2}}{-\frac{p^2}{Q} k^- - k_\perp^2} \frac{1}{-k^-} \times \frac{1}{-\cancel{\frac{p^2}{Q} k^-} - \vec k_\perp^{\,2} - \cancel{Q\frac{p^2}{Q}} + \cancel{\frac{p^2}{Q} k^-} + \cancel{p^2}}$$

$$+ \frac{2\pi i}{2(2\pi)^2} \int_{-Q}^{0} dk^- \int \frac{d^d k_\perp}{(2\pi)^d} \frac{\vec k_\perp^{\,2}}{k^-} \frac{1}{-k^-} \frac{1}{Q\frac{k_\perp^2}{k^-} + \frac{p^2}{Q} k^- + p^2} \frac{1}{\frac{k_\perp^2}{k^-} + \frac{p^2}{Q}} \Big\}$$

$$= + \frac{g^2 C_F \mu^{2\epsilon}}{2\pi} \bar u \gamma v \Big\{ \int_{-\infty}^{-Q} \frac{dk^-}{k^-} \int \frac{d^d k_\perp}{(2\pi)^d} \frac{\cancel{\vec k_\perp^{\,2}}}{\vec k_\perp^{\,2} + \frac{p^2}{Q} k^-} \frac{1}{-\cancel{\vec k_\perp^{\,2}}}$$

$$+ \int_{-Q}^{0} \frac{dk^-}{Q} \int \frac{d^d k_\perp}{(2\pi)^d} \frac{\vec k_\perp^{\,2}}{\vec k_\perp^{\,2} + \frac{p^2}{Q} k^-} \frac{1}{\vec k_\perp^{\,2} + \frac{p^2}{Q^2}(k^-)^2 + \frac{p^2 k^-}{Q}} \Big\}$$

$$= \frac{g^2 C_F \mu^{2\epsilon}}{2\pi} \bar u \gamma v \Big\{ - \int_{-\infty}^{-Q} \frac{dk^-}{k^-} \frac{1}{(4\pi)^{1-\epsilon}} \Gamma(\epsilon) \left(\frac{p^2}{Q} k^-\right)^{-\epsilon}$$

$$+ \int_0^1 dz \int_0^1 dx \underbrace{\int \frac{d^d k_\perp}{(2\pi)^d} \frac{\vec k_\perp^{\,2}}{[\vec k_\perp^{\,2} - p^2 z + p^2 x z^2]^2}}_{= \frac{1}{(4\pi)^{1-\epsilon}} (1-\epsilon) \frac{\Gamma(1-1+\epsilon)}{\Gamma(2)} (-p^2 z + p^2 x z^2)^{-\epsilon}} \Big\}$$

$$= g^2 \frac{C_F (4\pi\mu^2)^\epsilon}{8\pi^2} \Gamma(\epsilon) \Big\{ - \left(\frac{p^2}{Q}\right)^{-\epsilon} \frac{(k^-)^{-\epsilon}}{-\epsilon}\Big|_{-\infty}^{-Q}$$

$$+ \int_0^1 dz \int_0^1 dx\, (1-\epsilon) (-p^2)^{-\epsilon} z^{-\epsilon} (1 - xz)^{-\epsilon} \Big\} \bar u \gamma v$$

$$= \frac{\alpha_s C_F}{2\pi} \bar u \gamma v \left(\frac{4\pi\mu^2}{-p^2}\right)^\epsilon \Gamma(\epsilon) \Big\{ \frac{1}{\epsilon} + (1-\epsilon) \underbrace{\int_0^1 dz \int_0^1 dx\, z^{-\epsilon} (1-xz)^{-\epsilon}}_{y = xz} \Big\}$$

$$= \int_0^1 dz \int_0^z \frac{dy}{z} z^{-\epsilon} (1-y)^{-\epsilon} = \int_0^1 dz\, z^{-1-\epsilon} \frac{-1}{1-\epsilon} (1-y)^{1-\epsilon}\Big|_0^z$$

$$= -\int_0^1 dz \frac{z^{-1-\epsilon}}{1-\epsilon} \left[(1-z)^{1-\epsilon} - 1\right] = -\frac{1}{1-\epsilon}\left(\frac{\Gamma(-\epsilon)\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} - \frac{1}{-\epsilon} z^{-\epsilon}\Big|_0^1 \right)$$

$$= -\frac{1}{1-\epsilon}\left(\frac{\Gamma(-\epsilon)\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} + \frac{1}{\epsilon} \right)$$

$$= \frac{\alpha_s C_F}{2\pi} \bar u \gamma v \left(\frac{4\pi\mu^2}{-p^2}\right)^\epsilon \Gamma(\epsilon) \Big\{ \frac{1}{\epsilon} - \frac{\Gamma(-\epsilon)\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} - \frac{1}{\epsilon} \Big\}$$

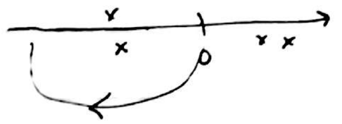
$$\overset{\overline{MS}}{=} \frac{\alpha_s C_F}{2\pi} \bar u \gamma v \Big\{ -\frac{1}{\epsilon^2} - \frac{1}{\epsilon} - 2 + \frac{\pi^2}{12} - \frac{1}{\epsilon} \ln \frac{\mu^2}{p^2} - \frac{1}{2} \ln^2 \frac{\mu^2}{p^2} - \ln \frac{\mu^2}{p^2} \Big\}$$

$$= A_{\bar n}$$

soft:

$$A_s = +2ig^2\mu^{2\epsilon} C_F\, \bar u \gamma_\mu v \int \frac{dk^+ dk^-}{2}\frac{d^d k_\perp}{(2\pi)^d}\frac{1}{k^+k^- - k_\perp^2 + i\epsilon}\,\frac{1}{k^+ + \frac{p^2}{Q} + i\epsilon}\,\frac{1}{-k^- + \frac{p^2}{Q} + i\epsilon}$$

poles in $k^+ = \frac{k_\perp^2 - i\epsilon}{k^-}\,,\ -\frac{p^2}{Q} - i\epsilon$ $\left(k^- = \frac{k_\perp^2 - i\epsilon}{k^+}\,,\ -\frac{p^2}{Q} - i\epsilon \right.$



k^-

$$= +2ig^2\mu^{2\epsilon} C_F\, \bar u \gamma_\mu v\ \frac{1}{2}\,\frac{-2\pi i}{(2\pi)^2}\int_{-\infty}^{0} dk^- \int \frac{d^d k_\perp}{(2\pi)^d}\,\frac{1}{-\frac{p^2}{Q}k^- - k_\perp^2}\,\frac{1}{-k^- + \frac{p^2}{Q}}$$

$$= -\frac{g^2 C_F \mu^{2\epsilon}}{2\pi}\,\bar u \gamma_\mu v \int_0^\infty dk^-\,\frac{1}{k^- + \frac{p^2}{Q}}\int \frac{d^d k_\perp}{(2\pi)^d}\,\frac{1}{k_\perp^2 - \frac{p^2}{Q}k^-}$$

$$= -\frac{g^2 C_F \mu^{2\epsilon}}{2\pi}\,\bar u \gamma_\mu v \int_0^\infty dk^-\,\frac{1}{k^- + \frac{p^2}{Q}}\,\frac{1}{(4\pi)^{1-\epsilon}}\,\Gamma(\epsilon)\left(-\frac{p^2}{Q}k^-\right)^{-\epsilon}$$ $\to \underline{\epsilon_{UV}}$

$$= -\frac{g^2 C_F}{8\pi^2}(4\pi\mu^2)^\epsilon\,\bar u \gamma_\mu v\ \Gamma(\epsilon)\left(-\frac{p^2}{Q}\right)^{-\epsilon}\underbrace{\int_0^\infty dk^-\,\frac{(k^-)^{-\epsilon}}{k^- + p^2/Q}}_{\left(\frac{p^2}{Q}\right)^{-\epsilon}\pi \csc \pi\epsilon}$$

$$= -\frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2 Q^2}{-p^4}\right)^\epsilon \bar u \gamma_\mu v\ \Gamma(\epsilon)\left(\frac{1}{\epsilon} + \frac{\pi^2}{6}\epsilon\right)$$

$$\overset{\overline{MS}}{=} -\frac{\alpha_s C_F}{2\pi}\,\bar u \gamma_\mu v\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\frac{\mu^2 Q^2}{-p^4} + \frac{1}{2}\ln^2\frac{\mu^2 Q^2}{-p^4} + \frac{\pi^2}{4}\right)$$

$$\Rightarrow A_n + A_{\bar n} + A_s = \frac{\alpha_s C_F}{2\pi}\,\bar u \gamma_\mu v\left\{ +\frac{1}{\epsilon^2} + \frac{2}{\epsilon} + \frac{1}{\epsilon}\left(\ln\frac{\mu^2 Q^2}{-p^4} + 2\ln\frac{\mu^2}{p^2}\right) - \frac{1}{2}\ln^2\frac{\mu^2 Q^2}{-p^4} + \ln^2\frac{\mu^2}{p^2} + 2\ln\frac{\mu^2}{p^2} + 4 - \frac{\pi^2}{6} - \frac{\pi^2}{4}\right\}$$

$$= \frac{\alpha_s C_F}{2\pi}\,\bar u \gamma^\mu v\left\{ +\frac{1}{\epsilon_{UV}^2} + \frac{2}{\epsilon_{UV}} - \frac{1}{\epsilon_{UV}}\ln\frac{-Q^2}{\mu^2} + 4 - \frac{5\pi^2}{12} + \frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \ln^2\frac{Q^2}{p^2} - 2\ln\frac{\mu^2}{p^2}\right\}$$

$$\left[\ \frac{1}{2}\ln^2\frac{\mu^2 Q^2}{p^4} - \ln^2\frac{\mu^2}{p^2} = \frac{1}{2}\ln^2\frac{\mu^2}{Q^2} + \frac{1}{2}\ln^2\frac{Q^4}{p^4} + \cancel{\ln\frac{\mu^2}{Q^2}\ln\frac{Q^4}{p^4}} - \ln^2\frac{\mu^2}{Q^2} - \ln\frac{Q^2}{p^2} - \cancel{2\ln\frac{\mu^2}{Q^2}\ln\frac{Q^2}{p^2}} = -\frac{1}{2}\ln^2\frac{\mu^2}{Q^2} + 2\ln^2\frac{Q^2}{p^2}\ \right]$$

wf renorm. $\quad \times \left(\frac{1}{\sqrt{Z_2}}\right)^2 = 1 - \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon_{UV}} + 1 - \ln\frac{-p^2}{\mu^2}\right)$

$$\Rightarrow V_{SCET} = \bar{u}\gamma_\mu v \left\{ 1 + \frac{\alpha_s C_F}{2\pi}\left[+ \frac{1}{\epsilon_{UV}^2} + \frac{3}{2\epsilon_{UV}} - \frac{1}{\epsilon_{UV}}\ln\frac{-Q^2}{\mu^2} + 4 - \frac{5\pi^2}{12} - \frac{1}{2} \right.\right.$$

$$\left.\left. + \frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \ln^2\frac{Q^2}{p^2} - 2\ln\frac{-p^2}{\mu^2} + \frac{1}{2}\ln\frac{-p^2}{\mu^2} \right]\right\}$$

$$= \bar{u}\gamma_\mu v \left\{ 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\epsilon_{UV}^2} + \frac{3}{2\epsilon_{UV}} - \frac{1}{\epsilon_{UV}}\ln\frac{-Q^2}{\mu^2} + \frac{7}{2} - \frac{5\pi^2}{12} \right.\right.$$

$$\left.\left. + \frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \ln^2\frac{Q^2}{p^2} - \frac{3}{2}\ln\frac{-p^2}{\mu^2} \right]\right\}$$

$$= C_2 \cdot V_{QCD} \qquad \boxed{Z_2 = 1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon}\ln\frac{\mu^2}{-Q^2}\right)}$$

$$\Rightarrow C_2 = 1 - \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{2} - \frac{5\pi^2}{12} + \frac{1}{2} + \frac{\pi^2}{3} \right.$$

$$+ \frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \ln^2\frac{Q^2}{p^2} - \frac{3}{2}\ln\frac{-p^2}{\mu^2}$$

$$\left. + \ln^2\frac{Q^2}{p^2} + \frac{3}{2}\ln\frac{p^2}{Q^2} \right]$$

$$\boxed{C_2 = 1 - \frac{\alpha_s C_F}{2\pi}\left[4 - \frac{\pi^2}{12} + \frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \frac{3}{2}\ln\frac{Q^2}{-\mu^2} \right]}$$ ✓