QCD in Collisions with Polarized Beams

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Summary of lecture six

- With the existing data from lepton-hadron and hadron-hadron collisions with polarized beams, we have a good idea on the quark/gluon helicity contribution to proton's spin

- Transversity and tensor charge are fundamental QCD quantities!

- But, EIC is a ultimate QCD machine, and absolutely needed:
  1) to discover and explore the quark/gluon structure and properties of hadrons and nuclei,
  2) to search for hints and clues of color confinement, and
  3) to measure the color fluctuation and color neutralization

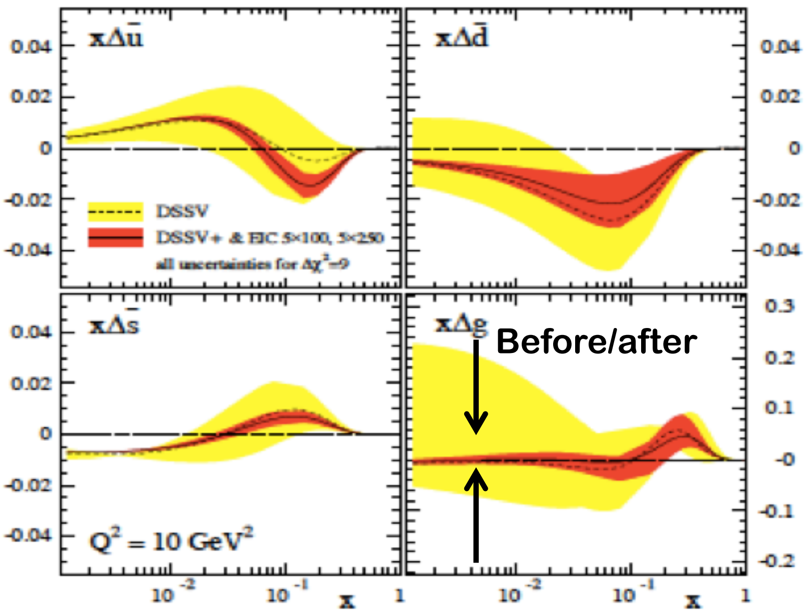
In particular, EIC can determine the helicity contribution to proton's spin, and to answer the question if there is a need for orbital contribution

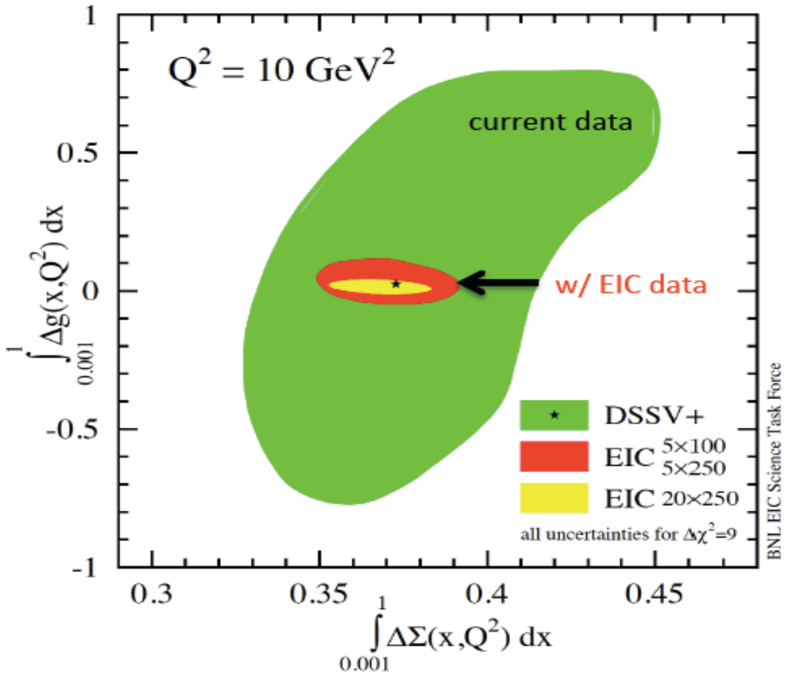
Thanks!

The Future: Challenges & opportunities

❑ One-year of running at EIC:

Wider Q^2 and x range including low x at EIC!





No other machine in the world can achieve this!

❑ Ultimate solution to the proton spin puzzle:

- ✧ ***Precision measurement of Δg(x) – extend to smaller x regime***
- ✧ ***Orbital angular momentum contribution – measurement of GPDs!***

Transverse spin phenomena in QCD

Double Transverse-Spin Asymmetry (A_{TT})

Probe the transversity distribution: $\delta q(x)$

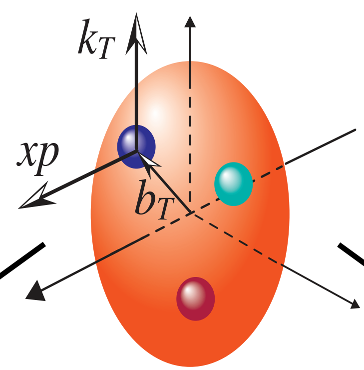
Drell-Yan – low rate

Single Transverse-Spin Asymmetry (SSA)

$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

Chance to go beyond the collinear approximation to explore hadron's 3D structure!

Probe parton's confined transverse motion!



Imagine parton's spatial distribution!

$\int d^2b_T$ $\quad$ $\int d^2k_T$

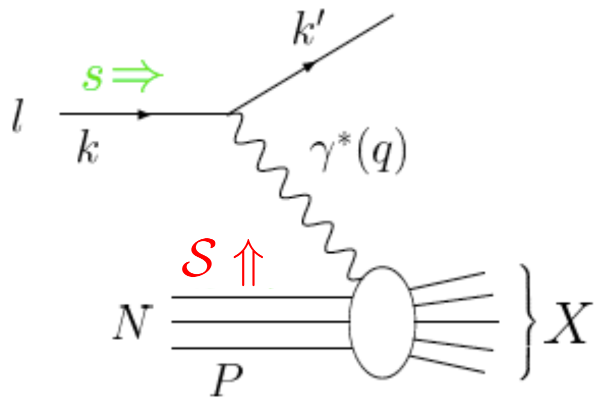
TMDs $f(x,k_T)$ $\quad$ $f(x,b_T)$ **GPDs**

Two scales

Single transverse spin asymmetry

- 40 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions

Parity and Time-reversal invariance

- In quantum field theory, physical observables are given by matrix elements of quantum field operators

- Consider two quantum states: $|\alpha\rangle \quad |\beta\rangle$

- Parity transformation:

$$|\alpha_P\rangle \equiv U_P\,|\alpha\rangle \qquad\qquad |\beta_P\rangle \equiv U_P\,|\beta\rangle$$

$$\langle\alpha_P|\beta_P\rangle = \langle\alpha|\,U_P^\dagger\,U_P\,|\beta\rangle = \langle\alpha|\beta\rangle$$

- Time-reversal transformation:

$$|\alpha_T\rangle \equiv V_T\,|\alpha\rangle \qquad\qquad |\beta_T\rangle \equiv V_T\,|\beta\rangle$$

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\,V_T^\dagger\,V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

Parity and Time-reversal invariance

❑ **Parton fields under P and T transformation:**

$$U_P\,\psi(y_0,\vec{y})\,U_P^{-1} = \gamma^0\,\psi(y_0,-\vec{y})$$

$$V_T\,\psi(y_0,\vec{y})\,V_T^{-1} = (i\gamma^1\gamma^3)\,\psi(-y_0,\vec{y})$$

$$\boxed{\mathcal{J} = i\,\gamma^1\,\gamma^3}$$

$$\Rightarrow\quad \boxed{\begin{aligned}&\langle P,\vec{s}_\perp|\,\bar{\psi}(0)\,\Gamma_i\,\psi(y^-)\,|P,\vec{s}_\perp\rangle\\ &= \langle P,-\vec{s}_\perp|\,\bar{\psi}(0)\left[\mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger\right]\psi(y^-)\,|P,-\vec{s}_\perp\rangle\end{aligned}}$$

❑ **Quark correlations contribute to polarized X-sections:**

$$T_i(x;\vec{s}_\perp) = -T_i(x;-\vec{s}_\perp) \quad\Rightarrow\quad \mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger = -\Gamma_i$$

$$\boxed{\Gamma_i = \gamma^\mu\gamma_5,\quad \sigma^{\mu\nu}\quad \text{(or}\quad \sigma^{\mu\nu}\,(i\gamma_5)}$$

$$\Gamma_i = I,\quad i\gamma_5,\quad \gamma^\mu$$ **contribute to spin-avg X-sections:**

A_N for inclusive DIS

- DIS cross section: $\sigma(\vec{s}_\perp) \propto L^{\mu\nu} W_{\mu\nu}(\vec{s}_\perp)$

- Leptionic tensor is symmetric: $L^{\mu\nu} = L^{\nu\mu}$

- Hadronic tensor: $W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp | j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$

- Polarized cross section:

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\ [W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)]$$

- Vanishing single spin asymmetry:

$$\boxed{\begin{aligned} A_N = 0 \quad \Leftrightarrow \quad & \langle P, \vec{s}_\perp | j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle \\ & \stackrel{?}{=} \langle P, -\vec{s}_\perp | j^\dagger_\nu(0)\, j_\mu(y)\, | P, -\vec{s}_\perp \rangle \end{aligned}}$$

A_N for inclusive DIS

- **Define two quantum states:**

$$\langle\beta| \equiv \langle P, \vec{s}_\perp|\, j_\mu^\dagger(0)\, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P, \vec{s}_\perp\rangle$$

- **Time-reversed states:**

$$|\alpha_T\rangle = V_T\,|P, \vec{s}_\perp\rangle = |-P, -\vec{s}_\perp\rangle$$

$$\begin{aligned}|\beta_T\rangle &= V_T\left[j_\mu^\dagger(0)\, j_\nu(y)\right]^\dagger |P, \vec{s}_\perp\rangle \\ &= \left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right)|-P, -\vec{s}_\perp\rangle\end{aligned}$$

- **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\, V_T^\dagger\, V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\begin{aligned}\Longrightarrow\ &\langle -P, -\vec{s}_\perp|\left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right)|-P, -\vec{s}_\perp\rangle \\ &= \langle P, \vec{s}_\perp|\, j_\mu^\dagger(0)\, j_\nu(y)\,|P, \vec{s}_\perp\rangle\end{aligned}$$

A_N for inclusive DIS

- **Parity invariance:**

$$\boxed{1 = U_P^{-1} U_P = U_P^{\dagger} U_P}$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^\dagger(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) | -P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^\dagger(y) V_T^{-1} U_P^{-1}\right) \left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}\right) | P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(-y) \, j_\mu(0) \, | P, -\vec{s}_\perp \rangle$$

Translation invariance:

$$\Downarrow$$

$$\boxed{\begin{aligned} &\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(0) \, j_\mu(y) \, | P, -\vec{s}_\perp \rangle \\ &= \langle P, \vec{s}_\perp | \, j_\mu^\dagger(0) \, j_\nu(y) \, | P, \vec{s}_\perp \rangle \end{aligned}}$$

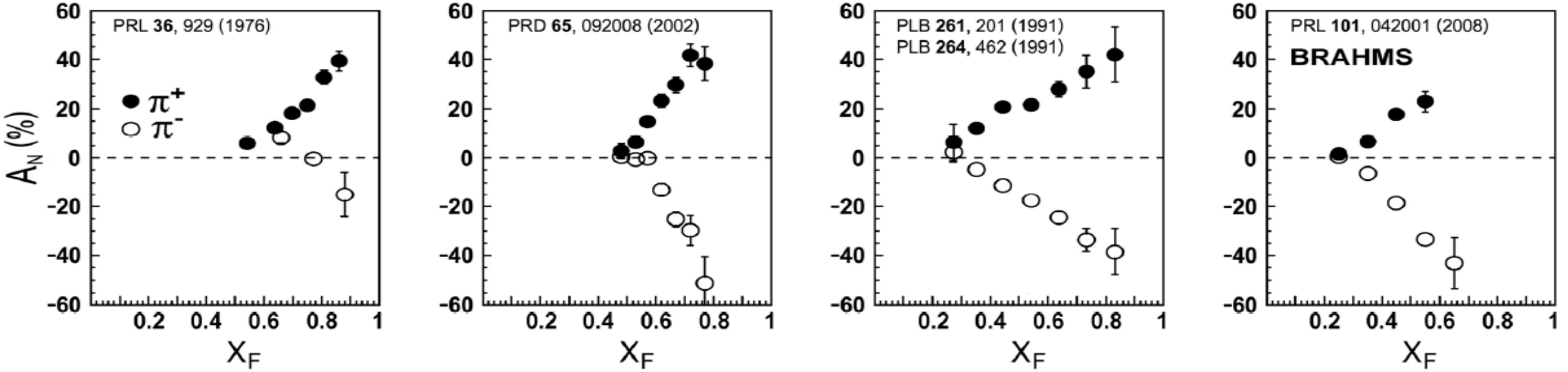
- **Polarized cross section:**

$$\boxed{\begin{aligned} \Delta\sigma(\vec{s}_\perp) &\propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right] \\ &= L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp) \right] = 0 \end{aligned}}$$

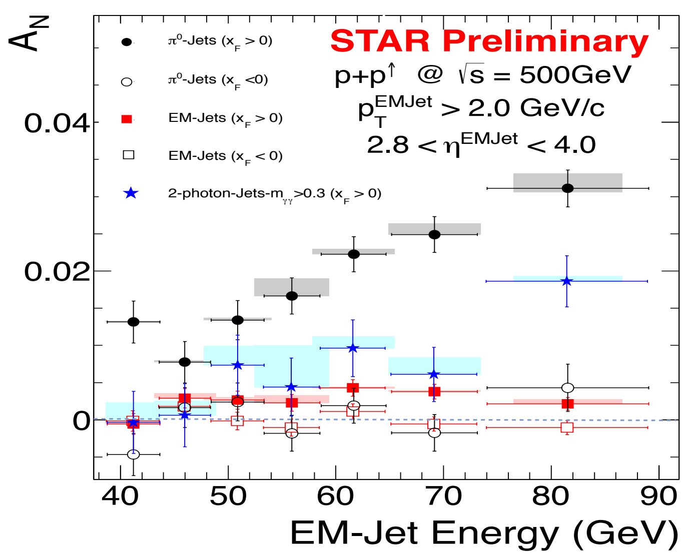
A_N in hadronic collisions

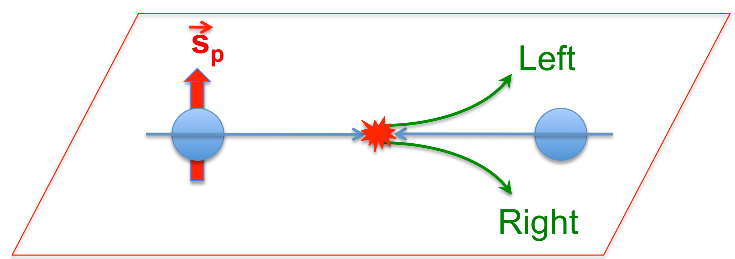
- A_N - consistently observed for over 35 years!

ANL – 4.9 GeV | BNL – 6.6 GeV | FNAL – 20 GeV | BNL – 62.4 GeV



- Survived the highest RHIC energy:





$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

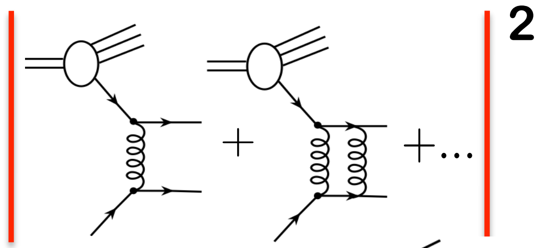
Do we understand this?

Do we understand it?

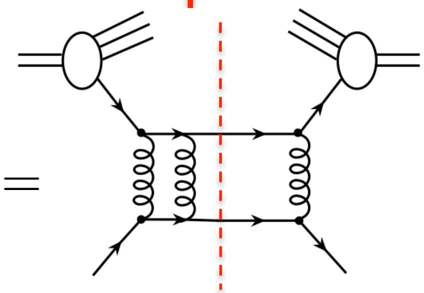
Kane, Pumplin, Repko, PRL, 1978

❑ Early attempt:

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) =$



$\propto \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

❑ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

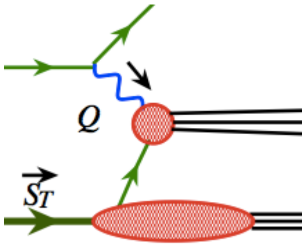
❑ Vanish without parton's transverse motion:

➡ **A direct probe for parton's transverse motion,**

Spin-orbital correlation, QCD quantum interference

Current understanding of TSSAs

- Symmetry plays important role:

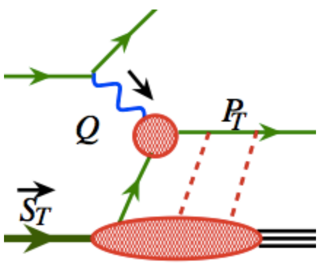


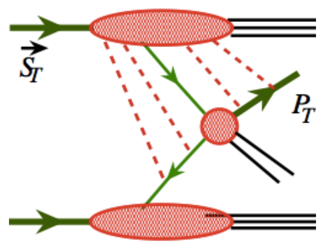
Inclusive DIS
Single scale
Q

Parity
Time-reversal

→ $A_N = 0$

- One scale observables – $Q >> \Lambda_{QCD}$:

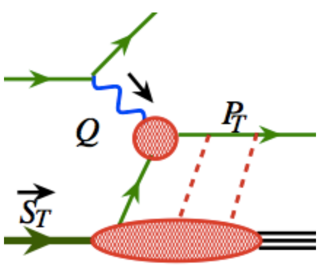


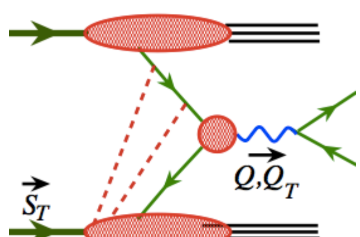


→ Collinear factorization / Twist-3 distributions

SIDIS: $Q \sim P_T$ DY: $Q \sim P_T$; Jet, Particle: P_T

- Two scales observables – $Q_1 >> Q_2 \sim \Lambda_{QCD}$:





→ TMD factorization / TMD distributions

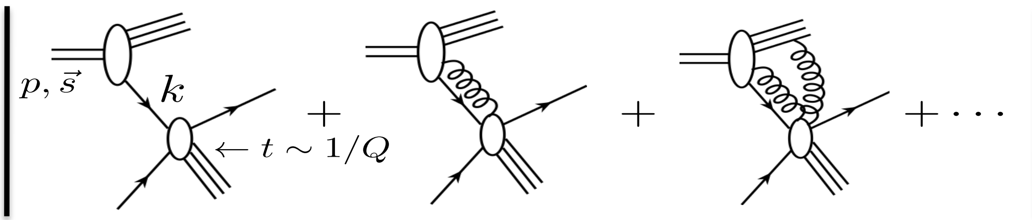
SIDIS: $Q>>P_T$ DY: $Q>>P_T$ or $Q<<P_T$

Brodsky et al. explicit calculation with m_q=\=0

How collinear factorization generates TSSA?

❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto$



$\Big|^2 \left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – Expansion

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

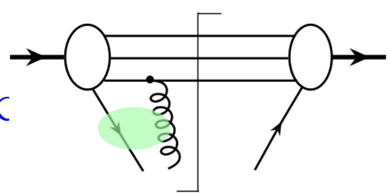
Too large to compete!

Three-parton correlation

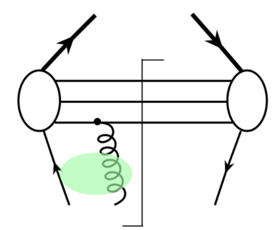
❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$ 

Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$ 

Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Needed Phase: Integration of "dx" using unpinched poles

Twist-3 distributions relevant to A_N

❑ Twist-2 distributions:

- **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

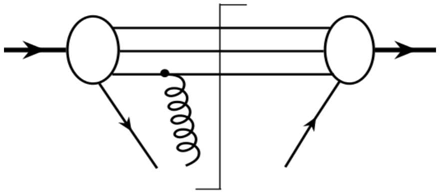
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- **Polarized PDFs:**

$$\Delta q(x) \propto \langle P, S_{\|}|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_{\|}\rangle$$

$$\Delta G(x) \propto \langle P, S_{\|}|F^{+\mu}(0)F^{+\nu}(y)|P, S_{\|}\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:



No probability interpretation!

$$\widetilde{\mathcal{T}}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{\mathcal{T}}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{\mathcal{T}}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{\mathcal{T}}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

❑ Twist-3 fragmentation functions:

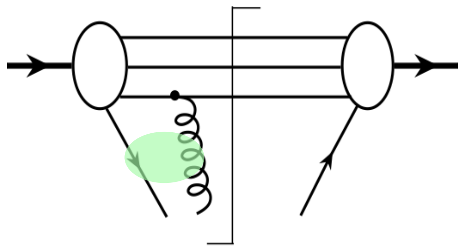
See Kang, Yuan, Zhou, 2010, Kang 2010

"Interpretation" of twist-3 correlation functions

❑ Measurement of direct QCD quantum interference:

Qiu, Sterman, 1991, …

$$T^{(3)}(x,x,S_\perp) \propto$$



Interference between a single active parton state and an active two-parton composite state

❑ "Expectation value" of QCD operators:

$$\langle P,s|\overline{\psi}(0)\gamma^+\psi(y^-)|P,s\rangle \longrightarrow \langle P,s|\overline{\psi}(0)\gamma^+\left[\epsilon_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P,s\rangle$$

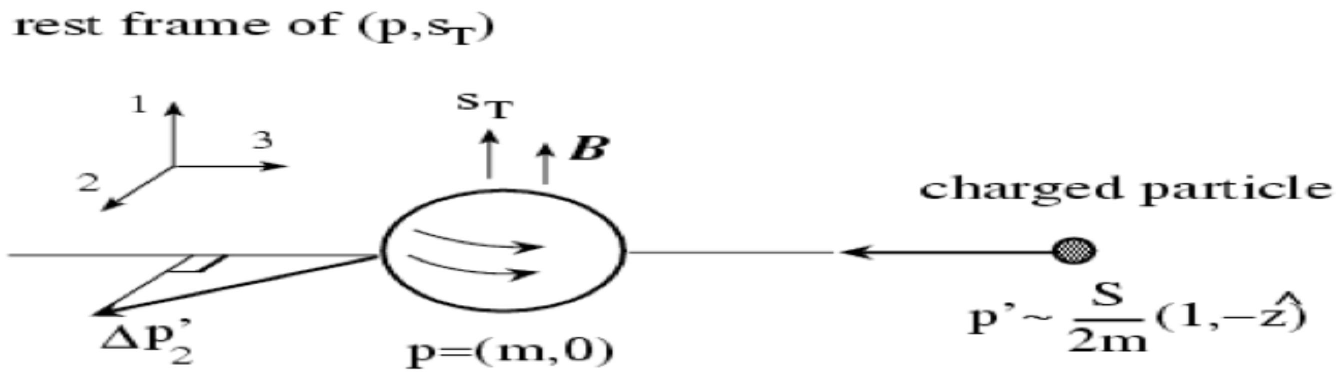
$$\langle P,s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P,s\rangle \longrightarrow \langle P,s|\overline{\psi}(0)\gamma^+\left[i\,g_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P,s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

Qiu, Sterman, 1998

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

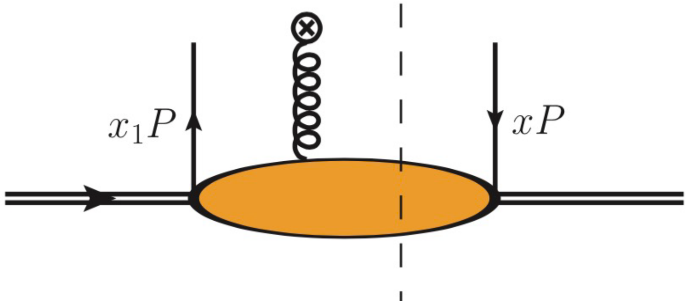
$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

- The total change: $\boxed{\Delta p_2' = e\int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)}$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Collinear twist-3 contribution to A_N

$$
\begin{aligned}
d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\
&= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
&+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
&+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$



SGP

$T_{FT}(x,x)$

$\downarrow$

Sivers-type function

SFP

$T_{FT}(0,x), \ldots$

$G_{FT}(0,x), \ldots$

Also tri-gluon correlators at SG

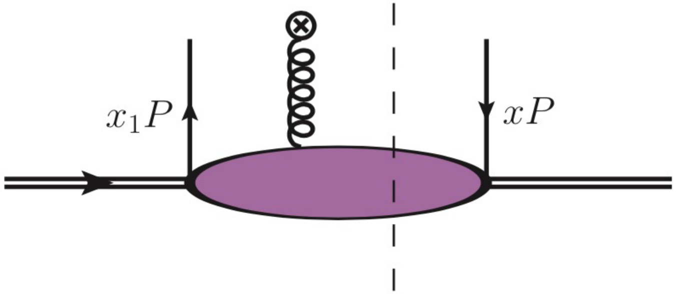
Collinear twist-3 contribution to A_N

$$
\begin{aligned}
d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\
&= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
&+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
&+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

SGP	SFP
$T_{FT}(x,x)$	$T_{FT}(0,x), \ldots$
$H_{FU}(x,x)$	$H_{FU}(0,x), \ldots$

$\downarrow$

Boer-Mulders-type function



Collinear twist-3 contribution to A_N

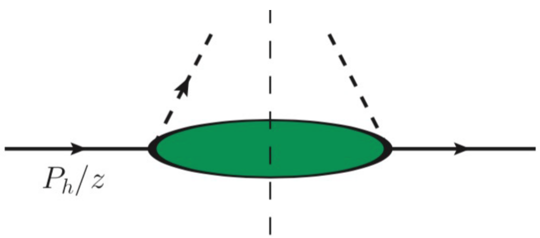
$$d\Delta\sigma(s_T) \equiv d\sigma(s_T) - d\sigma(-s_T)$$
$$= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$
$$+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}$$
$$+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}$$

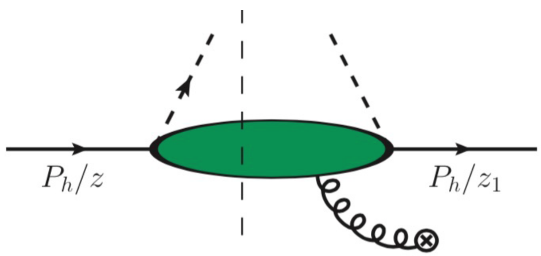
SGP	SFP
$T_{FT}(x,x)$	$T_{FT}(0,x), \ldots$
$H_{FU}(x,x)$	$H_{FU}(0,x), \ldots$

$\hat{H}(z), H(z), \hat{H}_{FU}(z,z_1), \ldots$

$\downarrow$

Collins-type function





Collinear twist-3 contribution to A_N

$$
\begin{aligned}
d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\
&= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
&+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
&+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

SGP	SFP
$T_{FT}(x,x)$	$T_{FT}(0,x), \ldots$
$H_{FU}(x,x)$	$H_{FU}(0,x), \ldots$

$\hat{H}(z), H(z), \hat{H}_{FU}(z, z_1), \ldots$

❑ Early work (before 2013):

Assumed that SGP (Sivers-type) dominates the twist-3 contribution to TSSAs in:

$$p^\uparrow + p \to \pi(x_F, p_T) + X$$

Qiu, Sterman (1991, 98)

$$
E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')
$$
$$
\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[T_{a,F}(x,x) - x\left(\frac{d}{dx} T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s}, \hat{t}, \hat{u})
$$

✧ Growth in x_F

✧ Slow fall off in p_T

Collinear twist-3 contribution to A_N

$$
\begin{aligned}
d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\
&= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
&\quad \boxed{+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}} \rightarrow \\
&\quad + H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

Negligible
Kanazawa & Koike (2000)

Collinear twist-3 contribution to A_N

$$d\Delta\sigma(s_T) \equiv d\sigma(s_T) - d\sigma(-s_T)$$

$$= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$

$$\boxed{+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}} \rightarrow$$

Negligible
Kanazawa & Koike (2000)

$$\boxed{+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}} \rightarrow$$

Important
Metz & Pitonyak (2013)

❑ Twist-3 fragmentation contribution:

$$\frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\,\epsilon_{\perp\mu\nu}\, S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'}\, \frac{1}{x'S + T/z}\, \frac{1}{-x\hat{u} - x'\hat{t}}$$

$$\times \frac{1}{x}\, h_1^a(x)\, f_1^b(x') \left\{ \left(\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz} \right) S_{\hat{H}}^i + \frac{1}{z}\, H^{C/c}(z)\, S_H^i \right.$$

$$\left. + 2z^2 \int \frac{dz_1}{z_1^2}\, PV \frac{1}{\frac{1}{z} - \frac{1}{z_1}}\, \hat{H}_{FU}^{C/c,\Im}(z, z_1)\, \frac{1}{\xi}\, S_{\hat{H}_{FU}}^i \right\}$$

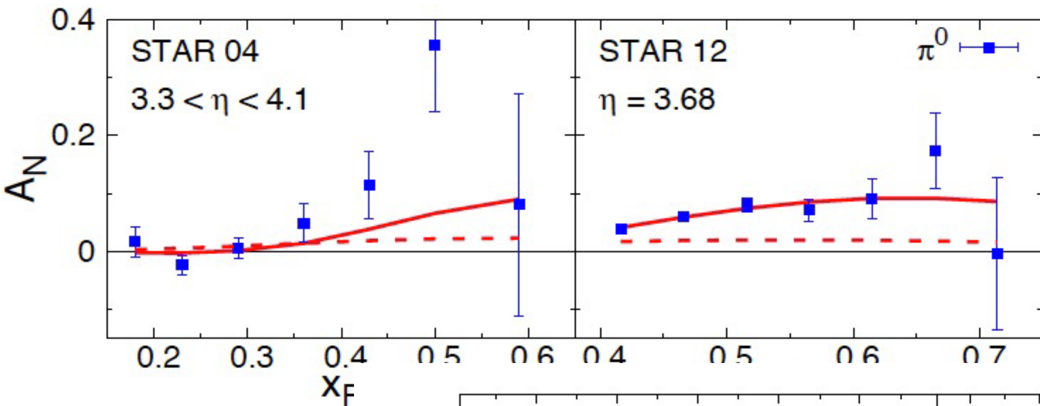
$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{\Im}(z, z_1) = H(z) + 2z\hat{H}(z)$$ **3-parton correlator**

$$\hat{H}(z) = H_1^{\perp(1)}(z)$$ **Collins-type function**

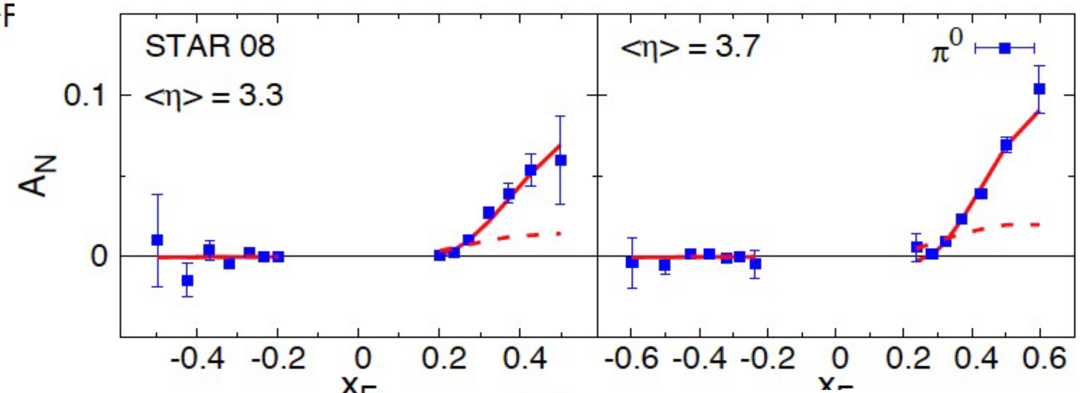
Collinear twist-3 contribution to A_N

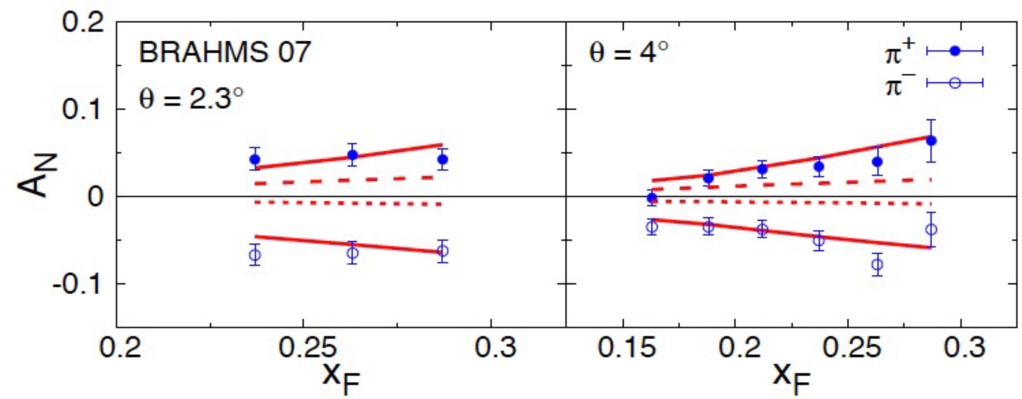
❑ Fragmentation + QS (fix through Sivers function):

Kanazawa, Koike, Metz, Pitonyak
PRD 89(RC) (2014)



χ^2/d.o.f. = 1.03





Total

NO 3-parton FF

Multi-gluon correlation functions

❑ Diagonal tri-gluon correlations:

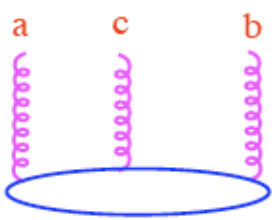
Ji, PLB289 (1992)

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-)\right] F^{\alpha +}(y_1^-) | P, s_\perp \rangle$$

❑ Two tri-gluon correlation functions – color contraction:

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$



Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

❑ D-meson production at EIC:

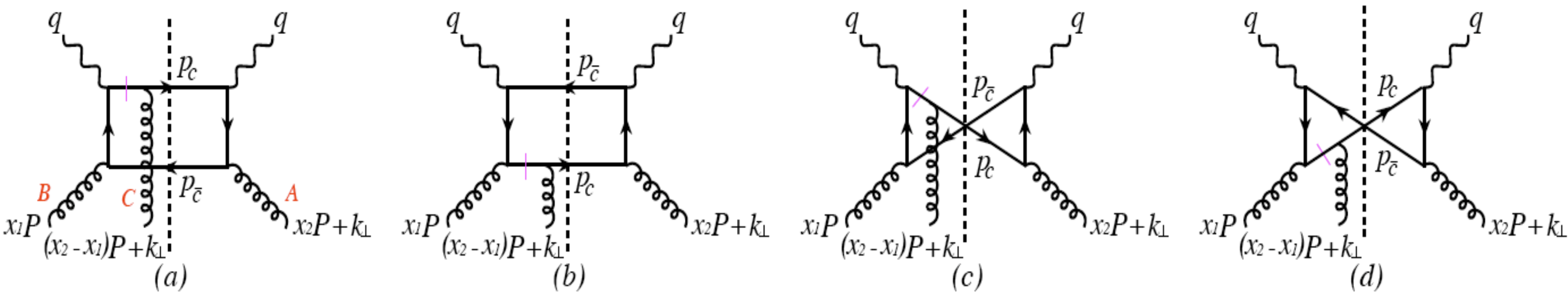
✧ Clean probe for gluonic twist-3 correlation functions

✧ $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

D-meson production at EIC

Kang, Qiu, PRD, 2008

❑ Dominated by the tri-gluon subprocess:



- ✧ Active parton momentum fraction cannot be too large
- ✧ Intrinsic charm contribution is not important
- ✧ Sufficient production rate

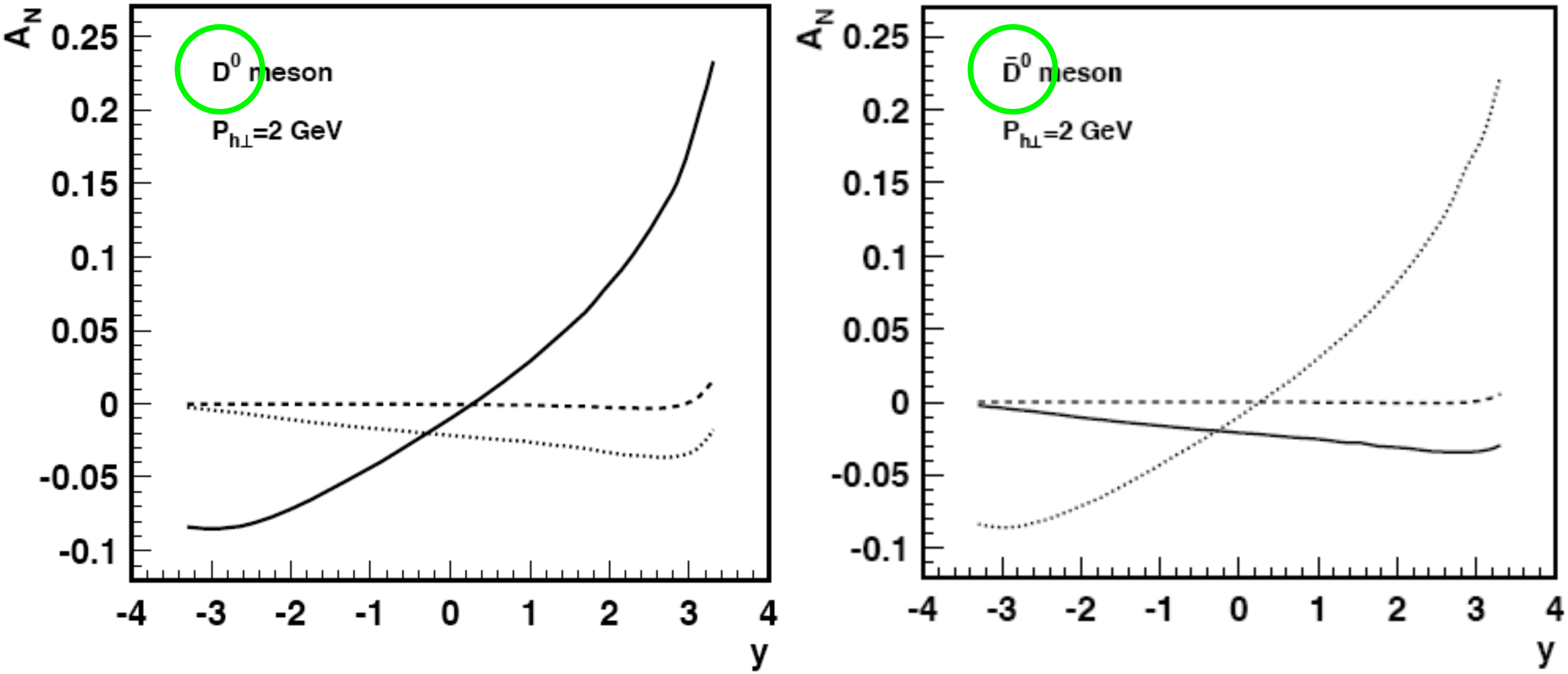
❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp)-\sigma(-s_\perp)}{\sigma(s_\perp)+\sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

SSA is directly proportional to tri-gluon correlation functions

SSA of D-meson production at RHIC

- Rapidity: $\sqrt{s} = 200\ \text{GeV} \quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3\ \text{GeV}$



Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = T_G^{(d)}$

Dashed: $(2)\ \lambda_f = \lambda_d = 0 \qquad T_G^{(f)} = T_G^{(d)} = 0$

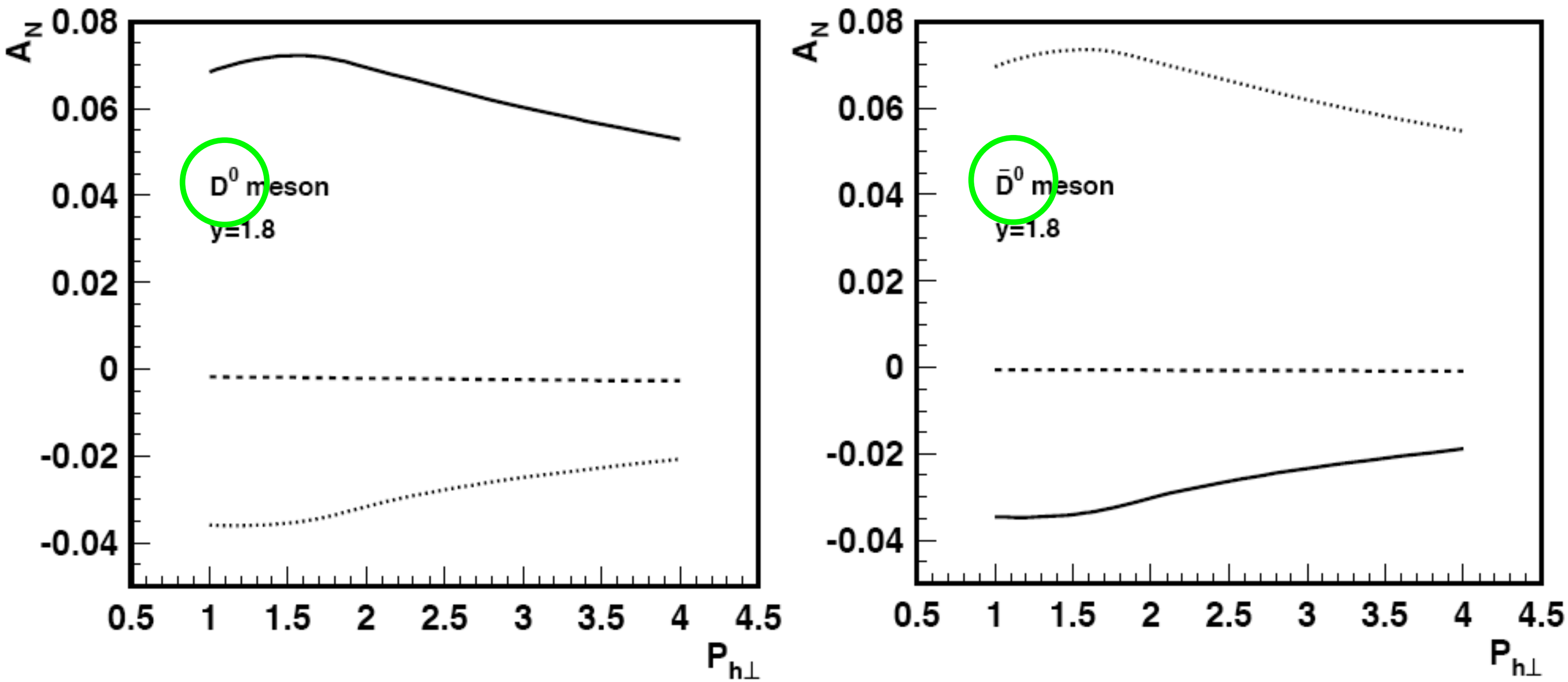
Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = -T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

SSA of D-meson production at RHIC

- **P_T dependence:** $\sqrt{s} = 200$ GeV $\quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3$ GeV



Solid: $(1)\ \lambda_f = \lambda_d = 0.07$ GeV $\quad T_G^{(f)} = T_G^{(d)}$

Dashed: $(2)\ \lambda_f = \lambda_d = 0 \quad T_G^{(f)} = T_G^{(d)} = 0$

Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07$ GeV $\quad T_G^{(f)} = -T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

Test QCD at twist-3 level

Kang, Qiu, 2009

❑ Scaling violation – "DGLAP" evolution:

$$
\mu_F^2 \frac{\partial}{\partial \mu_F^2}
\underbrace{\begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}}_{(x,\, x+x_2,\, \mu,\, s_T)}
=
\underbrace{\begin{pmatrix}
K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\
K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\
K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\
K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\
K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\
K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G}
\end{pmatrix}}_{(\xi,\, \xi+\xi_2;\, x,\, x+x_2,\, \alpha_s)}
\underbrace{\otimes}_{\int d\xi \int d\xi_2}
\begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}
$$

❑ Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

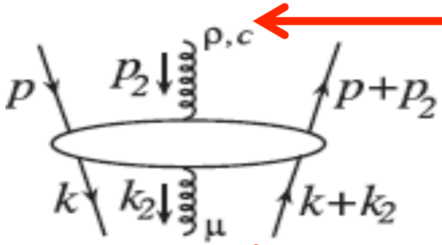
DGLAP for f_2: $\dfrac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\dfrac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\dfrac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

Evolution kernels – an example

Kang, Qiu, 2009

❑ Quark to quark:

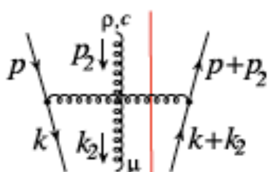


$$\mathcal{P}^{(\mathrm{LC})}_{q,F} = \frac{1}{2}\gamma\cdot P\left(\frac{-1}{\xi_2}\right)(i\epsilon^{s_T\rho n\bar{n}})\tilde{\mathcal{C}}_q$$

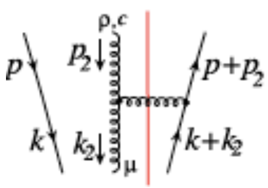
Cut vertex and projection operator in LC gauge

$$\mathcal{V}^{\mathrm{LC}}_{q,F} = \frac{\gamma^+}{2P^+}\delta\left(x-\frac{k^+}{P^+}\right)x_2\delta\left(x_2-\frac{k_2^+}{P^+}\right)(i\epsilon^{s_T\sigma n\bar{n}}[-g_{\sigma\mu}]\mathcal{C}_q$$

❑ Feynman diagram calculation:

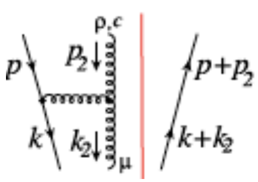


$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi_2)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[C_F-\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1+z^2}{1-z}\right)$$



$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi-x)\frac{1}{\xi_2}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\frac{2x+\xi_2}{x+\xi_2}\right)$$

$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\ \mathcal{T}_{q,F}(x,x)$$



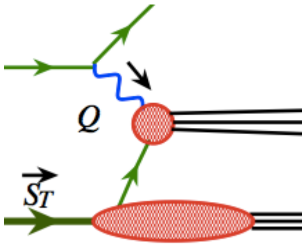
$$\int d\xi \int d\xi_2\ \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\ \delta(\xi+\xi_2-x)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\frac{1+z}{1-z}\right)$$

$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\ \mathcal{T}_{q,F}(x,x)$$

+ virtual loop diagrams

Current understanding of TSSAs

- **Symmetry plays important role:**

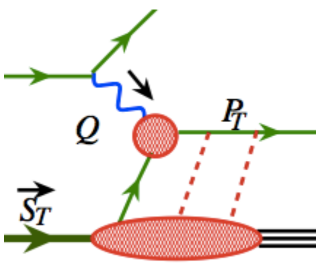


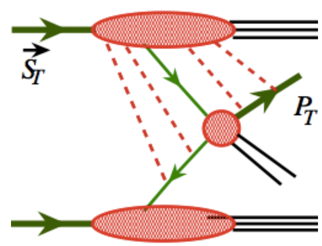
Inclusive DIS
Single scale
Q

Parity
Time-reversal

$A_N = 0$

- **One scale observables – $Q >> \Lambda_{QCD}$:**



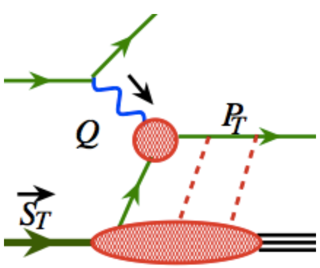


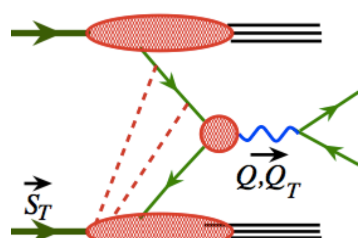
Collinear factorization
Twist-3 distributions

SIDIS: $Q \sim P_T$

DY: $Q \sim P_T$; Jet, Particle: P_T

- **Two scales observables – $Q_1 >> Q_2 \sim \Lambda_{QCD}$:**





TMD factorization
TMD distributions

SIDIS: $Q>>P_T$

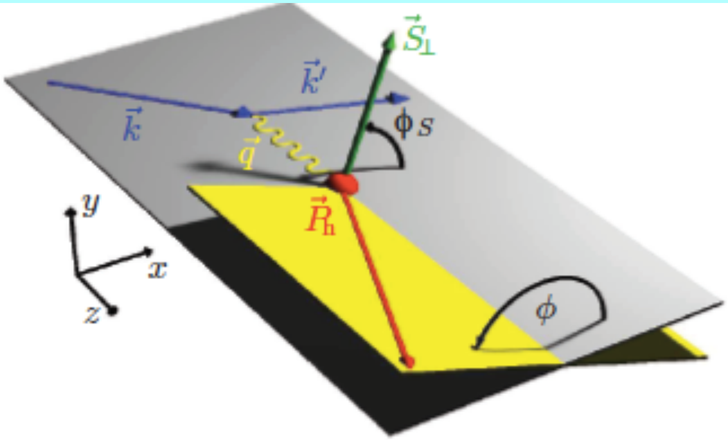
DY: $Q>>P_T$ or $Q<<P_T$

Brodsky et al. explicit calculation with m_q=\=0

Semi-inclusive DIS (SIDIS)

- **Process:**

$$e(k) + N(p) \longrightarrow e'(k') + h(P_h) + X$$



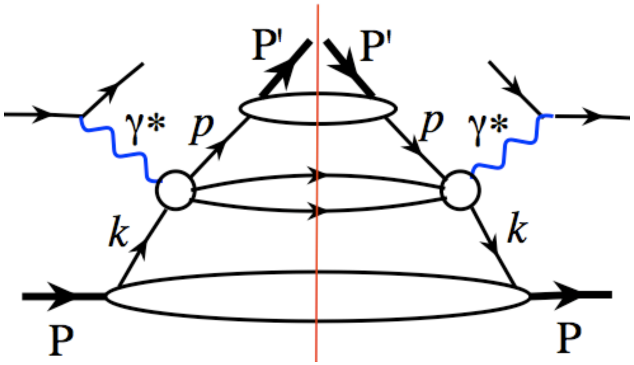
- **Natural event structure:**

  In the photon-hadron frame: $P_{h_T} \approx 0$

  Semi-Inclusive DIS is a natural observable with TWO very different scales

  $Q \gg P_{h_T} \gtrsim \Lambda_{\rm QCD}$ **Localized probe sensitive to parton's transverse motion**

- **Collinear QCD factorization holds if P_{hT} integrated:**



$$d\sigma_{\gamma^* h \to h'} \propto \phi_{f/h} \otimes d\hat{\sigma}_{\gamma^* f \to f'} \otimes D_{f' \to h'}(z)$$

$$z = \frac{P_h \cdot p}{q \cdot p} \qquad y = \frac{q \cdot p}{k \cdot p}$$

- **"Total c.m. energy":** $s_{\gamma^* p} = (p+q)^2 \approx Q^2 \left[\frac{1-x_B}{x_B}\right] \approx \frac{Q^2}{x_B}$

Single hadron production at low p_T

- ❑ **Unique kinematics - unique event structure:**

Briet frame: Large Q^2 virtual photon acts like a "wall"

High energy low p_T jet (or hadron) - ideal probe for parton's transverse motion!

- ❑ **Need for TMDs, if we observe $p_T \sim 1/\text{fm}$:**

$$\int d^4k_a \, \mathcal{H}(Q, p_T, k_a, k_b) \left(\frac{1}{k_a^2 + i\varepsilon}\right) \left(\frac{1}{k_a^2 - i\varepsilon}\right) \mathcal{T}(k_a, 1/r_0)$$

$$\approx \int \frac{dx}{x} d^2k_{a\perp} \, \mathcal{H}(Q, p_T, k_a^2 = 0, k_b) \left[\int dk_a^2 \left(\frac{1}{k_a^2 + i\varepsilon}\right) \left(\frac{1}{k_a^2 - i\varepsilon}\right) \mathcal{T}(k_a, 1/r_0)\right]$$

Can't set $k_T \sim 0$, since $k_T \sim p_T$

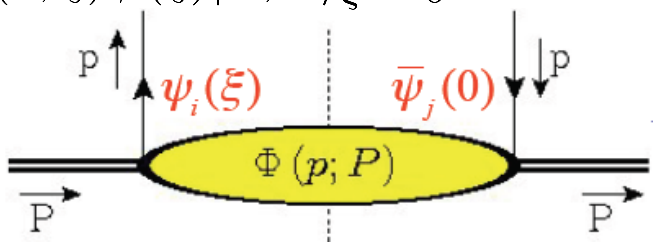
TMD distribution

Questions/issues for TMDs

❑ Non-perturbative definition:

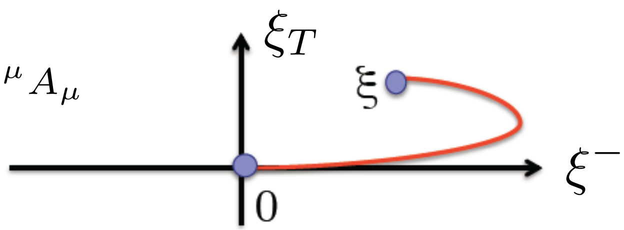
✧ **In terms of matrix elements of parton correlators:**

$$\Phi^{[U]}(x,p_T;n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,p\cdot\xi}\, \langle P,S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P,S\rangle_{\xi^+=0}$$



✧ **Depends on the choice of the gauge link:**

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ **Decomposes into a list of TMDs:**

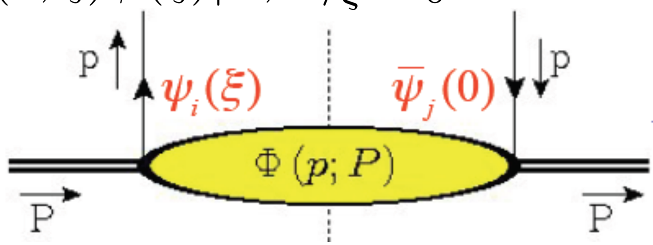
$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M} \Big\}\frac{\not{P}}{2},$$

Questions/issues for TMDs

❑ Non-perturbative definition:

✧ **In terms of matrix elements of parton correlators:**

$$\Phi^{[U]}(x,p_T;n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,p\cdot\xi}\, \langle P,S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P,S\rangle_{\xi^+=0}$$



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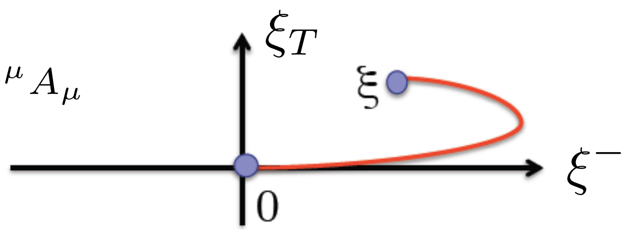


✧ **Decomposes into a list of TMDs:**

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

✧ **IF we knew proton wave function, this definition gives "unique" TMDs!**

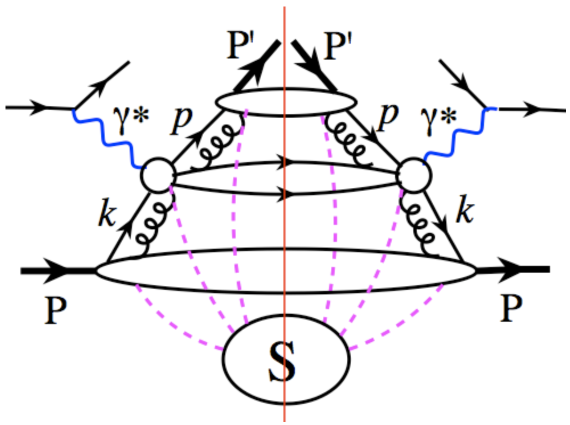
But, we do NOT know proton wave function (may calculate it using BSE?)

TMDs defined in this way are NOT direct physical observables!

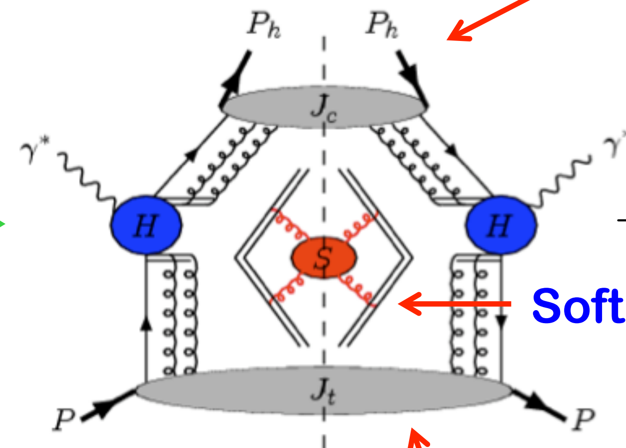
Questions/issues for TMDs

- **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:



TMD fragmentation



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

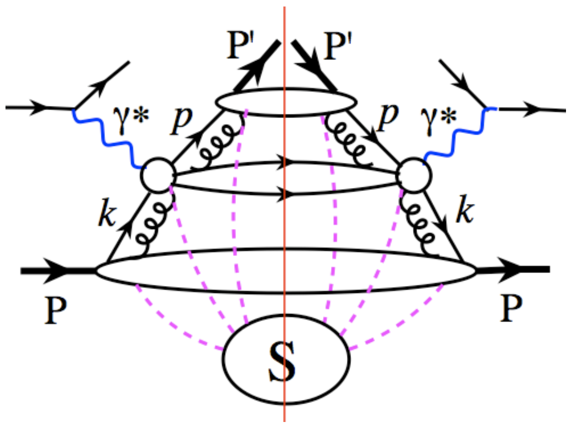
Soft factors

TMD parton distribution

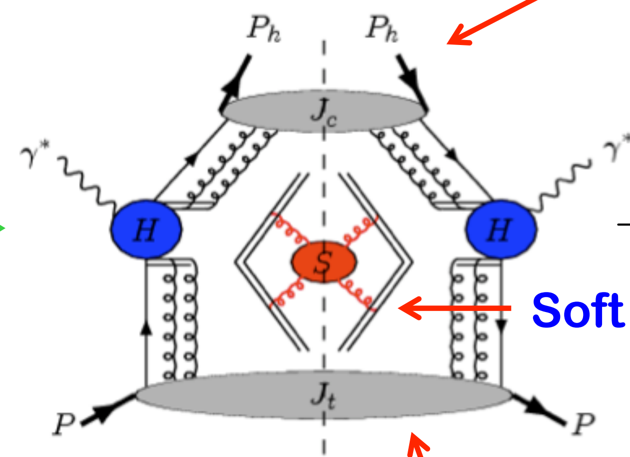
Definitions of TMDs

- **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:



TMD fragmentation



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}, \frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

- **Low P_{hT} – TMD factorization:**

$$\sigma_{\mathrm{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

- **High P_{hT} – Collinear factorization:**

$$\sigma_{\mathrm{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

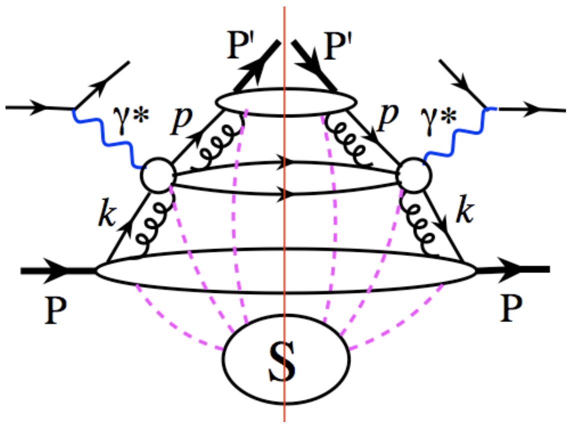
- **P_{hT} Integrated - Collinear factorization:**

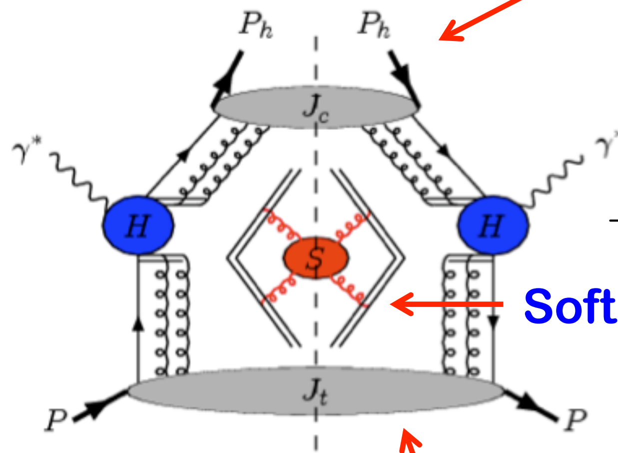
$$\sigma_{\mathrm{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

TMD fragmentation

Soft factors

TMD parton distribution

❑ Extraction of TMDs:

$$\sigma_{\mathrm{SIDIS}}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

Extracted TMDs are valid only when the $<p^2> << Q^2$

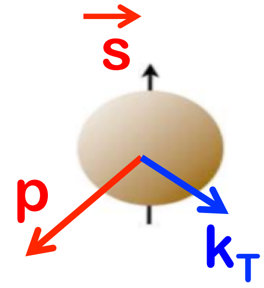
The Present: TMDs

❑ Power of spin – many more correlations:

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 =		$h_1^\perp$ = − Boer-Mulders
L		g_{1L} = − Helicity	$h_{1L}^\perp$ = −
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_1 = − Transversity $h_{1T}^\perp$ = −

Nucleon Spin — Quark Spin

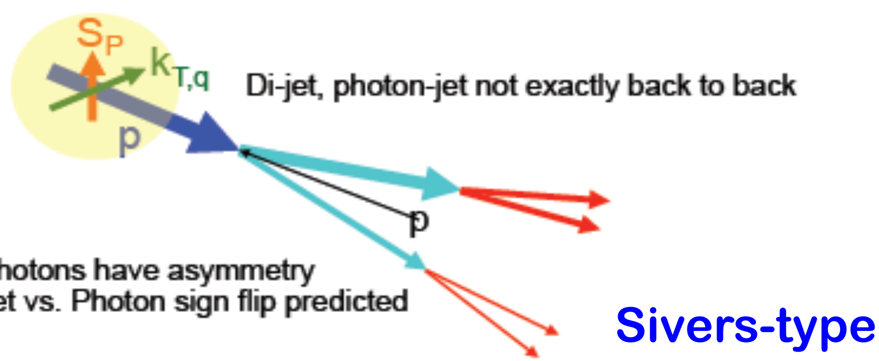
Similar for gluons

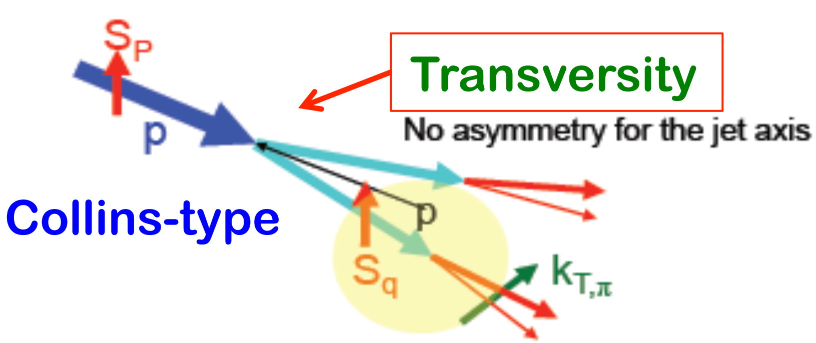


Require two Physical scales

More than one TMD contribute to the same observable!

❑ A_N – single hadron production:





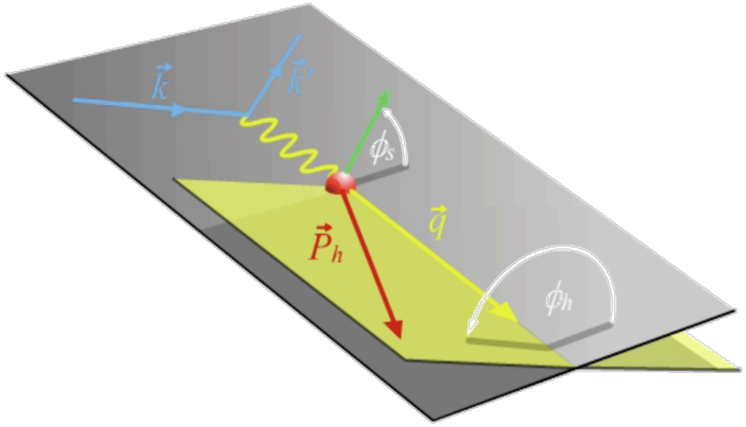
SIDIS is the best for probing TMDs

- Naturally, two scales & two planes:

$$A_{UT}(\varphi_h^l, \varphi_S^l) = \frac{1}{P}\frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}$$

$$= A_{UT}^{Collins}\sin(\phi_h + \phi_S) + A_{UT}^{Sivers}\sin(\phi_h - \phi_S)$$

$$+ A_{UT}^{Pretzelosity}\sin(3\phi_h - \phi_S)$$



- Separation of TMDs:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S)\rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S)\rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S)\rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

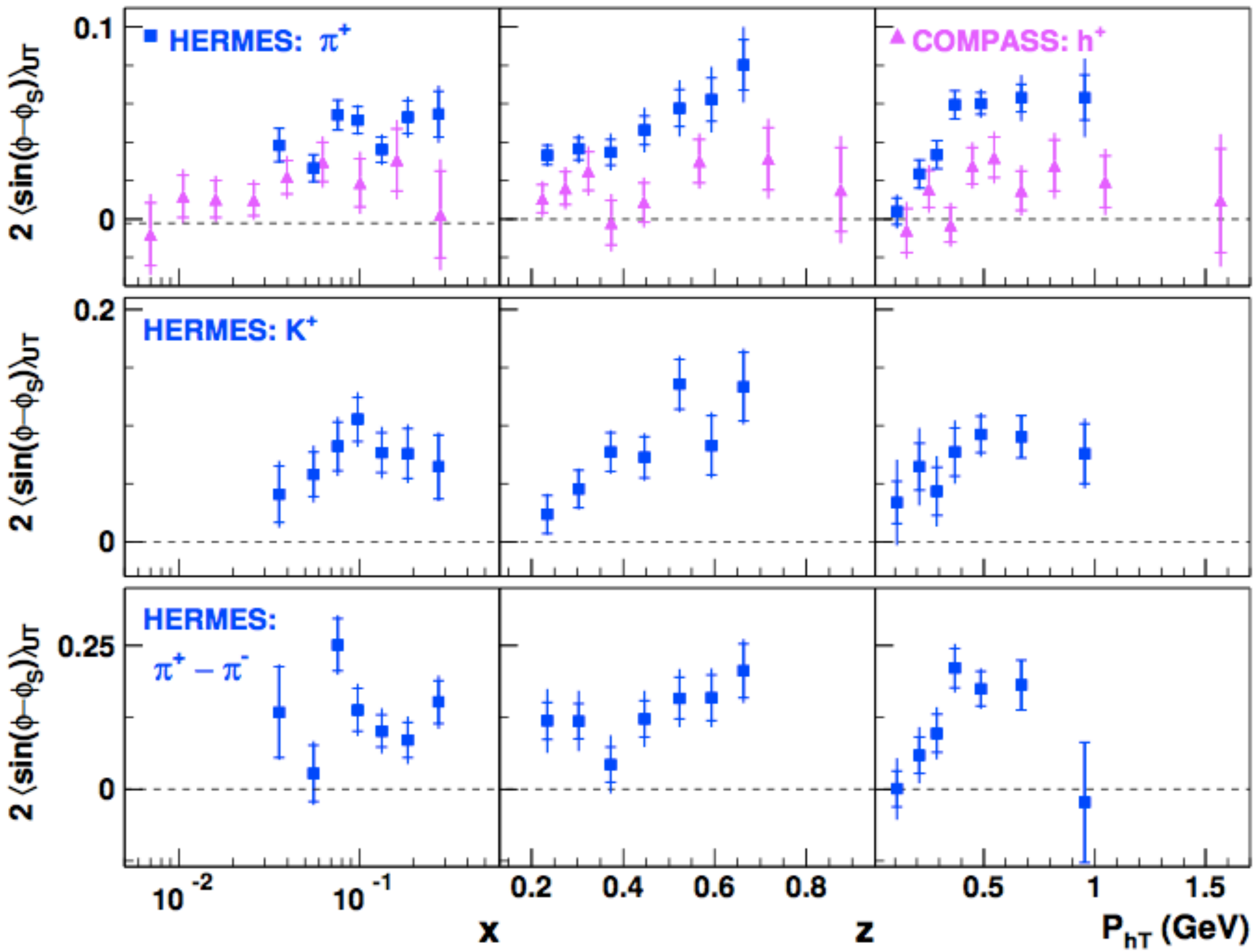
Collins frag. Func. from e^+e^- collisions

Hard, if not impossible, to separate TMDs in hadronic collisions

Using a combination of different observables (not the same observable): jet, identified hadron, photon, …

Sivers asymmetries from SIDIS

❑ **From SIDIS (HERMES and COMPASS) – low Q:**



Non-zero Sivers effects Observed in SIDIS!

Visible Q^2 dependence

Major theory Development In last year

Evolution equations for TMDs

J.C. Collins, in his book on QCD

- TMDs in the b-space:

$$\tilde{F}_{f/P^\uparrow}(x,\mathrm{b_T},S;\mu;\zeta_F)=\tilde{F}^{\mathrm{unsub}}_{f/P^\uparrow}(x,\mathrm{b_T},S;\mu;y_P-(-\infty))\sqrt{\frac{\tilde{S}_{(0)}(\mathrm{b_T};+\infty,y_s)}{\tilde{S}_{(0)}(\mathrm{b_T};+\infty,-\infty)\tilde{S}_{(0)}(\mathrm{b_T};y_s,-\infty)}}Z_F\,Z_2$$

- Collins-Soper equation:

Renormalization of the soft-factor

$$\frac{\partial\tilde{F}_{f/P^\uparrow}(x,\mathrm{b_T},S;\mu;\zeta_F)}{\partial\ln\sqrt{\zeta_F}}=\tilde{K}(b_T;\mu)\tilde{F}_{f/P^\uparrow}(x,\mathrm{b_T},S;\mu;\zeta_F)$$

$$\zeta_F=M_P^2x^2e^{2(y_P-y_s)}$$

$$\tilde{K}(b_T;\mu)=\frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{\tilde{S}(b_T;y_s,-\infty)}{\tilde{S}(b_T;+\infty,y_s)}\right)$$

Introduced to regulate the rapidity divergence of TMDs

- RG equations:

Wave function Renormalization

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu}=-\gamma_K(g(\mu))$$

Evolution equations are only valid when $b_T \ll 1/\Lambda_{QCD}$!

$$\frac{d\tilde{F}_{f/P^\uparrow}(x,\mathrm{b_T},S;\mu;\zeta_F)}{d\ln\mu}=\gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}_{f/P^\uparrow}(x,\mathrm{b_T},S;\mu;\zeta_F).$$

- Momentum space TMDs:

Need information at large b_T for all scale μ !

$$F_{f/P^\uparrow}(x,\mathrm{k_T},S;\mu,\zeta_F)=\frac{1}{(2\pi)^2}\int d^2\mathrm{b}_T\,e^{i\mathrm{k}_T\cdot\mathrm{b}_T}\,\tilde{F}_{f/P^\uparrow}(x,\mathrm{b_T},S;\mu,\zeta_F)$$

Evolution equations for Sivers function

Aybat, Collins, Qiu, Rogers, 2011

- Sivers function:

$$F_{f/P^{\uparrow}}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

- Collins-Soper equation:

Its derivative obeys the CS equation

$$\frac{\partial \ln \tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)$$

$$\tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)}{\partial b_T}$$

- RG equations:

$$\frac{d\tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2) \tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F)$$

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu)) \quad \Rightarrow \quad \frac{\partial \gamma_F(g(\mu); \zeta_F/\mu^2)}{\partial \ln \sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

- Sivers function in momentum space:

$$F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) = \frac{-1}{2\pi k_T} \int_0^{\infty} db_T \, b_T J_1(k_T b_T) \tilde{F}_{1T}^{\prime \perp f}(x, b_T; \mu, \zeta_F)$$

JI, Ma, Yuan, 2004
Idilbi, et al, 2004,
Boer, 2001, 2009,
Kang, Xiao, Yuan, 2011
Aybat, Prokudin, Rogers, 2012
Idilbi, et al, 2012,
Sun, Yuan 2013, …

Extrapolation to large b_T

- CSS b*-prescription:

Aybat and Rogers, arXiv:1101.5057
Collins and Rogers, arXiv:1412.3820

$$\tilde{F}_{f/P}(x, \mathrm{b}_T; Q, Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\mathrm{AA}}$$

$$\times \overbrace{\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\mathrm{BB}}$$

$$\times \boxed{\overbrace{\exp\left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \right\}}^{\mathrm{CC}}}$$

Nonperturbative "form factor"

$$b_* = \frac{b_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2/b_{\mathrm{max}}^2}} \qquad \text{with } b_{\mathrm{max}} \sim 1/2\ \mathrm{GeV}^{-1}$$

- Nonperturbative fitting functions

Various fits correspond to different choices for $g_{f/P}(x, b_T)$ and $g_K(b_T)$
e.g.

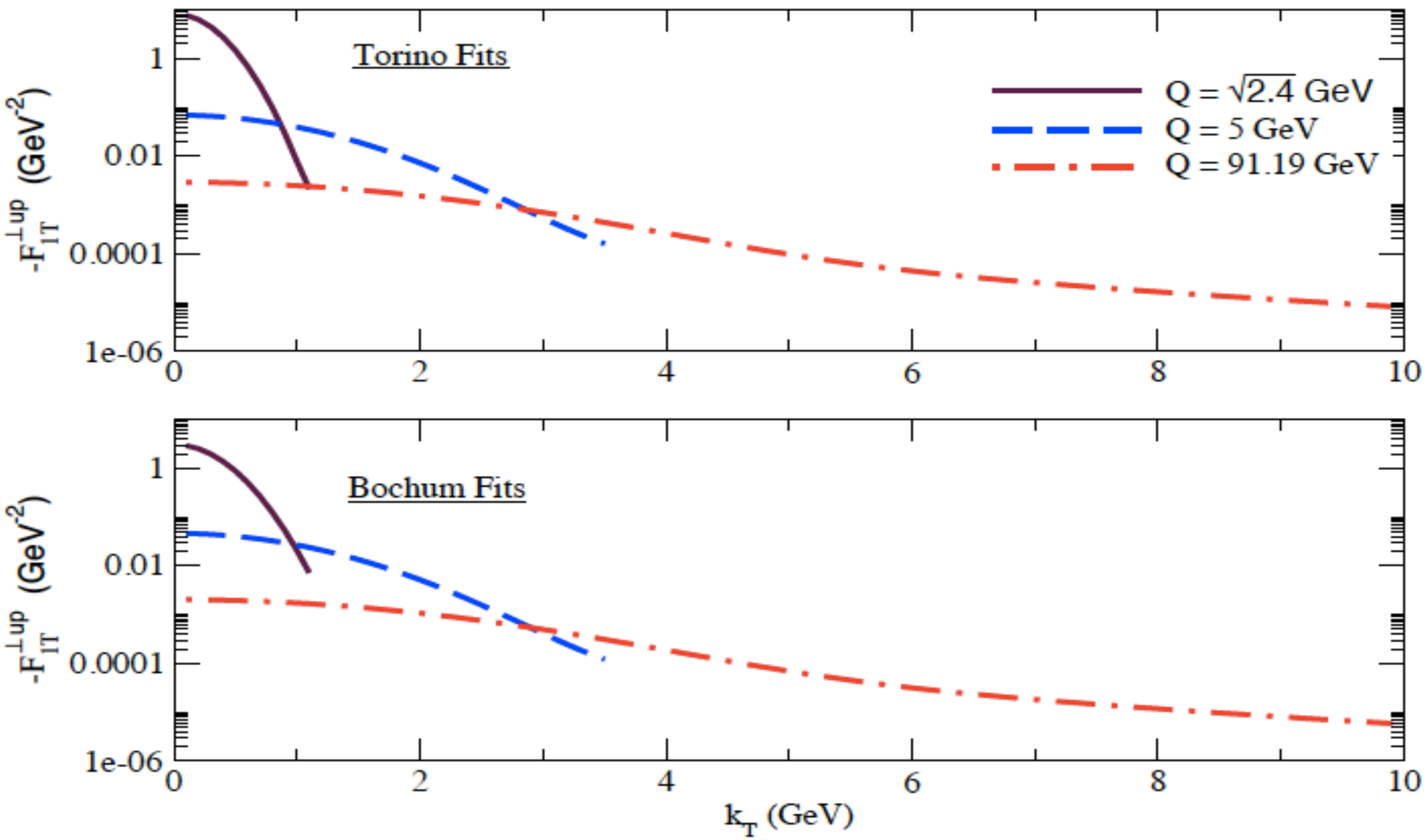
$$g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \equiv -\left[g_1 + g_2 \ln \frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

Different choice of g2 & b_* could lead to different over all Q-dependence!

Evolution of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

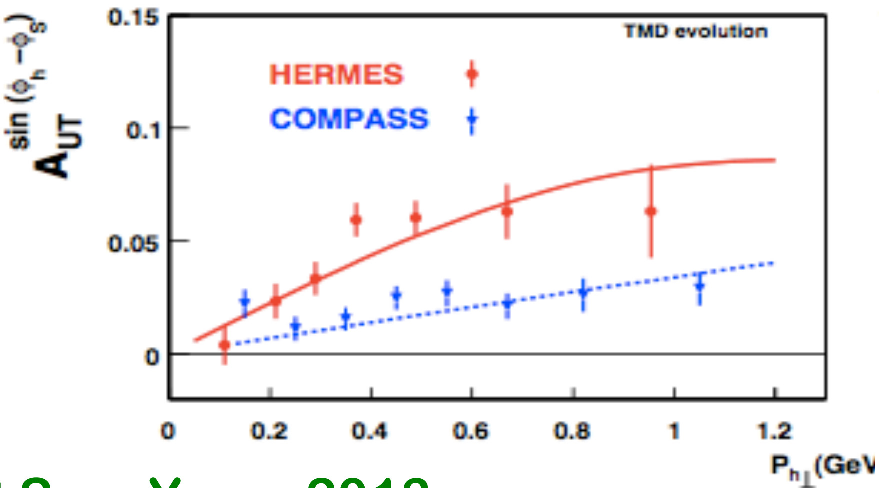
- Up quark Sivers function:

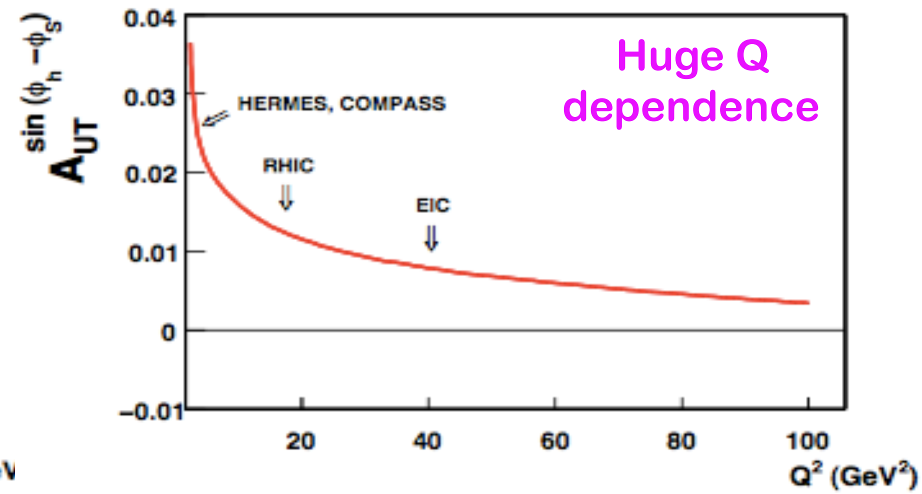


Very significant growth in the width of transverse momentum

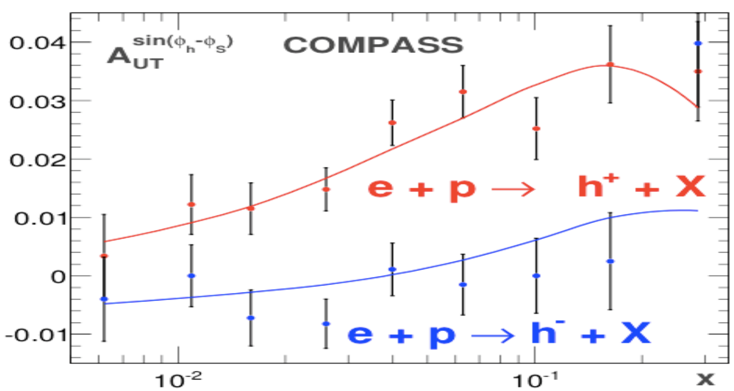
Different fits – different Q-dependence

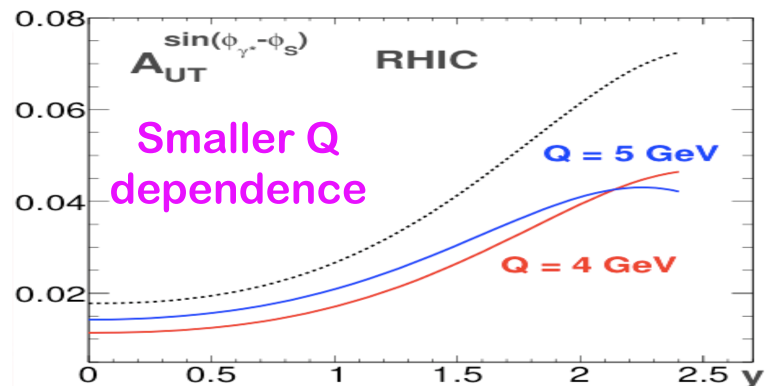
- Aybat, Prokudin, Rogers, 2012:





- Sun, Yuan, 2013:



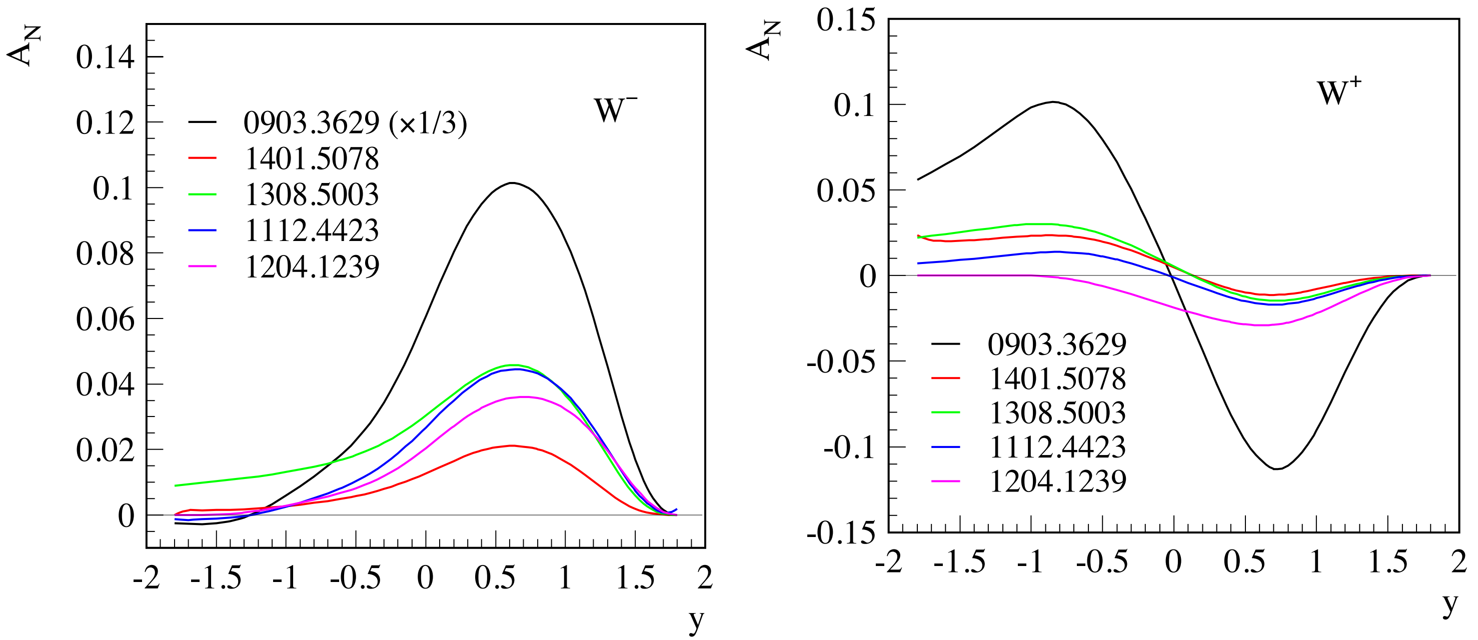


No disagreement on evolution equations!

Issues: Extrapolation to non-perturbative large b-region
Choice of the Q-dependent "form factor"

“Predictions” for A_N of W-production at RHIC?

- **Sivers Effect:**
  - Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons
  - QCD Prediction: Sign change of Sivers function from SIDIS and DY
- **Current “prediction” and uncertainty of QCD evolution:**



TMD collaboration proposal: Lattice, theory & Phenomenology

RHIC is the excellent and unique facility to test this (W/Z – DY)!

What happened?

- Sivers function:

Differ from PDFs!

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)=\frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T\, J_1(k_T b_T)\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

Need non-perturbative large b_T information for any value of Q! *$Q=\mu$*

- What is the "correct" Q-dependence of the large b_T tail?

$$\tilde{F}_{f/P}(x,\mathrm{b}_T;Q,Q^2) = \overbrace{\sum_j\int_x^1\frac{d\hat{x}}{\hat{x}}\tilde{C}_{f/j}(x/\hat{x},b_*;\mu_b^2,\mu_b,g(\mu_b))f_{j/P}(\hat{x},\mu_b)}^{\mathrm{AA}}$$

$$\times\ \overbrace{\times\exp\left\{\ln\frac{Q}{\mu_b}\tilde{K}(b_*;\mu_b)+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1)-\ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right\}}^{\mathrm{BB}}$$

$$\times\ \boxed{\overbrace{\exp\left\{g_{f/P}(x,b_T)+g_K(b_T)\ln\frac{Q}{Q_0}\right\}}^{\mathrm{CC}}}$$

Nonperturbative "form factor"

$$g_{f/P}(x,b_T)+g_K(b_T)\ln\frac{Q}{Q_0}\equiv-\left[g_1+g_2\ln\frac{Q}{2Q_0}+g_1g_3\ln(10x)\right]b_T^2$$

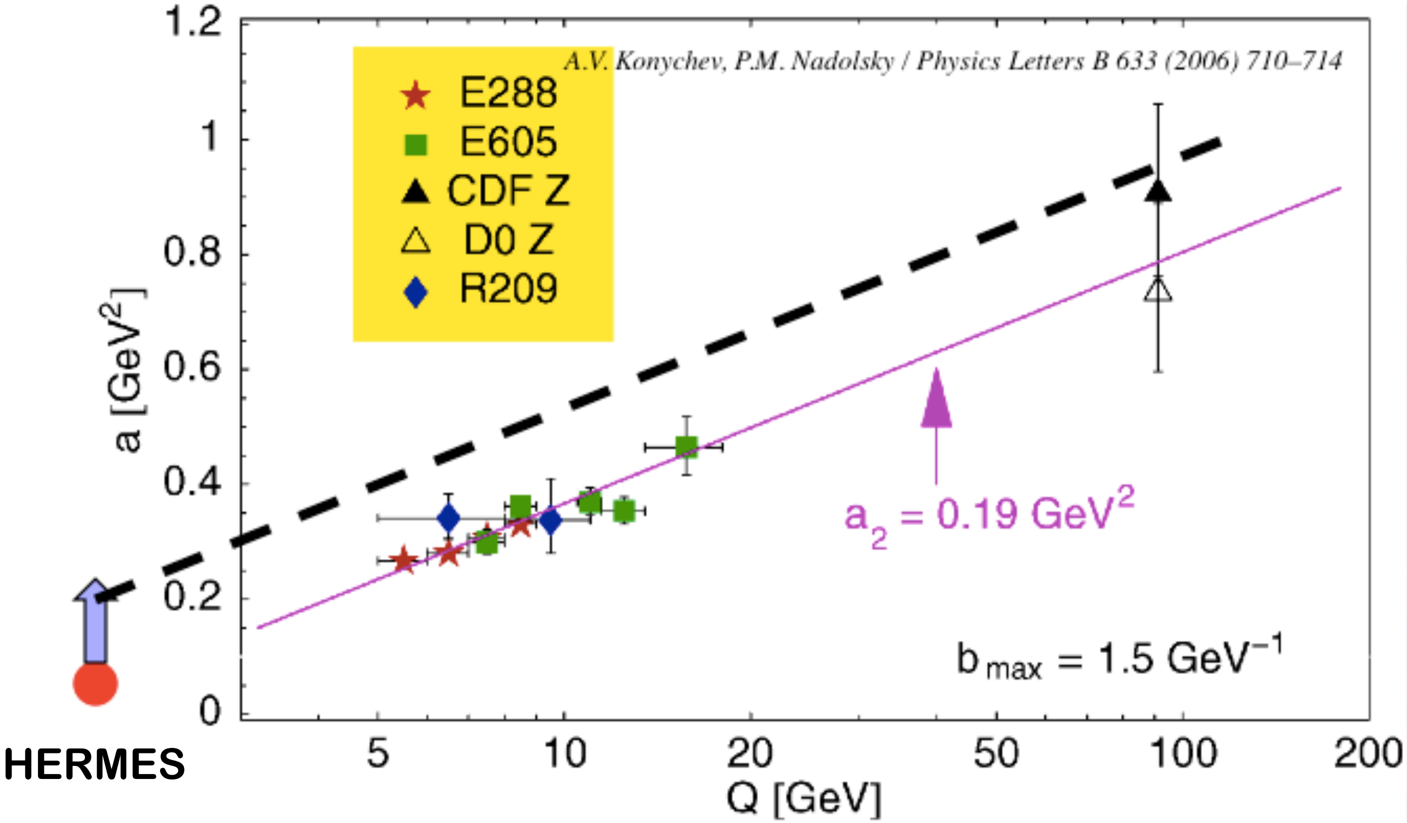
Is the log(Q) dependence sufficient? Choice of g_2 & b_* affects Q-dep.

The "form factor" and b_* change perturbative results at small b_T!

Q-dependence of the “form” factor

Konychev, Nadolsky, 2006

- ❑ **Q-dependence of the “form factor” :**



$$\mathcal{F}^{\mathrm{NP}}(b,Q) = a(Q^2)\, b^2$$

At Q ~ 1 GeV, $\ln(Q/Q_0)$ term may not be the dominant one!

$$\mathcal{F}^{\mathrm{NP}} \approx b^2(a_1 + a_2 \ln(Q/Q_0) + a_3 \ln(x_A x_B) + \ldots) + \ldots$$

Power correction? ***$(Q_0/Q)^n$-term?*** ***Better fits for HERMES data?***

Factorized Drell-Yan cross section

- TMD factorization ($q_\perp \ll Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^y \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- Collinear factorization ($q_\perp \sim Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

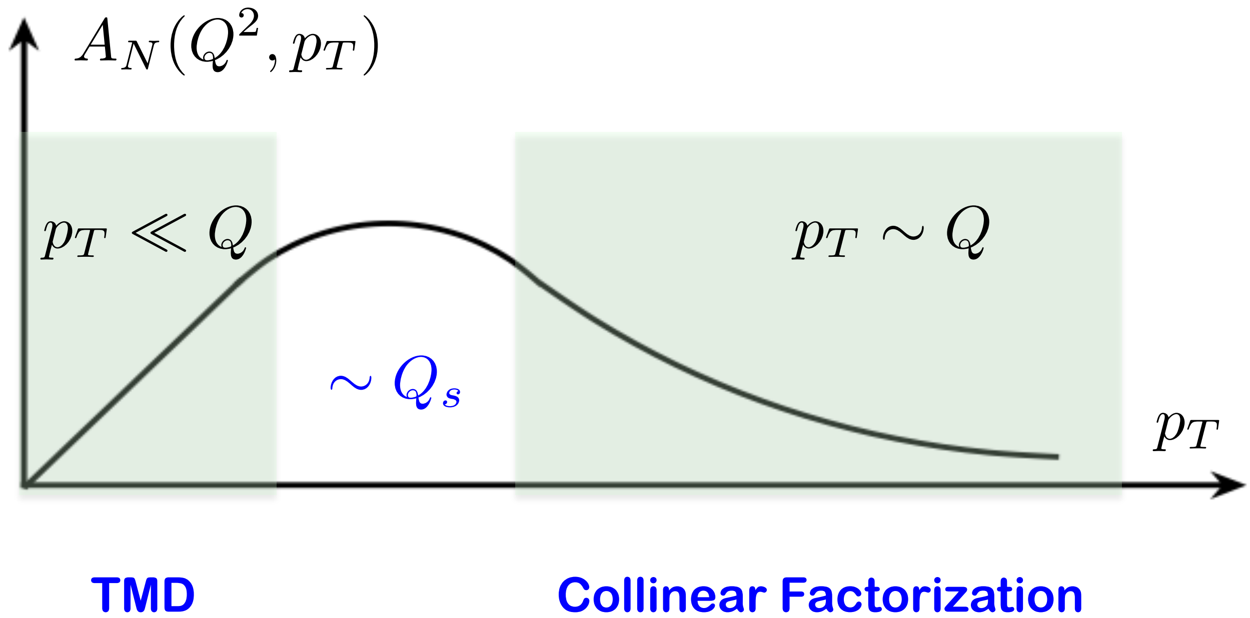
- Spin dependence:

The factorization arguments are independent of the spin states of the colliding hadrons

⇒ **same formula with polarized PDFs for γ^*,W/Z, H^0...**

Transition from low p_T to high p_T

❑ Two-scale becomes one-scale:



❑ TMD factorization to collinear factorization:

Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan

Two factorization are consistent in the overlap region: $\Lambda_{\rm QCD} \ll p_T \ll Q$

A_N finite – requires correlation of multiple collinear partons

No probability interpretation! New opportunities!

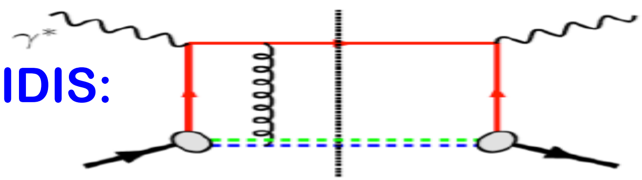
Broken universality for TMDs

❑ Definition:

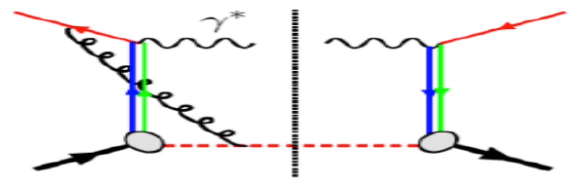
$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\boxed{\text{Gauge link}}\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

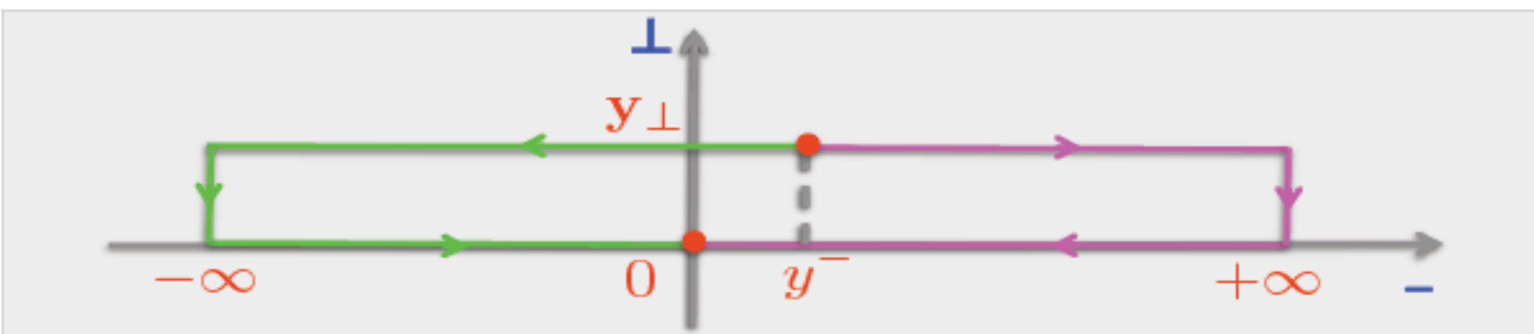
❑ Gauge links:

SIDIS:



DY:





❑ Process dependence:

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

Collinear factorized PDFs are process independent

Critical test of TMD factorization

- Parity – Time reversal invariance:

$$f_{q/h^\uparrow}^{\rm SIDIS}(x, \mathbf{k}_\perp, \vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x, \mathbf{k}_\perp, -\vec{S})$$

- Definition of Sivers function:

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x, k_\perp)\, \vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

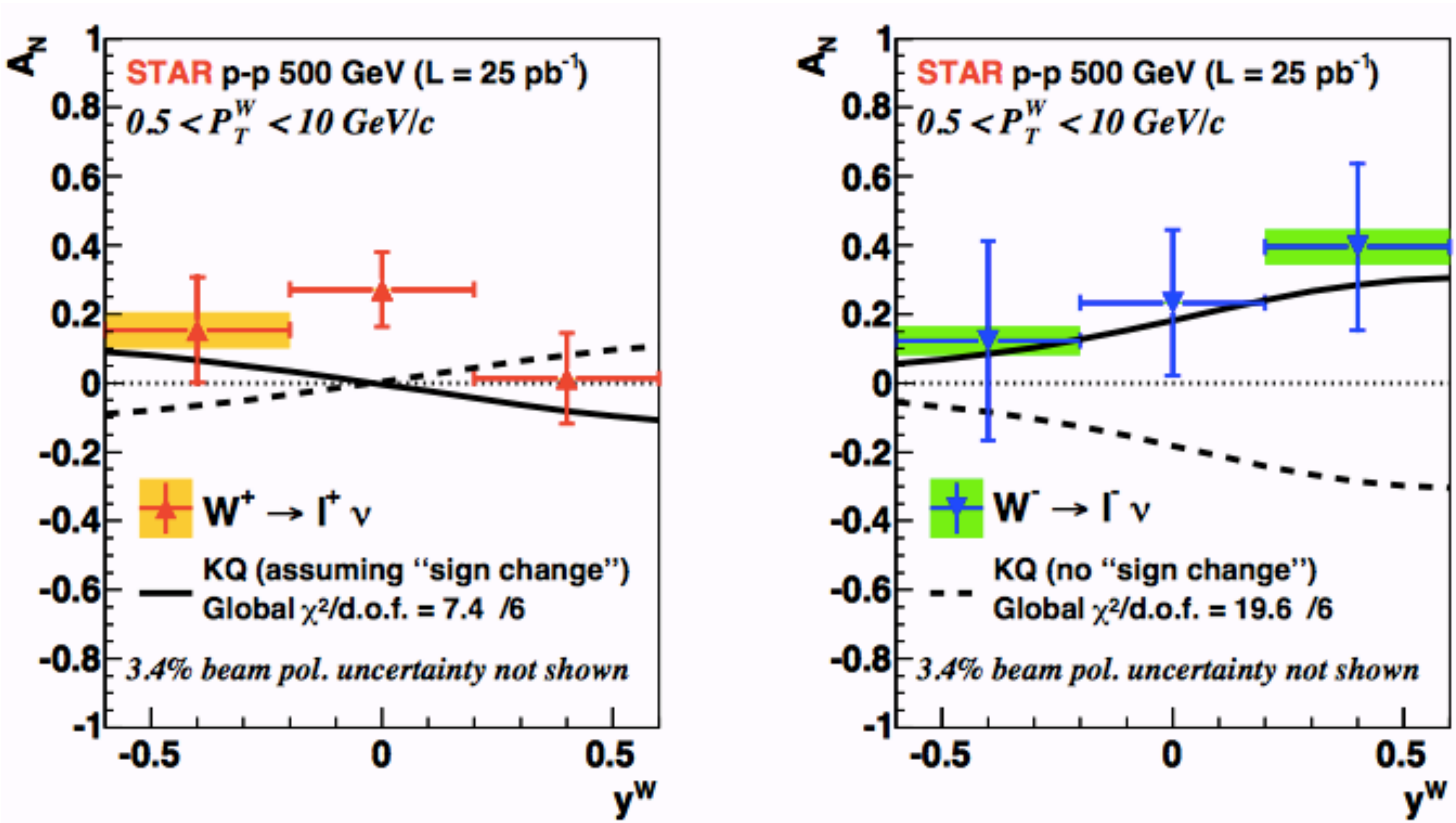
- Modified universality:

$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x, k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x, k_\perp)$$

Same applies to TMD gluon distribution

Spin-averaged TMD is process independent

A_N for W production at RHIC



Data from STAR collaboration on A_N for W-production are consistent with a sign change between SIDIS and DY

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)

Summary of lecture seven

- Single transverse-spin asymmetry in real, and is a unique probe for hadron's internal dynamics – Sivers, Collins, twist-3, … effects

- RHIC data seems to be consistent with the sign change of Sivers function, as predicted by QCD factorization

- But, the evolution of TMDs is still a very much open question! Better approach to non-perturbative inputs is needed!

- JLab12 and EIC should be able to provide much better data to help explore the confined motion of quarks/gluons

Thank you!

Backup slides

QCD and hadrons