# Energy Dependence of $pp$ and $pC$ CNI Analyzing Power

1. Basic formulas

- Hadronic proton-carbon scattering amplitude

$$g(q) = g_o(q) + \vec{\sigma} \cdot \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}\, g_s(q)$$

For small momentum transfer $q$

$$g_s(q) = \tau(s)\, \frac{q}{m_N} g_o(q)$$

- Corresponding "Coulomb" amplitudes

$$Z\alpha F^{em}(q)\left(\frac{1}{q^2} + \vec{\sigma} \cdot \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}\, \frac{\kappa}{2m_N q}\right)$$

$\kappa = 1.79$, the anomalous proton magnetic moment and $F^{em}(q)$ the carbon em form factor.

- Analyzing power

$$A_N(t,\tau) = \frac{2\,Im(g_s(q)g_0^*(q))}{|g_0(q)|^2 + |g_s(q)|^2}$$

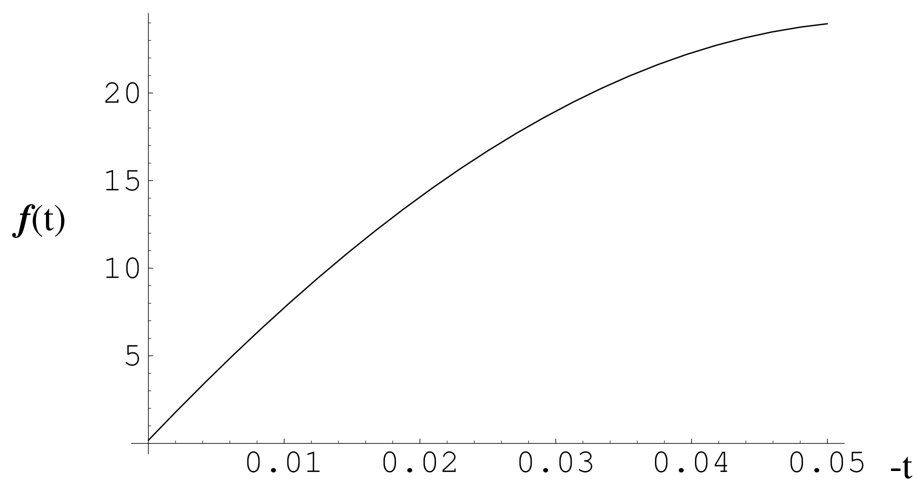
$$\frac{A_N(t,\tau)}{A_N(t,0)} = 1 - \frac{2}{\kappa}Re[\,\tau(s)] + \frac{2}{\kappa}Im[\,\tau(s)]f(t),$$

with

$$f(t) = \Big((1+\rho_{pC}^2(t))(t/t_c)(F_C^h(t)/F_C^{em}(t)) \\ -\rho_{pC}(t) - \delta_{pC}(t)\Big)/(1-\rho_{pC}(t)\delta_{pC}(t))$$

As usual $t = -q^2$, $\rho_{pC}(t) = Re(g_0(s,t))/Im(g_0(s,t))$, $\delta_{pC}$ is the Bethe phase and $t_c = -8\pi Z\alpha/\sigma_{tot}^{pC}$. $F_C^h$ is the hadronic "form factor" equal to $Im(g_0(s,t))/Im(g_0(s,0))$.

- $f(t)$ is calculable by Glauber methods (cf. Kopeliovich & Trueman, Phys Rev D 64 034004) and so $1 - \frac{2}{\kappa}Re[\,\tau(s)]$ and $\frac{2}{\kappa}Im[\,\tau(s)]$ can be each determined from the raw asymmetry if polarization $P$ is known. Otherwise $P$ enters as a common scale factor in each.



20
15
f(t)
10
5
0.01
0.02
0.03
0.04
0.05
-t

## 2. The Model

- is based on Regge fit to $pp$ scattering over wide energy range (cf. Cudell et al) which fixes non-flip parameters for the Pomeron (simple or multiple pole), a $C = -1$ vector meson (mainly $\omega$) and a $C = +1$ tensor meson (mainly $f_2$).

- The non-flip amplitude is

$$g_0(s,0) = g_P(s) + g_f(s) + g_\omega(s),$$

where the functions $g_R(s)$ have enery dependence and phase determined by standard Regge theory.

- The corresponding flip amplitude is determined by three real, energy independent constants

$$\begin{aligned} g_5(s,t) &= \tau(s)\frac{\sqrt{-t}}{m} g_0(s,t) \\ &= \frac{\sqrt{-t}}{m}\{\tau_P\, g_P(s) + \tau_f\, g_f(s) + \tau_\omega\, g_\omega(s)\}. \end{aligned}$$

so

$$\tau(s) = \{\tau_P \, g_P(s) + \tau_f \, g_f(s) + \tau_\omega \, g_\omega(s)\}/g_0(s,0)$$

From Berger et al (1978) the important properties of Regge pole contributions are displayed by

$$g^R_{\lambda'\lambda}(s,t) = (-t/m^2)^{|\lambda'-\lambda|}\beta^R_{\lambda'\lambda}(t)(1 \pm e^{-i\pi\alpha^R})s^{\alpha^R}$$

$$g_0 = g_{++}, \; g_s = g_{+-}$$

- The two constants in $\tau(21.7) = -0.213 - 0.054i$ determines two relations between the three constants $\tau_P, \tau_f, \tau_\omega$
- We need one more measurement to fix their values. If one measures the "shape" of the raw asymmetry over the CNI region *without knowing the value of $P$ at that energy* one can obtain the needed information:

$$S(p_L) = \frac{P Im[\tau(p_L)]}{P(\kappa/2 - Re[\tau(p_L))]} = \frac{Im[\tau(p_L)]}{\kappa/2 - Re[\tau(p_L)]}$$

## 3. Test of the method

- The program is then to measure the asymmetry at one energy where the polarization is known, say at $p_L = 21.7$ GeV/c, and to measure the shape at some other energy where the polarization may not be known, say $p_L = 100$ GeV/c. From these three numbers one can calculate the three numbers $\tau_P, \tau_f$ and $\tau_\omega$ and thereby, if the model is correct, obtain the polarization at 100 GeV or any other energy where the model is valid.

- The numbers for $\tau(21.7)$ are published and preliminary data at 100 GeV has been reported at conferences during the past year. From fitting this data we find $S(100) = -0.052$, and with this value we can work through this process to get tentative values for the Regge residues. We find

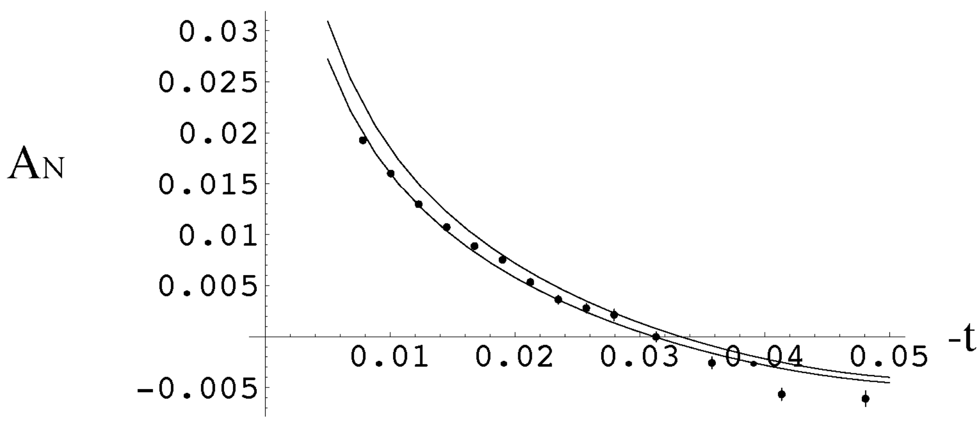
$$\begin{aligned} \tau_P &= -0.02 \\ \tau_f &= -0.43 \\ \tau_\omega &= 0.03 \end{aligned}$$

- Using this preliminary data we can also work out tentative errors on these values. We will see that only the $f$ spin-flip is significantly non-zero. Indeed, the often espoused assumption that the Pomeron spin-flip is zero is consistent with this determination.

- With these values we can tentatively calculated the polarization at 100 GeV in that experiment and find
$$P = 0.23.$$
The fact that this number is about $10\%$ below the value measure at 24 GeV may indicate a limitation on this method already.

- The newest 100 GeV data was taken in conjunction with the p-jet determination of the beam polarization so we have a very accurate and absolutely normalized $A_N$. Here it is along with the prediction using the Regge parameters just determined and the best fit to the data showing the same $10\%$ between the model and the data.

Comparison of recently measured $A_N$ at RHIC (from Osamu Jinnouchi) for pC scattering at 100 GeV/c with model prediction (upper curve) and best fit to data (lower curve).




Model $\tau = -0.130 - 0.053\, i$

Best fit $\tau = -0.017 - 0.049\, i$

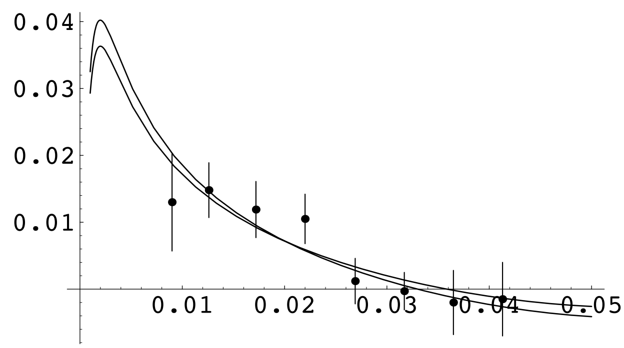
4. Regge couplings

- Here we give the revised Regge couplings using the new 100 GeV data and the shape of the E950 21.7 GeV data. A new revision is underway using the shape of the new 24 GeV data but not yet complete. We find

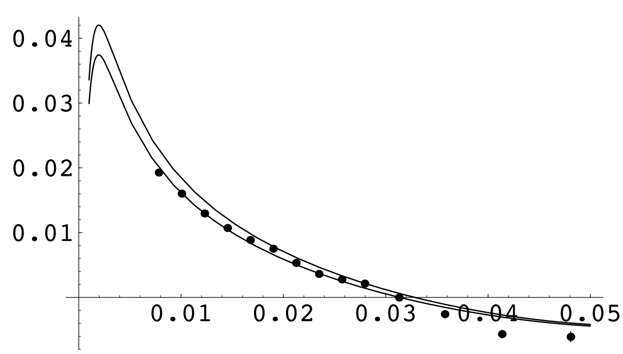
$$\tau_P = +.09$$
$$\tau_f = -0.30$$
$$\tau_\omega = 0.19$$

- The next figure shows a comparison between the E950 data, the original fit and this prediction. It looks pretty good, but examination of the crucial peak region again indicates a $10\%$ discrepancy. The following figure shows again the original error ellipses for the Regge residues with the new central values marked with x (errors not worked out yet). Agreement is O.K.. The new 24 GeV data looks at first sight to have a rather different shape from the earlier data and this might lead to bigger changes.

# Checking Regge model




21.7 GeV data and E950 fit with known
P(upper) and Regge prediction based on fit to
100 GeV data and 21.7 GeV shape (lower)




new 100 GeV data, best fit with known
P(lower) and Regge prediction based on E950 fit
and 100 GeV shape (upper)

5. New 24 GeV data

Just last week I was given the preliminary 24 GeV data from the latest RHIC run. The polarization was not directly measured at that energy so the plot shows the raw asymmetry $\epsilon$, which is enough to determine the shape. If we use this shape, which is very different from E950,

$$shape950 = -0.048, \; newshape = -0.036$$

the Regge couplings are changed to

$$\begin{aligned} \tau_P &= +.115 \\ \tau_f &= -0.336 \\ \tau_\omega &= 0.397 \end{aligned}$$

This determines $newP(24) = 0.35$. This is a little low given that the nominal polarization at 100 GeV from the jet is about 0.39, but it could be this low (Jinnouchi).

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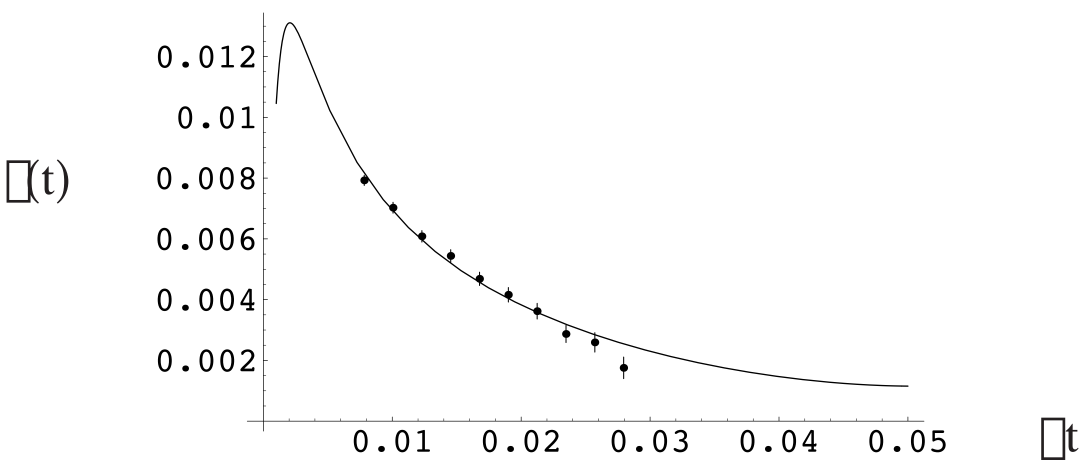
New (October 2004) 24 GeV data from RHIC for □(t)




best fit for shape = -0.036

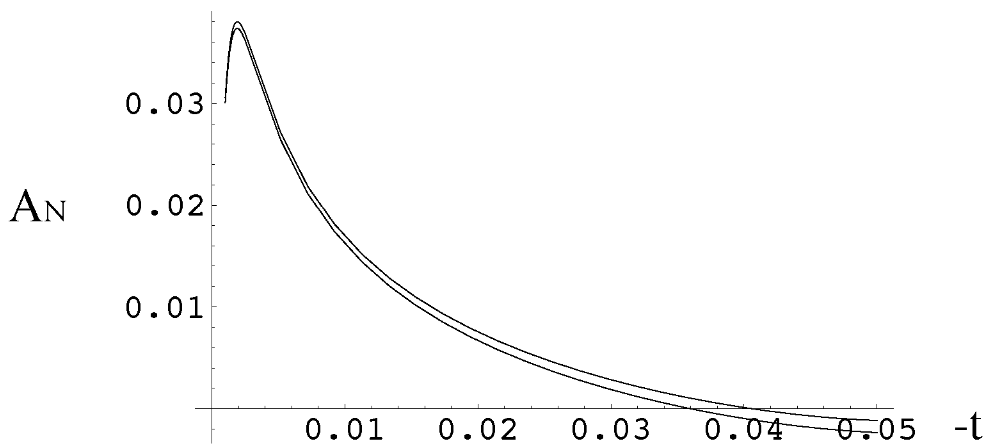
We can also use these numbers to predict the analyzing power at 250 GeV. This is shown for two cases : one with the new Regge couplings I just showed (upper) and the other a first look at the effect of the new 24 GeV data. Very little difference here. The corresponding tau values are

$$\text{new} \quad \tau(s) = 0.015 - 0.0407i$$

$$\text{new new} \quad \tau(s) = 0.025 - 0.0427i$$

Note the zero crossing moves in because the shape of the new new fit is larger than the shape of the "new" fit. The crossing occurs at the $t$-value when $f(t)$ in the analyzing power formula, about 20 and increasing in the range around 0.04, is equal to $1/S(p_L)$.

Analyzing power predicted for 250 GeV/c:
upper curve shows prediction prior to newest 24 GeV data,
lower curve uses the newest data to determine Regge couplings



6. $I = 1$ couplings and proton-proton elastic scattering

- Because $pp$ scattering involves the exchange of of $I = 1$ Regge poles, the $\rho$ and the $a_2$ in particular, we cannot simply use the results above to make predictions for this case. But we can use the beautiful new p-jet data and a couple of reasonable assumptions to achive this. We assume (1) at these energies the proton-proton and neutron-proton unpolarized scattering amplitudes are approximately equal and (2) the two $I = 1$ Regge poles are degenerate with the corresponding $I = 0$ Regge pole of the same Charge Conjugation parity, $C = -1$ for $\omega, \rho$ and $C = +1$ for $f, a_2$. Then we can describe $pp$ scattering in terms of 3-parameters: $\tau_+, \tau_-$ and the pomeron coupling $\tau_P$. Since we already know $\tau_P$, in some sense, from the $pC$ analysis and we can determine two parameters from the real and imaginary parts of $\tau$ obtained by fitting the p-jet data, we are in business.
- I fit the data that Sandro Bravar gave me to the standard CNI asymmetry formula. It is much simpler for $pp$ because the functions and the

parameters involved are all measured. The only uncertainty is the $\rho$ value at this energy which is not directly measured, but from fits over a big energy range it looks that one can reasonably interpolate the value $\rho = -0.082$. If you use a different value, like 0, you will get a different $\tau$. I obtain then

$$\tau_{pp}(100) = -0.065 - 0.0124i$$

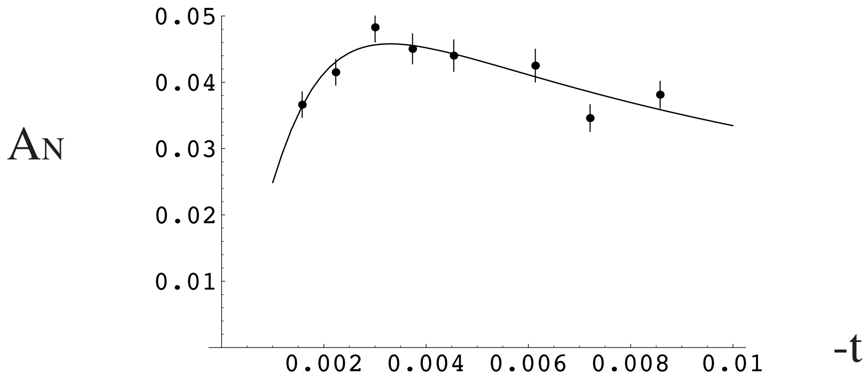
and from this and using $\tau_P = 0.09$ we find

$$\begin{aligned} \tau_P &= +.09 \\ \tau_+ &= -0.324 \\ \tau_- &= 1.06 \end{aligned}$$

I remind for the $I = 0$ Reggeons

$$\begin{aligned} \tau_P &= +.09 \\ \tau_f &= -0.30 \\ \tau_\omega &= 0.19 \end{aligned}$$

New p-jet data




□from p-jet fit = -0.0625 -0.011 i

□from E704 fit = 0.185 + 0.024 i

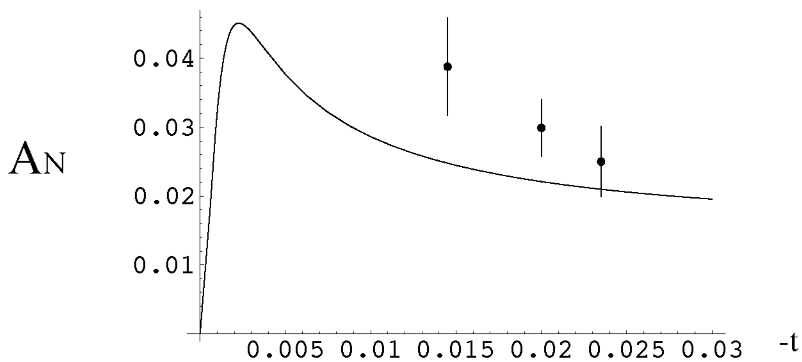
so the $f$ coupling is not much modified by the $a_2$ but the the $C = -1$ is changed by a factor of 10! This is not too much of a surprise: the ancient fit of Berger et al shows just such a pattern.

- Finally, we have also gotten some small-$t$ data from the pp2pp experiment, and equipped with these couplings we can calculate

$$\tau(s = 4 * 10^4) = 0.08 - 0.007i.$$

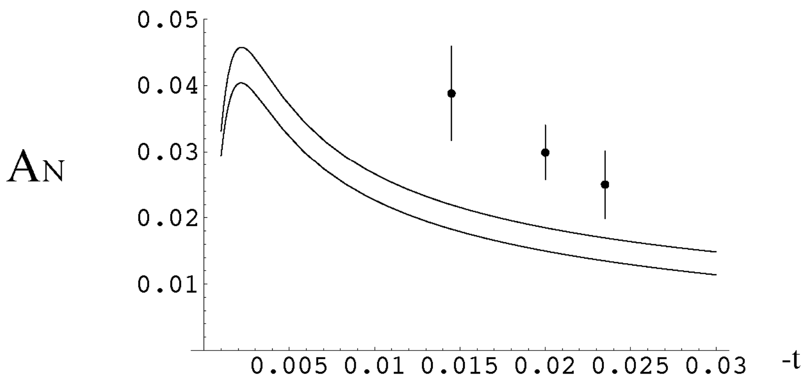
The last plot shows pp2pp data from colliding beams of protons at 100 GeV each together with calculated $A_N$ assuming (a) no hadronic flip-very low, (b) model prediction -even lower and (c) the best fit which gives an enormous spin-flip of $\tau = 1.2 - 0.042i$. I don't know what to make of this. All attempts to estimate a bound on this helicity, cf. Buttimore et al, come up with a number of about 0.15. More hard work is needed.

# Analyzing Power for pp2pp at $s=200^2$




no hadronic spin flip

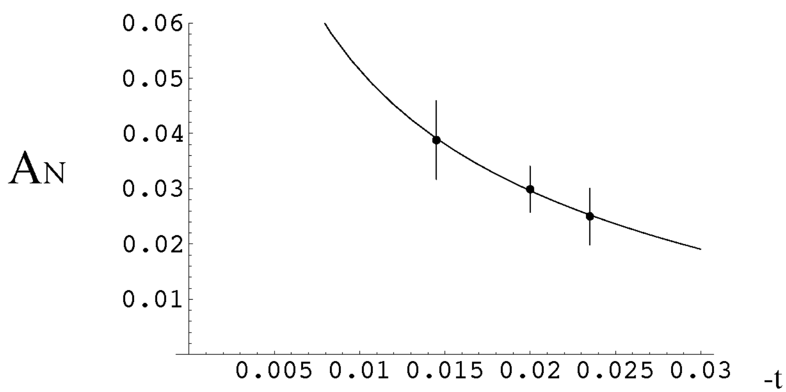
$\tau=0$




Model predictions

$\tau P$ = -0.02  $\tau= -0.025 - 0.0027\,i$

$\tau P$ =+0.09  $\tau= +0.076 - 0.0072\,i$




Best fit

$\tau= -1.2 -0.042\,i$