Tome 39 N° 1-6 JANVIER 1978

LE JOURNAL DE PHYSIQUE

Classification
Physics Abstracts
11.80 — 11.30
13.75C — 13.85D

FORMALISM OF NUCLEON-NUCLEON ELASTIC SCATTERING EXPERIMENTS

J. BYSTRICKY, F. LEHAR

D. Ph. P. E., CEN Saclay, BP n° 2, 91190 Gif sur Yvette, France

and

P. WINTERNITZ

Centre de Recherches Mathématiques, Université de Montréal, Montréal, P.Q., Canada

(*Reçu le 5 avril 1977, accepté le 20 septembre 1977*)

Résumé. — Nous présentons un exposé détaillé du formalisme de la diffusion élastique nucléon-nucléon en ajoutant de nouveaux résultats à ceux déjà connus. Nous passons en revue plusieurs représentations de la matrice de diffusion en tenant compte des principes de symétrie, notamment de la conservation de la parité, de l'invariance par renversement du temps, du principe de Pauli et de l'invariance isotopique. Les quantités expérimentales du système du centre de masse (c.m.s.) et du laboratoire (l.s.) sont exprimées en fonction des amplitudes de diffusion. Les relations entre ces quantités, découlant des symétries mentionnées ainsi que des relations entre les quantités du c.m.s. d'un côté et du l.s. de l'autre sont citées en détail. Nous discutons ensuite une relation générale décrivant la distribution angulaire dans la diffusion corrélée qui comprend toutes les quantités expérimentales existantes ; la formule pour chaque expérience choisie peut en être déduite en précisant les polarisations initiales et les pouvoirs analyseurs. Enfin, nous étudions les conséquences du principe de Pauli pour la diffusion de deux nucléons identiques. Nous exprimons les relations d'une part entre les quantités dans le c.m.s. mesurées aux angles de diffusion θ et $\pi - \theta$ et d'autre part entre les quantités dans le l.s. aux angles associés θ_1 et θ_2. Une attention particulière est prêtée aux angles $\theta = \pi/2$ du c.m.s. et $\theta_1 = \theta_2$ du l.s. Le contenu de l'article est susceptible d'intéresser des expérimentateurs et des phénoménologistes et plus spécialement ceux qui s'occupent de la reconstruction des amplitudes de diffusion à partir des données expérimentales.

Abstract. — A detailed exposition of the nucleon-nucleon elastic scattering formalism is presented, reviewing known results and adding some new ones. Several different representations of the scattering matrix are reviewed, paying attention to symmetry principles like parity conservation, time reversal invariance, the Pauli principle and iso-spin invariance. Experimental quantities in the centre-of-mass and laboratory systems are expressed in terms of scattering amplitudes. Relations between experimental quantities in each of these systems, following from the above mentioned symmetries, are spelt out in detail, as are relations between l.s. and c.m.s. quantities. A general formula for the angular distribution of correlated scattering is given and discussed. This formula involves all existing experimental quantities. It can be specialized to describe any chosen experiment by specifying the initial polarizations and final analyzing powers. Consequences of the Pauli principle for the scattering of identical nucleons are studied. Relations between c.m.s. quantities measured at the c.m.s. angles θ and $\pi - \theta$ or at l.s. angles θ_1 and θ_2 (scattering and recoil angle) are obtained. Special attention is paid to relations at $\theta = \pi/2$, i.e. $\theta_1 = \theta_2$. The material contained in this paper should be useful for experimentalists and for phenomenologists interested in the reconstruction of scattering amplitudes from data.

1. **Introduction.** — The purpose of this article is to provide a detailed study of the kinematics of nucleon-nucleon scattering. Since a large body of literature has already been devoted to this topic during the last 25 years or so, some parts of this article will have the character of a unifying review, while others contain new results (for some of the original work and previous reviews see refs. [1-15]).

In section 2 of this paper we discuss the nucleon-nucleon scattering matrix M, present and relate several different parametrizations of it and discuss the constraints on M following from invariance principles like parity conservation, time reversal invariance, the Pauli principle and isotopic invariance. In section 3 we define the concept of a *pure* experimental quantity (or a *pure* experiment), i.e. one that involves only spin projections onto certain basis vectors in momentum space. We list 256 different pure experiments in the centre of mass system and then find all the constraints on them, following from the invariance principles discussed in section 2. Nonlinear relations between c.m.s. experimental quantities are also discussed, as well as some inequalities imposed on them. Pure experiments in the laboratory system are considered in section 4. A general formula describing the angular distribution of correlated scattering for the case when both initial nucleons are polarized is presented. Simpler formulas relating various angular distributions to laboratory system components of polarization tensors are obtained from the general formula e.g. by assuming that one or both of the initial polarizations are zero and/or that one or both of the final polarizations are not detected. The laboratory frame components of the polarization tensors are expressed in terms of the scattering matrix in tables 5 and 6. The c.m.s. and laboratory system *pure* experimental quantities are related to each other in section 5, taking relativistic effects fully into account. Again, we first present a general formula for an arbitrary experimental quantity, then consider the case where 1, 2, 3 or 4 polarizations are involved. In section 6 we establish linear relations between laboratory system experimental quantities, following from the usual invariance principles. While these relations are not independent of similar ones in the c.m.s., their form, taking relativistic spin rotations into account, is considerably more complicated. In section 7 we discuss consequences of the Pauli principle. In addition to restricting the number of independent amplitudes from 6 to 5 in the scattering matrix and thus significantly restricting the number of independent experiments, the Pauli principle has further implications. Thus, for nn and pp scattering we present all symmetry relations between quantities measured at the c.m.s. angles θ and $\pi - \theta$, i.e. at laboratory system angles θ_1 and θ_2 (the scattering and recoil angles). Further, interesting relations for nn and pp scattering are obtained when $\theta = \pi/2$, i.e. $\theta_1 = \theta_2$, as well as relations between np and say nn experimental quantities. Some conclusions and future outlook are mentioned in section 8.

New results are contained in sections 3 to 7 and they mainly concern quantities involving polarized targets and especially the more complicated experimental quantities. While we give credit in cases when we use the results of other authors, we do not attempt to give anything like a complete bibliography of the field. We also make little effort to relate the formalism of this article to the numerous equivalent formalisms in the literature.

A few words on conventions and notations are in order.

Throughout the paper we use one set of basis vectors in momentum space in the centre of mass system and three different sets in the laboratory frame (relating to incident, scattered and recoil particles). One and the same normal to the scattering plane is used in all cases. While such usage is common to many workers in the field, it is strictly speaking not in agreement with the Basle convention [16]. Indeed, if the convention is applied exactly, then the polarizations of the scattered and recoil particles should be related to opposite normals. The target normal in the laboratory system is not well defined. In any case, it is a simple matter to transform formulas from the one-normal convention to a two-normal one.

We consistently use a four-subscript notation for experimental quantities : X_{pqik} where p and q refer to the scattered and recoil particle polarizations and i and k to the initial beam and target polarizations. If an initial particle is unpolarized or a final state polarization not analyzed, the corresponding subscript is set equal to zero. This notation should help avoid some common misunderstandings in the identification of experimental quantities. It also facilitates transitions between the one-normal and two-normal conventions and the establishment of relations between various quantities. The use of different letters for different experiments is now superfluous but for historical reasons we still use the letter I for intensities (cross sections), P for polarizations, A for asymmetries, D and K for depolarization and polarization transfer tensors, M and N for the contributions of two initial polarizations to the final polarizations of the scattered and recoil particle and C for polarization correlations.

2. **Nucleon-nucleon scattering matrix.** — For our purposes a convenient form of a nucleon-nucleon elastic matrix is [3, 5, 11]

$$M(\mathbf{k}_f, \mathbf{k}_i) = \frac{1}{2}\{ (a + b) + (a - b)(\boldsymbol{\sigma}_1, \mathbf{n})(\boldsymbol{\sigma}_2, \mathbf{n}) + (c + d)(\boldsymbol{\sigma}_1, \mathbf{m})(\boldsymbol{\sigma}_2, \mathbf{m}) + (c - d)(\boldsymbol{\sigma}_1, \mathbf{l})(\boldsymbol{\sigma}_2, \mathbf{l}) + e(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2, \mathbf{n}) \}. \quad (2.1)$$

Here the amplitudes a, b, c, d and e are complex functions of two variables, e.g. the centre of mass system (c.m.s.) energy k and the scattering angle θ. The c.m.s. basis vectors are :

$$\mathbf{l} = \frac{\mathbf{k}_f + \mathbf{k}_i}{|\mathbf{k}_f + \mathbf{k}_i|}, \qquad \mathbf{m} = \frac{\mathbf{k}_f - \mathbf{k}_i}{|\mathbf{k}_f - \mathbf{k}_i|}, \qquad \mathbf{n} = \frac{\mathbf{k}_i \times \mathbf{k}_f}{|\mathbf{k}_i \times \mathbf{k}_f|} \quad (2.2)$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are unit vectors in the direction of the incident and scattered particle momenta in the c.m.s. The spin matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ (the Pauli matrices) act on the first and the second nucleon wave functions, respectively. (The projection $(\boldsymbol{\sigma}, \mathbf{a})$ of a spin matrix $\boldsymbol{\sigma}$ on an arbitrary direction $\mathbf{a}$ will be written also as $(\boldsymbol{\sigma}, \mathbf{a}) = (\boldsymbol{\sigma}\mathbf{a}) = \sigma_a$.

In (2.1) we have already taken into account parity and time reversal invariance. We have also assumed that the particles are identical which is strictly valid for pp and nn scattering. For np scattering this assumes isotopic invariance of the nucleon-nucleon interaction. The scattering matrix for the elastic scattering of two nonidentical particles would contain a sixth term, namely

$$\frac{1}{2} f(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2, \mathbf{n}) . \tag{2.3}$$

Still assuming isotopic invariance, we can write the scattering matrices for pp, nn and np scattering in terms of two matrices M_0 and M_1 of the form (2.1), putting

$$M(\mathbf{k}_f, \mathbf{k}_i) = M_0 \left[\frac{1 - (\boldsymbol{\tau}_1, \boldsymbol{\tau}_2)}{4} \right] + M_1 \left[\frac{3 + (\boldsymbol{\tau}_1, \boldsymbol{\tau}_2)}{4} \right] \tag{2.4}$$

where $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are the nucleon isospin matrices, and M_0 and M_1 are isosinglet and isotriplet scattering matrices, respectively. Obviously we have

$$\begin{aligned} M(\mathrm{pp} \to \mathrm{pp}) &= M(\mathrm{nn} \to \mathrm{nn}) = M_1 , \\ M(\mathrm{np} \to \mathrm{np}) &= M(\mathrm{pn} \to \mathrm{pn}) = (M_1 + M_0)/2 , \\ M(\mathrm{np} \to \mathrm{pn}) &= M(\mathrm{pn} \to \mathrm{np}) = (M_1 - M_0)/2 . \end{aligned} \tag{2.5}$$

Formulas (2.4) and (2.5), unlike all the others in this paper, refer only to strong interaction scattering matrices, ignoring the electromagnetic interactions.

The generalized Pauli principle for the nucleons implies certain symmetry conditions for the amplitudes in (2.1), summarized in table I [5].

TABLE I

Symmetry properties following from the generalized Pauli principle

$\underline{T = 1}$	$\underline{T = 0}$
$a_1(\theta) = - a_1(\pi - \theta)$	$a_0(\theta) = a_0(\pi - \theta)$
$b_1(\theta) = - c_1(\pi - \theta)$	$b_0(\theta) = c_0(\pi - \theta)$
$c_1(\theta) = - b_1(\pi - \theta)$	$c_0(\theta) = b_0(\pi - \theta)$
$d_1(\theta) = d_1(\pi - \theta)$	$d_0(\theta) = - d_0(\pi - \theta)$
$e_1(\theta) = e_1(\pi - \theta)$	$e_0(\theta) = - e_0(\pi - \theta)$

Throughout this article we shall use the amplitudes a, b, c, d and e, although many different but equivalent parametrizations are often useful.

Hoshizaki [8] uses the scattering matrix

$$\begin{aligned} M(\mathbf{k}_f, \mathbf{k}_i) = a_{\mathrm{H}} &+ c_{\mathrm{H}}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2, \mathbf{n}) + m_{\mathrm{H}}(\boldsymbol{\sigma}_1, \mathbf{n})(\boldsymbol{\sigma}_2, \mathbf{n}) \\ &+ g_{\mathrm{H}}[(\boldsymbol{\sigma}_1, \mathbf{l})(\boldsymbol{\sigma}_2, \mathbf{l}) + (\boldsymbol{\sigma}_1, \mathbf{m})(\boldsymbol{\sigma}_2, \mathbf{m})] \\ &+ h_{\mathrm{H}}[(\boldsymbol{\sigma}_1, \mathbf{l})(\boldsymbol{\sigma}_2, \mathbf{l}) - (\boldsymbol{\sigma}_1, \mathbf{m})(\boldsymbol{\sigma}_2, \mathbf{m}] , \end{aligned} \tag{2.6}$$

so that

$$a_{\mathrm{H}} = \frac{1}{2}(a + b) , \quad c_{\mathrm{H}} = \frac{e}{2} , \quad m_{\mathrm{H}} = \frac{1}{2}(a - b) , \quad g_{\mathrm{H}} = \frac{c}{2} , \quad h_{\mathrm{H}} = -\frac{d}{2} \tag{2.7}$$

which implies

$$a = a_{\mathrm{H}} + m_{\mathrm{H}} , \quad b = a_{\mathrm{H}} - m_{\mathrm{H}} , \quad c = 2\, g_{\mathrm{H}} , \quad d = -2\, h_{\mathrm{H}} , \quad e = 2\, c_{\mathrm{H}} . \tag{2.8}$$

The so-called Wolfenstein amplitudes B, C, N, G, H [1] are defined as :

$$\begin{aligned} M(\mathbf{k}_f, \mathbf{k}_i) = BS + \{ & C(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2, \mathbf{n}) + N(\boldsymbol{\sigma}_1, \mathbf{n})(\boldsymbol{\sigma}_2, \mathbf{n}) \\ &+ \frac{1}{2} G[(\boldsymbol{\sigma}_1, \mathbf{m})(\boldsymbol{\sigma}_2, \mathbf{m}) + (\boldsymbol{\sigma}_1, \mathbf{l})(\boldsymbol{\sigma}_2, \mathbf{l})] \\ &+ \frac{1}{2} H[(\boldsymbol{\sigma}_1, \mathbf{m})(\boldsymbol{\sigma}_2, \mathbf{m}) - (\boldsymbol{\sigma}_1, \mathbf{l})(\boldsymbol{\sigma}_2, \mathbf{l})] \} \, T , \end{aligned} \tag{2.9}$$

where S and T are the spin-singlet and spin triplet projection operators, respectively :

$$S = \frac{1}{4}[1 - (\sigma_1, \sigma_2)] , \qquad T = \frac{1}{4}[3 + (\sigma_1, \sigma_2)] . \tag{2.10}$$

The Wolfenstein amplitudes are related to ours as follows :

$$B = b - c , \quad C = e/2 , \quad N = a , \quad G = a + b + c , \quad H = d \tag{2.11}$$

which implies

$$a = N , \quad b = (B - N + G)/2 , \quad c = (G - B - N)/2 , \quad d = H , \quad e = 2\,C . \tag{2.12}$$

The « singlet-triplet representation » matrix elements [17] are :

$$\begin{aligned}
M_{ss} &= b - c \\
M_{00} &= a + d\cos\theta \\
M_{11} &= \frac{1}{2}(a + b + c - d\cos\theta) \\
M_{10} &= -\frac{1}{\sqrt{2}}(d\sin\theta + ie) \\
M_{01} &= -\frac{1}{\sqrt{2}}(d\sin\theta - ie) \\
M_{1-1} &= \frac{1}{2}(-a + b + c + d\cos\theta) = M_{11} - M_{00} - \sqrt{2}(M_{10} + M_{01})\operatorname{cotg}\theta \\
M_{-1-1} &= M_{11} , \quad M_{-11} = M_{1-1} , \quad M_{0-1} = -M_{01} , \quad M_{-10} = -M_{10}
\end{aligned} \tag{2.13}$$

which implies

$$\begin{aligned}
a &= \frac{1}{2}(M_{11} + M_{00} - M_{1-1}) \\
b &= \frac{1}{2}(M_{11} + M_{ss} + M_{1-1}) \\
c &= \frac{1}{2}(M_{11} - M_{ss} + M_{1-1}) \\
d &= \frac{1}{2\cos\theta}(-M_{11} + M_{00} + M_{1-1}) = -\frac{1}{\sqrt{2}\sin\theta}(M_{10} + M_{01}) \\
e &= \frac{i}{\sqrt{2}}(M_{10} - M_{01}) .
\end{aligned} \tag{2.14}$$

Jacob and Wick [18] have developed the helicity formalism in which states are labelled by the spin projection λ onto the particle momentum (λ is the *helicity* quantum number). Since there are some ambiguities in the definitions of helicity states and amplitudes we shall specify our formalism here. Essentially it will coincide with that of Jacob and Wick [18], Martin and Spearman [19], Goldberger, Grisaru, MacDowel and Wong [15], Hoshizaki [8], etc. Other authors, e.g. Cohen-Tannoudji, Morel and Navelet [20] and Kotanski [21, 22] use somewhat different phase conventions (they omit the factor $(-1)^{s-\lambda}$ for particles 2 and 4).

We use the centre-of-mass system, consider the xz plane as the scattering plane, put the z-axis along the momentum of the incident particle and the y-axis along the normal $\mathbf{n}$. The helicity states of the incident and scattered particles are

$$\chi_{\lambda_1} = \begin{pmatrix} \frac{1}{2} + \lambda_1 \\ \frac{1}{2} - \lambda_1 \end{pmatrix} \tag{2.15}$$

and

$$\chi'_{\lambda_3} = \exp\left(-i\frac{\sigma_n}{2}\theta\right)\chi_{\lambda_3} = \begin{pmatrix} \left(\frac{1}{2}+\lambda_3\right)\cos\frac{\theta}{2} - \left(\frac{1}{2}-\lambda_3\right)\sin\frac{\theta}{2} \\ \left(\frac{1}{2}+\lambda_3\right)\sin\frac{\theta}{2} + \left(\frac{1}{2}-\lambda_3\right)\cos\frac{\theta}{2} \end{pmatrix}. \tag{2.16}$$

The helicity states of particles 2 and 4 are defined with a different phase as

$$\chi'_{\lambda_2} = (-1)^{\frac{1}{2}-\lambda_2}\exp\left(-i\frac{\sigma_n}{2}\pi\right)\chi_{\lambda_2} = (-1)^{\frac{1}{2}-\lambda_2}\begin{pmatrix} -\frac{1}{2}+\lambda_2 \\ \frac{1}{2}+\lambda_2 \end{pmatrix}. \tag{2.17}$$

and

$$\chi'_{\lambda_4} = (-1)^{\frac{1}{2}-\lambda_4}\exp\left[-i\frac{\sigma_n}{2}(\theta+\pi)\right]\chi_{\lambda_4} = (-1)^{\frac{1}{2}-\lambda_4}\begin{pmatrix} -\left(\frac{1}{2}+\lambda_4\right)\sin\frac{\theta}{2} - \left(\frac{1}{2}-\lambda_4\right)\cos\frac{\theta}{2} \\ \left(\frac{1}{2}+\lambda_4\right)\cos\frac{\theta}{2} - \left(\frac{1}{2}-\lambda_4\right)\sin\frac{\theta}{2} \end{pmatrix}. \tag{2.18}$$

The helicity λ for a nucleon is 1/2 if the spin projection is parallel to the momentum, $-1/2$ if it is antiparallel. The helicity amplitudes are denoted $\langle \lambda_3 \lambda_4 | M | \lambda_1 \lambda_2 \rangle$ and can be expanded into a partial wave sum as

$$\langle \lambda_3 \lambda_4 | M | \lambda_1 \lambda_2 \rangle = \frac{1}{2ik}\sum_J (2J+1)\langle \lambda_3 \lambda_4 | T^J(E) | \lambda_1 \lambda_2 \rangle d^J_{\lambda\mu}(\theta) \tag{2.19}$$

where $\lambda = \lambda_1 - \lambda_2$. $\mu = \lambda_3 - \lambda_4$ and $d^J_{\lambda\mu}(\theta)$ are Wigner rotation matrices [23] satisfying

$$d^J_{\lambda\mu}(\theta) = (-1)^{\lambda-\mu} d^J_{\mu\lambda}(\theta) = (-1)^{\lambda-\mu} d^J_{-\lambda-\mu}(\theta). \tag{2.20}$$

Parity conservation, time reversal invariance and the Pauli principle imply that

$$\begin{aligned} \langle -\lambda_3 - \lambda_4 | T^J(E) | -\lambda_1 - \lambda_2 \rangle &= \langle \lambda_3 \lambda_4 | T^J(E) | \lambda_1 \lambda_2 \rangle \\ \langle \lambda_1 \lambda_2 | T^J(E) | \lambda_3 \lambda_4 \rangle &= \langle \lambda_3 \lambda_4 | T^J(E) | \lambda_1 \lambda_2 \rangle \\ \langle \lambda_4 \lambda_3 | T^J(E) | \lambda_2 \lambda_1 \rangle &= \langle \lambda_3 \lambda_4 | T^J E | \lambda_1 \lambda_2 \rangle \end{aligned} \tag{2.21}$$

respectively. These relations for the partial wave helicity amplitudes together with (2.19) and (2.20) in turn imply that the total helicity amplitudes satisfy

$$\begin{aligned} \langle -\lambda_3 - \lambda_4 | M | -\lambda_1 - \lambda_2 \rangle &= (-1)^{\lambda_1-\lambda_2-\lambda_3+\lambda_4}\langle \lambda_3 \lambda_4 | M | \lambda_1 \lambda_2 \rangle \\ \langle \lambda_1 \lambda_2 | M | \lambda_3 \lambda_4 \rangle &= (-1)^{\lambda_1-\lambda_2-\lambda_3+\lambda_4}\langle \lambda_3 \lambda_4 | M | \lambda_1 \lambda_2 \rangle. \\ \langle \lambda_4 \lambda_3 | M | \lambda_2 \lambda_1 \rangle &= (-1)^{\lambda_1-\lambda_2-\lambda_3+\lambda_4}\langle \lambda_3 \lambda_4 | M | \lambda_1 \lambda_2 \rangle \end{aligned} \tag{2.22}$$

Taking these symmetry relations into account and indicating only the signs of the nucleon helicities, we put :

$$\begin{aligned} M_1 &\equiv \langle ++|M|++\rangle = \langle --|M|--\rangle \\ M_2 &\equiv \langle ++|M|--\rangle = \langle --|M|++\rangle \\ M_3 &\equiv \langle +-|M|+-\rangle = \langle -+|M|-+\rangle \\ M_4 &\equiv \langle +-|M|-+\rangle = \langle -+|M|+-\rangle \\ M_5 &\equiv \langle ++|M|+-\rangle = \langle -+|M|--\rangle = \langle --|M|+-\rangle = \\ &= \langle -+|M|++\rangle = -\langle --|M|-+\rangle = -\langle +-|M|++\rangle \\ &= -\langle ++|M|-+\rangle = -\langle +-|M|--\rangle. \end{aligned} \tag{2.23}$$

Substituting expression (2.1) for M and calculating the appropriate matrix elements we obtain the relations between the invariant c.m.s. amplitudes $a, \ldots, e$ and the helicity amplitudes $M_1, \ldots, M_5$. With our conventions we thus obtain

$$\begin{aligned}
M_1 &= \frac{1}{2}(a \cos\theta + b - c + d + ie \sin\theta) \\
M_2 &= \frac{1}{2}(a \cos\theta - b + c + d + ie \sin\theta) \\
M_3 &= \frac{1}{2}(a \cos\theta + b + c - d + ie \sin\theta) \\
M_4 &= \frac{1}{2}(- a \cos\theta + b + c + d - ie \sin\theta) \\
M_5 &= \frac{1}{2}(- a \sin\theta + ie \cos\theta)
\end{aligned} \tag{2.24}$$

Formulas (2.24) can be inverted to give

$$\begin{aligned}
a &= \frac{1}{2}[(M_1 + M_2 + M_3 - M_4)\cos\theta - 4 M_5 \sin\theta] \\
b &= \frac{1}{2}(M_1 - M_2 + M_3 + M_4) \\
c &= \frac{1}{2}(- M_1 + M_2 + M_3 + M_4) \\
d &= \frac{1}{2}(M_1 + M_2 - M_3 + M_4) \\
e &= -\frac{i}{2}[(M_1 + M_2 + M_3 - M_4)\sin\theta + 4 M_5 \cos\theta] .
\end{aligned} \tag{2.25}$$

For forward scattering, when $\theta = 0$, total angular momentum conservation implies that $e(0) = 0$, $a(0) - b(0) = c(0) + d(0)$. For helicity amplitudes this obviously implies that $M_4(0) = M_5(0) = 0$.

Obviously, infinitely many different types of nucleon-nucleon scattering amplitudes could be introduced and indeed a very large number exists in the literature. In addition to those introduced above we wish to consider two more, namely the transversity amplitudes [21, 22] and *exchange* amplitudes [24, 25], since both of these are used by various experimental groups.

The exchange amplitudes are useful since in the high energy limit they correspond to the exchange of definite quantum numbers (Regge poles) in the t channel (the amplitudes N_0, N_1, N_2 correspond to natural parity exchange, U_0 and U_2 to unnatural parity, the subscript denotes the amount of t-channel helicity flip). The exchange amplitudes are related to the s-channel helicity amplitudes and the $a, \ldots, e$ amplitudes by the relations

$$\begin{aligned}
N_0 &= \frac{1}{2}(M_1 + M_3) = \frac{1}{2}(a \cos\theta + b + ie \sin\theta) \\
N_1 &= M_5 \qquad\qquad\;\; = \frac{1}{2}(- a \sin\theta + ie \cos\theta) \\
N_2 &= \frac{1}{2}(M_4 - M_2) = \frac{1}{2}(- a \cos\theta + b - ie \sin\theta) \\
U_0 &= \frac{1}{2}(M_1 - M_3) = \frac{1}{2}(- c + d) \\
U_2 &= \frac{1}{2}(M_4 + M_2) = \frac{1}{2}(c + d) .
\end{aligned} \tag{2.26}$$

This can be inverted to give

$$\begin{aligned}
a &= (N_0 - N_2)\cos\theta - 2 N_1 \sin\theta \\
b &= N_0 + N_2 \\
c &= U_2 - U_0 \\
d &= U_2 + U_0 \\
e &= - i[(N_0 - N_2)\sin\theta + 2 N_1 \cos\theta] .
\end{aligned} \tag{2.27}$$

The transversity amplitudes T_{cdab} were introduced by Kotański [21, 22] in order to diagonalize crossing matrices. For nucleon-nucleon scattering they are related to the Jacob and Wick helicity amplitudes M_{cdab} by the relation

$$T_{cdab} = \sum_{a'b'c'd'} (-1)^{b+d+1} U^*_{cc'} U^*_{dd'} M_{c'd'a'b'} U_{a'a} U_{b'b} \tag{2.28}$$

with

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{2.29}$$

More explicitly, the five independent transversity amplitudes are given by

$$\begin{aligned}
T_1 &\equiv T_{++++} = \frac{1}{2}(M_1 + M_2 + M_3 - M_4 - 4\,iM_5) = (a + e)\exp(i\theta) \\
T_2 &\equiv T_{----} = \frac{1}{2}(M_1 + M_2 + M_3 - M_4 + 4\,iM_5) = (a - e)\exp(-i\theta) \\
T_3 &\equiv T_{+-+-} = \frac{1}{2}(M_1 - M_2 + M_3 + M_4) = b \\
T_4 &\equiv T_{++--} = \frac{1}{2}(-M_1 - M_2 + M_3 - M_4) = -d \\
T_5 &\equiv T_{+--+} = \frac{1}{2}(M_1 - M_2 - M_3 - M_4) = -c
\end{aligned} \tag{2.30}$$

Formulas (2.30) are in full agreement with those used in the Argonne National Laboratory (e.g. [26]).

3. **Experimental quantities in the centre-of-mass system.** — We shall introduce a four-subscript notation for all experimental quantities. The first and second subscript refer to the final state polarization of the scattered and recoil particle, respectively. The third and fourth subscript specify the initial polarization of the beam and target, respectively. In the c.m.s. the labels are denoted p, q, i and k, in this order. If an initial particle is unpolarized or the polarization of a final particle is not analyzed, the corresponding label is set equal to zero.

A « pure » experimental quantity (briefly a pure experiment) is by definition one involving only spin projections on basis vectors. The basis can be different for different particles but in the c.m.s. the system **l**, **m**, **n** will be used for all particles. In principle, 256 pure experiments can be defined as components of various tensors. These are summarized in table II and are defined as follows :

A.1. The unpolarized differential cross section

$$\sigma \equiv I_{0000} = \frac{1}{4}\,\mathrm{Tr}\, MM^+ .$$

TABLE II

Experimental quantities in the scattering of spin ½ particles

Measured Quantity		Unpolarized beam Unpolarized target *A*	Polarized beam Unpolarized target *B*	Unpolarized beam Polarized target *C*	Polarized beam Polarized target *D*
Differential cross-section	1	I_{0000}	A_{00i0}	A_{000k}	A_{00ik}
Polarization of scattered particles	2	P_{p000}	D_{p0i0}	K_{p00k}	M_{p0ik}
Polarization of recoil particles	3	P_{0q00}	K_{0qi0}	D_{0q0k}	N_{0qik}
Correlation of polarizations	4	C_{pq00}	C_{pqi0}	C_{pq0k}	C_{pqik}

A.2. Polarization of scattered particle

$$\sigma P_{p000} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} M M^{+} .$$

A.3. Polarization of recoil particle

$$\sigma P_{0q00} = \frac{1}{4} \operatorname{Tr} \sigma_{2q} M M^{+} .$$

B.1. Asymmetry in cross section due to polarized beam

$$\sigma A_{00i0} = \frac{1}{4} \operatorname{Tr} M \sigma_{1i} M^{+} .$$

C.1. Asymmetry in cross section due to polarized target

$$\sigma A_{000k} = \frac{1}{4} \operatorname{Tr} M \sigma_{2k} M^{+} .$$

A.4. Polarization correlation for initially unpolarized particles

$$\sigma C_{pq00} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} \sigma_{2q} M M^{+} .$$

B.2. Depolarization tensor for polarized beam

$$\sigma D_{p0i0} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} M \sigma_{1i} M^{+} .$$

B.3. Polarization transfer from beam to recoil particle

$$\sigma K_{0qi0} = \frac{1}{4} \operatorname{Tr} \sigma_{2q} M \sigma_{1i} M^{+} .$$

C.2. Polarization transfer from target to scattered particle

$$\sigma K_{p00k} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} M \sigma_{2k} M^{+} .$$

C.3. Depolarization tensor for polarized target

$$\sigma D_{0q0k} = \frac{1}{4} \operatorname{Tr} \sigma_{2q} M \sigma_{2k} M^{+} .$$

D.1. Asymmetry tensor for polarized beam and target

$$\sigma A_{00ik} = \frac{1}{4} \operatorname{Tr} M \sigma_{1i} \sigma_{2k} M^{+} .$$

B.4. Contribution to polarization correlation from polarized beam

$$\sigma C_{pqi0} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} \sigma_{2q} M \sigma_{1i} M^{+} .$$

C.4. Contribution to polarization correlation from polarized target

$$\sigma C_{pq0k} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} \sigma_{2q} M \sigma_{2k} M^{+} .$$

D.2. Contribution to the polarization of scattered particle from beam and target polarization

$$\sigma M_{p0ik} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} M \sigma_{1i} \sigma_{2k} M^{+} .$$

D.3. Contribution to recoil particle polarization from beam and target polarization

$$\sigma N_{0qik} = \frac{1}{4} \operatorname{Tr} \sigma_{2q} M \sigma_{1i} \sigma_{2k} M^{+} .$$

D.4. Contribution to polarization correlation from polarized beam and target

$$\sigma C_{pqik} = \frac{1}{4} \operatorname{Tr} \sigma_{1p} \sigma_{2q} M \sigma_{1i} \sigma_{2k} M^{+} .$$

For an arbitrary reaction of the type $\frac{1}{2} + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2}$ all 256 experiments could provide independent information. However, if parity conservation, the generalized Pauli principle and time reversal invariance are assumed, the number of independent experiments is greatly reduced.

Under space reflection vectors **l** and **m** change their signs, whereas **n** is conserved. The parity conservation thus implies that only experiments with an even number of l and m labels are non-zero.

The generalized Pauli principle (including isospin invariance for np scattering) requires an equality of two experiments related by interchanging beam with target and scattered with recoil particle states and momenta. It gives for a general pure experiment

$$X_{pqik} = (-1)^{[l]+[m]} X_{qpki} \tag{3.1}$$

where $[l]$ and $[m]$ are numbers of labels l and m, respectively, among p, q, i and k.

Parity conservation combined with the relation (3.1) gives

$$X_{pqik} = X_{qpki} . \tag{3.2}$$

The effect of time-reversal may be expressed by changing the signs of both momenta $\mathbf{k}_i$ and $\mathbf{k}_f$ as well as the signs of $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ and by interchanging the initial and final states and momenta. Therefore the basic vectors are transformed as

$$\mathbf{l} \to -\mathbf{l} , \qquad \mathbf{m} \to \mathbf{m} , \qquad \mathbf{n} \to -\mathbf{n} . \tag{3.3}$$

The time reversal invariance results in the relation

$$X_{pqik} = (-1)^{[m]} X_{ikpq} . \tag{3.4}$$

A very helpful method of demonstrating relations between different experimental quantities makes use of invariance under reflection in the scattering plane (the so called Bohr's rule [27]). For the nucleon-nucleon scattering matrix this invariance implies the identity

$$\sigma_{1n} \sigma_{2n} M \sigma_{1n} \sigma_{2n} = M \tag{3.5}$$

which can be verified directly using formula (2.1).

Let us discuss pure experiments in the c.m.s.

(0) The differential cross-section $\sigma \equiv I_{0000}$ is obviously a scalar with respect to all the discrete symmetries considered.

(1) *One-component tensors (axial vectors)* P_{p000}, P_{0q00}, A_{00i0} *and* A_{000k}. — Parity conservation implies that the only non-zero components are P_{n000}, P_{0n00}, A_{00n0} and A_{000n}. The Pauli principle implies $P_{n000} = P_{0n00}$ and $A_{00n0} = A_{000n}$. Finally, time reversal invariance gives : $P_{n000} = A_{00n0}$ and $P_{0n00} = A_{000n}$. Thus out of 12 different quantities 8 are equal to zero, the remaining 4 are equal to each other.

(2) *Two-component tensors* C_{pq00}, D_{p0i0}, K_{0qi0}, K_{p00k}, D_{0q0k} *and* A_{00ik}. — Parity conservation reduces 54 components to 30. The Pauli principle implies $C_{pq00} = C_{qp00}$, $D_{p0i0} = D_{0p0i}$, $K_{p00k} = K_{0pk0}$ and $A_{00ik} = A_{00ki}$, so that only 18 components remain. Finally time reversal invariance implies $C_{nn00} = A_{00nn}$, $C_{ll00} = A_{00ll}$. $C_{mm00} = A_{00mm}$, $C_{ml00} = -A_{00ml}$, $D_{m0l0} = -D_{l0m0}$ and $K_{0ml0} = -K_{0lm0}$, reducing the number of different experiments to 12.

(3) *Three-component tensors* C_{pqi0}, C_{pq0k}, M_{p0ik} *and* N_{0qik}. — Parity reduces the number of components from 108 to 52. The Pauli principle implies $C_{pqi0} = C_{qp0i}$ and $M_{p0ik} = N_{0pki}$. Time reversal invariance implies $C_{pqi0} = (-1)^{[m]} M_{i0pq}$. Thus we are left with 13 components. These can be further reduced by making use of the Bohr's rule (3.5), providing the relations : $C_{mmn0} = -C_{lln0}$, $C_{nmm0} = C_{nll0}$ and $C_{mnm0} = C_{lnl0}$ and relating C_{nnn0} to the polarization $C_{nnn0} = P_{n000}$. Finally, 9 components of the three-component tensors remain independent.

(4) *Four-component tensor* C_{pqik}. — Parity conservation reduces the number of non-zero components from 81 to 41. The Pauli principle provides the relation $C_{pqik} = C_{qpki}$ and time reversal invariance implies $C_{pqik} = (-1)^{[m]} C_{ikpq}$. We are thus left with 17 components. Using the Bohr's rule (3.5) we can reduce some components to components of lower-order tensors and also find two new relations among the components of C_{pqik}. These relations are :

$$\begin{aligned} C_{nnnn} &= 1, & C_{nlnl} &= D_{0m0m}, & C_{nmnm} &= D_{0l0l}, \\ C_{nlln} &= K_{0mm0}, & C_{nmmn} &= K_{0ll0}, & C_{llnn} &= -C_{mm00}, \\ C_{mmnn} &= -C_{ll00}, & C_{nmln} &= -K_{0lm0}, & C_{mlnn} &= C_{lm00}. \\ C_{mnln} &= -D_{l0m0}, \end{aligned} \quad (3.6)$$

and

$$C_{mmmm} = C_{llll}, \qquad C_{mmml} = -C_{lllm}. \quad (3.7)$$

Using the formula (2.1) directly we can find

$$\begin{aligned} \sigma_{2m} M \sigma_{2m} &= -\sigma_{1l} M \sigma_{1l} + \sigma_{1m} M \sigma_{1m} + \sigma_{2l} M \sigma_{2l} \\ -i\sigma_{2l} M \sigma_{1n} \sigma_{2m} &= \sigma_{1l} M \sigma_{1l} - i\sigma_{1l} M \sigma_{1m} \sigma_{2n} - \sigma_{2l} M \sigma_{2l} \end{aligned} \quad (3.8)$$

and

$$-i\sigma_{2m} M \sigma_{1n} \sigma_{2l} = -\sigma_{1l} M \sigma_{1l} + i\sigma_{1l} \sigma_{2n} M \sigma_{1m} + \sigma_{2l} M \sigma_{2l}.$$

By multiplying each of the equalities (3.8) by σ_{1l} from the left and by $\sigma_{1l} M^+$ from the right, we find three more linear relations amongst components of C_{pqik} and lower order tensors. These can be written e.g. as :

$$\begin{aligned} C_{lmlm} = C_{mlml} &= -1 + D_{n0n0} + C_{llll} \\ C_{llmm} = C_{mmll} &= 1 - A_{00nn} - C_{llll} \\ C_{lmml} = C_{mllm} &= -1 + K_{0nn0} + C_{llll}. \end{aligned} \quad (3.9)$$

Thus only two components of C_{pqik} carry new (linearly independent) information and we choose them to be C_{llll} and C_{lllm}.

Finally, we are left with 25 linearly independent quantities. Let us now express all non-zero experimental quantities in terms of the scattering amplitudes figuring in (2.1). The results are summarized in table III, where 122 non-zero c.m.s. experimental quantities are given explicitly. The remaining six components of C_{pqik} are given in formula (3.9).

Bilinear combinations of the amplitudes *a*, *b*, *c*, *d* and *e* expressed in terms of c.m.s. experimental quantities are given in table IV.

The only other independent experimental quantities are contained in the total cross-section. Indeed the total cross-section can be written as [28, 29]

$$\sigma_{\text{tot}} = \sigma_{0\text{tot}} + \sigma_{1\text{tot}}(\mathbf{P}_B\,\mathbf{P}_T) + \sigma_{2\text{tot}}(\mathbf{P}_B\,\mathbf{k})\,(\mathbf{P}_T\,\mathbf{k}) \quad (3.10)$$

where $\mathbf{P}_B$ and $\mathbf{P}_T$ are the beam and target polarizations and $\mathbf{k}$ is a unit vector in the direction of the beam. The terms $\sigma_{0\text{tot}}$, $\sigma_{1\text{tot}}$ and $\sigma_{2\text{tot}}$ can be obtained by measuring the total cross-section for appropriately polarized initial nucleons. They are related to the amplitudes via the optical theorem

$$\begin{aligned} \sigma_{0\text{tot}} &= \frac{2\pi}{k} \operatorname{Im}[a(0) + b(0)] \\ \sigma_{1\text{tot}} &= \frac{2\pi}{k} \operatorname{Im}[c(0) + d(0)] \\ \sigma_{2\text{tot}} &= -\frac{4\pi}{k} \operatorname{Im} d(0) \end{aligned} \quad (3.11)$$

where k is the wave number. The notation $\sigma_{1\text{tot}}$ and $\sigma_{2\text{tot}}$ should not be taken literally. Indeed these *cross-sections* can be positive, zero or negative and only σ_{tot} and $\sigma_{0\text{tot}}$ are positive definite.

TABLE III

Centre-of-mass experimental quantities in terms of scattering amplitudes

$$\sigma \equiv I_{0000} = \sigma C_{nnnn} = \frac{1}{2}\{ |a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 \}$$

$$\sigma C_{nn00} = \sigma A_{00nn} = \frac{1}{2}\{ |a|^2 - |b|^2 - |c|^2 + |d|^2 + |e|^2 \}$$

$$\sigma D_{n0n0} = \sigma D_{0n0n} = \frac{1}{2}\{ |a|^2 + |b|^2 - |c|^2 - |d|^2 + |e|^2 \}$$

$$\sigma K_{0nn0} = \sigma K_{n00n} = \frac{1}{2}\{ |a|^2 - |b|^2 + |c|^2 - |d|^2 + |e|^2 \}$$

$$\sigma C_{llll} = \sigma C_{mmmm} = \frac{1}{2}\{ |a|^2 + |b|^2 + |c|^2 + |d|^2 - |e|^2 \}$$

$$\sigma P \equiv \sigma P_{n000} = \sigma P_{0n00} = \sigma A_{00n0} = \sigma A_{000n} = \sigma C_{nnn0} = \sigma C_{nn0n} = \sigma M_{n0nn} = \sigma N_{0nnn} = \mathrm{Re}\, a^* e$$

$$\sigma C_{lllm} = \sigma C_{llml} = -\sigma C_{lmll} = -\sigma C_{mlll} = \sigma C_{lmmm} = \sigma C_{mlmm} = -\sigma C_{mmlm} = -\sigma C_{mmml} = \mathrm{Im}\, a^* e$$

$$\sigma C_{lnl0} = \sigma C_{mnm0} = \sigma C_{nl0l} = \sigma C_{nm0m} = \sigma M_{l0ln} = \sigma M_{m0mn} = \sigma N_{0lnl} = \sigma N_{0mnm} = \mathrm{Re}\, b^* e$$

$$\sigma D_{l0m0} = \sigma D_{0l0m} = -\sigma D_{m0l0} = -\sigma D_{0m0l} = \sigma C_{nlnm} = \sigma C_{lnmn} = -\sigma C_{mnln} = -\sigma C_{nmnl} = \mathrm{Im}\, b^* e$$

$$\sigma C_{nll0} = \sigma C_{nmm0} = \sigma C_{ln0l} = \sigma C_{mn0m} = \sigma M_{l0nl} = \sigma M_{m0nm} = \sigma N_{0lln} = \sigma N_{0mmn} = \mathrm{Re}\, c^* e$$

$$\sigma K_{0lm0} = \sigma K_{l00m} = -\sigma K_{m00l} = -\sigma K_{0ml0} = \sigma C_{nlmn} = \sigma C_{lnnm} = -\sigma C_{mnnl} = -\sigma C_{nmln} = \mathrm{Im}\, c^* e$$

$$\sigma C_{lln0} = -\sigma C_{mmn0} = \sigma C_{ll0n} = -\sigma C_{mm0n} = \sigma M_{n0ll} = -\sigma M_{n0mm} = \sigma N_{0nll} = -\sigma N_{0nmm} = -\mathrm{Re}\, d^* e$$

$$\sigma C_{lm00} = \sigma C_{ml00} = -\sigma A_{00lm} = -\sigma A_{00ml} = -\sigma C_{nnml} = -\sigma C_{nnlm} = \sigma C_{mlnn} = \sigma C_{lmnn} = \mathrm{Im}\, d^* e$$

$$\sigma D_{m0m0} = \sigma D_{0m0m} = \sigma C_{nlnl} = \sigma C_{lnln} = \mathrm{Re}\,(a^* b + c^* d)$$

$$\sigma C_{mnl0} = \sigma C_{nm0l} = -\sigma M_{l0mn} = -\sigma N_{0lnm} = \mathrm{Im}\,(a^* b + c^* d)$$

$$\sigma D_{l0l0} = \sigma D_{0l0l} = \sigma C_{nmnm} = \sigma C_{mnmn} = \mathrm{Re}\,(a^* b - c^* d)$$

$$\sigma C_{lnm0} = \sigma C_{nl0m} = -\sigma M_{m0ln} = -\sigma N_{0mnl} = -\mathrm{Im}\,(a^* b - c^* d)$$

$$\sigma K_{0mm0} = \sigma K_{m00m} = \sigma C_{nlln} = \sigma C_{lnnl} = \mathrm{Re}\,(a^* c + b^* d)$$

$$\sigma C_{nml0} = \sigma C_{mn0l} = -\sigma M_{l0nm} = -\sigma N_{0lmn} = \mathrm{Im}\,(a^* c + b^* d)$$

$$\sigma K_{0ll0} = \sigma K_{l00l} = \sigma C_{mnnm} = \sigma C_{nmmn} = \mathrm{Re}\,(a^* c - b^* d)$$

$$\sigma C_{nlm0} = \sigma C_{ln0m} = -\sigma M_{m0nl} = -\sigma N_{0mln} = -\mathrm{Im}\,(a^* c - b^* d)$$

$$\sigma C_{mm00} = \sigma A_{00mm} = -\sigma C_{nnll} = -\sigma C_{llnn} = \mathrm{Re}\,(a^* d + b^* c)$$

$$\sigma C_{lmn0} = \sigma C_{ml0n} = -\sigma M_{n0lm} = -\sigma N_{0nml} = -\mathrm{Im}\,(a^* d + b^* c)$$

$$\sigma C_{ll00} = \sigma A_{00ll} = -\sigma C_{mmnn} = -\sigma C_{nnmm} = -\mathrm{Re}\,(a^* d - b^* c)$$

$$\sigma C_{mln0} = \sigma C_{lm0n} = -\sigma M_{n0ml} = -\sigma N_{0nlm} = -\mathrm{Im}\,(a^* d - b^* c)$$

TABLE IV

Bilinear combinations of a, b, c, d, e in terms of the c.m.s. experimental quantities

$$
\begin{aligned}
|a|^2 &= \sigma/2\,\{ -1 + D_{n0n0} + K_{0nn0} + C_{nn00} + 2\,C_{llll} \} \\
|b|^2 &= \sigma/2\,\{ 1 + D_{n0n0} - K_{0nn0} - C_{nn00} \} \\
|c|^2 &= \sigma/2\,\{ 1 - D_{n0n0} + K_{0nn0} - C_{nn00} \} \\
|d|^2 &= \sigma/2\,\{ 1 - D_{n0n0} - K_{0nn0} + C_{nn00} \} \\
|e|^2 &= \sigma\,\{ 1 - C_{llll} \} \\
a^*\,b &= \sigma/2\,\{ D_{m0m0} + D_{l0l0} + i(C_{mnl0} - C_{lnm0}) \} \\
a^*\,c &= \sigma/2\,\{ K_{0mm0} + K_{0ll0} + i(C_{nml0} - C_{nlm0}) \} \\
a^*\,d &= \sigma/2\,\{ C_{mm00} - C_{ll00} - i(C_{lmn0} + C_{mln0}) \} \\
a^*\,e &= \sigma\,\{ P + iC_{lllm} \} \\
b^*\,c &= \sigma/2\,\{ C_{mm00} + C_{ll00} + i(C_{mln0} - C_{lmn0}) \} \\
b^*\,d &= \sigma/2\,\{ K_{0mm0} - K_{0ll0} + i(C_{nlm0} + C_{nml0}) \} \\
b^*\,e &= \sigma\,\{ C_{lnl0} + iD_{l0m0} \} \\
c^*\,d &= \sigma/2\,\{ D_{m0m0} - D_{l0l0} + i(C_{lnm0} + C_{mnl0}) \} \\
c^*\,e &= \sigma\,\{ C_{nll0} + iK_{0lm0} \} \\
d^*\,e &= \sigma\,\{ - C_{lln0} + iC_{lm00} \}
\end{aligned}
$$

In terms of helicity amplitudes we have

$$
\begin{aligned}
\sigma_{0\mathrm{tot}} &= \frac{2\pi}{k}\,\mathrm{Im}\,[M_1(0) + M_3(0)] \\
\sigma_{1\mathrm{tot}} &= \frac{2\pi}{k}\,\mathrm{Im}\,M_2(0) \\
\sigma_{2\mathrm{tot}} &= -\frac{2\pi}{k}\,\mathrm{Im}\,[M_1(0) + M_2(0) - M_3(0)]\,.
\end{aligned}
\tag{3.12}
$$

The quantities $\sigma_{0\mathrm{tot}}$, $\sigma_{1\mathrm{tot}}$ and $\sigma_{2\mathrm{tot}}$ are directly related to the singlet and triplet total cross-sections by

$$
\begin{aligned}
\sigma_{0\mathrm{tot}} &= \frac{1}{4}\,\sigma_s + \frac{1}{4}\,\sigma_{t,0} + \frac{1}{2}\,\sigma_{t,+1} \\
\sigma_{1\mathrm{tot}} &= \frac{1}{4}\,(\sigma_{t,0} - \sigma_s) \\
\sigma_{2\mathrm{tot}} &= \frac{1}{2}\,(\sigma_{t,+1} - \sigma_{t,0})
\end{aligned}
\tag{3.13}
$$

(with $\sigma_{t,-1} = \sigma_{t,+1}$).

In table III we have expressed 25 linearly independent experimental quantities in terms of 9 real parameters : the absolute values and relative phases of the amplitudes a, b, c, d and e. Obviously 16 independent nonlinear relations between the experimental quantities can be found, making use of trivial relations between complex numbers. If for a moment we put $x_1 = a$, $x_2 = b$, $x_3 = c$, $x_4 = d$ and $x_5 = e$, we can write down evident identities between the amplitudes

$$
(x_i\,x_j^*) = \frac{|x_i|^2 (x_1\,x_j^*)}{(x_1\,x_i^*)}
\tag{3.14}
$$

for $i = 2, 3, 4, 5$, $j = 1, 2, 3, 4, 5$ and $j \neq i$.

Regarded as equalities between the quadratic terms of the type $(x_i\,x_j^*)$, (3.14) represents an example of a full set of independent relations. Indeed, from (3.14) all other relations follow, such as

$$
(x_i\,x_j^*)\,(x_j\,x_k^*) = |x_j|^2 (x_i\,x_k^*)
\tag{3.15}
$$

or [30]

$$(x_l x_i^*) [(x_j x_k^*) - (x_k x_j^*)] + (x_l x_j^*) [(x_k x_i^*) - (x_i x_k^*)] + (x_l x_k^*) [(x_i x_j^*) - (x_j x_i^*)] = 0 \qquad (3.16)$$

and [31]

$$(x_i x_j^*) (x_j x_k^*) (x_k x_i^*) = | x_i |^2 | x_j |^2 | x_k |^2 \qquad (3.17)$$

for all $i, j, k, l = 1, 2, 3, 4, 5$.

The relations between observables can be written directly substituting into (3.15), (3.16) or (3.17) from table IV. In a similar way using table V we can obtain relations between laboratory quantities or even between the c.m.s. and laboratory experiments. Many such relations have been discussed in the literature [30, 31, 32] and we shall not dwell upon them here.

In a similar way we can find inequalities involving the experimental quantities. E.g. from

$$0 \leqslant | a \pm e |^2$$

we get

$$4 | P | \leqslant 1 + D_{n0n0} + K_{0nn0} + C_{nn00}$$

and

$$0 \leqslant | c \pm d |^2$$

implies

$$| D_{m0m0} - D_{l0l0} | \leqslant 1 - D_{n0n0} .$$

Further relations can be obtained analogously.

4. **Experimental quantities in the laboratory system.** — In any experiment only an angular distribution is measurable. In this section we will discuss the formula describing the angular distribution of correlated scattering for the case when both initial nucleons are polarized. This is the most general formula for elastic nucleon-nucleon scattering. It contains all possible experimental quantities and can be specialized to each case of interest by setting various initial or final polarizations equal to zero or choosing them in certain directions.

We introduce the symbols $\tilde{I}_i$ and $\tilde{P}_i$

$$\tilde{I}_i = \frac{1}{2} \mathrm{Tr}\, M_i M_i^+ , \qquad \tilde{I}_i \tilde{P}_i = \frac{1}{2} \mathrm{Tr}\, M_i(\boldsymbol{\sigma}_i, \mathbf{n}_i) M_i^+ \qquad (4.1)$$

$$(i = 1, 2)$$

for the cross-section and polarization in the scattering, on analyzer 1 for the scattered and 2 for the recoil particle with scattering matrices M_1 and M_2 and unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ in the directions of the normals to the first and second analyzing planes, respectively (we assume that the analyzing scatterings are performed on spinless nuclei). If the ith analyzer is absent, then we put $\tilde{P}_i = 0$ and M_i equal to the identity matrix which implies $\tilde{I}_i = 1$.

The general cross-section of correlated scattering is defined as

$$\Sigma_{P_B P_T}(\tilde{P}_1, \tilde{P}_2) = \mathrm{Tr}\, M_1 M_2 \rho M_2^+ M_1^+ , \qquad (4.2)$$

where the letters P_B and P_T indicate the initial beam and target polarizations and ρ is the nucleon-nucleon density matrix after the first, (i.e. studied) scattering. The dimension of (4.1) is the first second or third power of the cross-section if both $\tilde{P}_1$ and $\tilde{P}_2$ are zero, only one of them has non-zero value or both are different from zero, respectively.

Expanding the density matrix ρ in terms of the basic tensors, we obtain the general formula [32]

$$\begin{aligned} \Sigma_{P_B P_T}(\tilde{P}_1, \tilde{P}_2) = \tilde{I}_1 \tilde{I}_2 \sigma \{ & [1 + A_{00i0} P_{Bi} + A_{000k} P_{Tk} + A_{00ik} P_{Bi} P_{Tk}] + \\ & + \tilde{P}_1 [P_{p000} + P_{Bi} D_{p0i0} + P_{Tk} K_{p00k} + P_{Bi} P_{Tk} M_{p0ik}] n_{1p} \\ & + \tilde{P}_2 [P_{0q00} + P_{Bi} K_{0qi0} + P_{Tk} D_{0q0k} + P_{Bi} P_{Tk} N_{0qik}] n_{2q} \\ & + \tilde{P}_1 \tilde{P}_2 [C_{pq00} + P_{Bi} C_{pqi0} + P_{Tk} C_{pq0k} + P_{Bi} P_{Tk} C_{pqik}] n_{1p} n_{2q} \} . \end{aligned} \qquad (4.3)$$

Summation is understood throughout over repeated labels p, q, i and k. In practice the formula is useful in a treatment of scattering events by means of the maximum likelihood method. The angular distribution usually measured in an experiment is described by the ratio

$$\Sigma_{P_B P_T}(0, 0) / \Sigma_{00}(0, 0)$$

if $\tilde{P}_1 = \tilde{P}_2 = 0$ (i.e. for the experiments B.1, C.1 and D.1 of table II) and by

$$\Sigma_{P_B P_T}(\tilde{P}_1, \tilde{P}_2)/\tilde{I}_1 \tilde{I}_2 \Sigma_{P_B P_T}(0, 0)$$

in other cases. The differential cross-section (A.1 of table II) is an exception since in this experiment an absolute measurement is necessary.

Formula (4.3) is valid in any frame of reference, but we shall mainly use it in the laboratory one (i.e. with the stationary target), where the labels p, q, i and k will be replaced by a, b, c and d.

When discussing experiments in the laboratory system (l.s.) we shall use

$$\mathbf{k}, \mathbf{k}' \quad \text{and} \quad \mathbf{k}'' \tag{4.4}$$

i.e. unit vectors in the direction of the initial, scattered and recoil particle momenta in the l.s. ($\mathbf{k} = \mathbf{k}_i$). Further we use the transverse vectors

$$\mathbf{s} = \mathbf{n} \times \mathbf{k}, \qquad \mathbf{s}' = \mathbf{n} \times \mathbf{k}', \qquad \mathbf{s}'' = \mathbf{n} \times \mathbf{k}'', \tag{4.5}$$

where $\mathbf{n}$ is defined in (2.2).

In pure laboratory system experiments initial polarizations are specified to be along the directions $\mathbf{k}$, $\mathbf{s}$ or $\mathbf{n}$, the polarization of scattered particles is measured in the directions $\mathbf{k}'$, $\mathbf{s}'$ or $\mathbf{n}$ and that of the recoil particles in the directions $\mathbf{k}''$, $\mathbf{s}''$ or $\mathbf{n}$.

Note that in the presence of a magnetic field the spins of the scattered or recoil particles can be rotated before reaching the analysers (see comment at the end of this section).

Let us now consider individual cases of interest, making use of properties of experimental quantities, which will be established in section 6.

A. Unpolarized beam, unpolarized target : $P_B = P_T = 0$.

A.1. Final polarizations not analysed : $\tilde{P}_1 = \tilde{P}_2 = 0$, $\tilde{I}_1 = \tilde{I}_2 = 1$.

$$\Sigma_{00}(0, 0) = \sigma . \tag{4.6}$$

A.2. Polarization of scattered particles analysed : $\tilde{P}_2 = 0$, $\tilde{I}_2 = 1$.

$$\Sigma_{00}(\tilde{P}_1, 0) = \tilde{I}_1\, \sigma(1 + \tilde{P}_1\, P n_{1n}) . \tag{4.7}$$

A.3. Polarization of recoil particles analysed : $\tilde{P}_1 = 0$, $\tilde{I}_1 = 1$.

$$\Sigma_{00}(0, \tilde{P}_2) = \tilde{I}_2\, \sigma(1 + \tilde{P}_2\, P n_{2n}) . \tag{4.8}$$

A.4. Both final polarizations analysed.

$$\begin{aligned}\Sigma_{00}(\tilde{P}_1, \tilde{P}_2) = \tilde{I}_1 \tilde{I}_2\, \sigma(1 &+ [\tilde{P}_1\, n_{1n} + \tilde{P}_2\, n_{2n}]\, P + \\ &+ \tilde{P}_1 \tilde{P}_2 [C_{nn00}\, n_{1n}\, n_{2n} + C_{s's''00}\, n_{1s'}\, n_{2s''} + C_{s'k''00}\, n_{1s'}\, n_{2k''} + C_{k's''00}\, n_{1k'}\, n_{2s''} + C_{k'k''00}\, n_{1k'}\, n_{2k''}])\end{aligned} \tag{4.9}$$

B. Polarized beam, unpolarized target : $P_B \neq 0$, $P_T = 0$.

B.1. Final polarizations not analysed : $\tilde{P}_1 = \tilde{P}_2 = 0$, $\tilde{I}_1 = \tilde{I}_2 = 1$.

$$\Sigma_{P_B 0}(0, 0) = \sigma(1 + P P_{Bn}) . \tag{4.10}$$

B.2. Polarization of scattered particles analysed : $\tilde{P}_2 = 0$, $\tilde{I}_2 = 1$.

$$\begin{aligned}\Sigma_{P_B 0}(\tilde{P}_1, 0) = \tilde{I}_1\, \sigma \{ 1 + P P_{Bn} &+ \tilde{P}_1 [P n_{1n} + D_{n0n0}\, P_{Bn}\, n_{1n} + \\ &+ (D_{s'0s0}\, n_{1s'} + D_{k'0s0}\, n_{1k'})\, P_{Bs} + (D_{s'0k0}\, n_{1s'} + D_{k'0k0}\, n_{1k'})\, P_{Bk}] \} .\end{aligned} \tag{4.11}$$

B.3. Polarization of recoil particles analysed : $\tilde{P}_1 = 0$, $\tilde{I}_1 = 1$.

$$\begin{aligned}\Sigma_{P_B 0}(0, \tilde{P}_2) = \tilde{I}_2\, \sigma \{ 1 + P P_{Bn} &+ \tilde{P}_2 [P n_{2n} + K_{0nn0}\, P_{Bn}\, n_{2n} + \\ &+ (K_{0s''s0}\, n_{2s''} + K_{0k''s0}\, n_{2k''})\, P_{Bs} + (K_{0s''k0}\, n_{2s''} + K_{0k''k0}\, n_{2k''})\, P_{Bk}) \}\end{aligned} \tag{4.12}$$

B.4. Both final polarizations analysed.

The expression for $\Sigma_{P_B 0}(\tilde{P}_1, \tilde{P}_2)$ can be obtained from (4.3) by putting $P_T = 0$. Thus only P, D_{a0c0}, K_{0bc0}, C_{ab00} and C_{abc0} appear in the formula.

C. Unpolarized beam, polarized target : $P_B = 0, P_T \neq 0$.

C.1. Final polarizations not analysed : $\tilde{P}_1 = \tilde{P}_2 = 0, \tilde{I}_1 = \tilde{I}_2 = 1$.

$$\Sigma_{0P_T}(0, 0) = \sigma(1 + PP_{Tn}) . \tag{4.13}$$

C.2. Polarization of scattered particles analysed : $\tilde{P}_2 = 0, \tilde{I}_2 = 1$.

$$\Sigma_{0P_T}(\tilde{P}_1, 0) = \tilde{I}_1\, \sigma \{ 1 + PP_{Tn} + \tilde{P}_1[Pn_{1n} + K_{n00n}\, P_{Tn}\, n_{1n} + \\ + (K_{s'00s}\, n_{1s'} + K_{k'00s}\, n_{1k'})\, P_{Ts} + (K_{s'00k}\, n_{1s'} + K_{k'00k}\, n_{1k'})\, P_{Tk}) \} . \tag{4.14}$$

C.3. Polarization of recoil particles analysed : $\tilde{P}_1 = 0, \tilde{I}_1 = 1$.

$$\Sigma_{0P_T}(0, \tilde{P}_2) = \tilde{I}_2\, \sigma \{ 1 + PP_{Tn} + \tilde{P}_2[Pn_{2n} + D_{0n0n}\, P_{Tn}\, n_{2n} + \\ + (D_{0s''0s}\, n_{2s''} + D_{0k''0s}\, n_{2k''})\, P_{Ts} + (D_{0s''0k}\, n_{2s''} + D_{0k''0k}\, n_{2k''})\, P_{Tk}] \} . \tag{4.15}$$

C.4. Both final polarizations analysed.

The expression for $\Sigma_{0P_T}(\tilde{P}_1, \tilde{P}_2)$ is obtained from (4.3) by putting $P_B = 0$. It involves only P, K_{a00d}, D_{0b0d}, C_{ab00} and C_{ab0d}.

D. Polarized beam and target : $P_B \neq 0, P_T \neq 0$.

D.1. Final polarizations not analysed : $\tilde{P}_1 = \tilde{P}_2 = 0, \tilde{I}_1 = \tilde{I}_2 = 1$.

$$\Sigma_{P_BP_T}(0, 0) = \sigma[1 + P(P_{Bn} + P_{Tn}) + A_{00nn}\, P_{Bn}\, P_{Tn} + \\ + A_{00ss}\, P_{Bs}\, P_{Ts} + A_{00sk}(P_{Bs}\, P_{Tk} + P_{Bk}\, P_{Ts}) + A_{00kk}\, P_{Bk}\, P_{Tk}] . \tag{4.16}$$

D.2. Polarization of scattered particles analysed : $\tilde{P}_2 = 0, \tilde{I}_2 = 0$.
$\Sigma_{P_BP_T}(\tilde{P}_1, 0)$ is expressed in terms of P, A_{00cd}, D_{a0c0}, K_{a00d} and M_{a0cd}.
D.3. Polarization of recoil particles analysed : $\tilde{P}_1 = 0, \tilde{I}_1 = 1$.
$\Sigma_{P_BP_T}(0, \tilde{P}_2)$ is expressed in terms of P, A_{00cd}, D_{0b0d}, K_{0bc0} and N_{0bcd}.
D.4. Both final polarizations analysed.

In this case all terms in (4.3) survive and the general formula, while straightforward, is quite cumbersome. We shall not spell it out in its generality, but only consider some special cases of interest (specifying the directions of the polarizations). We concentrate on experiments yielding interesting components of the four-component tensor.

(i) $\mathbf{P}_B = P_B\, \mathbf{s}, \mathbf{P}_T = P_T\, \mathbf{s}, \tilde{\mathbf{P}}_1 = \tilde{P}_1\, \mathbf{n}_1 = \tilde{P}_{1s'}\, \mathbf{s}' + \tilde{P}_{1k'}\, \mathbf{k}', \tilde{\mathbf{P}}_2 = \tilde{P}_2\, \mathbf{n}_2 = \tilde{P}_{2s''}\, \mathbf{s}'' + \tilde{P}_{2k''}\, \mathbf{k}''$ [only the polarization components in the first scattering plane are analysed, i.e. $\mathbf{n}_1(\mathbf{n}_2)$ is a combination of $\mathbf{s}'$ and $\mathbf{k}'$ ($\mathbf{s}''$ and $\mathbf{k}''$)].

$$\begin{aligned} \Sigma_{P_BP_T}(\tilde{P}_1, \tilde{P}_2) = \tilde{I}_1\, \tilde{I}_2\, \sigma \{ 1 &+ A_{00ss}\, P_B\, P_T + \\ &+ \tilde{P}_1[P_B(D_{s'0s0}\, n_{1s'} + D_{k'0s0}\, n_{1k'}) + P_T(K_{s'00s}\, n_{1s'} + K_{k'00s}\, n_{1k'})] \\ &+ \tilde{P}_2[P_B(K_{0s''s0}\, n_{2s''} + K_{0k''s0}\, n_{2k''}) + P_T(D_{0s''0s}\, n_{2s''} + D_{0k''0s}\, n_{2k''})] \\ &+ \tilde{P}_1\, \tilde{P}_2[C_{s's''00}\, n_{1s'}\, n_{2s''} + C_{s'k''00}\, n_{1s'}\, n_{2k''} + C_{k's''00}\, n_{1k'}\, n_{2s''} + C_{k'k''00}\, n_{1k'}\, n_{2k''} \\ &+ P_B\, P_T(C_{s's''ss}\, n_{1s'}\, n_{2s''} + C_{s'k''ss}\, n_{1s'}\, n_{2k''} + C_{k's''ss}\, n_{1k'}\, n_{2s''} + C_{k'k''ss}\, n_{1k'}\, n_{2k''})] \} . \end{aligned} \tag{4.17}$$

(ii) $\mathbf{P}_B = P_B\, \mathbf{s}, \mathbf{P}_T = P_T\, \mathbf{k}, \tilde{\mathbf{P}}_1 = \tilde{P}_1\, \mathbf{n}_1 = \tilde{P}_{1s'}\, \mathbf{s}' + \tilde{P}_{1k'}\, \mathbf{k}', \tilde{\mathbf{P}}_2 = \tilde{P}_2\, \mathbf{n}_2 = \tilde{P}_{2s''}\, \mathbf{s}'' + \tilde{P}_{2k''}\, \mathbf{k}''$ ($\mathbf{n}_1$ and $\mathbf{n}_2$ are again in the first scattering plane).

$$\begin{aligned} \Sigma_{P_BP_T}(\tilde{P}_1, \tilde{P}_2) = \tilde{I}_1\, \tilde{I}_2\, \sigma \{ 1 &+ A_{00sk}\, P_B\, P_T + \\ &+ \tilde{P}_1[P_B(D_{s'0s0}\, n_{1s'} + D_{k'0s0}\, n_{1k'}) + P_T(K_{s'00k}\, n_{1s'} + K_{k'00k}\, n_{1k'})] \\ &+ \tilde{P}_2[P_B(K_{0s''s0}\, n_{2s''} + K_{0k''s0}\, n_{2k''}) + P_T(D_{0s''0k}\, n_{2s''} + D_{0k''0k}\, n_{2k''})] \\ &+ \tilde{P}_1\, \tilde{P}_2[C_{s's''00}\, n_{1s'}\, n_{2s''} + C_{s'k''00}\, n_{1s'}\, n_{2k''} + C_{k's''00}\, n_{1k'}\, n_{2s''} + C_{k'k''00}\, n_{1k'}\, n_{2k''} \\ &+ P_B\, P_T(C_{s's''sk}\, n_{1s'}\, n_{2s''} + C_{s'k''sk}\, n_{1s'}\, n_{2k''} + C_{k's''sk}\, n_{1k'}\, n_{2s''} + C_{k'k''sk}\, n_{1k'}\, n_{2k''})] \} . \end{aligned} \tag{4.18}$$

In all the above formulas it is sometimes useful to express the vector components in terms of the azimuthal angles Φ_1 and Φ_2 between the normal to the scattering plane and the two normals to the analysing planes. These satisfy :

$$\begin{aligned} \cos \Phi_1 &= (\mathbf{n}, \mathbf{n}_1), \qquad \sin \Phi_1 = (\mathbf{n}, \mathbf{n}_1 \times \mathbf{k}') = -(\mathbf{n}_1, \mathbf{s}') \\ \cos \Phi_2 &= (\mathbf{n}, \mathbf{n}_2), \qquad \sin \Phi_2 = (\mathbf{n}, \mathbf{n}_2 \times \mathbf{k}'') = -(\mathbf{n}_2, \mathbf{s}''). \end{aligned} \tag{4.19}$$

An important fact concerning all the above formulas must be kept in mind. In the absence of a magnetic field the scalar products $n_{1k'}$ and $n_{2k''}$ are zero, since the vectors $\mathbf{k}'$ and $\mathbf{k}''$ lie in the first and second analysing planes respectively. Thus, all components of polarization tensors involving $\mathbf{k}'$ or $\mathbf{k}''$ subscripts actually vanish from the measured distributions. In order to observe these components it is necessary to make use of magnetic fields in front of the analysers, rotating the polarizations. In particular, a magnetic field between the target and the analyser 1 (2) along the direction $\mathbf{s}'(\mathbf{s}'')$ will rotate the polarization of the scattered (recoil) particle in the $\mathbf{k}', \mathbf{n}$ ($\mathbf{k}'', \mathbf{n}$) plane. The scalar products n_{1n} and $n_{1k'}$ (n_{2n} and $n_{2k''}$) are then to be understood as cosines of the angles between the normals $\mathbf{n}_1$ ($\mathbf{n}_2$) and the direction to which the n and k' (n and k'') components of the scattered (recoil) particle polarization have been rotated by the magnetic field (after the scattering under consideration).

Laboratory experiments are expressed in table V in terms of amplitudes a, b, c, d, e and in table VI in terms of helicity amplitudes.

5. Relations between laboratory and c.m.s. quantities.

— Relativistic formulas for the differential cross-sections depend on the choice of the kinematic variables, are well known and will not be discussed here. We will now transform other laboratory experiments into the c.m.s. and express them in terms of combinations of the pure c.m.s. experimental quantities. Generally speaking, the relations can be written as

$$X^{\text{ls}}_{abcd} = X^{\text{cms}}_{pqik}\, a_{R_1 p}\, b_{R_2 q}\, c_i\, d_k \,. \tag{5.1}$$

A summation over repeated indices is to be understood. The symbols $a_{R_1 p}$ and $b_{R_2 q}$ are components of vectors $\mathbf{a}$ and $\mathbf{b}$ rotated through the relativistic spin rotation angles about the normal to the scattering plane, thus representing spin directions of the scattered and recoil particles in the c.m.s. if the directions in the l.s. are $\mathbf{a}$ and $\mathbf{b}$.

TABLE V

Laboratory experiments in terms of amplitudes a, b, c, d, e.

B.2.

$$\sigma D_{s'0s0} = \operatorname{Re} a^* b \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Re} c^* d \cos\left(\alpha - \frac{\theta}{2}\right) - \operatorname{Im} b^* e \sin\left(\alpha + \frac{\theta}{2}\right)$$

$$\sigma D_{s'0k0} = -\operatorname{Re} a^* b \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Re} c^* d \sin\left(\alpha - \frac{\theta}{2}\right) - \operatorname{Im} b^* e \cos\left(\alpha + \frac{\theta}{2}\right)$$

$$\sigma D_{k'0s0} = \operatorname{Re} a^* b \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Re} c^* d \sin\left(\alpha - \frac{\theta}{2}\right) + \operatorname{Im} b^* e \cos\left(\alpha + \frac{\theta}{2}\right)$$

$$\sigma D_{k'0k0} = \operatorname{Re} a^* b \cos\left(\alpha + \frac{\theta}{2}\right) - \operatorname{Re} c^* d \cos\left(\alpha - \frac{\theta}{2}\right) - \operatorname{Im} b^* e \sin\left(\alpha + \frac{\theta}{2}\right)$$

B.3.

$$\sigma K_{0s''s0} = -\operatorname{Re} a^* c \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Re} b^* d \cos\left(\beta - \frac{\theta}{2}\right) + \operatorname{Im} c^* e \sin\left(\beta + \frac{\theta}{2}\right)$$

$$\sigma K_{0s''k0} = \operatorname{Re} a^* c \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Re} b^* d \sin\left(\beta - \frac{\theta}{2}\right) + \operatorname{Im} c^* e \cos\left(\beta + \frac{\theta}{2}\right)$$

$$\sigma K_{0k''s0} = -\operatorname{Re} a^* c \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Re} b^* d \sin\left(\beta - \frac{\theta}{2}\right) - \operatorname{Im} c^* e \cos\left(\beta + \frac{\theta}{2}\right)$$

$$\sigma K_{0k''k0} = -\operatorname{Re} a^* c \cos\left(\beta + \frac{\theta}{2}\right) + \operatorname{Re} b^* d \cos\left(\beta - \frac{\theta}{2}\right) + \operatorname{Im} c^* e \sin\left(\beta + \frac{\theta}{2}\right)$$

C.2.

$$\sigma K_{s'00s} = \operatorname{Re} a^* c \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Re} b^* d \cos\left(\alpha - \frac{\theta}{2}\right) - \operatorname{Im} c^* e \sin\left(\alpha + \frac{\theta}{2}\right)$$

$$\sigma K_{s'00k} = -\operatorname{Re} a^* c \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Re} b^* d \sin\left(\alpha - \frac{\theta}{2}\right) - \operatorname{Im} c^* e \cos\left(\alpha + \frac{\theta}{2}\right)$$

$$\sigma K_{k'00s} = \operatorname{Re} a^* c \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Re} b^* d \sin\left(\alpha - \frac{\theta}{2}\right) + \operatorname{Im} c^* e \cos\left(\alpha + \frac{\theta}{2}\right)$$

$$\sigma K_{k'00k} = \operatorname{Re} a^* c \cos\left(\alpha + \frac{\theta}{2}\right) - \operatorname{Re} b^* d \cos\left(\alpha - \frac{\theta}{2}\right) - \operatorname{Im} c^* e \sin\left(\alpha + \frac{\theta}{2}\right)$$

C.3.

$$\sigma D_{0s''0s} = -\operatorname{Re} a^* b \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Re} c^* d \cos\left(\beta - \frac{\theta}{2}\right) + \operatorname{Im} b^* e \sin\left(\beta + \frac{\theta}{2}\right)$$

$$\sigma D_{0s''0k} = \operatorname{Re} a^* b \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Re} c^* d \sin\left(\beta - \frac{\theta}{2}\right) + \operatorname{Im} b^* e \cos\left(\beta + \frac{\theta}{2}\right)$$

$$\sigma D_{0k''0s} = -\operatorname{Re} a^* b \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Re} c^* d \sin\left(\beta - \frac{\theta}{2}\right) - \operatorname{Im} b^* e \cos\left(\beta + \frac{\theta}{2}\right)$$

$$\sigma D_{0k''0k} = -\operatorname{Re} a^* b \cos\left(\beta + \frac{\theta}{2}\right) + \operatorname{Re} c^* d \cos\left(\beta - \frac{\theta}{2}\right) + \operatorname{Im} b^* e \sin\left(\beta + \frac{\theta}{2}\right)$$

A.4.

$$\sigma C_{s's''00} = -\operatorname{Re} a^* d \cos(\alpha + \beta) - \operatorname{Re} b^* c \cos(\alpha - \beta) + \operatorname{Im} d^* e \sin(\alpha + \beta)$$
$$\sigma C_{k's''00} = -\operatorname{Re} a^* d \sin(\alpha + \beta) - \operatorname{Re} b^* c \sin(\alpha - \beta) - \operatorname{Im} d^* e \cos(\alpha + \beta)$$
$$\sigma C_{s'k''00} = -\operatorname{Re} a^* d \sin(\alpha + \beta) + \operatorname{Re} b^* c \sin(\alpha - \beta) - \operatorname{Im} d^* e \cos(\alpha + \beta)$$
$$\sigma C_{k'k''00} = \operatorname{Re} a^* d \cos(\alpha + \beta) - \operatorname{Re} b^* c \cos(\alpha - \beta) - \operatorname{Im} d^* e \sin(\alpha + \beta)$$

D.1.

$$\sigma A_{00ss} = \operatorname{Re} a^* d \cos\theta + \operatorname{Re} b^* c - \operatorname{Im} d^* e \sin\theta$$
$$\sigma A_{00sk} = \sigma A_{00ks} = -\operatorname{Re} a^* d \sin\theta - \operatorname{Im} d^* e \cos\theta$$
$$\sigma A_{00kk} = -\operatorname{Re} a^* d \cos\theta + \operatorname{Re} b^* c + \operatorname{Im} d^* e \sin\theta$$

B.4.

$$\sigma C_{s's''n0} = -\operatorname{Re} d^* e \cos(\alpha + \beta) - \operatorname{Im} a^* d \sin(\alpha + \beta) - \operatorname{Im} b^* c \sin(\alpha - \beta)$$
$$\sigma C_{k's''n0} = -\operatorname{Re} d^* e \sin(\alpha + \beta) + \operatorname{Im} a^* d \cos(\alpha + \beta) + \operatorname{Im} b^* c \cos(\alpha - \beta)$$
$$\sigma C_{s'k''n0} = -\operatorname{Re} d^* e \sin(\alpha + \beta) + \operatorname{Im} a^* d \cos(\alpha + \beta) - \operatorname{Im} b^* c \cos(\alpha - \beta)$$
$$\sigma C_{k'k''n0} = \operatorname{Re} d^* e \cos(\alpha + \beta) + \operatorname{Im} a^* d \sin(\alpha + \beta) - \operatorname{Im} b^* c \sin(\alpha - \beta)$$

$$\sigma C_{s'ns0} = \operatorname{Re} b^* e \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} a^* b \sin\left(\alpha + \frac{\theta}{2}\right) - \operatorname{Im} c^* d \sin\left(\alpha - \frac{\theta}{2}\right)$$

$$\sigma C_{k'ns0} = \operatorname{Re} b^* e \sin\left(\alpha + \frac{\theta}{2}\right) - \operatorname{Im} a^* b \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} c^* d \cos\left(\alpha - \frac{\theta}{2}\right)$$

$$\sigma C_{s'nk0} = -\operatorname{Re} b^* e \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} a^* b \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} c^* d \cos\left(\alpha - \frac{\theta}{2}\right)$$

$$\sigma C_{k'nk0} = \operatorname{Re} b^* e \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} a^* b \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} c^* d \sin\left(\alpha - \frac{\theta}{2}\right)$$

$$\sigma C_{ns''s0} = -\operatorname{Re} c^* e \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} a^* c \sin\left(\beta + \frac{\theta}{2}\right) + \operatorname{Im} b^* d \sin\left(\beta - \frac{\theta}{2}\right)$$

$$\sigma C_{nk''s0} = -\operatorname{Re} c^* e \sin\left(\beta + \frac{\theta}{2}\right) + \operatorname{Im} a^* c \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} b^* d \cos\left(\beta - \frac{\theta}{2}\right)$$

$$\sigma C_{ns''k0} = \operatorname{Re} c^* e \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} a^* c \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} b^* d \cos\left(\beta - \frac{\theta}{2}\right)$$

$$\sigma C_{nk''k0} = -\operatorname{Re} c^* e \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} a^* c \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} b^* d \sin\left(\beta - \frac{\theta}{2}\right)$$

C.4.

$$C_{k'k''0n} = -\, C_{s's''n0}\,, \qquad C_{k's''0n} = C_{s'k''n0}$$
$$C_{s's''0n} = -\, C_{k'k''n0}\,, \qquad C_{s'k''0n} = C_{k's''n0}$$

$$\sigma C_{s'n0s} = \operatorname{Re} c^* e \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} a^* c \sin\left(\alpha + \frac{\theta}{2}\right) - \operatorname{Im} b^* d \sin\left(\alpha - \frac{\theta}{2}\right)$$
$$\sigma C_{k'n0s} = \operatorname{Re} c^* e \sin\left(\alpha + \frac{\theta}{2}\right) - \operatorname{Im} a^* c \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} b^* d \cos\left(\alpha - \frac{\theta}{2}\right)$$
$$\sigma C_{s'n0k} = -\operatorname{Re} c^* e \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} a^* c \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} b^* d \cos\left(\alpha - \frac{\theta}{2}\right)$$
$$\sigma C_{k'n0k} = \operatorname{Re} c^* e \cos\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} a^* c \sin\left(\alpha + \frac{\theta}{2}\right) + \operatorname{Im} b^* d \sin\left(\alpha - \frac{\theta}{2}\right)$$
$$\sigma C_{ns''0s} = -\operatorname{Re} b^* e \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} a^* b \sin\left(\beta + \frac{\theta}{2}\right) + \operatorname{Im} c^* d \sin\left(\beta - \frac{\theta}{2}\right)$$
$$\sigma C_{nk''0s} = -\operatorname{Re} b^* e \sin\left(\beta + \frac{\theta}{2}\right) + \operatorname{Im} a^* b \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} c^* d \cos\left(\beta - \frac{\theta}{2}\right)$$
$$\sigma C_{ns''0k} = \operatorname{Re} b^* e \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} a^* b \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} c^* d \cos\left(\beta - \frac{\theta}{2}\right)$$
$$\sigma C_{nk''0k} = -\operatorname{Re} b^* e \cos\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} a^* b \sin\left(\beta + \frac{\theta}{2}\right) - \operatorname{Im} c^* d \sin\left(\beta - \frac{\theta}{2}\right)$$

D.2.

$$\sigma M_{n0ss} = -\,\sigma M_{n0kk} = \operatorname{Re} d^* e \cos\theta + \operatorname{Im} a^* d \sin\theta$$
$$\sigma M_{n0ks} = -\operatorname{Re} d^* e \sin\theta + \operatorname{Im} a^* d \cos\theta + \operatorname{Im} b^* c$$
$$\sigma M_{n0sk} = -\operatorname{Re} d^* e \sin\theta + \operatorname{Im} a^* d \cos\theta - \operatorname{Im} b^* c$$

$$M_{s'0ns} = C_{k'n0k}\,, \quad M_{k'0ns} = -\,C_{s'n0k}\,, \quad M_{s'0nk} = -\,C_{k'n0s}\,, \quad M_{k'0nk} = C_{s'n0s}$$
$$M_{s'0sn} = C_{k'nk0}\,, \quad M_{k'0sn} = -\,C_{s'nk0}\,, \quad M_{s'0kn} = -\,C_{k'ns0}\,, \quad M_{k'0kn} = C_{s'ns0}$$

D.3.

$$N_{0nss} = -\,N_{0nkk} = M_{n0ss}\,, \quad N_{0nks} = M_{n0sk}\,, \quad N_{0nsk} = M_{n0ks}$$
$$N_{0s''ns} = C_{nk''0k}\,, \quad N_{0k''ns} = -\,C_{ns''0k}\,, \quad N_{0s''nk} = -\,C_{nk''0s}\,, \quad N_{0k''nk} = C_{ns''0s}$$
$$N_{0s''sn} = C_{nk''k0}\,, \quad N_{0k''sn} = -\,C_{ns''k0}\,, \quad N_{0s''kn} = -\,C_{nk''s0}\,, \quad N_{0k''kn} = C_{ns''s0}$$

D.4.

$$\sigma(C_{s's''ss} + C_{k'k''ss}) = -\,(|b|^2 + |c|^2)\cos(\beta - \alpha)$$
$$\sigma(C_{s'k''ss} - C_{k's''ss}) = -\,(|b|^2 + |c|^2)\sin(\beta - \alpha)$$
$$\sigma(C_{s's''sk} + C_{k'k''sk}) = (|b|^2 - |c|^2)\sin(\beta - \alpha)$$
$$\sigma(C_{s'k''sk} - C_{k's''sk}) = -\,(|b|^2 - |c|^2)\cos(\beta - \alpha)$$

$$\sigma(C_{s's''ss} - C_{k'k''ss}) = -\,(|a|^2 - |e|^2)\cos(\alpha + \beta + \theta) - |d|^2\cos(\alpha + \beta - \theta) + 2\operatorname{Im} a^* e \sin(\alpha + \beta + \theta)$$
$$\sigma(C_{s'k''ss} + C_{k's''ss}) = -\,(|a|^2 - |e|^2)\sin(\alpha + \beta + \theta) - |d|^2\sin(\alpha + \beta - \theta) - 2\operatorname{Im} a^* e \cos(\alpha + \beta + \theta)$$
$$\sigma(C_{s's''sk} - C_{k'k''sk}) = (|a|^2 - |e|^2)\sin(\alpha + \beta + \theta) - {} - |d|^2\sin(\alpha + \beta - \theta) + 2\operatorname{Im} a^* e \cos(\alpha + \beta + \theta)$$
$$\sigma(C_{s'k''sk} + C_{k's''sk}) = -\,(|a|^2 - |e|^2)\cos(\alpha + \beta + \theta) + |d|^2\cos(\alpha + \beta - \theta) + 2\operatorname{Im} a^* e \sin(\alpha + \beta + \theta)\,.$$

TABLE VI

Laboratory experiments in terms of helicity amplitudes.

A.1.

$$\sigma = \frac{1}{2}(|M_1|^2 + |M_2|^2 + |M_3|^2 + |M_4|^2 + 4|M_5|^2)$$

A.2.

$$\sigma P = -\,\mathrm{Im}[M_5^*(M_1 + M_2 + M_3 - M_4)]$$

B.2.

$$\sigma D_{n0n0} = \mathrm{Re}\,(M_1^* M_3 - M_2^* M_4) + 2|M_5|^2$$

$$\sigma D_{s'0s0} = -\,\mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\sin\theta_1 + \mathrm{Re}\,(M_1^* M_3 + M_2^* M_4)\cos\theta_1$$

$$\sigma D_{s'0k0} = \mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\cos\theta_1 + \frac{1}{2}(|M_1|^2 - |M_2|^2 + |M_3|^2 - |M_4|^2)\sin\theta_1$$

$$\sigma D_{k'0s0} = -\,\mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\cos\theta_1 - \mathrm{Re}\,(M_1^* M_3 + M_2^* M_4)\sin\theta_1$$

$$\sigma D_{k'0k0} = -\,\mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\sin\theta_1 + \frac{1}{2}(|M_1|^2 - |M_2|^2 + |M_3|^2 - |M_4|^2)\cos\theta_1$$

B.3.

$$\sigma K_{0nn0} = -\,\mathrm{Re}\,(M_1^* M_4 - M_2^* M_3) + 2|M_5|^2$$

$$\sigma K_{0s''s0} = -\,\mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\sin\theta_2 - \mathrm{Re}\,(M_1^* M_4 + M_2^* M_3)\cos\theta_2$$

$$\sigma K_{0s''k0} = -\,\mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\cos\theta_2 + \frac{1}{2}(-|M_1|^2 + |M_2|^2 + |M_3|^2 - |M_4|^2)\sin\theta_2$$

$$\sigma K_{0k''s0} = \mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\cos\theta_2 - \mathrm{Re}\,(M_1^* M_4 + M_2^* M_3)\sin\theta_2$$

$$\sigma K_{0k''k0} = -\,\mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\sin\theta_2 - \frac{1}{2}(-|M_1|^2 + |M_2|^2 + |M_3|^2 - |M_4|^2)\cos\theta_2$$

C.2.

$$\sigma K_{s'00s} = -\,\mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\sin\theta_1 + \mathrm{Re}\,(M_1^* M_4 + M_2^* M_3)\cos\theta_1$$

$$\sigma K_{s'00k} = \mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\cos\theta_1 + \frac{1}{2}(-|M_1|^2 + |M_2|^2 + |M_3|^2 - |M_4|^2)\sin\theta_1$$

$$\sigma K_{k'00s} = -\,\mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\cos\theta_1 - \mathrm{Re}\,(M_1^* M_4 + M_2^* M_3)\sin\theta_1$$

$$\sigma K_{k'00k} = -\,\mathrm{Re}\,[M_5^*(-M_1 + M_2 + M_3 + M_4)]\sin\theta_1 + \frac{1}{2}(-|M_1|^2 + |M_2|^2 + |M_3|^2 - |M_4|^2)\cos\theta_1$$

C.3.

$$\sigma D_{0s''0s} = -\,\mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\sin\theta_2 - \mathrm{Re}\,(M_1^* M_3 + M_2^* M_4)\cos\theta_2$$

$$\sigma D_{0s''0k} = -\,\mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\cos\theta_2 + \frac{1}{2}(|M_1|^2 - |M_2|^2 + |M_3|^2 - |M_4|^2)\sin\theta_2$$

$$\sigma D_{0k''0s} = \mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\cos\theta_2 - \mathrm{Re}\,(M_1^* M_3 + M_2^* M_4)\sin\theta_2$$

$$\sigma D_{0k''0k} = -\,\mathrm{Re}\,[M_5^*(M_1 - M_2 + M_3 + M_4)]\sin\theta_2 - \frac{1}{2}(|M_1|^2 - |M_2|^2 + |M_3|^2 - |M_4|^2)\cos\theta_2$$

A.4.

$$\sigma C_{nn00} = \mathrm{Re}\,(M_1^* M_2 - M_3^* M_4) + 2|M_5|^2$$

$$\sigma C_{s's''00} = -\,1/2(|M_1|^2 + |M_2|^2 - |M_3|^2 - |M_4|^2)\sin\theta_1\sin\theta_2 - \mathrm{Re}\,(M_1^* M_2 + M_3^* M_4)\cos\theta_1\cos\theta_2 + \mathrm{Re}\,[M_5^*(M_1 + M_2 - M_3 + M_4)]\sin(\theta_1 - \theta_2)$$

$$\sigma C_{k's''00} = -\,1/2(|M_1|^2 + |M_2|^2 - |M_3|^2 - |M_4|^2)\cos\theta_1\sin\theta_2 + \mathrm{Re}\,(M_1^* M_2 + M_3^* M_4)\sin\theta_1\cos\theta_2 + \mathrm{Re}\,[M_5^*(M_1 + M_2 - M_3 + M_4)]\cos(\theta_1 - \theta_2)$$

$$\sigma C_{s'k''00} = 1/2(|M_1|^2 + |M_2|^2 - |M_3|^2 - |M_4|^2) \sin\theta_1 \cos\theta_2 - \mathrm{Re}\,(M_1^* M_2 + M_3^* M_4) \cos\theta_1 \sin\theta_2 + \mathrm{Re}\,[M_5^*(M_1 + M_2 - M_3 + M_4)] \cos(\theta_1 - \theta_2)$$

$$\sigma C_{k'k''00} = 1/2(|M_1|^2 + |M_2|^2 - |M_3|^2 - |M_4|^2) \cos\theta_1 \cos\theta_2 + \mathrm{Re}\,(M_1^* M_2 + M_3^* M_4) \sin\theta_1 \sin\theta_2 - \mathrm{Re}\,[M_5^*(M_1 + M_2 - M_3 + M_4)] \sin(\theta_1 - \theta_2)$$

D.1.

$$\sigma A_{00ss} = \mathrm{Re}\,(M_1^* M_2 + M_3^* M_4)$$
$$\sigma A_{00sk} = \sigma A_{00ks} = \mathrm{Re}\,[M_5^*(M_1 + M_2 - M_3 + M_4)]$$
$$\sigma A_{00kk} = -1/2(|M_1|^2 + |M_2|^2 - |M_3|^2 - |M_4|^2)$$

B.4.

$$\sigma C_{s's''n0} = \mathrm{Im}\,[M_5^*(M_1 + M_2 - M_3 + M_4) \cos(\theta_1 - \theta_2) + (M_1^* M_3 - M_2^* M_4) \cos\theta_1 \sin\theta_2 + (M_1^* M_4 - M_2^* M_3) \sin\theta_1 \cos\theta_2]$$

$$\sigma C_{k's''n0} = \mathrm{Im}\,[- M_5^*(M_1 + M_2 - M_3 + M_4) \sin(\theta_1 - \theta_2) - (M_1^* M_3 - M_2^* M_4) \sin\theta_1 \sin\theta_2 + (M_1^* M_4 - M_2^* M_3) \cos\theta_1 \cos\theta_2]$$

$$\sigma C_{s'k''n0} = \mathrm{Im}\,[- M_5^*(M_1 + M_2 - M_3 + M_4) \sin(\theta_1 - \theta_2) - (M_1^* M_3 - M_2^* M_4) \cos\theta_1 \cos\theta_2 + (M_1^* M_4 - M_2^* M_3) \sin\theta_1 \sin\theta_2]$$

$$\sigma C_{k'k''n0} = \mathrm{Im}\,[- M_5^*(M_1 + M_2 - M_3 + M_4) \cos(\theta_1 - \theta_2) + (M_1^* M_3 - M_2^* M_4) \sin\theta_1 \cos\theta_2 + (M_1^* M_4 - M_2^* M_3) \cos\theta_1 \sin\theta_2]$$

$$\sigma C_{s'ns0} = \mathrm{Im}\,[- M_5^*(M_1 - M_2 + M_3 + M_4) \cos\theta_1 - (M_1^* M_4 + M_2^* M_3) \sin\theta_1]$$
$$\sigma C_{k'ns0} = \mathrm{Im}\,[M_5^*(M_1 - M_2 + M_3 + M_4) \sin\theta_1 - (M_1^* M_4 + M_2^* M_3) \cos\theta_1]$$
$$\sigma C_{s'nk0} = \mathrm{Im}\,[- M_5^*(M_1 - M_2 + M_3 + M_4) \sin\theta_1 - (M_1^* M_2 - M_3^* M_4) \cos\theta_1]$$
$$\sigma C_{k'nk0} = \mathrm{Im}\,[- M_5^*(M_1 - M_2 + M_3 + M_4) \cos\theta_1 + (M_1^* M_2 - M_3^* M_4) \sin\theta_1]$$
$$\sigma C_{ns''s0} = \mathrm{Im}\,[M_5^*(- M_1 + M_2 + M_3 + M_4) \cos\theta_2 - (M_1^* M_3 + M_2^* M_4) \sin\theta_2]$$
$$\sigma C_{nk''s0} = \mathrm{Im}\,[M_5^*(- M_1 + M_2 + M_3 + M_4) \sin\theta_2 + (M_1^* M_3 + M_2^* M_4) \cos\theta_2]$$
$$\sigma C_{ns''k0} = \mathrm{Im}\,[- M_5^*(- M_1 + M_2 + M_3 + M_4) \sin\theta_2 - (M_1^* M_2 + M_3^* M_4) \cos\theta_2]$$
$$\sigma C_{nk''k0} = \mathrm{Im}\,[M_5^*(- M_1 + M_2 + M_3 + M_4) \cos\theta_2 - (M_1^* M_2 + M_3^* M_4) \sin\theta_2]$$

C.4.

$$\sigma C_{s'n0s} = \mathrm{Im}\,[- M_5^*(- M_1 + M_2 + M_3 + M_4) \cos\theta_1 - (M_1^* M_3 + M_2^* M_4) \sin\theta_1]$$
$$\sigma C_{k'n0s} = \mathrm{Im}\,[M_5^*(- M_1 + M_2 + M_3 + M_4) \sin\theta_1 - (M_1^* M_3 + M_2^* M_4) \cos\theta_1]$$
$$\sigma C_{s'n0k} = \mathrm{Im}\,[- M_5^*(- M_1 + M_2 + M_3 + M_4) \sin\theta_1 + (M_1^* M_2 + M_3^* M_4) \cos\theta_1]$$
$$\sigma C_{k'n0k} = \mathrm{Im}\,[- M_5^*(- M_1 + M_2 + M_3 + M_4) \cos\theta_1 - (M_1^* M_2 + M_3^* M_4) \sin\theta_1]$$

$$\sigma C_{ns''0s} = \mathrm{Im}\,[M_5^*(M_1 - M_2 + M_3 + M_4) \cos\theta_2 - (M_1^* M_4 + M_2^* M_3) \sin\theta_2]$$
$$\sigma C_{nk''0s} = \mathrm{Im}\,[M_5^*(M_1 - M_2 + M_3 + M_4) \sin\theta_2 + (M_1^* M_4 + M_2^* M_3) \cos\theta_2]$$
$$\sigma C_{ns''0k} = \mathrm{Im}\,[- M_5^*(M_1 - M_2 + M_3 + M_4) \sin\theta_2 + (M_1^* M_2 - M_3^* M_4) \cos\theta_2]$$
$$\sigma C_{nk''0k} = \mathrm{Im}\,[M_5^*(M_1 - M_2 + M_3 + M_4) \cos\theta_2 + (M_1^* M_2 - M_3^* M_4) \sin\theta_2]$$

D.2.

$$\sigma M_{n0ss} = -\sigma M_{n0kk} = -\mathrm{Im}\,[M_5^*(M_1 + M_2 - M_3 + M_4)]$$
$$\sigma M_{n0ks} = \mathrm{Im}\,(M_1^* M_4 - M_2^* M_3)$$
$$\sigma M_{n0sk} = -\mathrm{Im}\,(M_1^* M_3 - M_2^* M_4)$$

D.4.

$$\sigma(C_{s's''ss} + C_{k'k''ss}) = -\frac{1}{2}(|M_1 - M_2|^2 + |M_3 + M_4|^2) \cos(\theta_1 + \theta_2)$$

$$\sigma(C_{s'k''ss} - C_{k's''ss}) = -\frac{1}{2}(|M_1 - M_2|^2 + |M_3 + M_4|^2) \sin(\theta_1 + \theta_2)$$

$$\sigma(C_{s's''sk} + C_{k'k''sk}) = \mathrm{Re}\,[(M_1^* - M_2^*)(M_3 + M_4)]\sin(\theta_1 + \theta_2)$$

$$\sigma(C_{s'k''sk} - C_{k's''sk}) = \mathrm{Re}\,[(M_2^* - M_1^*)(M_3 + M_4)]\cos(\theta_1 + \theta_2)$$

$$\sigma(C_{s's''ss} - C_{k'k''ss}) = \left[4\,|M_5|^2 - \frac{1}{2}(|M_1 + M_2|^2 + |M_3 - M_4|^2)\right]\cos(\theta_2 - \theta_1) - 2\,\mathrm{Re}\,M_5^*(M_1 + M_2 + M_3 - M_4)\sin(\theta_2 - \theta_1)$$

$$\sigma(C_{s'k''ss} + C_{k's''ss}) = \left[4\,|M_5|^2 - \frac{1}{2}(|M_1 + M_2|^2 + |M_3 - M_4|^2)\right]\sin(\theta_2 - \theta_1) + 2\,\mathrm{Re}\,M_5^*(M_1 + M_2 + M_3 - M_4)\cos(\theta_2 - \theta_1)$$

$$\sigma(C_{s's''sk} - C_{k'k''sk}) = -\left[4\,|M_5|^2 - \mathrm{Re}\,(M_1^* + M_2^*)(M_3 - M_4)\right]\sin(\theta_2 - \theta_1) - 2\,\mathrm{Re}\,M_5^*(M_1 + M_2 + M_3 - M_4)\cos(\theta_2 - \theta_1)$$

$$\sigma(C_{s'k''sk} + C_{k's''sk}) = \left[4\,|M_5|^2 - \mathrm{Re}\,(M_1^* + M_2^*)(M_3 - M_4)\right]\cos(\theta_2 - \theta_1) - 2\,\mathrm{Re}\,M_5^*(M_1 + M_2 + M_3 - M_4)\sin(\theta_2 - \theta_1)\,.$$

The relativistic rotation angles are

$$\Omega_1 = \theta - 2\,\theta_1 = 2\,\alpha \qquad \text{i.e.} \quad \alpha = \frac{\theta}{2} - \theta_1$$

$$\Omega_2 = -\pi + \theta + 2\,\theta_2 = -\pi + 2\,\beta \quad \text{i.e.} \quad \beta = \frac{\theta}{2} + \theta_2 \tag{5.2}$$

for the scattered and recoil particle respectively, where θ is the c.m.s. scattering angle, and θ_1 and θ_2 are the l.s. scattering and recoil angles. The nonrelativistic case corresponds to

$$\alpha = 0\,, \qquad \beta = \frac{\pi}{2}\,. \tag{5.3}$$

Note that all angles θ, θ_1, θ_2, α, β and Ω_1 are not negative whereas $\Omega_2 \leqslant 0$. All vectors and angles involved are illustrated on figure 1. We easily find the relations

$$\mathbf{k}'_{R_1} = \mathbf{l}\cos\alpha + \mathbf{m}\sin\alpha\,, \qquad \mathbf{k}''_{R_2} = -\,\mathbf{l}\cos\beta - \mathbf{m}\sin\beta$$

$$\mathbf{s}'_{R_1} = -\,\mathbf{l}\sin\alpha + \mathbf{m}\cos\alpha\,, \qquad \mathbf{s}''_{R_2} = \mathbf{l}\sin\beta - \mathbf{m}\cos\beta \tag{5.4}$$

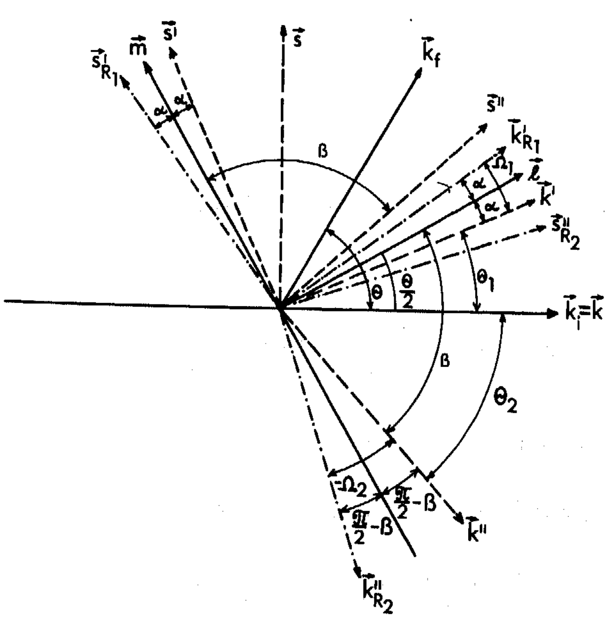

FIG. 1. — Nucleon-nucleon scattering kinematics in the centre-of-mass and laboratory systems. The angles indicated are : the c.m.s. scattering angle θ, the l.s. scattering and recoil angles θ_1 and θ_2. The relativistic rotation angles $\Omega_1 = 2\,\alpha$ and $\Omega_2 = -\pi + 2\,\beta$ for the scattered and recoil particles (in the nonrelativistic limit we have $\alpha = 0$, $\beta = \pi/2$). Unit vectors in the following directions are shown : the initial and final c.m.s. momentum $\mathbf{k}_i$ and $\mathbf{k}_f$, the directions $\mathbf{l} \sim \mathbf{k}_i + \mathbf{k}_f$, $\mathbf{m} \sim \mathbf{k}_f - \mathbf{k}_i$, the initial scattered and recoil particle l.s. momenta $\mathbf{k} = \mathbf{k}_i$, $\mathbf{k}'$ and $\mathbf{k}''$, the directions $\mathbf{s} = \mathbf{n} \times \mathbf{k}$, $\mathbf{s}' = \mathbf{n} \times \mathbf{k}'$ and $\mathbf{s}'' = \mathbf{n} \times \mathbf{k}''$, where $\mathbf{n} \sim \mathbf{k}_i \times \mathbf{k}_f$. Finally $\mathbf{k}'_{R_1}$, $\mathbf{s}'_{R_1}$, $\mathbf{k}''_{R_2}$ and $\mathbf{s}''_{R_2}$ are the above vectors $\mathbf{k}'$, $\mathbf{s}'$, $\mathbf{k}''$ and $\mathbf{s}''$ rotated through the angle Ω_1 (or Ω_2) for the scattered (recoil) particle.

and

$$\mathbf{k} = \mathbf{l} \cos \frac{\theta}{2} - \mathbf{m} \sin \frac{\theta}{2}$$
$$\mathbf{s} = \mathbf{l} \sin \frac{\theta}{2} + \mathbf{m} \cos \frac{\theta}{2} . \tag{5.5}$$

Formula (5.1) is written for the four-component tensor. The formulas for lower-order tensors are obtained by simply putting appropriate indices equal to zero and by omitting the corresponding vector components on the right hand side.

Each of the indices p, q, i and k in (5.1) is equal to 0, l, m or n [see (2.2)], a [b] is equal to 0, k', s' or n [0, k'', s'' or n] and each of c and d is equal to 0, k, s or n [see (4.4) and (4.5)].

Let us now consider individual cases from table II and derive relations between c.m.s. and l.s. experiments using the results of section 3.

A.2. *Polarization of scattered particles analysed* : $P^{\text{ls}} = P^{\text{cms}} = P$.

B.2. *Polarized beam, scattered particles analysed.* — The well-known Wolfenstein parameters $D_{a0c0} = D_{p0i0}\, a_{R_1 p}\, c_i$ are

$$D \equiv D_{n0n0} \quad \text{(l.s. and c.m.s. quantities are equal)}$$
$$R \equiv D_{s'0s0} = -D_{l0l0} \sin\alpha \sin\frac{\theta}{2} - D_{l0m0} \sin\left(\alpha + \frac{\theta}{2}\right) + D_{m0m0} \cos\alpha \cos\frac{\theta}{2}$$
$$A \equiv D_{s'0k0} = -D_{l0l0} \sin\alpha \cos\frac{\theta}{2} - D_{l0m0} \cos\left(\alpha + \frac{\theta}{2}\right) - D_{m0m0} \cos\alpha \sin\frac{\theta}{2}$$
$$R' \equiv D_{k'0s0} = D_{l0l0} \cos\alpha \sin\frac{\theta}{2} + D_{l0m0} \cos\left(\alpha + \frac{\theta}{2}\right) + D_{m0m0} \sin\alpha \cos\frac{\theta}{2}$$
$$A' \equiv D_{k'0k0} = D_{l0l0} \cos\alpha \cos\frac{\theta}{2} - D_{l0m0} \sin\left(\alpha + \frac{\theta}{2}\right) - D_{m0m0} \sin\alpha \sin\frac{\theta}{2} .$$

B.3. *Polarized beam, recoil particles analysed.* — The *transfer Wolfenstein parameters* satisfy $K_{0bc0} = K_{0qi0}\, b_{R_2 q}\, c_i$.

$$K \equiv K_{0nn0} \quad \text{(l.s. and c.m.s. quantities are equal)}$$
$$R_t \equiv K_{0s''s0} = K_{0ll0} \sin\beta \sin\frac{\theta}{2} + K_{0lm0} \sin\left(\beta + \frac{\theta}{2}\right) - K_{0mm0} \cos\beta \cos\frac{\theta}{2}$$
$$A_t \equiv K_{0s''k0} = K_{0ll0} \sin\beta \cos\frac{\theta}{2} + K_{0lm0} \cos\left(\beta + \frac{\theta}{2}\right) + K_{0mm0} \cos\beta \sin\frac{\theta}{2}$$
$$R'_t \equiv K_{0k''s0} = -K_{0ll0} \cos\beta \sin\frac{\theta}{2} - K_{0lm0} \cos\left(\beta + \frac{\theta}{2}\right) - K_{0mm0} \sin\beta \cos\frac{\theta}{2}$$
$$A'_t \equiv K_{0k''k0} = -K_{0ll0} \cos\beta \cos\frac{\theta}{2} + K_{0lm0} \sin\left(\beta + \frac{\theta}{2}\right) + K_{0mm0} \sin\beta \sin\frac{\theta}{2} .$$

C.2. *Polarized target, scattered particles analysed.* — We shall use small letters for the analogues of the Wolfenstein parameters in the case of polarized target and unpolarized beam. We have $K_{a00d} = K_{p00k}\, a_{R_1 p}\, d_k$ and we express K_{p00k} in terms of K_{0qi0}

$$k_t \equiv K = K_{n00n} \quad \text{(l.s. and c.m.c. quantities are equal)}$$
$$r_t \equiv K_{s'00s} = -K_{0ll0} \sin\alpha \sin\frac{\theta}{2} - K_{0lm0} \sin\left(\alpha + \frac{\theta}{2}\right) + K_{0mm0} \cos\alpha \cos\frac{\theta}{2}$$
$$a_t \equiv K_{s'00k} = -K_{0ll0} \sin\alpha \cos\frac{\theta}{2} - K_{0lm0} \cos\left(\alpha + \frac{\theta}{2}\right) - K_{0mm0} \cos\alpha \sin\frac{\theta}{2}$$
$$r'_t \equiv K_{k'00s} = K_{0ll0} \cos\alpha \sin\frac{\theta}{2} + K_{0lm0} \cos\left(\alpha + \frac{\theta}{2}\right) + K_{0mm0} \sin\alpha \cos\frac{\theta}{2}$$
$$a'_t \equiv K_{k'00k} = K_{0ll0} \cos\alpha \cos\frac{\theta}{2} - K_{0lm0} \sin\left(\alpha + \frac{\theta}{2}\right) - K_{0mm0} \sin\alpha \sin\frac{\theta}{2} .$$

C.3. *Polarized target, recoil particles analysed.* — We have $D_{0b0d} = D_{0q0k}\, b_{R_2 q}\, d_k$ and we express D_{0q0k} in terms of D_{p0i0}

$$d \equiv D = D_{n0n0} \quad \text{(l.s. and c.m.s. quantities are equal)}$$

$$r \equiv D_{0s''0s} = \quad D_{l0l0} \sin\beta \sin\frac{\theta}{2} + D_{l0m0} \sin\left(\beta + \frac{\theta}{2}\right) - D_{m0m0} \cos\beta \cos\frac{\theta}{2}$$

$$a \equiv D_{0s''0k} = \quad D_{l0l0} \sin\beta \cos\frac{\theta}{2} + D_{l0m0} \cos\left(\beta + \frac{\theta}{2}\right) + D_{m0m0} \cos\beta \sin\frac{\theta}{2}$$

$$r' \equiv D_{0k''0s} = -\, D_{l0l0} \cos\beta \sin\frac{\theta}{2} - D_{l0m0} \cos\left(\beta + \frac{\theta}{2}\right) - D_{m0m0} \sin\beta \cos\frac{\theta}{2}$$

$$a' \equiv D_{0k''0k} = -\, D_{l0l0} \cos\beta \cos\frac{\theta}{2} + D_{l0m0} \sin\left(\beta + \frac{\theta}{2}\right) + D_{m0m0} \sin\beta \sin\frac{\theta}{2}.$$

A.4. *Polarization correlation for initially unpolarized particles.* — We have $C_{ab00} = C_{pq00}\, a_{R_1 p}\, b_{R_2 q}$ so that

$$C_{nn00} = C_{s's''00} = -\, C_{ll00} \sin\alpha \sin\beta + C_{lm00} \sin(\alpha + \beta) - C_{mm00} \cos\alpha \cos\beta$$

(l.s. and c.m.s. quantities are equal)

$$C_{nn00} = C_{k's''00} = \quad C_{ll00} \cos\alpha \sin\beta - C_{lm00} \cos(\alpha + \beta) - C_{mm00} \sin\alpha \cos\beta$$

(l.s. and c.m.s. quantities are equal)

$$C_{nn00} = C_{s'k''00} = \quad C_{ll00} \sin\alpha \cos\beta - C_{lm00} \cos(\alpha + \beta) - C_{mm00} \cos\alpha \sin\beta$$

(l.s. and c.m.s. quantities are equal)

$$C_{nn00} = C_{k'k''00} = -\, C_{ll00} \cos\alpha \cos\beta - C_{lm00} \sin(\alpha + \beta) - C_{mm00} \sin\alpha \sin\beta$$

(l.s. and c.m.s. quantities are equal).

D.1. *Cross section for polarized beam and polarized target* [33]. — We have $A_{00cd} = A_{00ik}\, c_i\, d_k$:

$$A_{00ss} = C_{ll00} \sin^2\frac{\theta}{2} - C_{lm00} \sin\theta + C_{mm00} \cos^2\frac{\theta}{2}$$

$$A_{00sk} = A_{00ks} = \frac{1}{2}(C_{ll00} - C_{mm00}) \sin\theta - C_{lm00} \cos\theta$$

$$A_{00kk} = C_{ll00} \cos^2\frac{\theta}{2} + C_{lm00} \sin\theta + C_{mm00} \sin^2\frac{\theta}{2}.$$

B.4. *Polarized beam, both final polarizations analysed.* — We have $C_{abc0} = C_{pqi0}\, a_{R_1 p}\, b_{R_2 q}\, c_i$

$$C_{s's''n0} = \quad C_{lln0} \cos(\alpha + \beta) + C_{mln0} \cos\alpha \sin\beta + C_{lmn0} \sin\alpha \cos\beta$$

$$C_{k's''n0} = \quad C_{lln0} \sin(\alpha + \beta) + C_{mln0} \sin\alpha \sin\beta - C_{lmn0} \cos\alpha \cos\beta$$

$$C_{s'k''n0} = \quad C_{lln0} \sin(\alpha + \beta) - C_{mln0} \cos\alpha \cos\beta + C_{lmn0} \sin\alpha \sin\beta$$

$$C_{k'k''n0} = -\, C_{lln0} \cos(\alpha + \beta) - C_{mln0} \sin\alpha \cos\beta - C_{lmn0} \cos\alpha \sin\beta$$

$$C_{s'ns0} = \quad C_{lnl0} \cos\left(\alpha + \frac{\theta}{2}\right) + C_{mnl0} \cos\alpha \sin\frac{\theta}{2} - C_{lnm0} \sin\alpha \cos\frac{\theta}{2}$$

$$C_{k'ns0} = \quad C_{lnl0} \sin\left(\alpha + \frac{\theta}{2}\right) + C_{mnl0} \sin\alpha \sin\frac{\theta}{2} + C_{lnm0} \cos\alpha \cos\frac{\theta}{2}$$

$$C_{s'nk0} = -\, C_{lnl0} \sin\left(\alpha + \frac{\theta}{2}\right) + C_{mnl0} \cos\alpha \cos\frac{\theta}{2} + C_{lnm0} \sin\alpha \sin\frac{\theta}{2}$$

$$C_{k'nk0} = \quad C_{lnl0} \cos\left(\alpha + \frac{\theta}{2}\right) + C_{mnl0} \sin\alpha \cos\frac{\theta}{2} - C_{lnm0} \cos\alpha \sin\frac{\theta}{2}$$

$$C_{ns''s0} = -\, C_{nll0} \cos\left(\beta + \frac{\theta}{2}\right) - C_{nml0} \cos\beta \sin\frac{\theta}{2} + C_{nlm0} \sin\beta \cos\frac{\theta}{2}$$

$$C_{nk''s0} = -\, C_{nll0} \sin\left(\beta + \frac{\theta}{2}\right) - C_{nml0} \sin\beta \sin\frac{\theta}{2} - C_{nlm0} \cos\beta \cos\frac{\theta}{2}$$

$$C_{ns''k0} = \quad C_{nll0} \sin\left(\beta + \frac{\theta}{2}\right) - C_{nml0} \cos\beta \cos\frac{\theta}{2} - C_{nlm0} \sin\beta \sin\frac{\theta}{2}$$

$$C_{nk''k0} = -\, C_{nll0} \cos\left(\beta + \frac{\theta}{2}\right) - C_{nml0} \sin\beta \cos\frac{\theta}{2} + C_{nlm0} \cos\beta \sin\frac{\theta}{2}.$$

C.4. *Polarized target, both final polarizations analysed.* — We have $C_{ab0d} = C_{pq0k}\, a_{R_1p}\, b_{R_2q}\, d_k$ and express C_{pq0k} in terms of C_{pqi0}.

$$\begin{aligned}
C_{s'n0s} &= \quad C_{nll0} \cos\left(\alpha + \frac{\theta}{2}\right) + C_{nml0} \cos\alpha \sin\frac{\theta}{2} - C_{nlm0} \sin\alpha \cos\frac{\theta}{2} \\
C_{k'n0s} &= \quad C_{nll0} \sin\left(\alpha + \frac{\theta}{2}\right) + C_{nml0} \sin\alpha \sin\frac{\theta}{2} + C_{nlm0} \cos\alpha \cos\frac{\theta}{2} \\
C_{s'n0k} &= - C_{nll0} \sin\left(\alpha + \frac{\theta}{2}\right) + C_{nml0} \cos\alpha \cos\frac{\theta}{2} + C_{nlm0} \sin\alpha \sin\frac{\theta}{2} \\
C_{k'n0k} &= \quad C_{nll0} \cos\left(\alpha + \frac{\theta}{2}\right) + C_{nml0} \sin\alpha \cos\frac{\theta}{2} - C_{nlm0} \cos\alpha \sin\frac{\theta}{2} \\
C_{ns''0s} &= - C_{lnl0} \cos\left(\beta + \frac{\theta}{2}\right) - C_{mnl0} \cos\beta \sin\frac{\theta}{2} + C_{lnm0} \sin\beta \cos\frac{\theta}{2} \\
C_{nk''0s} &= - C_{lnl0} \sin\left(\beta + \frac{\theta}{2}\right) - C_{mnl0} \sin\beta \sin\frac{\theta}{2} - C_{lnm0} \cos\beta \cos\frac{\theta}{2} \\
C_{ns''0k} &= \quad C_{lnl0} \sin\left(\beta + \frac{\theta}{2}\right) - C_{mnl0} \cos\beta \cos\frac{\theta}{2} - C_{lnm0} \sin\beta \sin\frac{\theta}{2} \\
C_{nk''0k} &= - C_{lnl0} \cos\left(\beta + \frac{\theta}{2}\right) - C_{mnl0} \sin\beta \cos\frac{\theta}{2} + C_{lnm0} \cos\beta \sin\frac{\theta}{2} \\
C_{s's''0n} &= \quad C_{lln0} \cos(\alpha + \beta) + C_{lmn0} \cos\alpha \sin\beta + C_{mln0} \sin\alpha \cos\beta \\
C_{k's''0n} &= \quad C_{lln0} \sin(\alpha + \beta) + C_{lmn0} \sin\alpha \sin\beta - C_{mln0} \cos\alpha \cos\beta \\
C_{s'k''0n} &= \quad C_{lln0} \sin(\alpha + \beta) - C_{lmn0} \cos\alpha \cos\beta + C_{mln0} \sin\alpha \sin\beta \\
C_{k'k''0n} &= - C_{lln0} \cos(\alpha + \beta) - C_{lmn0} \sin\alpha \cos\beta - C_{mln0} \cos\alpha \sin\beta \,.
\end{aligned}$$

D.2. *Polarized beam and target, polarization of scattered particles analysed.* — We have

$$M_{a0cd} = M_{p0ik}\, a_{R_1p}\, c_i\, d_k$$

and express M_{p0ik} in terms of C_{pqi0}.

$$\begin{aligned}
M_{n0ss} &= - M_{n0kk} = - C_{lln0} \cos\theta - \frac{1}{2}(C_{mln0} + C_{lmn0}) \sin\theta \\
M_{n0ks} &= C_{lln0} \sin\theta + C_{mln0} \sin^2\frac{\theta}{2} - C_{lmn0} \cos^2\frac{\theta}{2} \\
M_{n0sk} &= C_{lln0} \sin\theta - C_{mln0} \cos^2\frac{\theta}{2} + C_{lmn0} \sin^2\frac{\theta}{2} \,.
\end{aligned}$$

D.3. *Polarized beam and target, polarization of recoil particles analysed.* — For the experiments N_{0bcd} see section 6.

D.4. *Polarized beam and target, both final polarizations analysed.* — For the relations see (6.30) and (6.31).

6. **Relations between experimental quantities in the laboratory system.** — The number of linearly independent experiments is the same in any frame of reference. The 25 linearly independent laboratory frame experiments can of course be chosen in many ways.

Contrary to the c.m.s., in the l.s. we use three different bases, which together with relativistic spin rotations complicates relations between experimental quantities following from parity conservation, the Pauli principle and time invariance. A relatively simple way to derive them is to use transformation relations between basis vectors in both systems. The inversion of (5.4) and (5.5) gives

$$\begin{aligned}
\mathbf{l} &= \quad \mathbf{k} \cos\frac{\theta}{2} + \mathbf{s} \sin\frac{\theta}{2} = \mathbf{k}'_{R_1} \cos\alpha - \mathbf{s}'_{R_1} \sin\alpha = - \mathbf{k}''_{R_2} \cos\beta + \mathbf{s}''_{R_2} \sin\beta \\
\mathbf{m} &= - \mathbf{k} \sin\frac{\theta}{2} + \mathbf{s} \cos\frac{\theta}{2} = \mathbf{k}'_{R_1} \sin\alpha + \mathbf{s}'_{R_1} \cos\alpha = - \mathbf{k}''_{R_2} \sin\beta - \mathbf{s}'_{R_2} \cos\beta \,.
\end{aligned} \tag{6.1}$$

We set them into the equalities derived in section 3 and transform the results into the laboratory system.

It is easy to see how the Bohr rule can be applied in the laboratory system. It implies that two experimental quantities are equal up to a sign if one results from the other by replacing the label 0 by n, n by 0, k (k' and k'') by s (s' and s'') and s (s' and s'') by k (k' and k''). The sign is equal to $(-1)^{1/2([s]_f-[s]_i+[k]_i-[k]_f)}$, where $[s]_i$ and $[s]_f$ indicate the number of s-type labels in the initial and final states and similarly for $[k]_i$ and $[k]_f$.

The parity conservation — as in the c.m.s. — implies that only experiments with an even number of k, k', k'', s, s' and s'' labels are non-zero.

The generalized Pauli principle together with the parity conservation give once more

$$X_{abcd} = X_{badc}\,. \tag{6.2}$$

Formula (6.2) relates pure laboratory system experiments if a and b are equal to 0 or n, c and d equal to 0, k, s or n (always with an even number of k and s labels). A substitution of (6.1) into (3.2) gives after a simple calculation

$$\begin{aligned} X_{k'bcd} &= -X_{bk''dc}\cos(\theta_1+\theta_2) + X_{bs''dc}\sin(\theta_1+\theta_2) \\ X_{s'bcd} &= -X_{bk''dc}\sin(\theta_1+\theta_2) - X_{bs''dc}\cos(\theta+\theta_2)\,, \end{aligned} \tag{6.3}$$

where $b = 0, n$ and $c, d = 0, k, s, n$. Notice that the second relation is a consequence of the first one and the Bohr rule. Using (3.2) and (6.1) again, as well as the Bohr rule, we obtain

$$\frac{X_{k'k''cd} + X_{s's''cd}}{X_{k's''cd} - X_{s'k''cd}} = -\frac{X_{k's''cc} - X_{s'k''cc}}{X_{k'k''cc} + X_{s's''cc}} = \tan(\theta_1+\theta_2) \tag{6.4}$$

where $(c, d) = (0, n), (n, 0), (k, s)$ or (s, k). In the nonrelativistic case (6.4) reduces to

$$X_{k's''cd} - X_{s'k''cd} = X_{k'k''cc} + X_{s's''cc} = 0\,.$$

Time reversal invariance implies relations of the type (3.4) in the laboratory system only if all labels are equal to 0 or n. Other combinations of subscripts give more complicated relations amongst pure experiments, such as :

$$\begin{aligned} \frac{X_{k'bsb} + X_{s'bkb}}{X_{k'bkb} - X_{s'bsb}} &= -\frac{X_{k'bkd} - X_{s'bsd}}{X_{k'bsd} + X_{s'bkd}} = \tan\theta_1 \\ \frac{X_{ak''ck} - X_{as''cs}}{X_{ak''cs} + X_{as''ck}} &= -\frac{X_{ak''as} + X_{as''ak}}{X_{ak''ak} - X_{as''as}} = \tan\theta_2 \end{aligned} \tag{6.5}$$

with $a, b, c, d = 0, n$, $a \neq c$ and $b \neq d$. Both lines are related by the Pauli principle (6.3).

Another consequence of time reversal invariance is

$$\begin{aligned} X_{k'bcs}\sin\theta_1 - X_{s'bcs}\cos\theta_1 &= X_{ck''sb}\sin\theta_2 + X_{cs''sb}\cos\theta_2 \\ X_{k'bcs}\cos\theta_1 + X_{s'bcs}\sin\theta_1 &= X_{ck''kb}\sin\theta_2 + X_{cs''kb}\cos\theta_2\,, \end{aligned} \tag{6.6}$$

where $b, c = 0, n$ and two further relations which can also be derived from (6.6) using the Bohr rule. Putting (6.3) into (6.6) we get four equalities related two by two by the Pauli principle

$$\begin{aligned} \frac{X_{k'bbs} + X_{s'bbk}}{X_{k'bbk} - X_{s'bbs}} &= -\frac{X_{k'bck} - X_{s'bcs}}{X_{k'bcs} + X_{s'bck}} = \tan\theta_1 \\ \frac{X_{ak''kd} - X_{as''sd}}{X_{ak''sd} + X_{as''kd}} &= -\frac{X_{ak''sa} + X_{as''ka}}{X_{ak''ka} - X_{as''sa}} = \tan\theta_2\,, \end{aligned} \tag{6.7}$$

where $a, b, c, d = 0, n$, $a \neq d$ and $b \neq c$.

Four further relations implied by time reversal invariance are

$$\begin{aligned} X_{k'k''cd} + X_{s's''cd} &= (X_{cdks} - X_{cdsk})\sin(\theta_1+\theta_2) - (X_{cdkk} + X_{cdss})\cos(\theta_1+\theta_2) \\ X_{k's''cd} - X_{s'k''cd} &= (X_{cdks} - X_{cdsk})\cos(\theta_1+\theta_2) + (X_{cdkk} + X_{cdss})\sin(\theta_1+\theta_2) \\ X_{k'k''cd} - X_{s's''cd} &= (X_{cdks} + X_{cdsk})\sin(\theta_2-\theta_1) - (X_{cdkk} - X_{cdss})\cos(\theta_2-\theta_1) \\ X_{k's''cd} + X_{s'k''cd} &= (X_{cdks} + X_{cdsk})\cos(\theta_2-\theta_1) + (X_{cdkk} - X_{cdss})\sin(\theta_2-\theta_1)\,, \end{aligned} \tag{6.8}$$

where c, $d = 0, n$. Notice, that if $c \neq d$ than $X_{cdkk} = - X_{cdss}$ by the Bohr rule and the Pauli principle, which simplifies the relations (6.8). On the other hand $c = d$ implies $X_{cdks} = X_{cdsk}$.

Let us discuss the individual classes of experiments.

(1) *One-component tensors*

$$P_{n000} = P_{0n00} = A_{00n0} = A_{000n} = P \tag{6.9}$$

as in the c.m.s.

(2) *Two-component tensors*

The Pauli principle (6.2), (6.3) and (6.4) implies

$$A_{00ks} = A_{00sk}\,, \qquad D_{0n0n} = D_{n0n0}\,, \qquad K_{n00n} = K_{0nn0} \tag{6.10}$$

$$\begin{aligned} D_{0k''0d} &= - D_{s'0d0} \sin (\theta_1 + \theta_2) - D_{k'0d0} \cos (\theta_1 + \theta_2) \\ D_{0s''0d} &= - D_{s'0d0} \cos (\theta_1 + \theta_2) + D_{k'0d0} \sin (\theta_1 + \theta_2) \\ K_{k'00d} &= K_{0s''d0} \sin (\theta_1 + \theta_2) - K_{0k''d0} \cos (\theta_1 + \theta_2) \\ K_{s'00d} &= - K_{0s''d0} \cos (\theta_1 + \theta_2) - K_{0k''d0} \sin (\theta_1 + \theta_2) \end{aligned} \tag{6.11}$$

for $d = s, k$ and

$$\frac{C_{k's''00} - C_{s'k''00}}{C_{k'k''00} + C_{s's''00}} = - \tan (\theta_1 + \theta_2)\,. \tag{6.12}$$

Time reversal invariance imposes further constraints, namely (3.4), (6.5) and (6.7) giving

$$C_{nn00} = A_{00nn} \tag{6.13}$$

$$\begin{aligned} \frac{D_{k'0s0} + D_{s'0k0}}{D_{k'0k0} - D_{s'0s0}} &= \frac{R' + A}{A' - R} = \tan \theta_1 \\ \frac{D_{0s''0k} + D_{0k''0s}}{D_{0s''0s} - D_{0k''0k}} &= \frac{a + r'}{r - a'} = \tan \theta_2 \\ \frac{K_{k'00s} + K_{s'00k}}{K_{k'00k} - K_{s'00s}} &= \frac{r'_t + a_t}{a'_t - r_t} = \tan \theta_1 \\ \frac{K_{0s''k0} + K_{0k''s0}}{K_{0s''s0} - K_{0k''k0}} &= \frac{A_t + R'_t}{R_t - A'_t} = \tan \theta_2\,. \end{aligned} \tag{6.14}$$

The relations (6.8) are simplified as

$$\begin{aligned} C_{k'k''00} + C_{s's''00} &= - (A_{00kk} + A_{00ss}) \cos (\theta_1 + \theta_2) \\ C_{k's''00} - C_{s'k''00} &= (A_{00kk} + A_{00ss}) \sin (\theta_1 + \theta_2) \\ C_{k'k''00} - C_{s's''00} &= 2\, A_{00ks} \sin (\theta_2 - \theta_1) - (A_{00kk} - A_{00ss}) \cos (\theta_2 - \theta_1) \\ C_{k's''00} + C_{s'k''00} &= 2\, A_{00ks} \cos (\theta_2 - \theta_1) + (A_{00kk} - A_{00ss}) \sin (\theta_2 - \theta_1)\,. \end{aligned} \tag{6.15}$$

Thus, we are left with 12 independent quantities, e.g. A_{00nn}, A_{00kk}, A_{00ss}, A_{00sk}, D_{n0n0}, three of the four quantities $D_{s'0s0}$, $D_{k'0k0}$, $D_{s'0k0}$, $D_{k'0s0}$; K_{0nn0} and three of the quantities $K_{0s''s0}$, $K_{0k''k0}$, $K_{0s''k0}$ and $K_{0k''s0}$.

(3) *Three-component tensors.* — Parity conservation implies that each tensor has at most 13 non-zero components. Consider first the tensor M_{a0cd}. Making use of the Bohr rule and (6.2) we find

$$M_{n0nn} = P\,, \qquad M_{n0kk} = - M_{n0ss}\,. \tag{6.16}$$

The generalized Pauli principle, together with time reversal invariance and the Bohr rule imply [see (6.5) and (6.7)]

$$\frac{M_{s'0sn} - M_{k'0kn}}{M_{k'0sn} + M_{s'0kn}} = \frac{M_{s'0ns} - M_{k'0nk}}{M_{k'0ns} + M_{s'0nk}} = \tan \theta_1\,. \tag{6.17}$$

Thus we are left with 9 linearly independent components of the polarization tensor M_{a0cd} in the laboratory frame.

The polarization of the recoil particle for both beam and target polarized is related by means of the generalized Pauli principle to the polarization of the scattered particle M_{a0cd}. Using (6.2) we get

$$N_{0nnn} = M_{n0nn} = P\,, \qquad N_{0ncd} = M_{n0dc} \tag{6.18}$$

and the inverse relations to (6.3) imply

$$\begin{aligned} N_{0k''nd} &= -\, M_{k'0dn} \cos(\theta_1 + \theta_2) - M_{s'0dn} \sin(\theta_1 + \theta_2) \\ N_{0s''nd} &= M_{k'0dn} \sin(\theta_1 + \theta_2) - M_{s'0dn} \cos(\theta_1 + \theta_2) \\ N_{0k''cn} &= -\, M_{k'0nc} \cos(\theta_1 + \theta_2) - M_{s'0nc} \sin(\theta_1 + \theta_2) \\ N_{0s''cn} &= M_{k'0nc} \sin(\theta_1 + \theta_2) - M_{s'0nc} \cos(\theta_1 + \theta_2)\,. \end{aligned} \tag{6.19}$$

The labels c and d in formulas (6.18) and (6.19) are equal to k or s. Thus all 13 non-zero components of N_{0bcd} are expressed in terms of M_{a0cd}.

The two polarization correlations tensors are related to each other. The Bohr rule gives

$$C_{ab0n} = -\, C_{[n\times a][n\times b]n0}\,, \tag{6.20}$$

where a and b run through (k', s') and (k'', s''), respectively and $[n \times v]$ is label corresponding to the direction of the vector product $[\mathbf{n} \times \mathbf{v}]$ for an arbitrary unit vector $\mathbf{v}$. In more detail (6.20) gives

$$\begin{aligned} C_{k'k''0n} &= -\, C_{s's''n0}\,, \qquad & C_{k's''0n} &= C_{s'k''n0} \\ C_{s'k''0n} &= C_{k's''n0}\,, \qquad & C_{s's''0n} &= -\, C_{k'k''n0}\,. \end{aligned} \tag{6.21}$$

Other components of C_{abi0} and C_{ab0i} are related by the Pauli principle (6.3).

In turn, the polarization correlation tensors can be related to M_{a0cd} and N_{0bcd}. Quite directly the Bohr rule gives

$$C_{nnn0} = P\,, \qquad C_{anc0} = M_{[n\times a]0[n\times c]n}\,, \qquad C_{nbc0} = N_{0[n\times b][n\times c]n}\,, \tag{6.22}$$

where a, b and c run through (s', k') (s'', k'') and (s, k), respectively. The last of equalities (6.22) together with (6.19) give

$$\begin{aligned} C_{ns''s0} &= -\, M_{s'0nk} \sin(\theta_1 + \theta_2) - M_{k'0nk} \cos(\theta_1 + \theta_2) \\ C_{ns''k0} &= M_{s'0ns} \sin(\theta_1 + \theta_2) + M_{k'0ns} \cos(\theta_1 + \theta_2) \\ C_{nk''s0} &= M_{s'0nk} \cos(\theta_1 + \theta_2) - M_{k'0nk} \sin(\theta_1 + \theta_2) \\ C_{nk''k0} &= -\, M_{s'0ns} \cos(\theta_1 + \theta_2) + M_{k'0ns} \sin(\theta_1 + \theta_2)\,. \end{aligned} \tag{6.23}$$

The remaining components of this tensor are mutually related by the Pauli principle (6.4)

$$\frac{C_{k'k''n0} + C_{s's''n0}}{C_{k's''n0} - C_{s'k''n0}} = \tan(\theta_1 + \theta_2) \tag{6.24}$$

and with M_{n0cd} by the time reversal invariance relation (6.8).

$$\begin{aligned} C_{k'k''n0} + C_{s's''n0} &= (M_{n0ks} - M_{n0sk}) \sin(\theta_1 + \theta_2) \\ C_{k's''n0} - C_{s'k''n0} &= (M_{n0ks} - M_{n0sk}) \cos(\theta_1 + \theta_2) \\ C_{k'k''n0} - C_{s's''n0} &= (M_{n0ks} + M_{n0sk}) \sin(\theta_2 - \theta_1) - 2\, M_{n0kk} \cos(\theta_2 - \theta_1) \\ C_{k's''n0} + C_{s'k''n0} &= (M_{n0ks} + M_{n0sk}) \cos(\theta_2 - \theta_1) + 2\, M_{n0kk} \sin(\theta_2 - \theta_1)\,. \end{aligned} \tag{6.25}$$

Thus we have directly or indirectly expressed all 52 non-zero components of the three-index tensors in terms of 9 linearly independent components of the tensor M_{a0cd}.

(4) *Four-component tensor.* — Parity conservation implies that only 41 components of C_{abcd} are non-zero, namely those with an even number of labels n : none, two or four. Using the Bohr rule we immediately reduce 25 of these components to components of lower-order tensors. Indeed

$$\begin{aligned} C_{nnnn} &= 1\,, \\ C_{abnn} &= -\, C_{[n\times a][n\times b]00}\,, \qquad & C_{nncd} &= -\, A_{00[n\times c][n\times d]}\,, \\ C_{nbcn} &= K_{0[n\times b][n\times c]0}\,, \qquad & C_{nbnd} &= D_{0[n\times b]0[n\times d]}\,, \\ C_{ancn} &= D_{[n\times a]0[n\times c]0}\,, \qquad & C_{annd} &= K_{[n\times a]00[n\times d]}\,. \end{aligned} \tag{6.26}$$

In (6.26) a and b run through (s', k') and (s'', k''), respectively and c, d run through k and s. The remaining 16 components are pairwise related by the Bohr rule

$$\begin{aligned} C_{k'k''kk} &= C_{s's''ss}, & C_{k'k''ks} &= -C_{s's''sk} \\ C_{k's''kk} &= -C_{s'k''ss}, & C_{k's''ks} &= C_{s'k''sk} \\ C_{s'k''kk} &= -C_{k's''ss}, & C_{s'k''ks} &= C_{k's''sk} \\ C_{s's''kk} &= C_{k'k''ss}, & C_{s's''ks} &= -C_{k'k''sk}. \end{aligned} \tag{6.27}$$

Thus, we are left with 8 components, e.g. those on the right hand sides of (6.27).

The Pauli principle (6.4) imposes two further constraints, namely

$$\frac{C_{k'k''sk} + C_{s's''sk}}{C_{k's''sk} - C_{s'k''sk}} = -\frac{C_{k's''ss} - C_{s'k''ss}}{C_{k'k''ss} + C_{s's''ss}} = \tan(\theta_1 + \theta_2). \tag{6.28}$$

One more independent relation between the six remaining components can be found using time invariance. Indeed, in the c.m.s. we have $C_{lmll} = -C_{lllm}$ which can be rewritten in the laboratory system using (6.1) and the Bohr rule as

$$\begin{aligned} &[(C_{s'k''sk} - C_{s's''ss}) \sin\theta_2 + (C_{s's''sk} + C_{s'k''ss}) \cos\theta_2] \cos\theta_1 = \\ &\qquad = [(C_{k'k''sk} - C_{k's''ss}) \sin\theta_2 + (C_{k's''sk} + C_{k'k''ss}) \cos\theta_2] \sin\theta_1. \end{aligned} \tag{6.29}$$

We are thus left with five components. Instead of looking for further relations in the laboratory frame we express eight cobinations of C_{abcd} components in terms of the c.m.s. quantities. Making use of the fact that C_{lmlm}, C_{llmm} and C_{lmml} are linear combinations of C_{llll} and lower-order tensors [see (3.9)] and including the Pauli principle (6.28) as well as the time reversal invariance (6.29) we find

$$\begin{aligned} C_{s's''ss} + C_{k'k''ss} &= (A_{00nn} - 1) \cos(\beta - \alpha) \\ C_{s'k''ss} - C_{k's''ss} &= (A_{00nn} - 1) \sin(\beta - \alpha) \\ C_{s's''sk} + C_{k'k''sk} &= -(K_{0nn0} - D_{n0n0}) \sin(\beta - \alpha) \\ C_{s'k''sk} - C_{k's''sk} &= (K_{0nn0} - D_{n0n0}) \cos(\beta - \alpha) \end{aligned} \tag{6.30}$$

$$\begin{aligned} C_{s's''ss} - C_{k'k''ss} = &- 2\,C_{llll} \cos(\alpha + \beta + \theta) + 2\,C_{lllm} \sin(\alpha + \beta + \theta) \\ &- (A_{00nn} - 1) \cos(\alpha + \beta) \cos\theta \\ &+ (K_{0nn0} + D_{n0n0} - 2) \sin(\alpha + \beta) \sin\theta \\ C_{s'k''ss} + C_{k's''ss} = &- 2\,C_{llll} \sin(\alpha + \beta + \theta) - 2\,C_{lllm} \cos(\alpha + \beta + \theta) \\ &- (A_{00nn} - 1) \sin(\alpha + \beta) \cos\theta \\ &- (K_{0nn0} + D_{n0n0} - 2) \cos(\alpha + \beta) \sin\theta \\ C_{s's''sk} - C_{k'k''sk} = &\; 2\,C_{llll} \sin(\alpha + \beta + \theta) + 2\,C_{lllm} \cos(\alpha + \beta + \theta) \\ &+ (A_{00nn} - 1) \cos(\alpha + \beta) \sin\theta \\ &+ (K_{0nn0} + D_{n0n0} - 2) \sin(\alpha + \beta) \cos\theta \\ C_{s'k''sk} + C_{k's''sk} = &- 2\,C_{llll} \cos(\alpha + \beta + \theta) + 2\,C_{lllm} \sin(\alpha + \beta + \theta) \\ &+ (A_{00nn} - 1) \sin(\alpha + \beta) \sin\theta \\ &- (K_{0nn0} + D_{n0n0} - 2) \cos(\alpha + \beta) \cos\theta. \end{aligned} \tag{6.31}$$

From (6.30) and (6.31) we see that only two combinations of C_{abcd} are actually independent of other laboratory experiments (they yield C_{llll} and C_{lllm}), as expected.

7. **The Pauli principle and nucleon-nucleon scattering.** — The nucleon-nucleon scattering matrix, as used in this article, is symmetric with respect to the interchange of particles 1 and 2. For proton-proton or neutron-neutron scattering this symmetry is a consequence of the particles being identical. For neutron-proton scattering the absence of a nonsymmetric term proportional to $(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2, \mathbf{n})$ is an additional assumption, related to the isotopic invariance of nuclear forces.

A direct consequence of this symmetry which is exploited through-out this article, is that the number of independent experiments (in any frame of reference) is greatly reduced (see sections 3 and 6).

Further consequences of the Pauli principle are obtained for genuinely identical particles (*pp* or *nn* scattering). The particle identity in either the final state or the initial state implies

$$X_{pqik}(\mathbf{k}_f, \mathbf{k}_i) = X_{qpik}(-\mathbf{k}_f, \mathbf{k}_i) = X_{pqki}(\mathbf{k}_f, -\mathbf{k}_i)\,. \tag{7.1}$$

All labels in the three terms of (7.1) refer to the basis given in (2.2) for the final and initial momenta in the $\mathbf{k}_i$ and $\mathbf{k}_f$ directions. However, the experimental quantities as defined in section 3 are labelled in the frames relative to the final and initial momentum directions actually considered, i.e. $-\mathbf{k}_f$, $\mathbf{k}_i$ for the second and $\mathbf{k}_f$, $-\mathbf{k}_i$ for the third term in (7.1). For this reason the following transformations should be made in addition to (7.1):

$$\mathbf{n} \to -\mathbf{n}\,, \quad \mathbf{l} \to -\mathbf{m}\,, \quad \mathbf{m} \to -\mathbf{l} \quad \text{for} \quad \mathbf{k}_f \to -\mathbf{k}_f$$

and

$$\mathbf{n} \to -\mathbf{n}\,, \quad \mathbf{l} \to \mathbf{m}\,, \quad \mathbf{m} \to \mathbf{l} \quad \text{for} \quad \mathbf{k}_i \to -\mathbf{k}_i\,.$$

Besides, $\mathbf{k}_f \to -\mathbf{k}_f$ as well as $\mathbf{k}_i \to -\mathbf{k}_i$ changes the scattering angle $\theta^{cms} = \theta$ to $\pi - \theta$.

In more detail, formulas (7.1) together with the relations of section 3 imply:

$$\begin{aligned}
\sigma(\theta) &= \sigma(\pi-\theta) & P(\theta) &= -P(\pi-\theta)\\
C_{nn00}(\theta) &= C_{nn00}(\pi-\theta)\,, & C_{ll00}(\theta) &= C_{mm00}(\pi-\theta)\\
C_{lm00}(\theta) &= C_{lm00}(\pi-\theta)\,, & &\\
D_{n0n0}(\theta) &= K_{0nn0}(\pi-\theta)\,, & D_{l0l0}(\theta) &= K_{0mm0}(\pi-\theta)\\
D_{m0m0}(\theta) &= K_{0ll0}(\pi-\theta)\,, & D_{l0m0}(\theta) &= K_{0ml0}(\pi-\theta)\\
C_{lln0}(\theta) &= C_{lln0}(\pi-\theta)\,, & C_{lmn0}(\theta) &= -C_{lmn0}(\pi-\theta)\\
C_{mln0}(\theta) &= -C_{mln0}(\pi-\theta)\,, & C_{lnl0}(\theta) &= -C_{nll0}(\pi-\theta)\\
C_{lnm0}(\theta) &= -C_{nml0}(\pi-\theta)\,, & C_{mnl0}(\theta) &= -C_{nlm0}(\pi-\theta)\\
C_{llll}(\theta) &= C_{llll}(\pi-\theta)\,, & C_{lllm}(\theta) &= -C_{lllm}(\pi-\theta)\\
C_{lmlm}(\theta) &= C_{lmml}(\pi-\theta)\,, & C_{llmm}(\theta) &= C_{llmm}(\pi-\theta)\,.
\end{aligned} \tag{7.2}$$

We recall that the labels l, m, n are always defined with respect to the experiment actually performed.

The above relations between c.m.s. experiments can be translated into relations amongst l.s. quantities measured at angles θ_1 and θ_2 (these being the l.s. scattering and recoil angles) by making use of (6.1). However, they can be obtained in a simpler manner using tables I and V and recalling that the transformation $\theta_1 \to \theta_2$ $(\theta \to \pi - \theta)$ corresponds to $\alpha \to \frac{\pi}{2} - \beta$ and $\beta \to \frac{\pi}{2} - \alpha$. Thus we obtain

$$\begin{aligned}
\sigma(\theta_1) &= \sigma(\theta_2)\,, & P(\theta_1) &= -P(\theta_2)\\
C_{nn00}(\theta_1) &= C_{nn00}(\theta_2)\,, & D_{n0n0}(\theta_1) &= K_{0nn0}(\theta_2)\\
C_{s's''00}(\theta_1) &= C_{s's''00}(\theta_2)\,, & A_{00ss}(\theta_1) &= A_{00ss}(\theta_2)\\
C_{s'k''00}(\theta_1) &= -C_{k's''00}(\theta_2)\,, & A_{00sk}(\theta_1) &= -A_{00sk}(\theta_2)\\
C_{k'k''00}(\theta_1) &= C_{k'k''00}(\theta_2)\,, & A_{00kk}(\theta_1) &= A_{00kk}(\theta_2)
\end{aligned} \tag{7.3}$$

$$\begin{aligned}
D_{s'0s0}(\theta_1) &= K_{0s''s0}(\theta_2)\,, & K_{s'00s}(\theta_1) &= D_{0s''0s}(\theta_2)\\
D_{s'0k0}(\theta_1) &= -K_{0s''k0}(\theta_2)\,, & K_{s'00k}(\theta_1) &= -D_{0s''0k}(\theta_2)\\
D_{k'0s0}(\theta_1) &= -K_{0k''s0}(\theta_2)\,, & K_{k'00s}(\theta_1) &= -D_{0k''0s}(\theta_2)\\
D_{k'0k0}(\theta_1) &= K_{0k''k0}(\theta_2)\,, & K_{k'00k}(\theta_1) &= D_{0k''0k}(\theta_2)\\
C_{s's''n0}(\theta_1) &= -C_{s's''n0}(\theta_2) & &\\
C_{k's''n0}(\theta_1) &= C_{s'k''n0}(\theta_2) & &\\
C_{k'k''n0}(\theta_1) &= -C_{k'k''n0}(\theta_2) & &\\
C_{s'ns0}(\theta_1) &= -C_{ns''s0}(\theta_2)\,, & C_{s'n0s}(\theta_1) &= -C_{ns''0s}(\theta_2)\\
C_{k'ns0}(\theta_1) &= C_{nk''s0}(\theta_2)\,, & C_{k'n0s}(\theta_1) &= C_{nk''0s}(\theta_2)\\
C_{s'nk0}(\theta_1) &= C_{ns''k0}(\theta_2)\,, & C_{s'n0k}(\theta_1) &= C_{ns''0k}(\theta_2)\\
C_{k'nk0}(\theta_1) &= -C_{nk''k0}(\theta_2)\,, & C_{k'n0k}(\theta_1) &= -C_{nk''0k}(\theta_2)
\end{aligned} \tag{7.3}$$

$$\begin{aligned}
M_{n0kk}(\theta_1) &= - M_{n0kk}(\theta_2) \\
M_{n0ks}(\theta_1) &= M_{n0sk}(\theta_2) \\
C_{s's''ss}(\theta_1) &= C_{s's''ss}(\theta_2)\,, & C_{k'k''sk}(\theta_1) &= - C_{k'k''sk}(\theta_2) \\
C_{k'k''ss}(\theta_1) &= C_{k'k''ss}(\theta_2)\,, & C_{s'k''ss}(\theta_1) &= - C_{k's''ss}(\theta_2) \\
C_{s'k''sk}(\theta_1) &= C_{k's''sk}(\theta_2)\,, & C_{s's''sk}(\theta_1) &= - C_{s's''sk}(\theta_2)\,.
\end{aligned} \tag{7.3}$$

Additional simple relations between experimental quantities are obtained for $\theta = \pi/2$, i.e. for $\theta_1 = \theta_2$. For *nn* (or *pp*) scattering the relations follow from the fact that $a_1(\pi/2) = 0$, $b_1(\pi/2) = - c_1(\pi/2)$ [see table I] (many of them can be obtained from (7.2) by putting $\pi - \theta = \theta$).

Thus we find in the c.m.s. [11]

$$\begin{aligned}
& P\left(\frac{\pi}{2}\right) = C_{lmn0}\left(\frac{\pi}{2}\right) = C_{mln0}\left(\frac{\pi}{2}\right) = C_{lllm}\left(\frac{\pi}{2}\right) = 0\,, \\
& C_{mm00}\left(\frac{\pi}{2}\right) = C_{ll00}\left(\frac{\pi}{2}\right), \quad D_{n0n0}\left(\frac{\pi}{2}\right) = K_{0nn0}\left(\frac{\pi}{2}\right), \quad D_{l0m0}\left(\frac{\pi}{2}\right) = K_{0ml0}\left(\frac{\pi}{2}\right), \\
& D_{l0l0}\left(\frac{\pi}{2}\right) = - D_{m0m0}\left(\frac{\pi}{2}\right) = - K_{0ll0}\left(\frac{\pi}{2}\right) = K_{0mm0}\left(\frac{\pi}{2}\right), \\
& C_{mnl0}\left(\frac{\pi}{2}\right) = C_{lnm0}\left(\frac{\pi}{2}\right) = - C_{nml0}\left(\frac{\pi}{2}\right) = - C_{nlm0}\left(\frac{\pi}{2}\right), \\
& C_{lnl0}\left(\frac{\pi}{2}\right) = - C_{nll0}\left(\frac{\pi}{2}\right), \quad C_{lmlm}\left(\frac{\pi}{2}\right) = C_{lmml}\left(\frac{\pi}{2}\right), \\
& 2\, D_{n0n0}\left(\frac{\pi}{2}\right) + C_{nn00}\left(\frac{\pi}{2}\right) + 2\, C_{llll}\left(\frac{\pi}{2}\right) = 1\,.
\end{aligned} \tag{7.4}$$

Since the Pauli principle implies $d_0(\pi/2) = e_0(\pi/2) = 0$ we also obtain relations between *nn* (or *pp*) and *np* experiments in the c.m.s. [11]

$$\begin{aligned}
\sigma^{nn}\left(\frac{\pi}{2}\right) C^{nn}_{lm00}\left(\frac{\pi}{2}\right) &= 4\,\sigma^{np}\left(\frac{\pi}{2}\right) C^{np}_{lm00}\left(\frac{\pi}{2}\right) \\
\sigma^{nn}\left(\frac{\pi}{2}\right) C^{nn}_{lln0}\left(\frac{\pi}{2}\right) &= 4\,\sigma^{np}\left(\frac{\pi}{2}\right) C^{np}_{lln0}\left(\frac{\pi}{2}\right) \\
\sigma^{nn}\left(\frac{\pi}{2}\right)\left[1 - C^{nn}_{llll}\left(\frac{\pi}{2}\right)\right] &= 4\,\sigma^{np}\left(\frac{\pi}{2}\right)\left[1 - C^{np}_{llll}\left(\frac{\pi}{2}\right)\right] \\
\sigma^{nn}\left(\frac{\pi}{2}\right)\left[1 + C^{nn}_{nn00}\left(\frac{\pi}{2}\right) - 2\,D^{nn}_{n0n0}\left(\frac{\pi}{2}\right)\right] &= 4\,\sigma^{np}\left(\frac{\pi}{2}\right)\left[1 + C^{np}_{nn00}\left(\frac{\pi}{2}\right) - D^{np}_{n0n0}\left(\frac{\pi}{2}\right) - K^{np}_{0nn0}\left(\frac{\pi}{2}\right)\right]
\end{aligned} \tag{7.5}$$

Relations equivalent to (7.4) for *nn* or *pp* scattering can also be written in the laboratory system. These relations can be obtained directly from (7.4) by performing the appropriate rotations or from the formulas of table V, remembering that $a_1(\pi/2) = 0$, $b_1(\pi/2) = - c_1(\pi/2)$.

The laboratory system relations hold for $\theta_1 = \theta_2 = \pi/4 - \alpha = \beta - \pi/4$ (i.e. $\theta = \pi/2$, $\alpha + \beta = \pi/2$). The l.s. scattering angle for which this occurs is given by

$$\cos \theta_1 = \cos \theta_2 = \left(\frac{s}{s + 4\,m^2}\right)^{1/2} \tag{7.6}$$

where s is the invariant total energy squared and m is the nucleon mass.

Let us first consider the scattering of identical nucleons. All relations (7.3) hold with $\theta_2 = \theta_1$. Additional relations are :

$$P = A_{00sk} = C_{s's''n0} = C_{k'k''n0} = M_{n0ss} = C_{s's''sk} = C_{k'k''sk} = 0 \tag{7.7a}$$

$$\begin{aligned}
D_{s'0s0} &= - D_{0s''0s} = - K_{s'00s} = K_{0s''s0}\,, \\
D_{s'0k0} &= D_{0s''0k} = - K_{s'00k} = - K_{0s''k0}\,, \\
D_{k'0s0} &= D_{0k''0s} = - K_{k'00s} = - K_{0k''s0}\,, \\
D_{k'0k0} &= - D_{0k''0k} = - K_{k'00k} = K_{0k''k0}\,;
\end{aligned}$$

$$\begin{aligned} A_{00kk} - A_{00ss} &= C_{s's''00} - C_{k'k''00} , \\ A_{00kk} + A_{00ss} &= C_{nn00} - 1 , \\ 2\, C_{s'k''00} &= (1 - C_{nn00}) \sin 2\theta_1 , \end{aligned}$$

$$\frac{D_{s'0k0}}{D_{k'0k0}} = - \frac{D_{k'0s0}}{D_{s'0s0}} = \tan \theta_1 , \qquad \frac{2\, C_{s'k''00}}{C_{s's''00} + C_{k'k''00}} = \tan 2\theta_1 , \tag{7.7b}$$

$$\begin{aligned} C_{s'ns0} &= - C_{s'n0s} = - C_{ns''s0} = C_{ns''0s} , \\ C_{k'ns0} &= - C_{k'n0s} = C_{nk''s0} = - C_{nk''0s} , \\ C_{s'nk0} &= - C_{s'n0k} = C_{ns''k0} = - C_{ns''0k} , \\ C_{k'nk0} &= - C_{k'n0k} = - C_{nk''k0} = C_{nk''0k} , \\ \frac{C_{s'ns0}}{C_{k'ns0}} &= - \frac{C_{k'nk0}}{C_{s'nk0}} = \tan \theta_1 , \end{aligned} \tag{7.7c}$$

$$\begin{aligned} C_{s's''ss} + C_{k'k''ss} &= - C_{s's''00} - C_{k'k''00} , \\ C_{s'k''ss} &= - C_{s'k''00} , \\ C_{k'k''ss} - C_{s's''ss} + 2\, C_{s'k''sk} &= 1 - 2\, D_{n0n0} + C_{nn00} . \end{aligned} \tag{7.7d}$$

For identical nucleons at $\theta_1 = \theta_2$ ($\theta = \pi/2$) only three amplitudes are independent, e.g. $b = b_1$, $d = d_1$ and $e = e_1$ (the subscript refers to isospin $T = 1$). Thus only 9 linearly independent experimental quantities exist. Indeed, a measurement of σ, C_{nn00} and of $D_{n0n0} = K_{n00n}$ will determine $|b|^2$, $|d|^2$ and $|e|^2$. Two independent components of the two index tensors, e.g. $D_{s'0s0}$ and $D_{s'0k0}$ will determine Re $b^* d$ and Im $b^* e$, and a measurement of any one of the quantities A_{00ss}, A_{00kk}, $C_{s's''00}$ or $C_{k'k''00}$ will yield Im $d^* e$. Finally, three components of the three-component tensors, e.g. $C_{s'ns0}$, $C_{s'nk0}$ and M_{n0ks} will provide us with Re $b^* e$, Im $b^* d$ and Re $d^* e$.

The formulas (7.5) relating nn (or *pp*) and *np* experimental quantities have their equivalents in the laboratory system. Four independent relations of this type can be written in various forms. We find a convenient set of relations to be the following :

$$\begin{aligned} \sigma^{nn}(C^{nn}_{s's''00} - C^{nn}_{k'k''00}) &= 4\, \sigma^{np}(C^{np}_{s's''00} - C^{np}_{k'k''00}) \\ \sigma^{nn}(C^{nn}_{s'k''n0} + C^{nn}_{k's''n0}) &= 4\, \sigma^{np}(C^{np}_{s'k''n0} + C^{np}_{k's''n0}) \\ \sigma^{nn}(1 + C^{nn}_{nn00} - 2\, D^{nn}_{n0n0}) &= 4\, \sigma^{np}(1 + C^{np}_{nn00} - D^{np}_{n0n0} - K^{np}_{0nn0}) \\ \sigma^{nn}(2\, D^{nn}_{n0n0} - C^{nn}_{s's''ss} + C^{nn}_{k'k''ss}) &= 4\, \sigma^{np}(D^{np}_{n0n0} + K^{np}_{0nn0} - C^{np}_{s's''ss} + C^{np}_{k'k''ss}) \end{aligned} \tag{7.8}$$

8. **Conclusions.** — In this article we have reviewed the kinematics of nucleon-nucleon scattering and filled in many gaps in the existing formalism. We have concentrated on phenomenological aspects only, i.e. the relations between experimental quantities and the scattering matrix and relations amongst experimental quantities themselves. The formulas presented in this paper should be useful for experimentalists studying nucleon-nucleon elastic scattering and for the practitioners of nucleon-nucleon amplitude analysis.

The results obtained make it possible to compare explicitly and exactly all experiments performed under different conditions to obtain the same physical information (like various components of the polarization rotation tensor D_{a0c0} for a polarized beam or D_{0b0d} for a polarized target, various components of the scattered or recoil particle polarization for initially polarized beams and targets, etc.). A reasonably complete list of relations between experimental quantities thus facilitates the use of any experimental data in the reconstruction of scattering amplitudes (e.g. via phase shift analysis). On the other hand, these relations make it possible to check various sets of experiments, usually performed in different laboratories, for consistency between them. Since the origin of most the relations can be traced back to various symmetries (parity, time reversal invariance and the Pauli principle), a test of the relations is also a test of the underlying principles.

Some new formulas are contained in all of sections 3 to 7, but we specially wish to mention the detailed study of the implications of the Pauli principle for the scattering of identical nucleons, presented in section 7. For *nn* (or *pp*) scattering, relations are given between experimental quantities measured at the c.m.s. angles θ and $\pi - \theta$ and l.s. angles θ_1 and θ_2 (θ_1 and θ_2 are the scattering and recoil angles). All constraints occurring for $\theta = \pi/2$, i.e. $\theta_1 = \theta_2$ are listed and also relations between certain *nn* and *np* quantities for the same angle.

In view of the interest in nucleon-nucleon interactions, the availability of accelerators in intermediate and high energy regions and the increasing use of polarized proton targets we think that this is the correct moment to present an explicit and complete exposition of the nucleon-nucleon formalism.

We would like to reemphasize that the entire contents of this article is a formalism, i.e. pure kinematics. As such it should be useful in any study of nucleon-nucleon scattering, either theoretical or experimental. In particular the entire formalism is relevant for any attempts to reconstruct the nucleon-nucleon amplitudes from data. This holds both for a direct reconstruction making use of some complete experiment, as defined by Puzikov, Ryndin and Smorodinskii [5] and for a reconstruction via phase shift analysis, Regge pole theory or any other expansion.

In the near future we plan to present some thoughts and results making use of the present formalism for reconstructing nucleon-nucleon scattering amplitudes from experiments. One of our interests here is the question of the uniqueness of such a reconstruction (be it a direct reconstruction or one via a phase shift analysis). A further program in which this formalism will be used concerns a simultaneous reconstruction for all energies (and angles) making use of previously developed two-variable expansions of scattering amplitudes (see the review [34] and the paper [35]).

Acknowledgments. — At the origin of this paper were stimulating suggestions by L. Van Rossum to whom we express our gratitude. We have appreciated discussions on the subject with D. Besset, M. Borghini, A. Junod, G. Greeniaus, R. Hess, C. Lechanoine, M. J. Moravcsik, J. Patera, Ch. Weddigen and D. Werren.

References

[1] Wolfenstein, L. and Ashkin, J., *Phys. Rev.* **85** (1952) 947.
Wolfenstein, L., *Phys. Rev.* **96** (1954) 1054.
Wolfenstein, L., *Phys. Rev.* **101** (1956) 427.
Wolfenstein, L., *Ann. Rev. Nucl. Sci.* **6** (1956) 43.

[2] Dalitz, R. H., *Proc. Phys. Soc.* **A 65** (1952) 175.

[3] Oehme, R., *Phys. Rev.* **98** (1954) 147, 216.

[4] Stapp, H. P., *Phys. Rev.* **103** (1956) 425.

[5] Puzikov, L. D., Ryndin, R. M. and Smorodinskii, Ya. A., *Zh. Eksp. Teor. Fiz.* **32** (1957) 592 [*Sov. Phys. JETP* **5** (1957) 489];
Bilenky, S. M., Lapidus, L. I. and Ryndin, R. M., *Usp. Fiz. Nauk* **84** (1964) 243 [*Sov. Phys. Usp.* **7** (1965) 721].

[6] Halzen, F. and Thomas, G. H., *Phys. Rev.* **D 10** (1974) 344.

[7] MacGregor, M. H., Moravcsik, M. J. and Stapp, H. P., *Ann. Rev. Nucl. Sci.* **10** (1960) 291.

[8] Hoshizaki, N., *Suppl. Progr. Theor. Phys.* **42** (1968) 1, 107.

[9] Csonka, P. L. and Moravcsik, M. J., *Phys. Rev.* **D 1** (1970) 1821.

[10] Lacombe, M., Thesis, Faculté des Sciences de l'Université de Paris, Centre d'Orsay, 1964.

[11] Lehar, F. and Winternitz, P., *Fortschr.* **15** (1967) 495 and *Czech. J. Phys.* **A 17** (1967) 158.

[12] Faissner, H., *Ergeb. Exakten Naturwiss.* **32** (1959) 180.

[13] Wilson, R., *The nucleon-nucleon interaction, Experimental and Phenomenological Aspects* (J. Wiley and Sons, New York) 1963.

[14] Reid, R. V. and Moravcsik, M. J., *Ann. Phys.* (N.Y.) **84** (1974) 535.

[15] Goldberger, M. L., Nambu, Y. and Oehme, R., *Ann. Phys.* (N.Y.) **2** (1957) 226.
Goldberger, M. L., Grisaru, M. T., MacDowell, S. W. and Wong, D. Y., *Phys. Rev.* **120** (1960) 2250.

[16] Basle convention, *Nucl. Phys.* **21** (1960) 696.

[17] Stapp, H. P., Ypsilantis, T. J. and Metropolis, N., *Phys. Rev.* **105** (1957) 302.

[18] Jacob, M. and Wick, G. C., *Ann. Phys.* (N.Y.) **7** (1959) 404.

[19] Martin, A. D. and Spearman, T. S., *Elementary particle theory* (J. Wiley and Sons, New York) 1970.

[20] Cohen-Tannoudji, G., Morel, A. and Navelet, H., *Ann. Phys.* (N.Y.) **46** (1968) 239.

[21] Kotański, A., *Acta Phys. Pol.* **29** (1966) 699.

[22] Kotański, A., *Acta Phys. Pol.* **30** (1966) 629.

[23] Brink, D. M. and Satchler, G. R., *Angular momentum* (Clarendon Press, Oxford) 1968.

[24] Leader, E. and Slansky, R. C., *Phys. Rev.* **148** (1966) 1491.

[25] Halzen, F. and Thomas, G. H., *Phys. Rev.* **D 10** (1974) 344.

[26] *Physics with polarized beams*, Report of the ANL Technical Advisory Panel, ANL-HEP-CP-75-73.

[27] Bohr, A., *Nucl. Phys.* **10** (1959) 486.

[28] Bilenky, S. M. and Ryndin, R. M., *Phys. Lett.* **6** (1963) 217.

[29] Phillips, R. J. N., *Nucl. Phys.* **43** (1963) 413.

[30] Bilenky, S. M., Lapidus, L. I. and Ryndin, R. M., Preprint JINR P-2227, Dubna 1965.

[31] Bourrely, C. and Soffer, J., Preprint 74/P.663, Marseille 1974 and *Phys. Rev.* **D 12** (1975) 2932.
Delaney, R. M. and Gammel, J. L., *Phys. Rev.* **D 12** (1975) 1978.

[32] Winternitz, P., Lehar, F. and Janout, Z., *Yad. Fiz.* **5** (1967) 201; [*Sov. J. Nucl. Phys.* **5** (1967) 141] and *Czech. J. Phys.* **B 18** (1968) 8.

[33] Raynal, J., *Nucl. Phys.* **28** (1961) 220.

[34] Kalnins, E. G., Patera, J., Sharp, R. T. and Winternitz, P. in « *Group Theory and its applications* », Vol. 3 (edited by E. Loebl, Academic Press) 1975.

[35] Bystricky, J., Lehar, F., Patera, J. and Winternitz, P., *Phys. Rev.* **D 13** (1976) 1276.