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LEPTO[1] 6.5 — A Monte Carlo Generator for Deep Inelastic Lepton-Nucleon Scattering

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Abstract. Physics and programming aspects are discussed for a Fortran 77 Monte Carlo program to simulate complete events in deep inelastic lepton-nucleon scattering. The parton level interaction is based on the standard model electroweak cross sections, which are fully implemented in leading order for any lepton of arbitrary polarization, and different parametrizations of parton density functions can be used. First order QCD matrix elements for gluon radiation and boson-gluon fusion are implemented and higher order QCD radiation is treated using parton showers. Hadronization is performed using the Lund string model, implemented in JETSET/PYTHIA. Rapidity gap events are generated through a model based on soft colour interactions.

1 Introduction

Deep inelastic lepton-nucleon scattering [1] has played an important role for probing the structure of the proton and understanding the fundamental electromagnetic, weak and strong interactions at the level of quarks and leptons. With the order of magnitude increase in the centre-of-mass energy now available through *ep* collisions in HERA, this line of research will continue to be at the forefront.

The basic lepton-quark scattering processes have well-defined cross section formulae within the electroweak standard model [2]. With the inclusion of parton density distributions and perturbative QCD corrections the problem of practical evaluations become quite complex and analytical calculations are only possible in simplified cases or through approximations. Normal numerical methods are in many cases possible, but Monte Carlo simulation is often preferable because of its generality and applicability to complex problems. In the case of the multiparticle hadronic final state, the only viable alternative is, in fact, the Monte Carlo method.

The present program, LEPTO, is a general and flexible Monte Carlo (MC) to simulate complete lepton-nucleon scattering events and integrate cross sections. It is based on the leading order electroweak cross sections for the underlying parton level scattering

[1] Information and code via WWW page `http://www3.tsl.uu.se/thep/lepto/`

processes. Thus, electroweak radiative corrections are not included, but can be simulated using the interface DJANGO [3] between LEPTO and HERACLES [4]. The main emphasis of LEPTO is rather on the hadronic part of the event. QCD corrections are therefore included using exact first order matrix elements and higher orders in the leading $\log Q^2$ parton cascade approach. The fragmentation of produced partons into observable hadrons is performed with the Lund string hadronization model [5]. An arbitrary configuration of a lepton and a nucleon can be defined with constraints on the scattering kinematics and the generated events can be transformed to different frames. The present version is the latest development in a series of earlier versions [6, 7] that have developed in stages and profitted greatly by feed-back from extensive comparisons with experimental data on the hadronic final state in leptoproduction. This has given valuable insights concerning the QCD processes of gluon radiation and boson-gluon fusion as well as the confinement induced hadronization of colour charged partons. The generally good agreement between data, in particular from the European Muon Collaboration and the neutrino experiments WA21 and WA25 at CERN, and the Monte Carlo shows the validity of the models and procedures used in the program. This can be illustrated by the results on longitudinal momentum spectra of different identified hadrons [8], transverse momentum properties, energy flows and jet structure [9]. The much higher energies available in *ep* collisions at HERA provides both qualitative and quantitative new information on QCD effects, such that detailed tests of MC models are now being performed [10, 11].

In the following a comprehensive description is given of the theoretical framework built into the program (section 2) as well as the various program components (section 3) and their usage (section 4).

2 Physics and MC implementation

2.1 Kinematics

The main kinematic relations given here are for the case of electron-proton scattering, $e + p \to \ell + H$ where ℓ is the scattered lepton and H the final hadron system, but are of course equally valid for any lepton beam or a neutron target. Let p_e, p_ℓ be the four-vectors of the incoming and scattered lepton, respectively, and P that of the incoming proton. Some basic kinematic relations are then (cf. [12, 13])

$$s \equiv (p_e + P)^2 \simeq 4E_eE_p \tag{1}$$

$$W^2 \equiv (q + P)^2 = Q^2\frac{1-x}{x} + m_p^2 \tag{2}$$

$$Q^2 \equiv -q^2 = -(p_e - p_\ell)^2 \simeq 4E_eE_\ell \sin^2\frac{\theta_\ell}{2} \tag{3}$$

$$m_p\nu \equiv P \cdot q \simeq 2E_p(E_e - E_\ell \cos^2\frac{\theta_\ell}{2}) \tag{4}$$

$$x \equiv \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2m_p\nu} \simeq \frac{E_eE_\ell \sin^2\frac{\theta_\ell}{2}}{E_p(E_e - E_\ell\cos^2\frac{\theta_\ell}{2})} \tag{5}$$

$$y \equiv \frac{P \cdot q}{P \cdot p_e} = \frac{\nu}{\nu_{max}} \simeq \frac{E_e - E_\ell\cos^2\frac{\theta_\ell}{2}}{E_e} \tag{6}$$

where the '$\simeq$' sign means that the lepton and nucleon masses are neglected, an approximation which is usually acceptable already at fixed target energies. Nevertheless, exact formulae are used in the program to avoid unnecessary approximations. Here, $\sqrt{s}$ is the total invariant mass and W the invariant mass of the hadronic system H. The exchanged vector boson, γ/Z^0 for neutral current interactions and $W^\pm$ for charged current interactions, carries the momentum transfer Q^2 and the variable ν is the energy of this current in the target rest frame. Finally, Bjorken-x and y are the convenient dimensionless scaling variables in the range $[0, 1]$.

For the overall event kinematics there are only two independent variables and hence by measuring, e.g., the energy (E_ℓ) and angle (θ_ℓ) of the scattered lepton (here defined with respect to the incoming electron direction), all other global variables can be calculated. For theoretical purposes, cross section formulae etc, the variables (x, y) or (x, Q^2) are usually used as the independent ones. No assumption has here been made about the structure of the proton, or about the final hadronic state. By *assuming* the quark-parton model, where the current couples to a quark with four-vector $p_i \equiv \xi(E_p, 0, 0, E_p)$ and *assuming* the initial and final quark to be massless one finds $\xi = x$. The Bjorken-x variable can then be interpreted as the momentum fraction of the proton which is carried by the struck quark. However, taking QCD corrections into account this relation no longer holds [14].

For ep colliders like HERA where the proton beam energy is much larger than the electron beam energy, the phase space is somewhat special and quite elongated in the proton beam direction (see Fig. 2 in [12]). Although the events are certainly not evenly distributed in the available phase space, they are in general very asymmetric, with most of the final state hadrons in the 'forward' direction along the incoming proton.

For most analyses the event kinematics need to be reconstructed. The differential cross sections, which are the basis of most analyses, are usually needed as functions of, e.g., x and Q^2. The event kinematics can be straightforwardly obtained from the formulae above if the scattered lepton can be well measured. However, this is not always the case due to instrumental effects or because it is a neutrino. These problems are accentuated at ep colliders where the acceptance is limited by the beam pipes and where charged current interactions give an undetectable neutrino as scattered lepton. This implies the importance of being able to use the hadronic part of the event to reconstruct the kinematical variables. In the naive quark-parton model (QPM) the scattered quark gives the necessary information, but in reality a number of smearing effects enter (QCD processes, fragmentation, mass effects, jet reconstruction). Considering the hadronic final state as a single system, whose internal structure is of no importance, and applying energy momentum conservation between this system and the scattered lepton leads to methods such as 'Jacquet–Blondel' [15, 16], 'double angle' [17] and 'Σ' [18]. These are particularly suited to ep colliders, since they minimize the effects of particles lost in the beam pipe.

2.2 Electroweak cross sections

In leading order electroweak theory [2] the differential neutral current (NC) cross section for the scattering of a charged lepton is (neglecting masses) given by

$$\frac{d^2\sigma_{NC}(e^\mp)}{dx\,dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[y^2xF_1(x,Q^2) + (1-y)F_2(x,Q^2) \pm \left(y - \frac{y^2}{2}\right)xF_3(x,Q^2)\right] \tag{7}$$

in terms of the nucleon structure functions F_1, F_2, F_3. Two of these are related through the Callan-Gross relation, $2xF_1 = F_2$, which holds for spin 1/2 quarks when neglecting masses, intrinsic transverse momenta and order α_s QCD effects. These effects are usually negligibly small, except at small-x [19, 20], but can optionally be included (cf. section 2.3). More important for the structure of the electroweak theory is to take the lepton beam polarization into account. For a left- (L) and right-handed (R) electron one has the differential cross section

$$\frac{d^2\sigma_{NC}(e^-_{L,R}p)}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[\left(1+(1-y)^2\right)F_2^{L,R}(x,Q^2) + \left(1-(1-y)^2\right)xF_3^{L,R}(x,Q^2)\right] \quad (8)$$

The structure functions are in the standard parton model given by

$$F_2^{L,R}(x,q^2) = \sum_q \left[xq(x,Q^2) + x\bar{q}(x,Q^2)\right]\ A_q^{L,R}(Q^2) \quad (9)$$

$$xF_3^{L,R}(x,q^2) = \sum_q \left[xq(x,Q^2) - x\bar{q}(x,Q^2)\right]\ B_q^{L,R}(Q^2) \quad (10)$$

where the sum is over all quark flavours and $q(\bar{q})$ denote the quark (antiquark) density distributions in the nucleon (cf. section 2.4). The coefficients are given by

$$A_q^{L,R}(Q^2) = e_q^2 - 2e_q(v_e \pm a_e)v_q P_Z + (v_e \pm a_e)^2(v_q^2 + a_q^2)P_Z^2 \quad (11)$$

$$B_q^{L,R}(Q^2) = \mp 2e_q(v_e \pm a_e)a_q P_Z \pm 2(v_e \pm a_e)^2 v_q a_q P_Z^2 \quad (12)$$

With $f = e, q$ for any fundamental fermion, e_f is here the electric charge ($e_e = -1$), $v_f = (T_{3f} - 2e_f \sin^2\theta_W)/\sin 2\theta_W$ and $a_f = T_{3f}/\sin 2\theta_W$ are the NC vector and axial vector couplings expressed in terms of the third component of the weak isospin ($T_{3e} = -\frac{1}{2}$) and the Weinberg angle θ_W. P_Z is the ratio of the Z and γ propagators $P_Z = Q^2/(Q^2 + M_Z^2)$. The corresponding cross sections for positrons, $e^+_{L,R}$, are obtained from the above electron formulae, eqs. (8)-(12), by the replacements

$$F_2^{L,R} \to F_2^{R,L}, \quad xF_3^{L,R} \to -xF_3^{R,L}\ , \quad (13)$$

i.e. a left(right)-handed electron corresponds to a right(left)-handed positron and an additional sign change in xF_3. The cross section for an arbitrarily polarized electron/positron beam is simply obtained as a linear combination of these pure left- and right-handed cross sections (cf. [19]).

The pure γ exchange term, i.e. the one without a P_Z dependence in eq. (11), dominates completely at low Q^2, and the cross section then takes the familiar form measured in fixed target electron and muon beam experiments

$$\frac{d^2\sigma_\gamma(ep)}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4}(1+(1-y)^2)F_2^{em}(x,Q^2) \quad (14)$$

where the electromagnetic structure function is given by

$$F_2^{em}(x,Q^2) = \sum_q e_q^2\left[xq(x,Q^2) + x\bar{q}(x,Q^2)\right] \quad (15)$$

With increasing Q^2 first the γ/Z^0 interference term (linear in P_Z) and then the pure weak term (quadratic in P_Z) become important and finally dominate the cross section at large Q^2, see e.g. Fig. 3 in [12].

The differential cross sections for charged current (CC) ep interactions are given by

$$\frac{d^2\sigma_{CC}(e^-p)}{dx\,dQ^2} = \frac{(1-\lambda)\pi\alpha^2}{4\sin^4\theta_W\,(Q^2+M_W^2)^2}\sum_{i,j}\left[\left|V_{u_id_j}\right|^2 u_i + (1-y)^2\left|V_{u_jd_i}\right|^2 \bar{d}_i\right] \quad (16)$$

$$\frac{d^2\sigma_{CC}(e^+p)}{dx\,dQ^2} = \frac{(1+\lambda)\pi\alpha^2}{4\sin^4\theta_W\,(Q^2+M_W^2)^2}\sum_{i,j}\left[\left|V_{u_id_j}\right|^2 \bar{u}_i + (1-y)^2\left|V_{u_jd_i}\right|^2 d_i\right] \quad (17)$$

where $V_{u_id_j}$ are elements of the Kobayashi-Maskawa matrix, u_i and d_j denote the parton density functions for the up-type and down-type quark flavours, respectively, and i,j are family indices. The $e^\pm$ beam polarization is denoted by λ (± 1 for a right/left-handed state). Considering only four massless quark flavours (u,d,s,c) and using the unitarity relation $\sum_j \left|V_{u_id_j}\right|^2 = \sum_j \left|V_{u_jd_i}\right|^2 = 1$ one obtains for any lepton with fixed helicity

$$\frac{d^2\sigma_{CC}(\ell p)}{dx\,dQ^2} \simeq \frac{G_F^2}{\pi}\left(1+\frac{Q^2}{M_W^2}\right)^{-2}\begin{cases}(u+c)+(1-y)^2(\bar{d}+\bar{s}) & for \quad \ell = e_L^-,\ \bar{\nu}\\ (\bar{u}+\bar{c})+(1-y)^2(d+s) & for \quad \ell = e_R^+,\ \nu \\ 0 & for \quad \ell = e_R^-,\ e_L^+\end{cases} \quad (18)$$

Here, $G_F = \pi\alpha/(\sqrt{2}\sin^2\theta_W M_W^2)$ is the Fermi coupling constant, M_W the W-boson mass and u denotes the u-quark density $u(x,Q^2)$ etc.

The above formulae apply equally well to muon scattering, whereas for neutrino scattering some rearrangements are required. For charged current scattering, eq. (16) apply for a $\bar{\nu}$ with $\lambda = -1$ and eq. (17) for a ν with $\lambda = +1$, giving the results in eq. (18). For neutral current neutrino scattering a similar correspondence applies, but the electron electroweak couplings must also be changed to the neutrino ones. Hence, only the pure Z^0 contribution remains in eqs. (11,12) with $\nu - Z^0$ vector and axial vector couplings, v_ν and a_ν [21].

When simulating the kinematic variables according to the above differential cross sections one may choose the two independent variables depending on the process. The choice x, Q^2 for NC and x, y for CC is suitable and adopted in the program, but this can be changed (see LST(1) in common LEPTOU) to other combinations that may improve effiency, e.g., by better reflecting strong kinematic cuts. The two variables are first chosen according to a function that can be directly generated without any rejections, i.e. its primitive function can be obtained and inverted analytically. By chosing this function to represent the strongest variation of the cross section formulae, the remaining part can be taken into account by a simple rejection technique whose efficiency will be higher the smaller the variation in this remaining function. Formally, a function

$$h_v(v) = a_0 + \frac{a_1}{v} + \frac{a_2}{v^2} + \frac{a_3}{v^3} \quad (19)$$

is introduced for each variable $v = x, y, Q^2, W^2$ and the cross section is rewritten in the form (using variables x, Q^2 as a definite example)

$$d\sigma = \left\{h_x(x)dx\ h_{Q^2}(Q^2)dQ^2\right\}\left\{\frac{d\sigma/dxdQ^2}{h_x h_{Q^2}}\right\} \quad (20)$$

where the random variables can be chosen exactly according to the expression in the first bracket and the remaining factor is used for the weighting procedure. The latter simply means that the chosen x, Q^2 point is rejected if it results in a value for the second factor which is smaller than a random number (uniformly chosen between zero and unity) times the maximum of that factor with respect to the two variables (x, Q^2 in this case). Actually, any estimate larger than the maximum will do, but efficiency improves the closer to the true maximum it is. This estimate is, however, a constant for any fixed interaction and kinematic region and can therefore be obtained in the initialization phase of the Monte Carlo program, see subroutine LINIT. Thus new points are generated using the first factor until accepted by weighting with the second factor and hence the resulting events, or phase space points, will have no weight associated with them (or more correctly, they all have unit weight).

The parameters a_i in the functions h_v are in the program represented by OPT$v(i)$ in common LOPTIM and are given process dependent default values in subroutine LINIT to optimize program speed under normal conditions. This means that the functions h_v reflect the dominant variation of the matrix element from propagators etc. Since this may change with kinematics, e.g. dominance of pure Z^0 exchange at very large Q^2, more optimal values may be found under certain conditions.

In the initialization phase (subroutine LINIT) the simulation variables are defined and their effective limits calculated from applied cuts. The optimization parameters are set and the maximum needed for the weighting is found using an adaptation of MINUIT [22]. Further, the total cross section can be obtained at this stage by numerical integration over the kinematic variables. In the simulation phase, see subroutine LEPTO, phase space points are chosen from the cross section (in subroutine LEPTOX) as discussed above. This Monte Carlo sampling is also used to provide an estimate of the cross section for the process being simulated, see PARL(24) in common LEPTOU. Since the result is updated with each generated event the accuracy depends on the generated statistics as $\sim 1/\sqrt{N}$.

2.3 Longitudinal structure function

Target mass effects and the longitudinal structure function, defined by

$$F_L(x,Q^2) = \left(1 + 4\frac{x^2 m_p^2}{Q^2}\right) F_2(x,Q^2) - 2xF_1(x,Q^2) \tag{21}$$

can be included as an option (see LST(11)) using the formalism for photon exchange. The cross section in eq. (14) is then modified to [21]

$$\begin{aligned} \frac{d^2\sigma_\gamma(ep)}{dx\,dQ^2} &= \frac{4\pi\alpha^2}{xQ^4}\left[\left(1-y-y^2\frac{x^2 m_p^2}{Q^2}\right)F_2(x,Q^2) + y^2\,xF_1(x,Q^2)\right] && (22)\\ &= \frac{2\pi\alpha^2}{xQ^4}\left[\left(1+(1-y)^2\right)F_2(x,Q^2) - y^2\tilde{F}_L(x,Q^2)\right] && (23)\end{aligned}$$

where all mass effects have been absorbed into $\tilde{F}_L$ which consists of the three terms

$$\tilde{F}_L(x,Q^2) = F_L^{QCD}(x,Q^2) + F_L^{TM}(x,Q^2) + F_L^{HT}(x,Q^2) \tag{24}$$

The QCD contribution (which is leading twist) is to order α_s given by [23]

$$F_L^{QCD}(x,Q^2) = \frac{4\alpha_s(Q^2)}{3\pi}x^2\int_x^1\frac{dy}{y^3}F_2(y,Q^2) + \frac{2\alpha_s(Q^2)}{\pi}\sum_q e_q^2\,x^2\int_x^1\frac{dy}{y^3}\left(1-\frac{x}{y}\right)yg(y,Q^2) \tag{25}$$

The first term originates from the gluon radiation diagram and the second one from the photon-gluon fusion process, where the sum runs over the quark flavours (taken as u, d, s, c in the program). F_L^{QCD} gives an important contribution at small-x, where the gluon term dominates and F_L may hence be used to extract the gluon distribution [20]. The target mass correction to $\mathcal{O}(m_p^2/Q^2)$ is given by [23, 24]

$$F_L^{TM}(x,Q^2) = 4\frac{m_p^2}{Q^2}x^3\int_x^1 \frac{dy}{y^2}F_2(y,Q^2) - 2\frac{m_p^2}{Q^2}x^2F_2(x,Q^2) \tag{26}$$

where also the mass term in eq. (22) has been included. Finally the (dynamical) higher twist contribution to $\mathcal{O}(1/Q^2)$ can be written as [24, 23]

$$F_L^{HT}(x,Q^2) = 8\frac{\kappa^2}{Q^2}F_2(x,Q^2) \tag{27}$$

where the scale of this twist-4 contribution is given by the parameter κ^2 (cf. PARL(19)) with a value around 0.03 GeV^2 obtained [24] from SLAC data.

In the numerical implementation of these equations the evaluation of the integrals in eqs. (25,26) can be performed for each event at its proper x and Q^2-value. Since this tends to be time-consuming another option (see LST(11)) is to initially (in LINIT) set up a grid in x, Q^2 with their values and then perform a linear interpolation to the desired x, Q^2 when simulating events. The latter method is preferrable and gives almost the same precision under normal conditions. The inclusion of F_L only affects the distribution in x, Q^2, i.e. eq. (22) and thereby the cross section, but not the generation of the hadronic part of the event. For the first order QCD matrix element corrections, which are still generated as usual (cf. section 2.5), this is not fully consistent but should be an adequate approximation for inclusive properties of the hadronic final state.

2.4 Parton density distributions

To define the parton content of the proton for the cross section formulae above the parton density functions $q(x,Q^2)$, $\bar{q}(x,Q^2)$ and $g(x,Q^2)$ are needed. These give the probability to find a quark or antiquark of specific flavours, or a gluon, respectively, carrying a fraction x of the proton momentum when probing the proton with a momentum transfer Q^2. Several parametrizations of these distributions have been obtained using data, in particular from lepton scattering experiments, and with Q^2-dependence according to the perturbative QCD evolution equations [25]. The fit to the data provides the x-dependence and the QCD parameter Λ. The choice among many available parton density parametrisations in PYTHIA 5.7 [26] and in PDFLIB [27] is made through the switches LST(15) and LST(16) in common LEPTOU.

The possibility to scatter on intrinsic charm and bottom quarks in the nucleon is included by an option, see LST(15). The hypothesis of intrinsic charm quarks in the proton was introduced [28] as an attempt to understand a large discrepancy between early charm hadroproduction data and leading order perturbative QCD (pQCD) calculations. This discrepance has largely disappeared as more data have been collected and next-to-leading order (NLO) pQCD calculations have been made. Still, however, there are certain aspects of charm production data which are difficult to understand within the pQCD framework, but are natural if the intrinsic charm hypothesis is basically correct (see [29] and references therein). Intrinsic charm (IC) corresponds to a Fock-state decomposition of the proton wave function, $|p\rangle = \alpha|uud\rangle + \beta|uudc\bar{c}\rangle + ...$, with a small, but finite, probability β^2 (PARL(12)) for an intrinsic $c\bar{c}$ pair as a quantum fluctuation. The normalization of this component is the key unknown, although it should decrease as $1/m_Q^2$. Originally, a 1% probability was assumed, but later investigations based on EMC data on the charm structure function $F_2^c(x, Q^2)$ [30] indicate a somewhat smaller but non-vanishing level; $\sim 0.3\%$ [31] and $(0.86 \pm 0.60)\%$ [32].

From the IC model one obtaines an effective charm quark density [29]

$$c_{IC}(x) = \beta^2 1800 x^2 \left\{ \frac{1}{3}(1-x)(1+10x+x^2) + 2x(1+x)\ln x \right\} \tag{28}$$

which gives a charactersitic hard momentum spectrum with $\langle x_c \rangle = 2/7$. The Q^2 dependence from normal leading log GLAP equations have been calculated for IC [31], but can be taken into account through a simple extension of the parameterisation in Eq. (28) [29]. This quark density is then used in the electroweak cross section fomulae to simulate the scattering on such an intrinsic charm (or bottom) quark. One should note that the intrinsic charm quark density is *not* the same as the normal charm quark density present in conventional parametrizations. The latter starts with no charm being present at small Q^2 and develops a charm sea through perturbative QCD processes, e.g. GLAP evolution, resulting in a soft momentum distribution. In contrast, intrinsic charm is present already in the non-perturbative proton wave function giving the characteristic hard momentum fraction distribution above.

The scattered intrinsic (anti)charm quark may radiate gluons through initial and final state parton showers as usual. Adding conventional Lund model hadronization gives a charmed hadron in the current fragmentation region and the remaining partner (anti)charm quark in the proton remnant gives a charmed hadron in the target fragmentation region. For phenomenological studies with this model, see [29].

2.5 First order QCD processes

The leading order parton level process is $V^\star q \to q$, where $V^\star$ is the exchanged virtual boson γ/Z or W. In first order QCD the gluon radiation or QCD Compton process, $V^\star q \to qg$, and the boson-gluon fusion (BGF) process, $V^\star g \to q\bar{q}$, appear and can be included with their matrix elements (ME) [33, 34, 35, 36]. Quark masses are not included in these ME, but a threshold factor is applied for boson-gluon fusion into heavy quarks. For more accurate heavy quark simulations the AROMA Monte Carlo [37] may be used.

The first order ME are rather complicated functions involving three new degrees of freedom corresponding to energy, polar angle and azimuthal angle of one final parton (the other is then determined by energy-momentum conservation). In terms of the more suitable variables [34] $x_p = x/\xi$ and $z_q = P \cdot p_q / P \cdot q$, where ξ is the momentum fraction that the incoming parton take of the proton and p_q the four-momentum of, e.g., the final quark, the cross sections are five-fold differential

$$\frac{d^5\sigma}{dx\, dQ^2\, dx_p\, dz_q\, d\phi} \tag{29}$$

Here ϕ is the parton azimuthal angle with respect to the lepton scattering plane (in the γp cms) and its distribution has the form

$$d\sigma = d\sigma_0 + d\sigma_1 \cos\phi + d\sigma_2 \cos 2\phi \tag{30}$$

where $d\sigma_i$ depends on the other four variables. When averaging over ϕ, only the first term contributes and the ϕ-dependence can therefore often be neglected, but in dedicated analyses it can be observed [38]. The invariant mass squared of the two emerging partons (qg or $q\bar{q}$) is labelled $\hat{s}$.

The matrix elements have soft and collinear divergences that may be partly cancelled by virtual corrections and partly absorbed in the parton density functions. The QCD-Compton cross section $\hat{\sigma}_{qg}$ diverges as $1/(1-x_p)(1-z_q)$ and the boson gluon fusion $\hat{\sigma}_{q\bar{q}}$ as $1/z_q(1-z_q)$. To avoid these singularities in a Monte Carlo simulation procedure, it is common practice to impose a cut-off on the matrix elements. In LEPTO, several different cut-off procedures are available through LST(20). In the W scheme it is required that each pair of emerging partons (including the proton remnant) have a minimum invariant mass, expressed as $s_{ij} = (p_i + p_j)^2 > y_{cut} W^2$. As a new default in the present version of the program, we use a variation of the 'mixed scheme' [36]. In this '$z\hat{s}$' scheme the applied cuts are $z_{q,min} < z_q < 1 - z_{q,min}$ and $\hat{s} > \hat{s}_{min}$, with $z_{q,min}$ and $\hat{s}_{min}$ set by PARL(8) and PARL(9), respectively. As discussed in [39], this gives some advantages compared to previously used cut-off schemes. In particular, it gives a better division of the available phase space into a region for hard emission, which is best described by exact matrix elements, and the soft and collinear regions where higher orders are important and can be taken into account by leading log parton showers (cf. section 2.6).

To decide on an event-by-event basis whether to generate one of the first order event types (qg- or $q\bar{q}$-event) rather than the leading order process (q-event), the probability for each event type must be available as a function of the kinematic variables x, Q^2. The q-event probability is taken as $P_q = 1 - P_{qg} - P_{q\bar{q}}$ and the probabilities P_{qg} and $P_{q\bar{q}}$ are defined as the integral of the relevant first order matrix element over the above three variables, divided by the overall differential cross section $d\sigma/dxdQ^2$. The integration over ϕ is trivial and the variable z_q can be integrated analytically leaving only the x_p-integration, which involves parton density parametrizations, to be performed numerically using an adaptive Gaussian method. If the cut-off is chosen too low the calculated probabilities for the first order processeses $P_{qg} + P_{q\bar{q}}$ can be larger than unity at the given x and Q^2. In this case the cutoff is increased, the y_{cut} in the W scheme and the z_{min} or the $\hat{s}_{min}$ in the $z\hat{s}$ scheme, until $1-$PARL(13)$<$ $P_{qg} + P_{q\bar{q}} < 1$. The effective cut-off actually used in a generated event is stored in PARL(27).

To save computer time the probabilities for qg- and $q\bar{q}$-events can be calculated at the initialization time (LINIT) and stored on a 'grid' in the x, W or x, y plane. In the following event generation phase the probabilities at any x, Q^2 (or x, y) point, chosen according to section 2.2, are then obtained by linear interpolation on this grid. The switch LST(19) regulates the use of such grids and provides an option with a grid that automatically adjusts to the kinematical region chosen by the user. There is also a possibility to calculate these integrals for each event, and thereby avoid interpolation errors.

Having chosen to generate a first order event based on the probabilities P_q, P_{qg} and $P_{q\bar{q}}$, the internal variables x_p, z_q and ϕ are generated in turn from the matrix element formulae where the subsequent variables are integrated out. Given the values of all five variables the four-momenta of the scattered lepton and partons can be calculated.

2.6 QCD parton shower evolution

In order to take higher than first order QCD effects into account the parton shower (PS) approach has been implemented as described in detail in ref. [14]. This has the advantage that arbitrarily high orders in α_s can be simulated, but only in the leading $\log Q^2$ approximation as opposed to the exact treatment in fixed order ME. Higher order effects are important at high energies where multiple parton emission can give rise to multijet events as well as affect the internal properties, such as hardness and width, of a jet [15, 40] and the overall structure of the event in terms of, *e.g.*, energy flows.

In DIS the quark struck by the electroweak boson can emit partons both before and after the boson vertex giving rise to initial and final state parton showers, respectively. A parton close to mass-shell in the incoming nucleon can initiate a parton emission cascade (or shower) where in each branching one parton becomes increasingly off-shell with a space-like virtuality ($m^2 < 0$) and the other is on-shell or has a time-like virtuality ($m^2 > 0$). This initial state space-like shower results in a space-like quark which interacts with the electroweak boson that turns it into an outgoing quark which is either on-shell or has a time-like virtuality. In the latter case a final state, time-like shower will result where the off-shell mass is reduced by branching into daughter partons with decreasing off-shell masses and decreasing opening angles. This shower continues until all partons are (essentially) on-shell. Any parton with a time-like virtuality from the initial state shower will develop similarly. The general behaviour of initial and final state showers are similar since they are both based on the branching processes $q \to qg$, $g \to gg$ and $g \to q\bar{q}$ as described by the GLAP equations [25] in the leading $logQ^2$ approximation of perturbative QCD.

The final state radiation is analogous to parton radiation in $e^+e^- \to q\bar{q}$ and is theoretically well developed and tested against data. The routine LUSHOW in JETSET [26] is therefore used for all time-like showers. The evolution is based on the Sudakov form factor, which expresses the probability that a parton does not branch between some initial maximum virtuality and some minimum value. From this one can find the mass of the decaying parton, the energy fractions in the branching and the flavours of the daughter partons. The process is iterated with a reduced virtuality until all parton virtualities are below some cutoff m_0^2 around 1 GeV^2. The technical details are given in [14], but it should be noted that coherence in soft gluon emission is taken into account through angular ordering (decreasing opening angles in subsequent branches) and that $p_\perp^2 \simeq z(1-z)m^2$ is used as argument in α_s as suggested by studies of coherence effects.

The initial state radiation is performed using the 'backwards' evolution scheme [41] where the shower is constructed from the hard electroweak interaction backwards with decreasing virtualities down to the on-shell parton from the incoming nucleon. A modified version [14] of the routine PYSSPA in PYTHIA [42] is used for the space-like shower. This is a more complicated process, e.g. since the nucleon parton density functions must be taken into account (which tend to reduce the amount of radiation). In addition it is not so well tested by the more 'messy' hadron collision data. When combining the initial and final state radiation to get the complete model, special precautions have been taken [14] to preserve energy-momentum conservation and keep the normal definitions of the kinematic variables for the electroweak scattering such that they will be obtained from the scattered lepton as usual, in particular that Bjorken-x is preserved.

The amount and hardness of the initial and final radiation depends on the off-shellness of the struck parton just before and the partons after the boson vertex. These virtualities are chosen, using the Sudakov form factor, between the lower cut-off and a maximum value to be given by the energy or momentum transfer scale in the process. The default procedure is to add the parton shower to a matrix element event. The possible final states are then q, qg and $q\bar{q}$ events. In the case of a q event the maximum scales are set by the matrix element cut-off (e.g. $y_{cut}W^2$ in the W-scheme) since emissions harder than the cut-off would be double-counting. In the case of a qg or $q\bar{q}$ event the maximum virtuality scale for the final state shower is set to $\hat{s}$. As maximum virtuality for the backwards initial state shower it is natural to use the mass-squared of the quark propagator just before the boson vertex. This is given by the known four-vectors of the exchanged boson and the two final partons from the matrix element, but depending on the underlying Feynman diagram in the amplitude different combinations are possible leaving some remaining ambiguity. The largest of these possible virtualities is being used (cf. subroutine LSCALE).

In the case where the parton shower is used without the matrix element there is a large ambiguity in which maximum scale should be used. Whereas only one scale is present in e^+e^-, any function of Q^2 and W^2 may be possible in DIS. Although Q^2 and W^2 can often be of similar magnitude, cf. eq. (2), at small x-values W^2 is much larger than Q^2 giving rise to drastically different amounts of radiation [43]. Given this uncertainty, different scales are available (cf. LST(9)) based on different motivations. The phase space limit is given by W^2, such that a smaller scale (Q^2) would cut off the tail of high-$p_\perp$ parton emission. Refering to the first order ME parton level one finds $\langle p_\perp^2\rangle \sim Q^2(1-x)$ for $x \to 1$ and $\langle p_\perp^2\rangle \sim Q^2 ln(1/x)$ for $x \to 0$ [33]. A suitable interpolation between these limiting behaviours is the choice $Q^2(1-x)max(1, \ln\frac{1}{x})$ which is the default choice when the matrix elements are not used.

The parton shower approach has some shortcomings due to its approximate nature. The separation of initial and final state parton emission implies the neglect of interference terms between the two and is not gauge invariant. The use of the leading logarithm approximation means that the emission of partons that are soft or close to the directions of the emitting partons should be well described, while the emission of hard partons at large angles could be mistreated. Therefore, the rate of events with extra hard partons that give rise to separate jets, i.e. multijet events, need not be well accounted for. The use of matrix elements is preferable for these purposes and is therefore the default option. Starting with the ME the hard emission is generated and extra, but softer emissions are then added using PS. An advantage of this procedure is, of course, that the hard parton emission is properly treated using the first order ME.

2.7 Nucleon remnant and hadronization

The remnant system is the target nucleon 'minus' the parton entering the hard scattering system (initial parton showers and matrix elements). This interacting parton can be either a valence quark, a sea-quark or a gluon.

When the interacting parton is a valence quark the nucleon remnant is simply a diquark composed of the two left-over valence quarks as spectators. In the Lund model [5] a colour triplet string is stretched between the colour triplet charged struck quark and the diquark which is a colour antitriplet. This system is then hadronized in the usual way [5, 26] by the production of quark-antiquark and diquark-antidiquark pairs from the energy in the field, leading to hadron production. The proton remnant diquark is not a single entity;

its two quarks may go into a leading baryon but they can also be separated to produce a leading meson followed by a baryon.

In case the interacting parton is a sea quark (q_s) or antiquark the nucleon remnant contains the corresponding antiquark or quark in addition to the three valence quarks (q_v). This more complicated four-quark system $q_v q_v q_v \bar{q}_s$ or $q_v q_v q_v q_s$ must be taken into account to conserve the flavour quantum numbers.

In the conventional way (default in LEPTO version 6.2 and earlier) the following treatment has been used. If $\bar{q}_s = \bar{u}$ or $\bar{d}$ it is cancelled against a corresponding valence quark leaving a simple diquark system to be treated as above. For other flavours of $\bar{q}_s$ it is joined with a valence quark of arbitrary flavour into a meson ($M = q_v \bar{q}_s$). The $\bar{q}_s$ is assumed to have no specific dynamic properties such that this splitting process into a meson and a diquark should be similar to normal hadronization. The meson is then given a fraction z of the remnants energy-momentum ($E + p_z$) along the beam direction from a probability distribution $P(z)$ (cf. LST(14)) and only a small Gaussian $p_\perp$ (cf. PARL(14)). The left-over diquark, with longitudinal momentum given by $1 - z$ and equal but opposite $p_\perp$, forms a string system with the scattered quark and hadronization proceeds as usual. If an antiquark ($\bar{q}_s$) was scattered the remnant is a four-quark system $q_v q_v q_v q_s$ which is treated similarly to the previous case. Here, the corresponding quark (q_s) is combined with a random diquark giving a baryon ($B = q_v q_v q_s$) leaving the remaining valence quark to form a string system with the scattered antiquark. The split of the remnant is as before, taking account of the masses in the distribution for z (cf. LST(14)).

In LEPTO 6.3 a modified treatment of sea quarks in the remnant was introduced which is now default (cf. LST(35)). The essential difference is that the sea quark partners ($\bar{q}_s$) are treated dynamically and also u and d quarks can be considered as sea quarks. The interacting quark is assigned to be a valence or sea quark from the relative size of the corresponding parton distributions $q_v(x_1, Q_1^2)$ and $q_s(x_1, Q_1^2)$, where x_1 is the momentum fraction of the quark 'leaving' the proton and Q_1^2 is the relevant scale (typically the cutoff Q_0^2 of the initial state parton shower). In case of a valence quark the previous treatment is used, but in case of a sea quark a new treatment is used. The left-over partner $\bar{q}_s$ is given a longitudinal momentum fraction from the Altarelli-Parisi splitting function $P(g \to q\bar{q})$ and the transverse momentum follows from the masses of the partons in the splitting. Essentially the same results are obtained if the longitudinal momentum fraction is chosen from the corresponding sea quark momentum distribution. The former approach is presently used since this allows the mechanism to be simply implemented in the initial state parton shower routine as an additional, but non-perturbative, $g \to q\bar{q}$ process. This partner sea quark will then be at the end-point of a string and not, as previously, go directly into a hadron together with another spectator parton. Depending on the momentum of the partner sea quark, this new string may extend more or less into the central region and through hadronization contribute to the particle and energy flow in the forward region. In particular, the transverse forward energy flow will be enhanced [44, 39] and improve the agreement with HERA data.

In boson-gluon fusion the removed gluon leaves the three valence quarks in a colour octet state. This remnant is split into a quark and a diquark, chosen with random flavours, which form two separate strings with the antiquark and quark, respectively, produced in the fusion process. Again the split of the remnant involves the same longitudinal momentum sharing and a Gaussian transverse momentum. For the order α_s gluon radiation process (qg-event) the string is stretched from the scattered quark via the gluon to the target remnant.

In the parton shower case, the backwards evolution always results in one parton being removed from the nucleon as in the above cases such that the same procedures can be applied. The additional partons emitted in the PS case will, however, lead to a more complicated string configuration. The string follows the colour flow of the parton shower such that it starts from a colour triplet quark and goes via a number of colour octet gluons, which are kinks on the string, before ending up on a colour antitriplet antiquark or diquark. Where quark-antiquark pairs have been produced in the shower, the colour flow will be broken resulting in a termination of the first string piece and the start of a new one. The string system may thus be divided into subunits which then hadronize separately.

The ME and PS emissions may give a varying number of soft or collinear partons, depending on the details of the cut-offs. Although such partons cannot be observed as separate jets, they may give a 'softening' and 'fattening' of jets. The Lund string model is particularly suitable in this context, since it provides a stability in the sense that the hadron level result will not depend strongly on the presence of extra soft partons. Rather, one obtains a smooth transition to a configuration without them [45, 46]. The independent hadronization model, available as an option in [26] does not have the same property and is therefore not recommendable.

In this context one should also note that the two-string configuration for sea-quark initiated processes provides a desirable continuity between the two-string gluon-initiated $q\bar{q}$-events and the one-string quark-initiated q-events. Depending on the partner sea-quark momentum, the corresponding string will extend more or less into the central region in rapidity. The hadronization of this extra string will contribute to the particle multiplicity and energy flow in this region [44, 39].

The parameters for the hadronization process in JETSET [26] are obtained from fits to e^+e^- data and are assumed to be the same in DIS based on jet universality. Nevertheless, they depend on which QCD effects are explicitly included in the Monte Carlo simulation. The default values are suitable when higher orders are taken into account via parton showers, whereas with first order ME alone the hadronization should be made slightly 'softer' and 'wider' to account for the additional parton emission not simulated explicitly.

2.8 Soft colour interactions and rapidity gaps

The rapidity gap events discovered in deep inelastic scattering at HERA [47] are usually interpreted in terms of pomeron exchange models [48]. Although this seems to work reasonably well phenomenologically, there is no satisfactory understanding of the pomeron and its interactions mechanism. As an alternative, we have introduced a model [44, 39] based on soft colour interactions (SCI) that give rise to rapidity gap events without using the concept of a pomeron.

At small Bjorken-x ($10^{-4} - 10^{-2}$), where the rapidity gap events occur, the events are frequently initiated by a gluon from the proton. This can either be directly from the boson gluon fusion matrix element or after the initial state parton shower, including a possible split in the sea quark treatment. In the conventional string hadronization model this gives two separate strings from the q and $\bar{q}$ to the proton remnant spectator partons, where the gluons from the parton shower are kinks on the string, thereby causing particle production over the whole rapidity region in between. The new hypothesis introduced here is that additional non-perturbative soft colour interactions may occur. These have small momentum transfers, below the scale Q_0^2 defining the limit of pQCD, and do not significantly change momenta from the perturbative phase. However, SCI will change the colour of the partons involved and thereby change the colour topology as represented by the strings. Thus, it is proposed [44] that the perturbatively produced quarks and gluons can interact softly with the colour medium of the proton as they propagate through it. This should be a natural part of the processes in which 'bare' perturbative partons are 'dressed' into non-pertubative quarks and gluons and the formation of the confining colour flux tube in between them.

Lacking a proper understanding of such non-perturbative QCD processes, a simple model is used to describe and simulate these interactions. All partons from the hard interaction (electroweak + pQCD) plus the remaining quarks in the proton remnant constitute a set of colour charges. Each pair of charges can make a soft interaction changing only the colour and not the momenta, which may be viewed as soft non-perturbative gluon exchange. As the process is non-perturbative the exchange probability cannot be calculated so instead it is described by a parameter, PARL(7). The number of soft exchanges will vary event-by-event and change the colour topology of the events such that, in some cases, colour singlet subsystems arise separated in rapidity. In the Lund model this corresponds to a modified string stretching and rapidity gaps may arise when a gap at the parton level is not spanned by a string, as illustrated in [44]. In particular, when the hard process starts with a gluon from the proton, leaving a colour octet remnant and giving a colour octet hard scattering system, a soft colour exchange between the two octet systems can give two colour singlets separated in rapidity.

SCI may thereby give a string system, including a valence diquark from the proton remnant, which has a small invariant mass. Such systems are not optimally treated in JETSET regarding production of one- or two-particle final states and taking isospin constraints into account. We have therefore constructed a new treatment (implemented in subroutine LSMALL) for such systems giving two particles if kinematically possible and otherwise one. When one particle is made, the isospin restriction from the effective isospin singlet exchange in SCI is taken into account to prevent Δ production. The invariant mass of such small systems is roughly given by the constituent masses of the partons in the proton remnant and the transverse momentum in the remnant split. With the default contituent masses ($m_q = 325$ MeV and $m_{qq} = 650$ MeV) and width of the transverse momentum in the remnant split (350 MeV), the invariant mass of the consituents in the proton remnant exceeds the mass of the proton itself. Therefore the possibility has been introduced to reduce the constituent masses of the remnant partons when the target remnant is more complicated than a simple diquark (cf. PARL(20)).

SCI also gives rise to other string topologies without gaps, but where the string goes back and forth in rapidity between the partons. In such cases, there will be more particles and energy per unit of rapidity. This contributes to a better description of, *e.g.*, the forward energy flow observed at HERA as discussed in [39].

3 Description of program components

The present program, LEPTO version 6.5, is a backwards compatible update of versions 6.1 [7] to 6.4. It is a further development of earlier versions [6], but is not backwards compatible with those due to the changed conventions in JETSET version 7 now being used. Previous knowledge of earlier versions is not necessary, but is helpful since the main structure and use of the program has been kept similar. The code is written completely in standard FORTRAN77 and should therefore run on any computer with such a compiler. Single precision is normally used, but double precision is being used when required.

3.1 Subroutines and functions

The following routines should or may be called by the user:

SUBROUTINE **LINIT**(LFILE,LEPIN,PLZ,PPZ,INTER)

Purpose: to initialize the event generation procedure and, optionally, integrate cross section.

Arguments:

LFILE : logical file number containing weights for first order QCD, see LST(8).

$= 0$: the weights are calculated but not save on file, no file is used.

< 0: the weights are calculated and stored on file number –LFILE.

> 0: the weights are read from file number LFILE.

Remarks: When using weights from a file, the same conditions (interaction, cuts etc) must hold as when the weights were calculated. The relevant quantities are checked and a mismatch results in an error message, cf. LST(3). There is no strong reason, in this upgraded program, to store weights on a file since their calculation is fast enough to be repeated in each run.

LEPIN : type of lepton, i.e. $11 = e^-$, $12 = \nu_e$, $13 = \mu^-$, $14 = \nu_\mu$ and negative values for the coresponding antiparticles, i.e. JETSET [26] code.

PLZ, PPZ : momentum (GeV/c) for incoming lepton and nucleon, respectively, along the z-axis (if both non-zero, i.e. colliding beams, they must have opposite signs). Colliding beams which are not along a common axis, or variable beam energies, are possible using LST(17).

INTER : type of interaction to be simulated.

=1: electromagnetic (EM), i.e. γ exchange.

=2: weak charged current (CC), i.e. $W^\pm$ exchange.

=3: weak neutral current, i.e. Z^0 exchange.

=4: neutral current (NC), i.e. γ/Z^0 exchange.

Procedure: Various constants are calculated, effective limits on kinematic variables are derived from the cuts and, depending on the interaction, integration variables are chosen and parameters are set. For the Monte Carlo rejection technique to give unweighted events, the maximum of the differential cross section (having factored out a suitable function that can be simulated exactly, see section 2.2) is found using an adaptation of MINUIT [22]. The cross section is calculated using numerical integration (LST(10)) and stored (PARL(23)). Optionally, grids with probabilities for first order QCD processes are set up (LST(8)) and grids for including the longitudinal structure function (LST(11)).

SUBROUTINE **LEPTO**

Purpose: to administer the generation of one event of the kind specified by the last LINIT call.

Procedure: Beam energies are taken from LINIT (optionally from LUJETS, see LST(17)). A phase space point x, Q^2 is chosen according to the differential cross section (using LEPTOX), and the parton level system, optionally with QCD corrections (cf. LST(8)) is set up. Hadronization and decays are performed, via LUEXEC, and the event is transformed to the selected frame (LST(5),LST(6)). The Monte Carlo estimate of the cross section, PARL(24), is updated with each event.

Remarks: Under some conditions, an error may have occured and the event should be rejected, see LST(21).

SUBROUTINE **LFRAME**(IFRAME,IPHI)

Purpose: to transform the event between different frames.

Arguments:

IFRAME : specifies the desired frame (as for LST(5)).

=1: hadronic CM frame, z-axis along exchanged boson.

=2: lepton-nucleon CM frame, z-axis along lepton.

=3: lab system as defined by last user call to LINIT.

=4: as 3, but z-axis along exchanged boson.

IPHI : specifies whether to include a random rotation for the azimuthal angle, ϕ, of the lepton scattering plane.

=0: no rotation, scattering plane is $x - z$ plane. A possible earlier rotation is undone.

=1: random rotation in ϕ, made in lepton-nucleon CM frame. Not made for IFRAME=1.

Remark: The present frame is stored in LST(28), LST(29) and is updated by LFRAME. Transforming the event with other routines can therefore cause errors in a following LFRAME call.

SUBROUTINE **LPRWTS**(NSTEP)

Purpose: to print a table of the QCD weights in common LGRID, i.e. the probabilities for q-, qg- and $q\bar{q}$-events in first order QCD. Only the values on each NSTEP point in the x, W grid is printed, i.e. NSTEP=1 prints all grid points.

SUBROUTINE **LWBB**(ENU)

Purpose: to give energy (ENU in GeV) of a (anti-)neutrino chosen from a simple parametrization (defined by DATA statement within the routine) of a wide band beam. The energy is actually chosen from the beam spectrum weighted with the energy to take into account the linear rise with energy of the cross section, which LEPTO does not account for. This routine is a simple example to be replaced by a more realistic beam energy distribution if necessary.

Remark: For running with variable beam energies, see LST(17).

SUBROUTINE **LTIMEX**(TIME)

Purpose: to get cpu execution time (in seconds) since start of job. Interface to machine dependent routine, by default TIMEX (Z007 in the CERN library) which can simply be changed, or TIME set to zero since timing information is not essential (although useful).

SUBROUTINE **LNSTRF**(X,Q2,XPQ)

Purpose: to give the parton distribution functions per nucleon for a nucleus defined by PARL(1) and PARL(2). Arguments as in LYSTFU which is called. This routine is called internally, but can also be used separately after initialization by LINIT.

SUBROUTINE **LYSTFU**(KF,X,Q2,XPQ)

Purpose: to give the parton distribution functions for the target hadron through an interface to subroutine PYSTFU in PYTHIA 5.7 [26], complemented with options for intrinsic charm and beauty quarks in the nucleon.

Arguments:

KF : particle flavour, e.g. 2212=p, 2112=n, -2212=$\bar{p}$, -2112=$\bar{n}$.

X : momentum fraction carried by the parton.

Q2 : momentum transfer scale Q^2.

XPQ : array (-6:6) that on return contains the values of the momentum-weighted parton densities, i.e. $x \cdot q(x, Q^2)$ and $x \cdot g(x, Q^2)$. Index: 0=g, 1=d, 2=u, 3=s, 4=c, 5=b, 6=t and -1=$\bar{d}$, -2=$\bar{u}$, -3=$\bar{s}$, -4=$\bar{c}$, -5=$\bar{b}$, -6=$\bar{t}$.

Remarks: Different parametrizations of structure functions are implemented and can be selected by LST(15) and LST(16). This routine is called internally but can also be used separately after initialization by LINIT.

In the following list all subroutines (S) and functions (F) are briefly described. The order is as they appear in the code and reflects the flow in the program. Further details about their purpose and procedures used are given by comments in the code. All routine names start with characteristic letters to indicate origin and avoid name clashes. L, or D for real functions, is for LEPTO routines in general; FL for routines related to longitudinal structure function; LY for modified PYTHIA routines [42]; LM for modified MINUIT routines [22]; GADAP, RIW and DV for routines related to different integration procedures.

Routine		*Purpose*
LTIMEX	(S)	to give execution time since start of job, see above.
LEPTOD		block data to give default values to all switches and parameters.
LINIT	(S)	to initialize program package, see above.
LEPTO	(S)	to administer the generation of an event, see above.
LEPTOX	(S)	called by LEPTO to generate kinematic variables, within the applied cuts, according to the differential cross section giving unweighted events. Optionally include F_L. Update Monte Carlo estimate of cross section, PARL(24), select lepton helicity and target nucleon.
LKINEM	(F)	called by LEPTOX to calculate various kinematic variables and optionally (LST(2)) to reject event if outside given kinematic limits in CUT.
LQCDPR	(S)	called by LEPTO to obtain probabilities for first order QCD processes, either by linear interpolation of weights stored on a grid set up in LINIT, or direct calculation event-by-event, see LST(19).

LQEV (S) called from LEPTO to generate parton system for q-event, i.e. without QCD processes.

LQEVAR entry in LQEV used by ARIADNE [49] for some boson-gluon fusion events.

LQGEV (S) called from LEPTO to generate parton system for qg-event, i.e. first order QCD gluon radiation.

LQQBEV (S) called from LEPTO to generate parton system for $q\bar{q}$-event, i.e. first order QCD boson-gluon fusion.

LXP (S) called from LQGEV and LQQBEV to generate a value of x_p from the relevant QCD matrix element folded with parton density functions (for given x, Q^2, but z_q and ϕ integrated out).

LZP (S) called from LQGEV and LQQBEV to generate a value of z_q, from QCD matrix element (for given x, Q^2, x_p, but ϕ integrated out).

LQMCUT (F) to apply cuts on QCD parton configuration to take quark mass effects into account and ensure ability to apply string hadronization.

LAZIMU (S) called from LQGEV and LQQBEV to generate the azimuthal angle ϕ, of parton plane w.r.t. lepton scattering plane, from first order QCD matrix elements eq. (30), for given x, Q^2, x_p, z_q.

DSIGMA (F) differential cross section $d\sigma/dx_p$ for first order QCD processes.

DSIGM2 (F) modified DSIGMA with a variable substitution to speed up integration

DQCD (F) first order QCD differential cross sections $d\sigma/dx_p dz_q$ (section 2.5) [34].

DQCDI (F) first order QCD differential cross section $d\sigma/dx_p$ obtained after analytical integration over z_p and factoring out $1/(1-x_p)$.

LFLAV (S) selects flavour of struck quark and outgoing quark (with flavour mixing in CC) and defines the corresponding parton system from the nucleon remnant, also applies threshold factor for charm and heavier quarks.

LREMH (S) gives energy-momentum fraction for target remnant split, cf. LST(14); also selects remnant flavours (not for PS case).

LPRIKT (S) generates magnitude and azimuthal angle for Gaussian primordial $k_\perp$ of parton in nucleon, see PARL(3).

LFRAME (S) transforms event to different frames, see above.

LWBB (S) selects energy from a neutrino wide band beam, see above.

LWEITS (S) integrates first order QCD matrix elements to set up grid in x, W or x, y of weights for gluon radiation and boson-gluon fusion and finds maxima used for QCD simulation, see LINIT and common LGRID.

LPRWTS (S) prints QCD weights, see above.

LSIGMX (S) called by MINUIT routines to calculate differential electroweak cross section divided by the optimization function to obtain the maximum for the weighting procedure.

LXSECT (S) called from LINIT to integrate cross section using GADAP, RIWIAD or DIVONNE, see LINIT and LST(10).

RIWIBD (S) substitutes block data for RIWIAD [50], print flag changed.

DVNOPT (S) substitutes block data for DIVONNE [51], print flag changed.

DFUN (F) integrand for DIVONNE integration, calls RIWFUN.

RIWFUN (F) integrand for RIWIAD integration, calls DCROSS.

DCROSS (F) differential electroweak cross section used as integrand for numerical integration, see LXSECT. Integration variables given by LST(1), function is zero outside the cuts in array CUT.

DLOWER, DUPPER (F) lower and upper limit on second kinematic variable (y,Q^2 or W^2) given a value of the first variable (x) and the cuts. Used together with DCROSS for cross section integration.
FLTABL (S) called from LINIT to tabulate integrals in x, Q^2-grid for the longitudinal structure function, see LST(11).
FLIPOL (S) called from LEPTOX to obtain longitudinal structure function from interpolation on grid from FLTABL, see LST(11).
FLINTG (S) called from LEPTOX to obtain longitudinal structure function by integration event-by-event, see LST(11).
FLQINT (F) integrand for quark contribution to QCD longitudinal structure function.
FLGINT (F) integrand for gluon contribution to QCD longitudinal structure function.
FLTINT (F) integrand for target mass correction, see LST(11).
LSCI (S) generates soft colour interactions (SCI), see section 2.8.
LECSWI,LEASWI (S) switches colour and anticolour pointers for SCI.
LSMALL (S) creates hadrons from small mass colour singlet systems including diquarks.
LSHOWR (S) administer parton cascade evolution added to a q-event.
LMEPS (S) administer parton cascade evolution added to a qg- or $q\bar{q}$-event from first order QCD matrix elements.
LSCALE (S) gives scale for maximum virtuality in parton showers, see LST(9).
LYSSPA (S) simulate initial state parton cascade evolution, modification [14] of routine in [42].
LYREMN, LYSPLI (S) treatment of target remnant and primordial $k_\perp$ when using parton showers, modifications [14] of routines in [42].
LMCMND,LMINTO,LMIDAT,LMINEW,LMPRIN,LMPINT,LMRAZZ,LMSIMP modified MINUIT routines [22] to find maximum of cross section.
GADAP, GADAP2, GADAPF one- and two-dimensional adaptive Gaussian integration routines [52].
LNSTRF (S) gives parton distributions in a nucleus, see above.
LYSTFU (S) interface to subroutine PYSTFU in PYTHIA 5.7 [26] to give parton distribution functions, see above.

3.2 Common blocks

Most of the communication between the user and the program is via the switches and parameters in the common blocks. The user need mainly be concerned with common LEPTOU since all others are essentially for internal use. All variables are given sensible default values in block data LEPTOD, as shown by (D=...) below. These values may be changed by the user to modify the behaviour of the program. Note, however, that this should usually be done before calling LINIT and that some of the parameters are interrelated. Variables whose name start with D are in double precision. The generated event is stored in common LUJETS, described in [26].

COMMON /**LEPTOU**/ CUT(14),LST(40),PARL(30),X,Y,W2,Q2,U

Purpose: contains input switches (LST(1)–LST(20),LST(34),LST(35)) and input parameters (PARL(1)–PARL(20)) to specify physics, kinematic cuts and numerical procedures, as well as output flags (LST(21)–LST(40)) and output variables (PARL(21)–PARL(30)). Overwriting default values should be made before calling LINIT.

Parameters:

CUT(1), CUT(2) : (D= $10^{-4}, 1.$) lower and upper limit of Bjorken-x variable.
CUT(3), CUT(4) : (D= $0., 1.$) lower and upper limit of y variable.
CUT(5), CUT(6) : (D= $4., 10^8$) lower and upper limit of Q^2 (GeV^2).
CUT(7), CUT(8) : (D= $5., 10^8$) lower and upper limit of W^2 (GeV^2).
CUT(9), CUT(10) : (D= $1., 10^8$) lower and upper limit of variable ν (GeV).
CUT(11), CUT(12) : (D=$1., 10^8$) lower and upper limit of scattered lepton energy (GeV, in frame defined by LINIT call).
CUT(13), CUT(14) : (D=$0., 3.1416$) lower and upper limit of lepton scattering angle (in radians), with respect to incoming lepton in frame defined by LINIT call.

Remarks: These cuts are applied already when choosing kinematic variables, before evaluating cross section formulae and structure functions, and will therefore be more efficient than applying cuts later on in the users program. The cross section estimates take these cuts into account. CUT(11)–CUT(14) are less efficient, since they refer to a special frame and the user should, if possible, translate them into cuts in x, y, Q^2.

LST(1) : (D=0) choice of the two independent variables to be used for simulation and numerical integration of cross section.
=0: program makes a 'best' choice for simulation efficiency depending on which process is to be generated (1 if γ-exchange included, else 2) cf. LST(31).
=1: x and Q^2
=2: x and y
=3: x and W^2

LST(2) : (D=1) choice of simulation and applying cuts in array CUT. (Used internally also with negative values.)
=1: kinematic variables, LST(1), chosen from differential cross section and cuts applied.
=2: variables (x, y) supplied by user via LEPTOU, cuts applied.
=3: variables (x, y) supplied by user via LEPTOU, cuts not applied.

LST(3) : (D=5) regulates output and error handling.
=0: no output, execution not stopped on error.
=1: only warnings printed, execution not stopped on error.
=2: as 1, but execution stopped on error.
=3: as 2, and output at first initialization, no MINUIT output.
=4: as 3, but output at all initialization calls.
=5: full output, i.e. as 4 and MINUIT output.

LST(4) : (D=1) regulates information in the event record. To be given as $I_{lepton} + 10 \times I_{shower}$, where $I_{lepton} = 0/1$ inactives/actives the scattered lepton (K(i,1)=21/1) and $I_{shower} = 0/1$ excludes/includes intermediate partons in the parton showers.

LST(5) : (D=3) choice of frame for the event.
=1: hadronic CM frame, z-axis along exchanged boson.
=2: lepton-nucleon CM frame, z-axis along lepton.
=3: lab system as defined by last user call to LINIT.
=4: as 3, but z-axis along exchanged boson.

LST(6) : (D=1) regulates the azimuthal angle, ϕ, of the lepton scattering plane.
=0: no ϕ-rotation, scattering plane is $x - z$ plane.
=1: random ϕ-rotation, performed in lepton-nucleon cms for LST(5)≥ 2.

LST(7) : (D=1) regulates completeness of Monte Carlo simulation (to speed up program when only partial information is needed).
= −1: only kinematic variables generated.
= 0: kinematic variables and parton level event generated, optionally including QCD effects (cf. LST(8)). Hadronization can be made later by calling LUEXEC.
= 1: full event generated, i.e. as 0 plus hadronization and decays.

LST(8) : (D=12) simulation of QCD effects in hadronic final state.
=0: QCD switched off.
=1: first order QCD matrix elements (ME) for gluon radiation and boson-gluon fusion.
=2: QCD parton cascade evolution from initial and final quark.
=3: QCD parton cascade evolution from initial quark only.
=4: QCD parton cascade evolution from final quark only.
=5: QCD switched off, but target remnant treatment as in cascade case.
=9: set by ARIADNE [49] when simulating parton emission in the colour dipole model [53].
=12–15: as 2–5, but PS added on event from first order matrix elements.
Remarks: See PARL(8),PARL(9) for cut-off on ME. Without PS one may decrease the cutoff to maximize QCD emission.

LST(9) : (D=5) scale in parton showers, i.e. maximum virtuality (mass-squared) of parton initiating the shower. Only used when QCD ME are not included, i.e. LST(8)=2,3,4. See further section 2.6.
=1: Q^2
=2: W^2
=3: $W * Q$
=4: $Q^2(1-x)$, which is $\sim \langle p_\perp^2 \rangle$ from ME at large x
=5: $Q^2(1-x)\, max(1, ln\frac{1}{x})$, which is a combination of $\langle p_\perp^2 \rangle$-dependence from ME at large and small x
=6: $x_0 W^2$, to represent the mass-squared of the hadronic system whithout the (non-perturbative) spectator. A valence-like parton distribution $f(x_0, Q_0^2) \sim (1-x_0)^a$ $(a = 4)$ is used to chose x_0 representing an original parton.
=9: $W^{4/3}$, i.e. similar as in the colour dipole cascade model [49].

LST(10) : (D=1) numerical integration of cross section in LINIT.
=0: not performed.
=1: performed using GADAP [52], i.e. adaptive Gaussian method.
=2: performed using RIWIAD [50], i.e. adaptive Monte Carlo method.
=3: performed using DIVONNE [51], automatic invocation.
=4: performed using DIVONNE [51], detailed invocation.
Remarks: Integration variables are defined by LST(1), integration region by the cuts in array CUT and required accuracy by PARL(15). Result stored in PARL(23). Under normal conditions the simpler GADAP routine is accurate enough.

LST(11) : (D=0) inclusion of longitudinal structure function, target mass and higher twist corrections, see section 2.3, for γ and γ/Z exchange. To be set as LQCD+10*LTM+100*LHT where LQCD, LTM and LHT are 0 or 1 to exclude or include contributions from QCD, Target Mass and Higher Twist, respectively. The QCD and target mass parts involve integrals which are evaluated at initialization and stored on an x, Q^2 grid used for interpolation when simulating events. In this mode, the cross section estimates PARL(23) and PARL(24) include F_L. By setting LQCD or LTM equal 2, the corresponding contribution are evaluated by integration for each event at the proper x, Q^2 point, which gives a more accurate result. In this case, the result is included in PARL(24) but not in PARL(23) since nested integrations is too time consuming.

LST(12) : (D=4) maximum flavour used in sea structure function parametrizations.

LST(13) : (D=5) heaviest quark flavour allowed in boson-gluon fusion. A threshold factor is applied to compensate for neglected quark masses in the matrix elements.

LST(14) : (D=4) treatment of target remnant after removing interacting parton, see section 2.7.

=0: remnant approximated by anti-parton of removed parton, i.e. by $q, \bar{q}, g$ for removed $\bar{q}, q, g$. No baryon is produced.

=1: for removed valence quark the remnant is a diquark hadronizing into a baryon with the Lund model. For a removed gluon, sea quark (q_s), sea antiquark the remnant is, respectively, a $q_v q_v q_v$, $q_v q_v q_v \bar{q}_s$, $q_v q_v q_v q_s$ which is split into $q_v q_v + q_v$, $q_v q_v + M(q_v \bar{q}_s)$, $q_v + B(q_v q_v q_s)$. The (lighter) part of the remnant containing one random flavour valence quark q_v, takes the energy-momentum fraction z given by $P(z) = 2(1-z)$, i.e. $\langle z \rangle = 1/3$.

=2: as 1, but with $P(z) = (a+1)(1-z)^a$ with a chosen such that $\langle z \rangle = 1/(a+2) = m/(m+M)$ where m (M) is the mass of the light (heavy) remnant subsystem.

=3: as 2, but using the 'Peterson' function $P(z) = N/(z(1-1/z-c/(1-z))^2)$ with $c = (m/M)^2$.

=4: using LUZDIS, i.e. the fragmentation function chosen in JETSET.

LST(15) : (D=9) choice of parton distribution functions, $xq(x, Q^2)$, $x\bar{q}(x, Q^2)$ and $xg(x, Q^2)$, for the target hadron, see section 2.4 and LST(16).

=0: parton density choice and parameters are controlled directly through parameters MSTP(51), MSTP(52), MSTP(57), MSTP(58) and PARP(51) in common PYPARS in PYTHIA 5.7 [26].

=1: Eichten-Hinchliffe-Lane-Quigg set 1 [54].

=2: Eichten-Hinchliffe-Lane-Quigg set 2 [54].

=3: Duke-Owens set 1 [55].

=4: Duke-Owens set 2 [55].

=5: CTEQ2M (best $\overline{\text{MS}}$ fit) [56].

=6: CTEQ2MS (singular at small-x) [56].

=7: CTEQ2MF (flat at small-x) [56].

=8: CTEQ2ML (large Λ)) [56].

=9: CTEQ2L (best leading order fit) [56].

=10: CTEQ2D (best DIS fit) [56].

=-4: Intrinsic charm quarks only, see section 2.4; to be used with overall normalisation PARL(12), primordial transverse momentum PARL(3)$\simeq m_c \simeq$ 1 GeV, LST(19)=-1 (no grid) and the cuts CUT(5) (min Q^2) and CUT(7) (min W^2) such that charm production is kinematically allowed.

=-5: Intrinsic bottom quarks only, approximated by taking the same distribution as for intrinsic charm, but renormalized by $m_c^2/m_b^2 \simeq 0.1$. Comments for intrinsic charm apply with m_b instead of m_c.

Remarks: Except for intrinsic charm and beauty, the parton densities are obtained from subroutine PYSTFU in PYTHIA 5.7 [26]; with parameters set to MSTP(51)=LST(15), MSTP(52)=LST(16) and MSTP(58)=LST(12), unless LST(16)=0 when the user directly controls the PYSTFU parameters. To evaluate structure functions separately, see subroutines LYSTFU above.

LST(16) : (D=1) choice of proton parton-distribution-function library, i.e. MSTP(52) in PYTHIA 5.7 [26].
- =1: internal PYTHIA according to LST(15).
- =2: PDFLIB (version 4 or later) [27], with with NGROUP and NSET to be given as MSTP(51)=1000*NGROUP+NSET.

LST(17) : (D=0) regulates varying energies of initial particles from event to event.
- =0: fixed energies as specified in LINIT.
- =1: energies allowed to vary; momenta should be given in P(i,j) with i=1,2 for lepton, nucleon and j=1...5 for p_x, p_y, p_z, E, m. See also LWBB. Note: this option is not fully tested, use ME only with LST(19)=-1.

LST(18) : (D=2) running of electromagnetic coupling α and choice of W and Z masses.
- =0: α fixed to value at $Q^2 = 0$ given in PARL(16), Z and W masses given independently by PMAS(23), PMAS(24) in /LUDAT2/.
- =1: as 0, but W, Z masses calculated from standard model using $\sin\theta_W, \alpha, G_F$ and radiative corrections; i.e. PARL(5), PARL(16), PARL(17), PARL(18)
- =2: as 1, but α varying with Q^2 using ULALEM in [26], see MSTU(101).

LST(19) : (D=-10) regulates use of grid to store probabilities for qg- and $q\bar{q}$-events.
- =-10: the probabilites are obtained from an automatic grid, but close to the saturation limit they are recalculated to avoid interpolation errors.
- =-1: no grid, necessary integrals calculated for each event, more time consuming but higher precision.
- =0: user defined grid to be read in free format, see comments in subroutine LWEITS.
- =1: grid suitable for lepton beam energy $<$ 300 GeV on fixed target.
- =2: grid suitable for lepton beam energy $<$ 1000 GeV on fixed target.
- =3: grid suitable for *ep* collisions in HERA.
- =4: grid suitable for *ep* collisions in LEP+LHC.
- =10: grid chosen automatically after the kinematic region specified by user.

LST(20) : (D=5) scheme for cut-offs against divergences in the QCD matrix elements [36], *cf.* section 2.5 and PARL(8),PARL(9). (m_{ij} is the invariant mass of any parton pair).
- =1: W^2 (or JADE) scheme, $m_{ij}^2 > y_{cut}W^2$
- =2: Q^2 scheme, $m_{ij}^2 > y_{cut}Q^2$
- =3: mixed scheme, i.e. $m_{ij}^2 > c_1 Q^2$ for partons i, j that are not spectators and $2p_i \cdot P > c_2 Q^2/x = c_2(W^2 + Q^2)$ else. Virtuality scale for matched parton showers in q-event is $c_1 Q^2$.
- =4: as 3, but with virtuality $c_2 Q^2/x$ for initial state parton showers.
- =5: $z - \hat{s}$ scheme, $\hat{s} > \hat{s}_{min}$ and $z_q < z_{q,min} < 1 - z_q$. To regulate the divergence, z_q is changed, see PARL(27).
- =6: as 5, but $\hat{s}_{min}$ is changed to regulate the divergence, see PARL(27).

LST(21) : error flag, =0 for properly generated event. Nonzero value indicates incorrect event that should be rejected; may occur in case of user supplied kinematic variables or variable beam energies, cf. LST(2) and LST(17).

LST(22) : specifies chosen target nucleon in current event.
=1: proton.
=2: neutron.
LST(23) : specifies process simulated.
=1: electromagnetic (EM), i.e. γ exchange.
=2: weak charged current (CC), i.e. $W^{\pm}$ exchange.
=3: weak neutral current, i.e. Z^0 exchange.
=4: neutral current (NC), i.e. γ/Z^0 exchange.
LST(24) : specifies first order QCD process in current event.
=1: q-event, i.e. no first order QCD.
=2: qg-event, i.e. gluon radiation in first order QCD.
=3: $q\bar{q}$-event, i.e. boson-gluon fusion in first order QCD.
LST(25) : specifies flavour of struck quark in current event: 1=d, 2=u, 3=s, 4=c, 5=b, –1=$\bar{d}$, –2=$\bar{u}$, –3=$\bar{s}$, –4=$\bar{c}$, –5=$\bar{b}$.
LST(26) : entry line in event record of outgoing struck quark. In parton shower case, quark at boson vertex before final state shower.
LST(27) : signals split of non-trivial nucleon remnant, cf. LST(14).
=0: no split, simple diquark or LST(14)=0.
=1: split into parton and particle, $qq+M$ or $q+B$, occurs when sea (anti)quark removed through the interaction.
=2: split into quark and diquark, $q+qq$, occurs when a gluon is removed.
LST(28) : specifies the frame in which the current event is given with code as for LST(5), cf. LFRAME.
LST(29) : specifies azimuthal angle rotation with code as for LST(6), cf. LFRAME.
LST(30) : specifies chosen helicity of beam lepton in current event.
$= -1$: left-handed.
$= +1$: right-handed.
LST(31) : specifies chosen variables for the simulation/integration.
=1: x and Q^2.
=2: x and y.
=3: x and W^2.
LST(32) : Internal flag, 0 or 1 for simulation and integration, respectively.
LST(33) : Reserved for internal test.
LST(34) : (D=2) switch for soft colour interactions; see section 2.8.
=0: off
=1: SCI possible within all pairs of emerging partons.
=2: SCI only within parton pairs where at least one parton comes from the target nucleon remnant, thus excluding pairs with both partons from the perturbative emissions.
LST(35) : (D=1) switch for new sea quark treatment; see section 2.7.
=0: off
=1: on, the virtuality of the seaquark is given by the cut-off in the initial state parton shower.
=2: on, the virtuality of the seaquark is chosen from a Gaussian distribution of width PARL(10).
LST(36)–LST(40) : unused at present.

PARL(1) : (D=1.) number of nucleons in target nucleus, i.e. A.
PARL(2) : (D=1.) number of protons in target nucleus, i.e. Z.

PARL(3) : (D=0.44 GeV) width of Gaussian distribution for the primordial transverse momentum $k_\perp$ of partons in the nucleon.

PARL(4) : (D=0.75) probability that a ud-diquark in the target remnant has spin and isospin equal zero, i.e. I=S=0.

PARL(5) : (D=0.2319) $\sin^2\theta_W$ (Weinberg angle) [21].

PARL(6) : (D=0.) polarization of lepton beam; should be set to $1-2P_L=2P_R-1$ where $P_{L,R}$ is the probability of left(right)-handed helicity, i.e. $-1(+1)$ corresponds to a fully left(right)-handed polarized beam.

PARL(7) : (D=0.5) probability for soft colour interactions between parton pairs, see section 2.8.

PARL(8),PARL(9) : (D=0.04,4) cut-offs against divergences in the QCD matrix elements, cf. section 2.5 and LST(20). Suitable values are given in parenthesis.
LST(20)=1,2: PARL(8)= y_{cut} and PARL(9)=$\min m_{ij}$ in GeV. (0.005,2)
LST(20)=3,4: PARL(8)= c_2 and PARL(9)= c_1. (0.05,2)
LST(20)=5,6: PARL(8)= $z_{q,min}$ and PARL(9)= $\hat{s}_{min}$ in GeV^2. (0.04,4)

Remarks: These are starting values when integrating first order QCD matrix elements, but the effective cut used, PARL(27), is automatically increased in order that the QCD probabilities do not exceed unity.

PARL(10) : (D=0.44 GeV) width of Gaussian distribution for the virtuality of the seaquark in the seaquark treatment when LST(35)=2.

PARL(11) : (D=0.01) required relative accuracy for one-dimensional integration, used for first order QCD matrix element weights and longitudinal structure function integrals.

PARL(12) : (D=0.01) probability β^2 for an intrinsic charm quark-antiquark pair in the proton (cf. section 2.4).

PARL(13) : (D=0.1) internal parameters used for adjustment of y_{cut} for integration of QCD matrix elements.

PARL(14) : (D=0.35 GeV) width of Gaussian distribution in transverse momentum when a non-trivial target remnant is split into two particles, cf. LST(27).

PARL(15) : (D=0.01) required relative accuracy for two-dimensional integration to get total cross section, cf. LST(10).

PARL(16) : (D= 7.29735×10^{-3}) finestructure constant α [21], cf. LST(18).

PARL(17) : (D= 1.16639×10^{-5} GeV^{-2}) weak Fermi constant G_F [21].

PARL(18) : (D=0.044) Δr from radiative corrections [21].

PARL(19) : (D=0.03) scale κ^2 in GeV^2 for higher twist correction [24], see eq. (27).

PARL(20) : (D=0.1 GeV) in case of a more complicated nucleon remnant than a simple diquark, the constituent masses of quarks and diquarks in the remnant are decreased with PARL(20) and 2*PARL(20) respectively.

PARL(21) : $2P\cdot k$, where P is proton and k lepton four-vectors, equals invariant mass squared, s, when masses are neglected.

PARL(22) : $2P\cdot q$, where P is proton and q is boson four-vectors.

PARL(23) : cross section in *pb*, corresponding to the kinematic region allowed by the cuts in array CUT, obtained by numerical integration in LINIT call, cf. LST(10). If LQCD=2 or LTM=2, cf. LST(11), the corresponding contribution to F_L is not included.

PARL(24) : Monte Carlo estimate of the cross section in *pb* associated with the generated event sample, taking CUT into account. Set to zero by LINIT call and updated with each event generated, hence accuracy improves with statistics as $1/\sqrt{N}$ and only final value should be used.

PARL(25) : value of α_s in current event.

PARL(26) : value of Λ obtained from structure functions, which is used in α_s and in the initial state parton shower.

PARL(27) : depending on LST(20), present value of the y_{cut}-, z_q- or $\hat{s}$-cut for first order QCD. These are given by PARL(8) and PARL(9), but modified internally to prevent QCD weights to exceed unity.

PARL(28),PARL(29),PARL(30) : values of x_p, z_q and ϕ in first order massless QCD matrix elements, section 2.5, for current event if it is a qg- or $q\bar{q}$-event, see LST(24). For a $q-$event they are set to 1.0, 1.0 and 0.0.

X : Bjorken-x, i.e. $x = Q^2/2P \cdot q$.

Y : standard y variable, i.e. $y = P \cdot q/P \cdot p_e$.

W2 : mass-square of hadronic system, i.e. $W^2 = (P+q)^2$.

Q2 : momentum transfer squared, i.e. $Q^2 = -q^2 = -(p_e - p_\ell)^2$.

U : energy transfer variable $\nu = P \cdot q/m_p$.

Remark: for details see section 2.1.

COMMON /**LFLMIX**/ CABIBO(4,4)

Purpose: Contains the Cabbibo-Kobayashi-Maskawa matrix elements squared for flavour mixing. First index corresponds to the up-quark flavours u, c, t, h and second index to the down-quark flavours d, s, b, l. The default values are [21] CABIBO/.95,.05,2*0.,.05,.948,.002,2*0.,.002,.998,4*0.,1./

COMMON /**LOPTIM**/ OPTX(4),OPTY(4),OPTQ2(4),OPTW2(4),COMFAC

Purpose: parameters to optimize simulation efficiency, see section 2.2. Default values are good for normal usage, but improvements may be possible under, e.g., unusual kinematic conditions. For changes, see their detailed meaning given by comments in the code of subroutine LEPTOX.

COMMON /**LBOOST**/ DBETA(2,3),STHETA(2),SPHI(2),PB(5),PHIR

Purpose: rotation and boost parameters for transforming event between frames. DBETA(i,j), STHETA(i) and SPHI(i) are the boosts, polar and azimuthal angles for transforming from lepton-nucleon cms to lab (i=1) and from hadronic cms to lepton-nucleon cms (i=2). For these cases, the order of the transformations should be first rotation in θ, then in ϕ followed by the boost where j=1,2,3 corresponds to x, y, z components. Should only be used via subroutine LFRAME to set flags properly, cf. LST(5) and LST(6).

COMMON /**LGRID**/ NXX,NWW,XX(31),WW(21),PQG(31,21,3),PQQB(31,21,2),
& QGMAX(31,21,3),QQBMAX(31,21,2),YCUT(31,21),XTOT(31,21),NP

Purpose: information for simulating first order QCD processes.

Remarks: Probabilities PQG and PQQB for gluon radiation and boson-gluon fusion, i.e. qg- and $q\bar{q}$-events, are stored on a grid in x, W (or x, y for LST(19)=10) with NXX, NWW points defined by XX and WW. Indices are for x, W (y) and helicity contributions. The cut on parton pair masses y_{ij} is also stored and maxima for the Monte Carlo rejection technique. The grid content is set up by LWEITS and can be printed by LPRWTS. See LST(19) and section 2.5 for physics and methods.

COMMON /**FLINFO**/ RFLQ,RFLG,RFLM,RFLT

Purpose: gives the relative contributions from the different parts (quark, gluon, mass, higher twist) of the longitudinal structure funtion to the differential cross section at the x, Q^2 point of the current event, cf. section 2.3.

COMMON /**LYPARA**/ IPY(80),PYPAR(80),PYVAR(80)

Purpose: parameters for parton cascade routines. This is common PYPARA from PYTHIA 4.8 and described in detail in [42]. Only those parameters used with LEPTO in parton cascade mode are commented here (default values as in PYTHIA 4.8 if not given).

Parameters:

IPY(8) : set to LST(12) in LINIT.

IPY(13) : set to zero in LINIT if LST(8)=3,5,13,15.

IPY(14) : set to zero in LINIT if LST(8)=4,5,14,15.

IPY(11),IPY(15),IPY(34),IPY(40)–IPY(42),IPY(47),IPY(48) : are used.

PYPAR(8),PYPAR(11)–PYPAR(16) : are used.

PYPAR(21) : (D=PARL(26)) Λ_{QCD} in initial state parton shower.

PYPAR(22) : (D=1. GeV^2) cutoff for initial state parton shower.

PYPAR(23),PYPAR(24) : are used.

PYPAR(25),PYPAR(26) : (D=2*1.) multiplies the chosen scale, cf. LST(9), to give maximum virtuality for final and initial state showers, respectively.

PYPAR(27) : (D=1.) multiplies virtuality Q^2 for α_s and structure functions in initial state showers.

PYVAR(1)–PYVAR(5) : are used.

COMMON /**LUJETS**/ N,K(4000,5),P(4000,5),V(4000,5)
COMMON /**LUDAT1**/ MSTU(200),PARU(200),MSTJ(200),PARJ(200)

Purpose: LUJETS contains the record of the generated event and is essential for using the results. LUDAT1 contains switches and parameters that are, e.g., essential to control final state parton showers, α_s evaluation and hadronization. These common blocks are described in the JETSET manual [26].

Short description of all common blocks:

Common	*Purpose*
LEPTOU	main common block for user control of program, see above.
LFLMIX	quark flavour mixing parameters (KM-matrix), see above.
LOPTIM	parameters to optimize simulation efficiency, see above.
LBOOST	boost and rotation parameters, see above.
LGRID	grid for first order QCD event simulation, see above.
LINTER	internal parameters and variables; charges, couplings, cross section weights.
LINTRL	internally used; basic system in different frames, effective limits on kinematic variables.
LPFLAG	internal for output control.
LINTEG	internal counters for integrand evaluations.
FLGRID	grid with integrals for longitudinal structure function.
FLINFO	relative size of longitudinal structure function contributions, see above.
LYPARA	parameters for parton cascade evolution [42], see above.
LYPROC,LYINT1	internal for parton showers [42].
LMINUI,LMINUC	input parameters and character names for MINUIT [22].

LM.... internally used in adaptation of MINUIT routines.
GADAP1 internal in GADAP integration routines.
PARAMS,ANSWER input/output for RIWIAD integration [50].
STORE,STORE1,OPTION,RANDOM,INTERN internal for RIWIAD [50].
BNDLMT,SAMPLE,PRINT input for DIVONNE integration [51].
ARDAT1 parameters in the ARIADNE MC for dipole radiation [49].
LUDAT1,LUDAT2 switches, parameters, particle data in JETSET [26].
PYPARS switches, parameters used for parton densities in PYTHIA 5.7 [26].
LUJETS contains generated event, see [26].

4 Usage and availability

LEPTO 6.5 should be loaded together with JETSET 7.4 and PYTHIA 5.7 [26] (the latter to access parton density parametrizations). The ordinary gamma function, GAMMA(X), is called and must be supplied (usually available in FORTRAN77). Access to the CERN library is not necessary, but gives access to the PDFLIB, RIWIAD and DIVONNE program packages as well as the TIMEX routine. The program is a slave system, which the user must call from his own steering program.

Information about the program, its update history, source code, example jobs etc. can be obtained on request from the authors or directly via the WWW on the LEPTO home page, `http://www3.tsl.uu.se/thep/lepto/`.

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